Introduction to Logistic Regression

Ivan Corneillet

Data Scientist

Learning Objectives

After this lesson, you should be able to:

- Build a logistic regression classification model using *sklearn*
- Describe the logit and sigmoid functions, odds and odds ratios, as well as how they relate to logistic regression
- Evaluate a model using metrics such as classification accuracy/error

Announcements and Exit Tickets

Q & A

Review

k-NN | Pros and cons

- Pros
  - Intuitive and simple to explain
  - Training phase is fast
  - Non-parametric (does not presume a “form” of the “decision boundary”)
  - Easily capture non-linearity

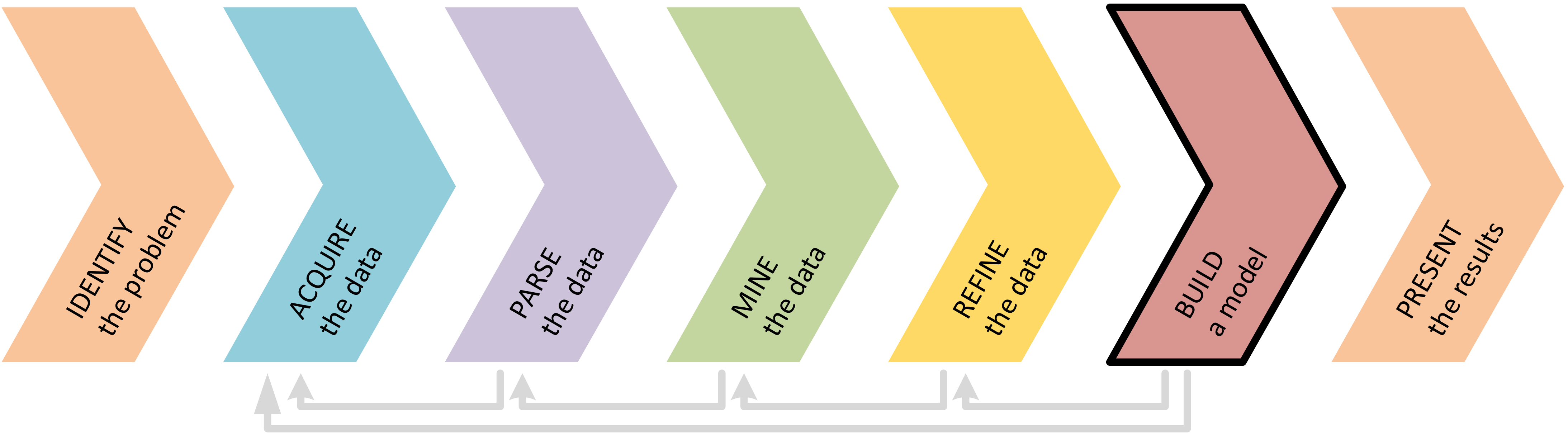
- Cons
  - Not interpretable
  - Prediction phase can be slow when n (number of observations) is large
  - Very sensitive to feature scaling; need to standardize the data
  - Sensitive to irrelevant features
  - Cannot be used if you have sparse data and feature space with dimension ≥ 4

Today

Today, we are focusing on logistic regression

Research Design and Data Analysis	Research Design	Data Visualization in *pandas*	Statistics	Exploratory Data Analysis in *pandas*
Foundations of Modeling	Linear Regression	Classification Models	Evaluating Model Fit	Presenting Insights from Data Models
Data Science in the Real World	Decision Trees and Random Forests	Time Series Data	Natural Language Processing	Databases

Today, we keep our focus on the BUILD a model step but with a focus on logistic regression



Here’s what’s happening today:

- Announcements and Exit Tickets
- Review
- ❻ Build a Model | Logistic Regression
  - How logistic regression relates to linear regression
  - “Retrofitting” linear regression into logistic regression
  - Interpreting the logistic regression coefficients
  - Iris dataset and Codealong on the Iris dataset
- Lab – Introduction to Logistic Regression
- Review
- Exit Tickets

Pre-Work

Pre-Work

Before this lesson, you should already be able to:

- Implement a linear model (`LinearRegression`) with *sklearn*
- Define the concept of coefficients
- Recall metrics for accuracy and misclassification

Logistic Regression

Why is logistic regression so valuable to know?

