Collaborative Filtering

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- Search or Content based Method
- User-Based Collaborative Filtering
- Item-to-Item Collaborative Filtering
- Using Google's PageRank
- Memory-Based Algorithms (Breese et al, UAI98)

Collaborative Filtering

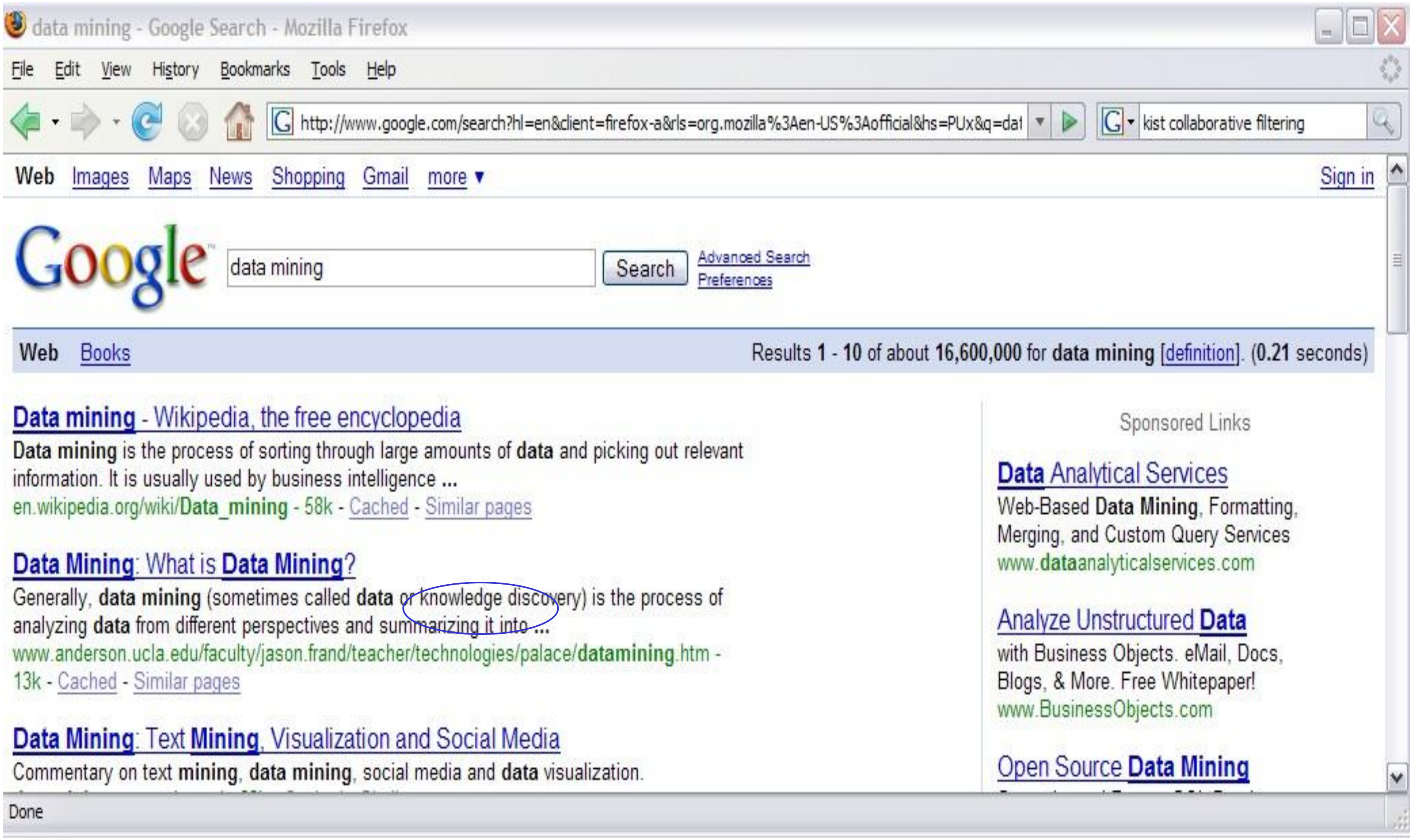
	Star Wars	Hoop Dreams	Contact	Titanic
Joe	5	2	5	4
John	2	5		3
Al	2	2	4	2
Nathan	5	1	5	?

