Physics I: Classical Mechanics

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1 Chapter 2 - One Dimensional Motion

1.1 Displacement, Time, and Average Velocity

1. All objects are placed on a coordinate system, and viewed as a particle, or a single point of negligible size and shape
2. The displacement is equal to the vector from the initial point to the final point, written $\Delta x = x_f - x_i$, measured in meters (m)
   (a) Displacement is typically drawn as a function of time on an x-t graph, such that the secant line from two points is the average velocity within
   (b) Distance is defined as the scalar quantity of the movement of the particle in the time interval
3. Velocity is the rate of change of position with respect to time, in m/s
4. $v_{average} = \frac{\Delta x}{\Delta t}$, such that Δt is the scalar change in time during the movement
   (a) Speed is defined as the scalar distance over change in time, or $s = \frac{d}{\Delta t}$
   (b) Δt is typically represented as going from 0 to t, such that Δt is often represented at t

1.2 Instantaneous Velocity

1. Instantaneous velocity is the velocity at a specific point in time or position
   (a) v ¡ 0 is moving backward, v ¿ 0 forward, and v = 0 is not moving
   (b) Instantaneous speed is the instantaneous velocity, as a scalar quantity
2. $v_{instantaneous} = \frac{dx}{dt}$
   (a) Can be drawn on a graph as the slope of the secant line at a specific time and point
3. Velocity can be drawn on a motion diagram, drawing a line for the x-axis, then a specific point in time on the line, with an instantaneous velocity vector drawn on it at that time

1.3 Average and Instantaneous Acceleration

1. Acceleration is the rate of change of velocity with respect to time, in m/s^2
   (a) On a v-t graph, it is interpreted the same as velocity is on an x-t graph
   (b) On an x-t graph, it is viewed as the concavity of the graph, equal to 0 at a point of inflection, or where there is no concavity
   (c) Can be drawn on a motion diagram showing the change in velocity from one point in time to another, similarly to how velocity is drawn
   (d) On an a-t graph, Δv is the area under the curve to the x-axis
2. $a_{average} = \frac{\Delta v}{t}$
3. $a_{instantaneous} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

1.4 Motion with Constant Acceleration

1. Drawn by a straight line on an a-t graph, or a parabola on an x-t graph such that there is a constant rate of change of velocity
2. $v_f = v_i + at$
3. $v_{average} = \frac{v_i + v_f}{2}$

4. $\Delta x = v_i t + \frac{at^2}{2}$
5. $v_f^2 - v_i^2 = 2a\Delta x$

1.5 Freely Falling Bodies

1. Particles falling without any force other than gravity acting on them are said to be in free fall, and move with constant acceleration of g, which is 9.8 m/s^2

1.6 Motion with Non-Constant Acceleration

1. $\Delta v = \int_0^t a(t)dt$
2. $\Delta x = \int_0^t v(t)dt$
3. $\Delta d = \int_0^t |v(t)|dt$
4. To get proper functions and values, the boundary conditions must always be used on integrals, rather than simply used as indefinite integrals

2 Chapter 3 - Multidimensional Motion

2.1 Position, Velocity, and Acceleration Vectors

1. Position Vector $(\vec{r}) = x\vec{i} + y\vec{j} + z\vec{k}$
2. $\Delta\vec{r} = \Delta x\vec{i} + \Delta y\vec{j} + \Delta z\vec{k}$
3. Speed (s) = —v—
4. $\vec{v} = \frac{d\vec{r}}{dt}$
5. Velocity can be thought of in two perpendicular components and a parallel component, such that the perpendicular signify the change in direction, while the parallel signifies the change in speed
6. $a = \frac{d\vec{v}}{dt}$

2.2 Projectile Motion

1. Projectiles are a body with initial velocity, such that the only force acting on it is air resistance and gravitational acceleration
2. Idealized projectiles move with gravitational acceleration that is constant, ignoring air resistance, earth rotation, and earth curvature
   (a) Ideal projectiles follow their parabolic 2D trajectory path
3. It is measured, so that $a_x = 0$ and $a_y = -g = 9.81 m/s^2$, with initial velocity (v_0), so $v_{0x} = v_0 cos\theta$, and $v_{0y} = v_0 sin\theta$, typically using the starting point as the origin
4. Air resistance pushes against all directions of motion, such that the ideal angle for maximum distance is $< 45^o$, and max height and range are less than expected
5. Range = $\frac{2sin(\theta)cos(\theta)v^2}{g}$
6. Height = $\frac{v^2 sin^2(\theta)}{2g}$
7. Flight Time = $\frac{2vsin\theta}{g}$