- It addresses many commercially valuable classification problems, such as:
  - Fraud detection (e.g., payments, e-commerce)
  - Churn prediction (marketing)
  - Medical diagnoses (e.g., is the test positive or negative?)
  - and many, many others…

Logistic Regression

How logistic regression relates to linear regression

Logistic regression is a generalization of the linear regression model to classification problems

- The name is somewhat misleading
  - "Regression" comes from fact that we fit a linear model to the feature space
  - But it is really a technique for classification, not regression

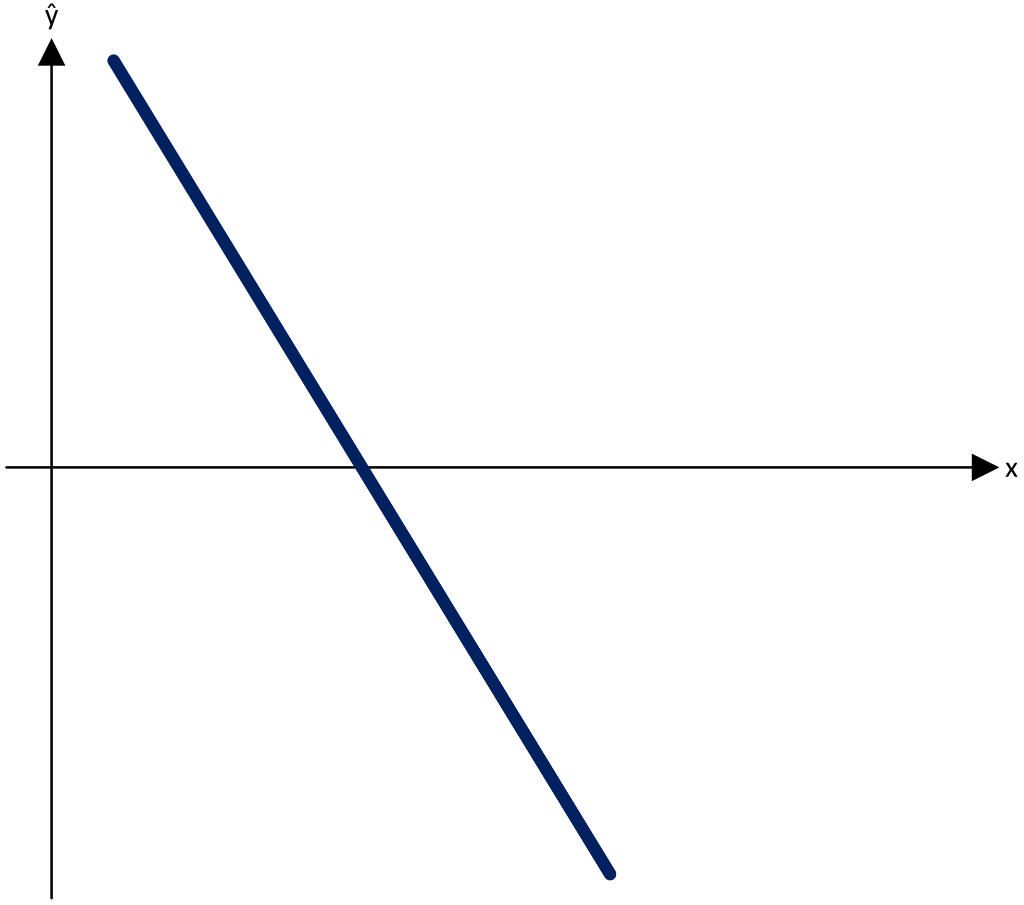
- We use a linear model, similar to linear regression, in order to solve if an item *belongs* or *does not* belong to a class model
  - It is a binary classification technique: $y = \{0, 1\}$
  - Our goal is to classify correctly two types of examples:
    - Class 0, labeled as 0, e.g., *"belongs"*
    - Class 1, labeled as 1, e.g., *"does not belong"*

With linear regression, $\hat{y}$ is in $]-\infty;\ +\infty[$, not $[0; 1]$. How do we fix this for logistic regression?