Recommender systems: Systems that evaluate quality based on the preferences of others with a similar point of view

Collaborative Filtering: *The problem of collaborative filtering is to predict how well a user will like an item that he has not rated given a set of historical preference judgments for a community of users.*

- Predict the opinion the user will have on the different items
- Recommend the 'best' items based on the user's previous likings and the opinions of like-minded users whose ratings are similar

Collaborative Filtering in our life



Collaborative Filtering in our life

Search or Content based Method

- Given the user's purchased and rated items, constructs a search query to find other popular items
- For example, same author, artist, director, or similar keywords/subjects
- Impractical to base a query on all the items

User-Based Collaborative Filtering

Some issues with User based collaborative filtering

- Complexity grows linearly with the number of customers and items
- The sparsity of recommendations on the data set
  - Even active customers may have purchased well under 1% of the products

Item-to-Item Collaborative Filtering

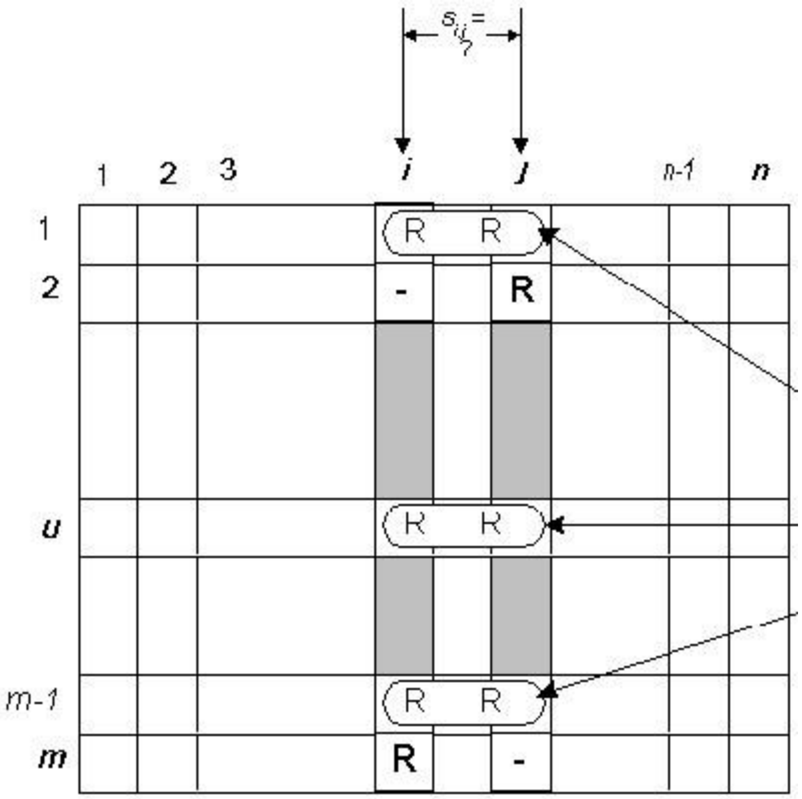
- Rather than matching the user to similar customers, build a similar-items table by finding that customers tend to purchase together
- Amazon.com used this method
- Scales independently of the catalog size or the total number of customers
- Acceptable performance by creating the expensive similar-item table offline

Item-to-Item CF Algorithm

```
For each item in product catalog, I1
  For each customer C who purchased I1
    For each item I2 purchased by
      customer C
      Record that a customer purchased I1
      and I2
  For each item I2
    Compute the similarity between I1 and I2
```

- O(N^2M) as worst case, O(NM) in practical

Item-to-Item CF Algorithm Similarity Calculation



Computed by looking into co-rated items only.

