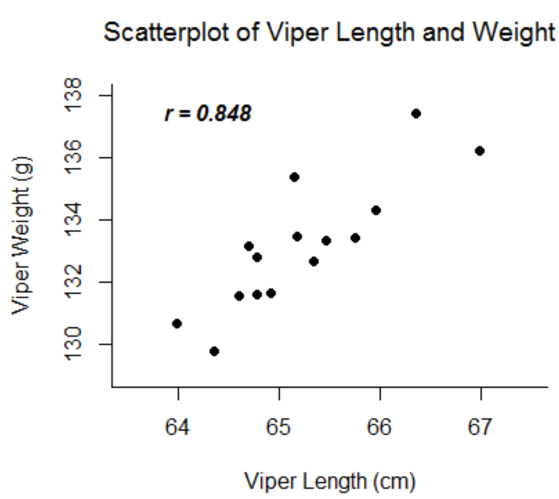
Pearson Correlation and Linear Regression

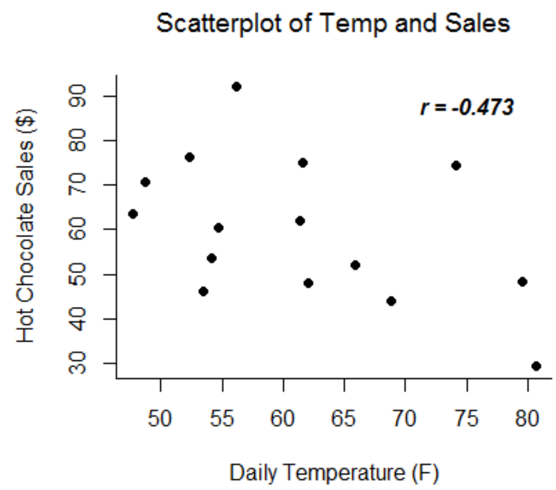
A correlation or simple linear regression analysis can determine if two numeric variables are significantly linearly related. A correlation analysis provides information on the **strength** and **direction** of the linear relationship between two variables, while a simple linear regression analysis estimates parameters in a linear equation that can be used to **predict** values of one variable based on the other.

Correlation

The Pearson correlation coefficient, r, can take on values between -1 and 1. The further away r is from zero, the stronger the linear relationship between the two variables. The sign of r corresponds to the direction of the relationship. If r is positive, then as one variable increases, the other tends to increase. If r is negative, then as one variable increases, the other tends to decrease. A perfect linear relationship (r=-1 or r=1) means that one of the variables can be perfectly explained by a linear function of the other.

Examples:





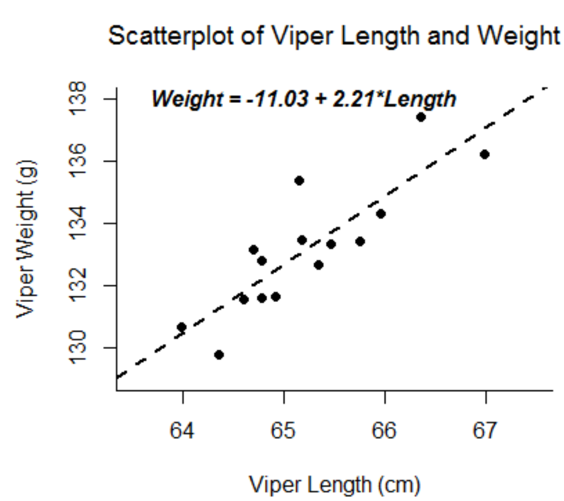
Linear Regression

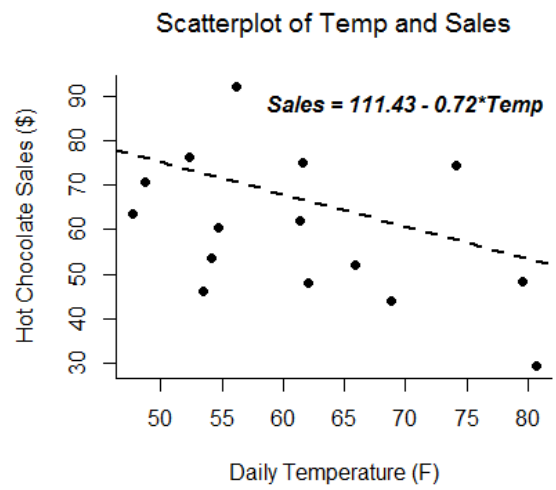
A linear regression analysis produces estimates for the **slope** and **intercept** of the linear equation predicting an outcome variable, *Y*, based on values of a predictor variable, *X*. A general form of this equation is shown below:

$$Y = b_0 + b_1 \cdot X$$

The intercept, b_0, is the predicted value of *Y* when *X*=0. The slope, b_1, is the average change in *Y* for every one unit increase in *X*. Beyond giving you the strength and direction of the linear relationship between *X* and *Y*, the slope estimate allows an interpretation for how *Y* changes when *X* increases. This equation can also be used to predict values of *Y* for a value of *X*.

Examples:





Inference

Inferential tests can be run on both the correlation and slope estimates calculated from a random sample from a population. Both analyses are *t*-tests run on the null hypothesis that the two variables are not linearly related. If run on the same data, a correlation test and slope test provide the same test statistic and *p*-value.

Assumptions:

- Random samples
- Independent observations
- The predictor variable and outcome variable are linearly related (assessed by visually checking a scatterplot).
- The population of values for the outcome are normally distributed for each value of the predictor (assessed by confirming the normality of the residuals).
- The variance of the distribution of the outcome is the same for all values of the predictor (assessed by visually checking a residual plot for a funneling pattern).