- The key variable in any regression problem is the outcome variable $\hat{y}$ given the covariate x

$$\hat{y} = X \cdot \hat{\beta}$$

- With linear regression, $\hat{y}$ takes values in $]-\infty;\ +\infty[$
- However, with logistic regression, $\hat{y}$ takes values in the unit interval $[0; 1]$



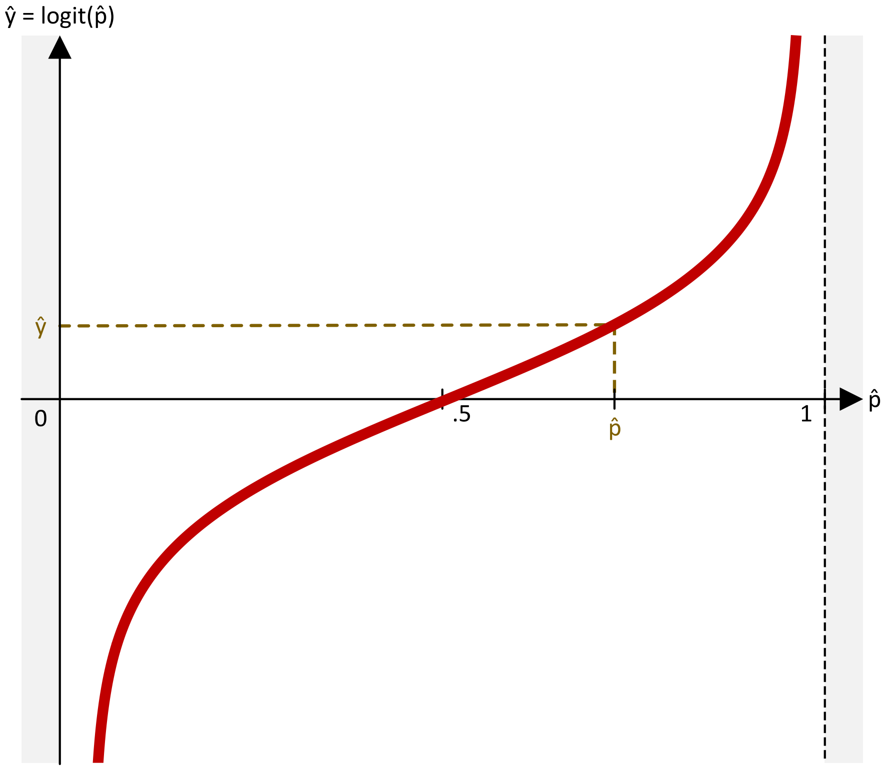
Logistic Regression

"Retrofitting" linear regression into logistic regression

We “retrofit” linear regression in logistic regression with a transformation called the *logit* function (a.k.a., the *log-odds* function) and its inverse, the *logistic* function (a.k.a., *sigmoid* function)

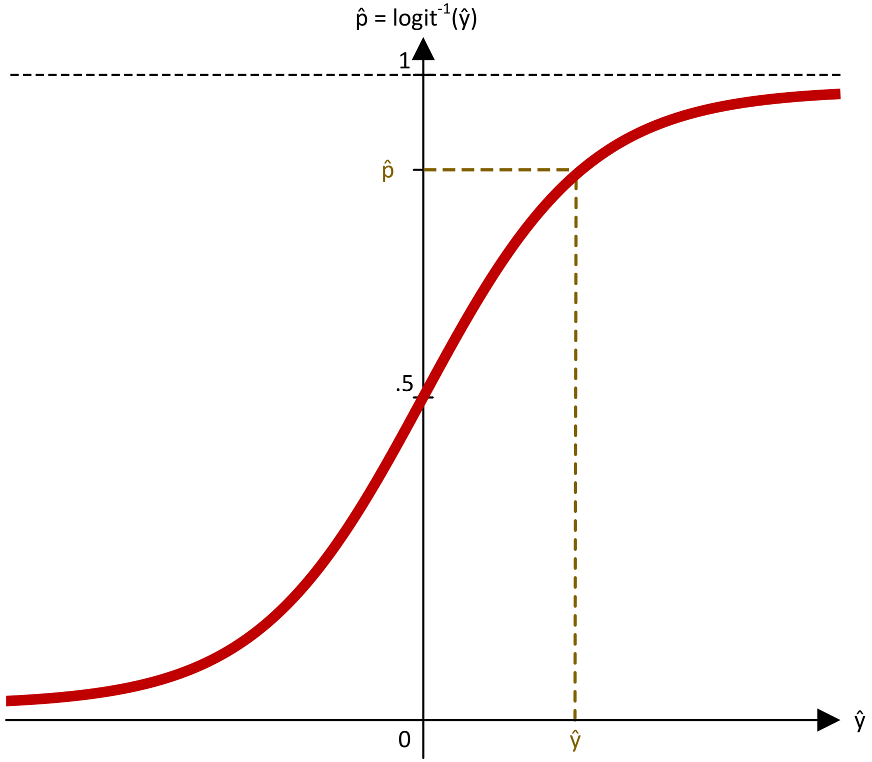
$logit$ maps $\hat{p}$ ($[0; 1]$) to $\hat{y}$ ($]-\infty;\ +\infty[$)