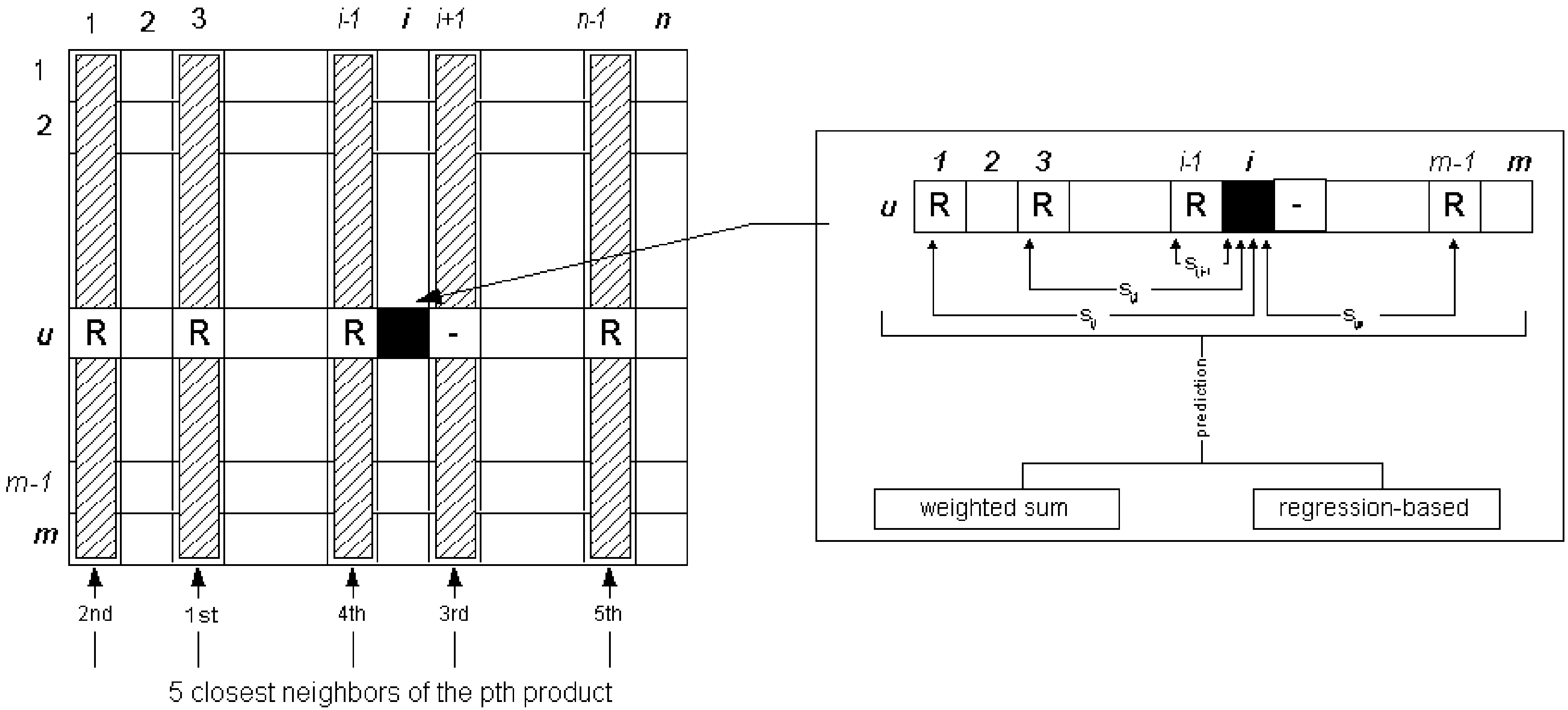
These co-rated pairs are obtained from different users.

Item-to-Item CF Algorithm Similarity Calculation

- For similarity between two items i and j,

$$sim(i,j) = \frac{\sum_{u \in U}(R_{u,i} - \bar{R_u})(R_{u,j} - \bar{R_u})}{\sqrt{\sum_{u \in U}(R_{u,i} - \bar{R_u})^2}\sqrt{\sum_{u \in U}(R_{u,j} - \bar{R_u})^2}}.$$