Hypotheses:

H_o: The two variables are not linearly related.
H_a: The two variables are linearly related.

Relevant Equations:

Degrees of freedom: $df = n-2$

$$r = \frac{\sum z_x z_y}{n-1}$$

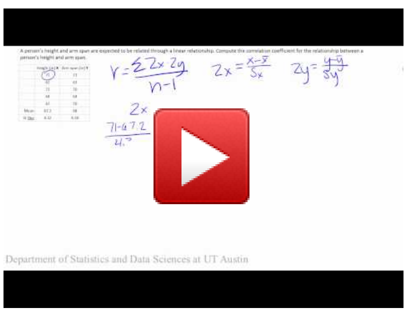
$$b_1 = r \cdot \frac{s_y}{s_x}$$

$$b_0 = \bar{Y} - b_1 \cdot \bar{X}$$

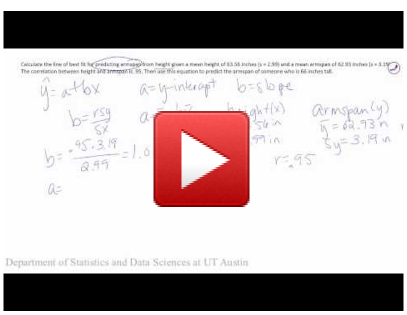
Example 1: Hand calculation

These videos investigate the linear relationship between people's heights and arm span measurements.

Correlation:



Regression:



Sample conclusion: Investigating the relationship between armspan and height, we find a large positive correlation (*r*=.95), indicating a strong positive linear relationship between the two variables. We calculated the equation for the line of best fit as *Armspan*=-1.27+1.01*(Height)*. This indicates that for a person who is zero inches tall, their predicted armspan would be -1.27 inches. This is not a possible value as the range of our data will fall much higher. For every 1 inch increase in height, armspan is predicted to increase by 1.01 inches.

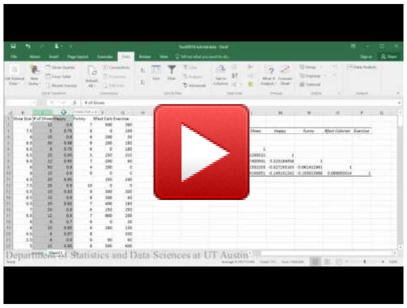
Example 2: Performing analysis in Excel 2016 on

Some of this analysis requires you to have the add-in Data Analysis ToolPak in Excel enabled.

Dataset used in videos

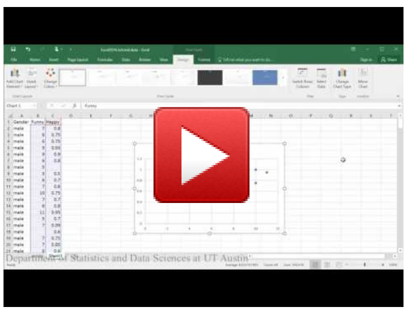
Correlation matrix and *p*-value:

PDF directions corresponding to video



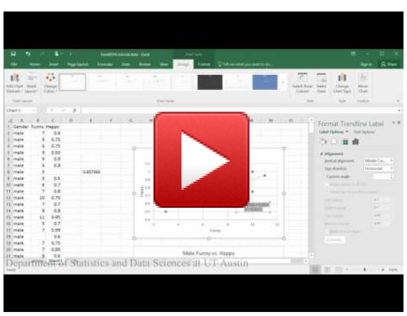
Creating scatterplots:

PDF directions corresponding to video



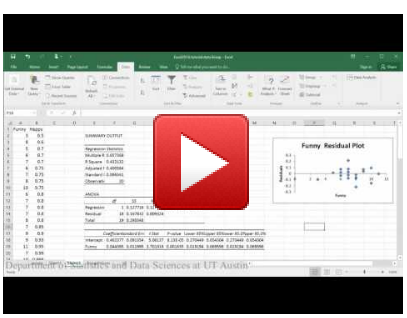
Linear model (first half of tutorial):

PDF directions corresponding to video



Creating residual plots:

PDF directions corresponding to video



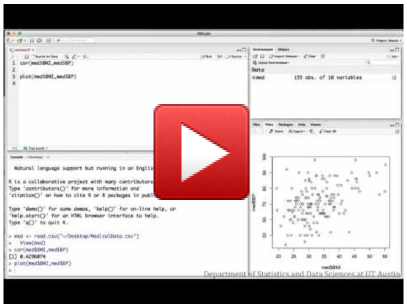
Sample conclusion: In evaluating the relationship between how happy someone is and how funny others rated them, the scatterplot indicates that there appears to be a moderately strong positive linear relationship between the two variables, which is supported by the correlation coefficient (r = .65). A check of the assumptions using the residual plot did not indicate any problems with the data. The linear equation for predicting happy from funny was *Happy*=.04+0.46*(Funny).* The y-intercept indicates that for a person whose funny rating was zero, their happiness is predicted to be .04. Funny rating does significantly predict happiness such that for every 1 point increase in funny rating the males are predicted to increase by .46 in happiness (t = 3.70, p = .002).

Example 3: Performing analysis in R

The following videos investigate the relationship between BMI and blood pressure for a sample of medical patients.

Dataset used in videos

Correlation:
R script file used in video



Regression:
R script file used in video