$$logit(\hat{p}) = ln\left(\frac{\hat{p}}{1-\hat{p}}\right) = \hat{y}$$



$\pi = logit^{-1}$ maps $\hat{y}$ ($]-\infty;\ +\infty[$) to $\hat{p}$ ($[0; 1]$)

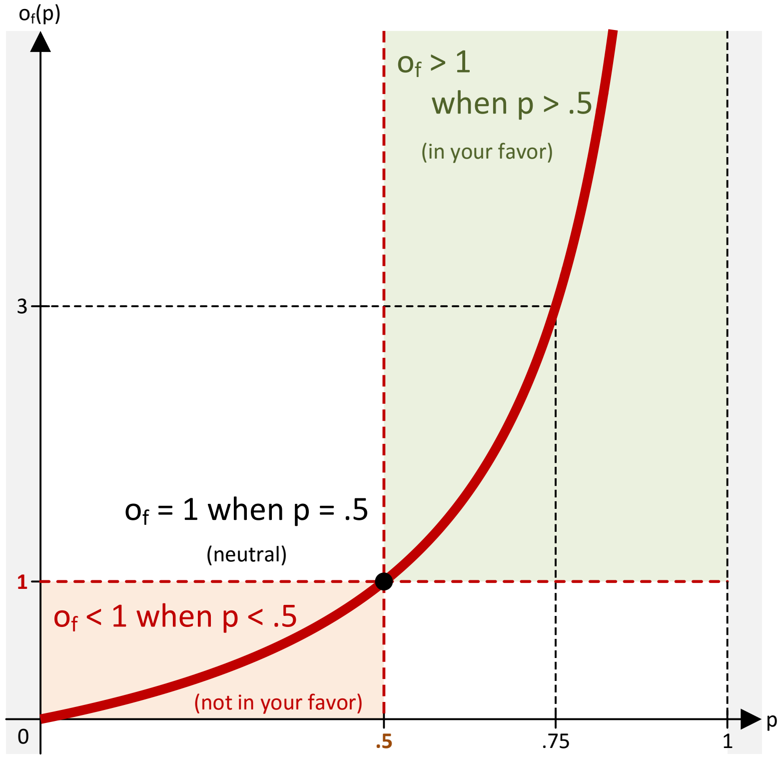
$$\pi(\hat{y}) = \frac{e^{\hat{y}}}{e^{\hat{y}}+1} = \hat{p}$$