Item-to-Item CF Algorithm Prediction Computation



- Recommend items with high-ranking based on similarity

Item-to-Item CF Algorithm Prediction Computation

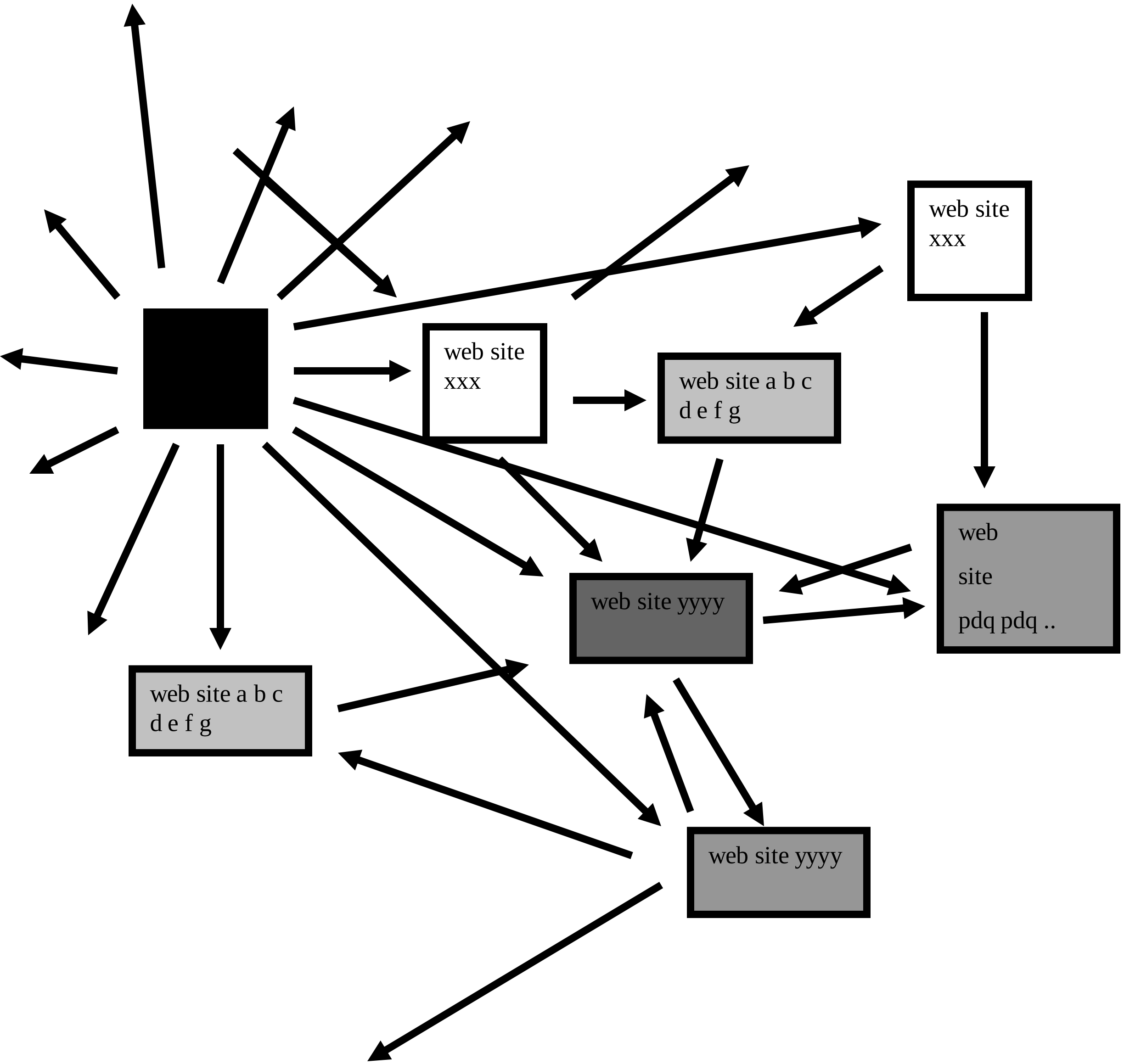
- Weighted Sum to capture how the active user rates the similar items

$$P_{u,i} = \frac{\sum_{\text{all similar items, N}} (s_{i,N} * R_{u,N})}{\sum_{\text{all similar items, N}} (|s_{i,N}|)}$$

- Regression to avoid misleading in the sense that two similarities may be distant yet may have very high similarities

$$\bar{R'_N} = \alpha \bar{R}_i + \beta + \epsilon$$

Google's PageRank



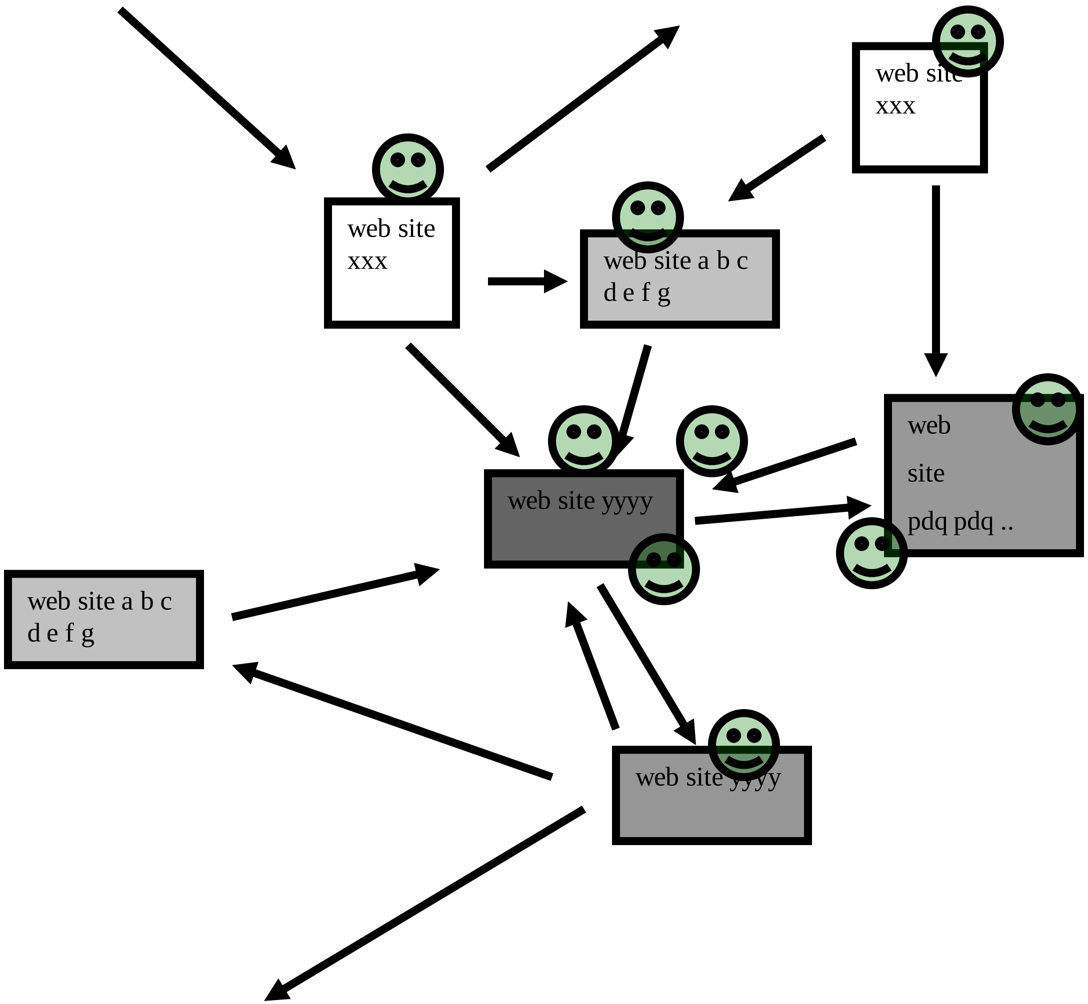
Inlinks are "good" (recommendations)

Inlinks from a "good" site are better than inlinks from a "bad" site

but inlinks from sites with many outlinks are not as "good"...

"Good" and "bad" are relative.