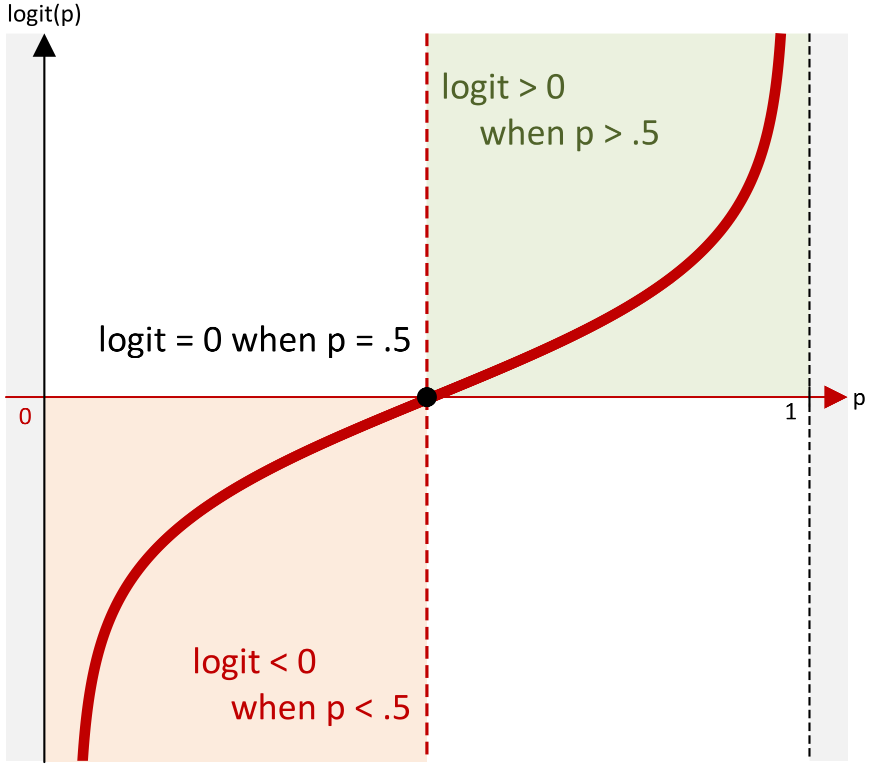


Why is the *logit* function also called the *log-odds* function?

$$\underbrace{o_f}_{\text{odds (in favor)}} = \frac{\overbrace{p}^{\text{probability that the event (with probability } p\text{) happens}}}{\underbrace{1-p}_{\text{probability that the event } \textbf{does not happen}}}$$