Google's PageRank

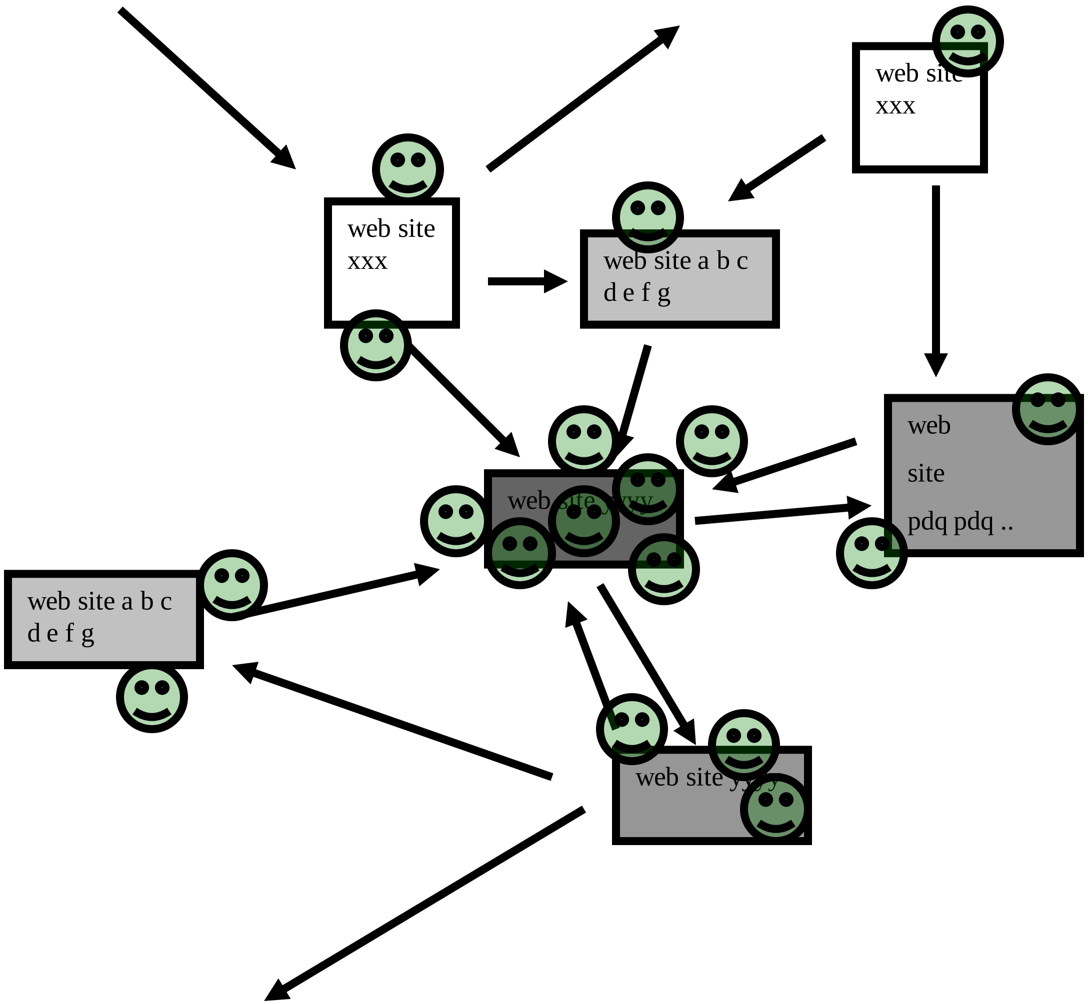


Imagine a "pagehopper" that always either

- follows a random link, or
- jumps to random page

Google's PageRank

(Brin & Page, http://www-db.stanford.edu/~backrub/google.html)



Imagine a "pagehopper" that always either

- follows a random link, or
- jumps to random page

PageRank ranks pages by the amount of time the pagehopper spends on a page:

- or, if there were many pagehoppers, PageRank is the expected "crowd size"

Memory-Based Algorithms (Breese et al, UAI98)

- $v_{i,j}$ = vote of user i on item j
- I_i = items for which user i has voted
- Mean vote for i is

$$\overline{v}_i = \frac{1}{|I_i|} \sum_{j \in I_i} v_{i,j}$$

- Predicted vote for "active user" a is weighted sum

$$p_{a,j} = \overline{v}_a + \kappa \sum_{i=1}^{n} \underbrace{w(a,i)}_{\text{weights of } n \text{ similar users}} (v_{i,j} - \overline{v}_i)$$

normalizer

weights of n similar users

Memory-Based Algorithms (Breese et al, UAI98)

- K-nearest neighbor

$$w(a,i) = \begin{cases} 1 & \text{if } i \in \text{neighbors}(a) \\ 0 & \text{else} \end{cases}$$

- Pearson correlation coefficient (Resnick '94, Grouplens):

$$w(a,i) = \frac{\sum_j (v_{a,j} - \overline{v}_a)(v_{i,j} - \overline{v}_i)}{\sqrt{\sum_j (v_{a,j} - \overline{v}_a)^2 \sum_j (v_{i,j} - \overline{v}_i)^2}}$$

- Cosine distance (from IR)

$$w(a,i) = \sum_j \frac{v_{a,j}}{\sqrt{\sum_{k \in I_a} v_{a,k}^2}} \frac{v_{i,j}}{\sqrt{\sum_{k \in I_i} v_{i,k}^2}}$$

Memory-Based Algorithms (Breese et al, UAI98)

- Cosine with "inverse user frequency" $f_i = log(n/n_j)$, where n is number of users, n_j is number of users voting for item j

$$w(a,i) = \frac{\sum_j f_j \sum_j f_j v_{a,j} v_{i,j} - (\sum_j f_j v_{a,j})(\sum_j f_j v_{i,j}))}{\sqrt{UV}}$$

where

$$U = \sum_j f_j (\sum_j f_j v_{a,j}^2 - (\sum_j f_j v_{a,j})^2)$$

$$V = \sum_j f_j (\sum_j f_j v_{i,j}^2 - (\sum_j f_j v_{i,j})^2)$$

Memory-Based Algorithms (Breese et al, UAI98)

- Evaluation:
  - split users into train/test sets
  - for each user *a* in the test set:
    - split *a's* votes into observed *(I)* and to-predict *(P)*
    - measure average absolute **deviation** between predicted and actual votes in *P*
    - predict votes in *P*, and form a **ranked list**
    - assume (a) utility of *k*-th item in list is $max(v_{a,j}-d,0)$, where *d* is a "default vote" (b) probability of reaching rank *k* drops exponentially in *k*. Score a list by its expected utility R_a
  - average R_a over all test users

Memory-Based Algorithms (Breese et al, UAI98)

soccer score

	EachMovie, Rank Scoring			
Algorithm	Given2	Given5	Given10	AllBut1
CR+	**41.60**	42.33	**41.46**	**23.16**
VSIM	**42.45**	**42.12**	**40.15**	22.07
BC	38.06	36.68	34.98	21.38
BN	28.64	30.50	33.16	23.49
POP	30.80	28.90	28.01	13.94
RD	*0.75*	*0.75*	*0.78*	*0.78*

Why are these numbers **worse**?

golf score

	EachMovie, Absolute Deviation			
Algorithm	Given2	Given5	Given10	AllBut1
CR	**1.257**	1.139	**1.069**	**0.994**
BC	1.127	1.144	1.138	1.103
BN	1.143	1.154	1.139	1.066
VSIM	**2.113**	**2.177**	**2.235**	**2.136**
RD	*0.022*	*0.023*	*0.025*	*0.043*

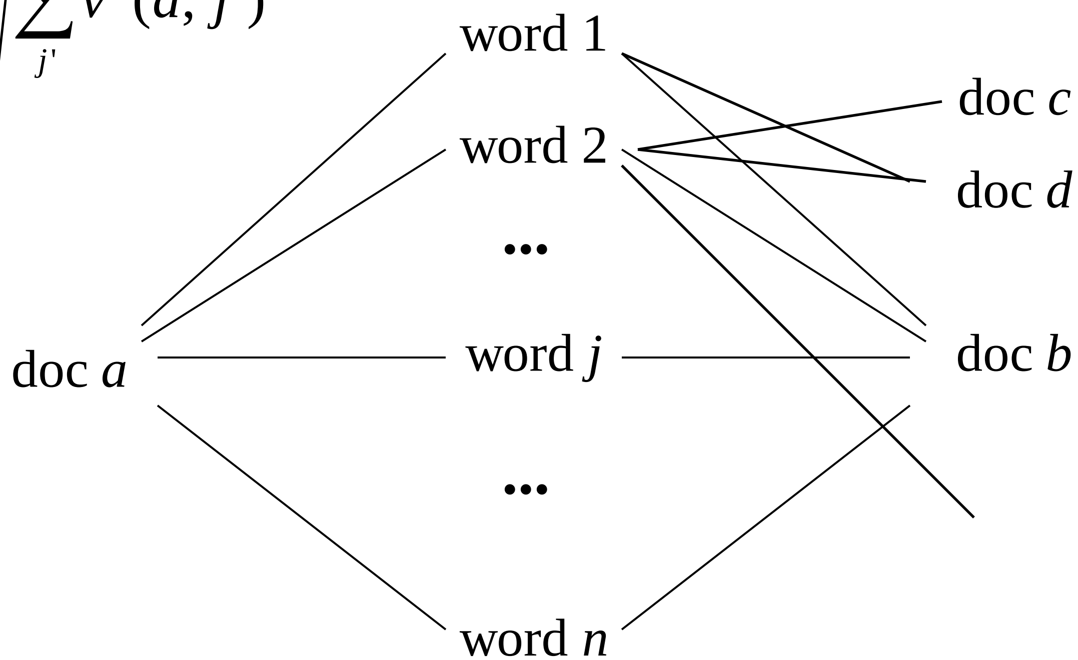
Visualizing Cosine Distance

similarity of doc a to doc b = $sim(a,b) = \sum_{\text{word } i} \frac{v(a, j)}{\sqrt{\sum_{j'} v^2(a, j')}} \cdot \frac{v(b, j)}{\sqrt{\sum_{j'} v^2(b, j')}}$

Let $\vec{A} = <\ldots, v(a, j), \ldots>$

Let $\vec{A}' = \frac{\vec{A}}{\|\vec{A}\|} = \frac{\vec{A}}{\sqrt{\sum_{j'} v^2(a, j')}}$