$$logit(p) = ln(o_f) = ln\left(\frac{p}{1-p}\right)$$



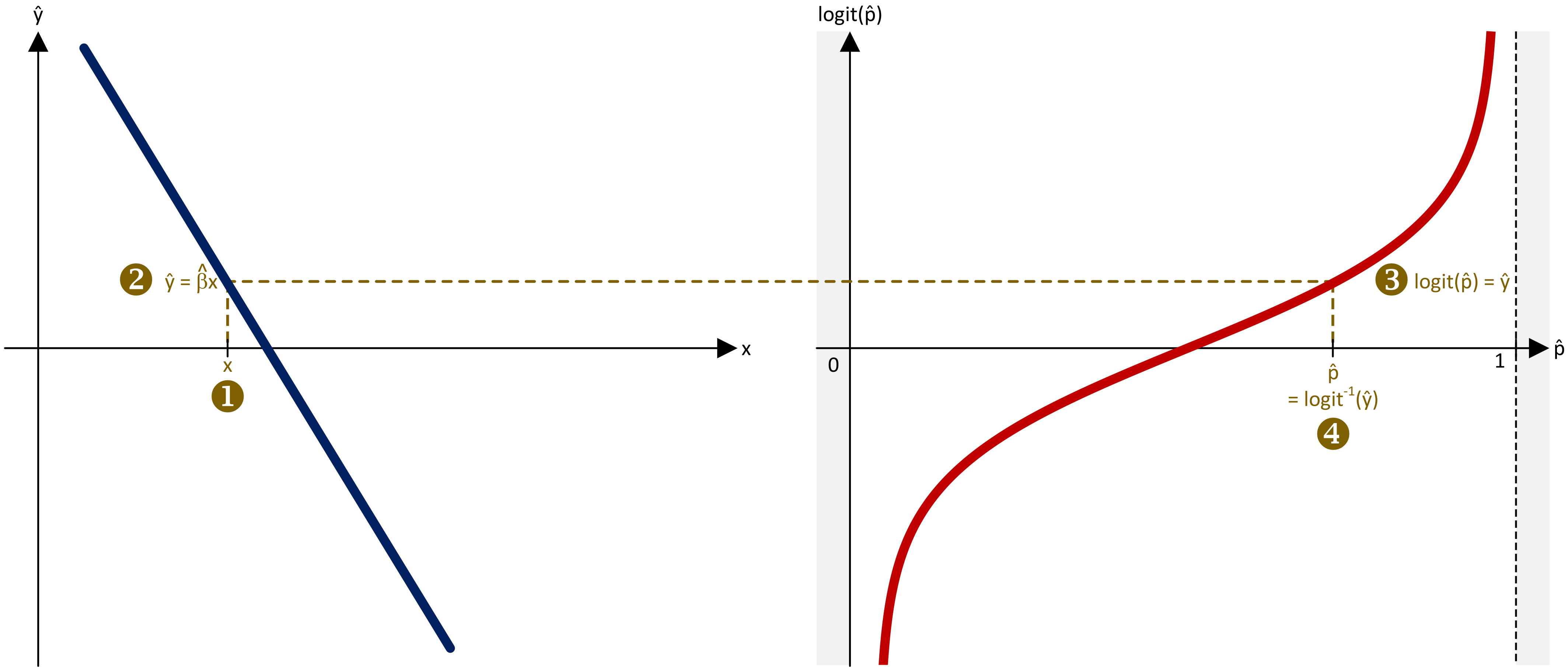


Logistic Regression

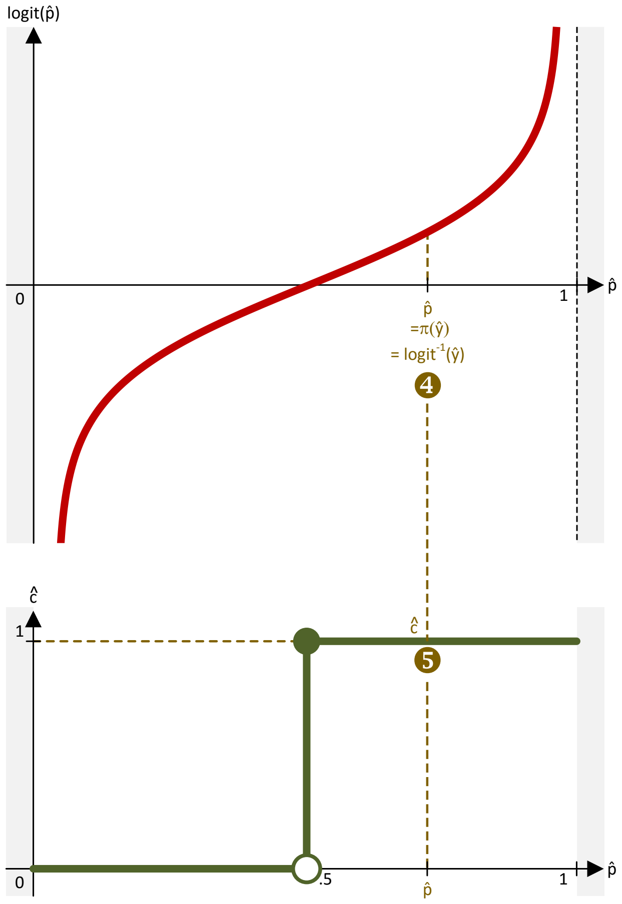
- Putting together $\hat{y} = \hat{\beta}x$ and $\hat{p} = \pi(\hat{y})$ (really, mapping $\hat{y}$ back to $\hat{p}$), we get

$$\hat{p} = \pi\left(\hat{\beta}x\right) = \frac{e^{\widehat{\beta}x}}{e^{\widehat{\beta}x} + 1} = \frac{1}{1 + e^{-\widehat{\beta}x}}$$

$$\hat{p} = logit^{-1}(\hat{y}) = logit^{-1}(\hat{\beta}x) = \frac{1}{1 + e^{-\hat{\beta}x}}$$



Finally, probabilities are “snapped” to class labels (e.g., by thresholding at the 50% level)