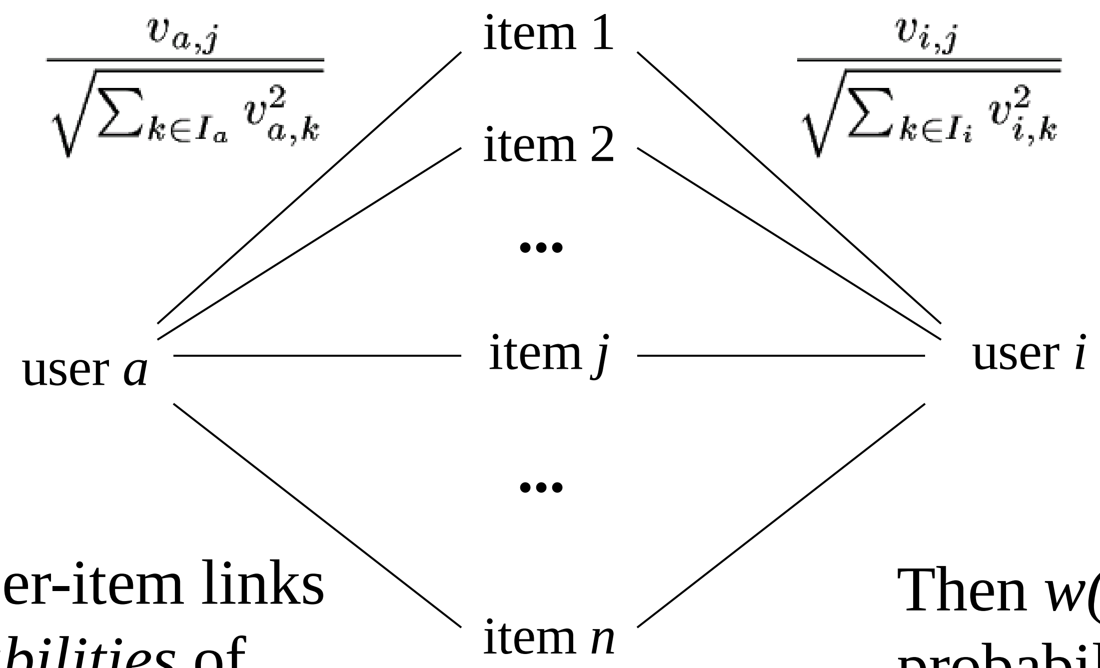
$= A' \cdot B'$



Visualizing Cosine Distance

distance from user *a* to user *i* = $w(a,i) = \sum_j \frac{v_{a,j}}{\sqrt{\sum_{k \in I_a} v_{a,k}^2}} \frac{v_{i,j}}{\sqrt{\sum_{k \in I_i} v_{i,k}^2}}$

$$\frac{v_{a,j}}{\sqrt{\sum_{k \in I_a} v_{a,k}^2}} \qquad \frac{v_{i,j}}{\sqrt{\sum_{k \in I_i} v_{i,k}^2}}$$

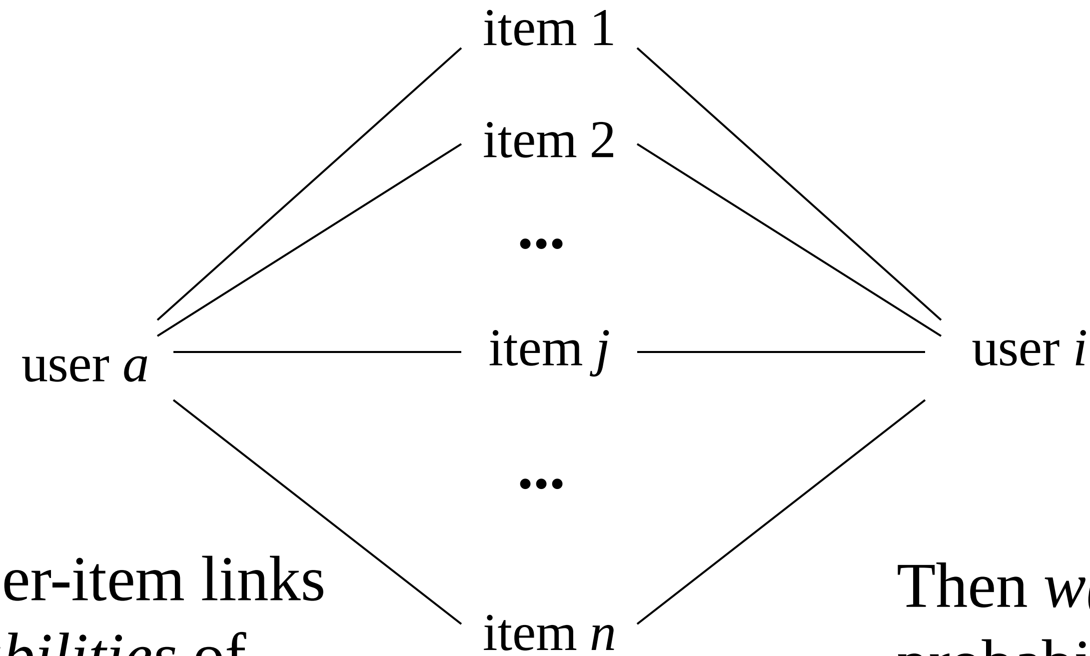


Suppose user-item links were *probabilities* of following a link

Then *w(a,i)* is probability of *a* and *i* “meeting”

Visualizing Cosine Distance

Approximating Matrix Multiplication for Pattern Recognition Tasks, Cohen & Lewis, SODA 97—explores connection between cosine distance/inner product and random walks



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