Logistic Regression

Interpreting the logistic regression coefficients

Interpreting the logistic regression coefficients

- With linear regressions, $\widehat{\beta_j}$ represents the change in y for a change in unit of x_j

$$ln\left(\frac{\hat{p}}{1-\hat{p}}\right) = \hat{\beta}x = \widehat{\beta_0} + \widehat{\beta_1} \cdot x_1 + \cdots + \widehat{\beta_k} \cdot x_k$$

- With logistic regressions, $\widehat{\beta_j}$ represents the **log-odds** change in y for a change in unit of x_j

- This also means that $e^{\widehat{\beta_j}}$ represents the multiplier change in **odds** in y for a change in unit of x_j

$$\frac{\widehat{odds}(x_j+1)}{\widehat{odds}(x_j)} = \frac{e^{\hat{y}(x_j+1)}}{e^{\hat{y}(x_j)}} = e^{\hat{y}(x_j+1)-\hat{y}(x_j)} = e^{(\cancel{\boxtimes}+\cancel{\widehat{\beta_j}\cdot x_j}+\cancel{\otimes})-(\cancel{\boxtimes}+\widehat{\beta_j}\cdot(\cancel{x_j}+1)+\cancel{\otimes})} = e^{\widehat{\beta_j}}$$

DS

Logistic Regression

Activity | Interpreting the logistic regression coefficients

Activity | Interpreting the logistic regression coefficients

EXERCISE

DIRECTIONS (5 minutes)

1. Suppose we are interested in mobile purchasing behavior. Let y be a class label denoting purchase/no purchase, and x a feature denoting whether a phone is an iPhone or not. After performing a logistic regression, we get $\beta_1 =$ .693. What does this mean?
2. When finished, share your answers with your table

DELIVERABLE

Answers to the above question

Activity | Interpreting the logistic regression coefficients (cont.)

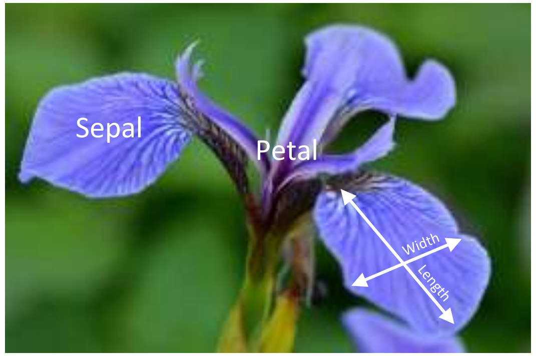
EXERCISE

1. In this case, the odds ratio change is $e^{\beta_1} = e^{.693} = 2$, meaning the likelihood of purchase is twice as high if the phone is an iPhone

Iris Dataset, Take 2

Review | Iris dataset

Iris Setosa



Iris Versicolor

Iris Virginica

Source: Flickr

- 3 classes of Irises (*Setosa*, *Versicolor*, and *Virginica*)
- 4 attributes
  - Sepal length and width
  - Petal length and width
- 50 instances of each class

Iris Dataset, Take 2

Codealong – Logistic Regression

Logistic Regression

Pros and Cons

Logistic Regression | Pros and cons

- Pros
  - Fit is fast
  - Output is a (posterior) probability which is easy to interpret

- Cons
  - Limited to binary classification (but sklearn provides a multiclass implementation; use ensemble under the hood)

Logistic Regression

Further Readings

Further Readings

- ISLR
  - Logistic Regression (section 4.3, pp. 130 – 138)
- ESLII
  - Logistic Regression (section 4.4, pp. 119 – 129)

Lab

Introduction to Logistic Regression

Review

Review

You should now be able to:

- Build a Logistic regression classification model using *sklearn*
- Describe the logit and sigmoid functions, odds, and odds ratios as well as how they relate to logistic regression
- Evaluate a model using metrics such as classification accuracy/error

Q & A

Next Class

Flexible Class Session #2 | Machine Learning Modeling

Learning Objectives

After this next lesson, you should be able to:

- Review Steps ❺ Refine the Data and ❻ Build a Model and more specifically
  - Linear Modeling (OLS)
  - Classification Modeling (k-NN and Logistic Regressions)
- Have fun doing Data Science!

Exit Ticket

Don’t forget to fill out your exit ticket here