2.3 Circular Motion

1. Cartesian vectors can be converted to polar form, such that $\vec{r}$ and $\vec{\theta}$ are the unit vectors, where $\vec{r}$ is coming from the origin, while $\vec{\theta}$ is perpendicular, such that it is tangential to the circle, both dependent on the angle from the origin (ø), while r is a fixed vector length
   (a) $\vec{r} = \vec{i}cos(\theta) + \vec{j}sin(\theta)$
   (b) $vec\theta = \vec{i}(-sin(\theta)) + \vec{j}cos(\theta)$
   (c) $\vec{r}$ (position vector) $= r\vec{r}$ by polar conversion ($x = rcos\theta$, $y = rsin\theta$)
   (d) Thus, $\vec{v} = r\vec{\theta}\frac{d\theta}{dt} = v\vec{\theta}$, where v is tangential velocity, such that if $\theta = \omega t$ where ω is angular velocity, then $\vec{v} = r\omega\vec{\theta}$
   (e) $\vec{a} = \vec{\theta}\frac{dv}{dt} + v\frac{d\theta}{dt}$, where the first part is tangential acceleration, and $\vec{a}_c = -v(\frac{d\theta}{dt}\vec{r} = -v\omega\vec{r}$, where if $\theta = \omega t$, then $\vec{a}_c = \frac{-v^2}{r}$
2. Uniform circular motion is moving around a circle with constant speed, such that the acceleration is purely perpendicular
3. $a_{radial} = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2}$
   (a) T is the time for one revolution around the circle, while R is the radius
   (b) This acceleration is called centripetal acceleration, due to going toward the center of the circle
4. Nonuniform circular motion is motion around a circle with non constant speed, such that radial acceleration is still the same, but there is also tangential (parallel) acceleration due to the change in speed
   (a) Radial acceleration changes based on the speed at that moment
   (b) $a_{tan} = \frac{d|v|}{dt}$

2.4 Relative Velocity

1. The appearance of velocity relative to the observer is called relative velocity, determined by the movement of the frame of reference of the observer
   (a) $x_{P/A} = x_{P/B} + x_{B/A}$, meaning the position of P relative to frame A is equal to the position of P relative to B plus the position of B relative to frame A
   (b) Thus, $\frac{dx_{P/A}}{dt} = \frac{dx_{P/B}}{dt} + \frac{dx_{B/A}}{dt}$, and $v_{P/A} = v_{P/B} + v_{B/A}$
2. If $\vec{v}$ is the constant velocity of the origin, such that origin O moves to O', then $\vec{r} = \vec{v}t + \vec{r}'$
   (a) If $\vec{v}_r$ is the velocity of the point, $\vec{v}_r'$ (velocity of the point relative to O') $= \vec{v}_r - \vec{v}$, while acceleration is the same
3. This is referred to as the Galilean Velocity Transformation
   (a) At frame of reference speeds nearing the speed of light, real velocity can be greater than c, which is impossible, leading to the Relativistic Transformation

3 Chapter 4 - Newton's Laws of Motion

3.1 Force

1. Force is a vector quantity that one body or the environment exerts on another, measured in Newtons (N), or $kg * m/s^2$

(a) Measured with a spring balance, which stretches a spring to measure the force with a meter, or an inverse, which compresses to measure

2. Contact force is a force through direct contact between the bodies, including normal and tension force
   (a) Normal force is exerted by any surface a body is in contact with, perpendicular to the surface, only existing through direct contact
3. Tension force is the force through a body in tension (being pulled from both sides), such as a frictionless rope, acting on the body attached in the direction the attached body is being pulled
   (a) Tension forces is constant throughout a string, going in each direction being pulled, so that a block connected to a pulley has double the force, though a string separating into other strings has different forces on each
   (b) Tension force is equal to the force being exerted on the object through the rope, if there is no responsive force on that exerting
4. Spring force is the force exerted by the spring being stretched by some mass, such that F = kx, where x is the displacement from equilibrium
   (a) The spring constant of springs in series is the reciprocal of the reciprocal sum, while parallel springs are the sum of the forces
5. Long range forces are forces which act on separate bodies, such as electromagnetic force
   (a) Weight is the gravitational force, exerted by the planet
6. Net force on an object is equal to the vector sum of individual forces, allowing individual forces to be replaced by their component vectors
7. Free body diagrams use vectors coming out from a particle to show all external forces acting on a specific body, to help understand the net force on the body

3.2 Newton's First Law

1. A body acted on by no net force moves with constant velocity, and zero acceleration, thus staying at rest or at motion, due to the inertia of matter
   (a) A body is at equilibrium if the net force is 0
   (b) Assumes each body can be represented as a point particle
2. The law is valid with an inertial frame of reference, such that the reference itself doesn't have a force acting on it
   (a) If a non-inertial reference has force acting on it, the relative velocity to the reference would change, though it would not change relative to another inertial reference
   (b) The Earth is approximately inertial, though not, due to rotation and revolution around the Sun

3.3 Newton's Second Law

1. $\sum F = ma$, such that $\sum F$ is the sum of external forces (forces exerted by other bodies on the body in question)
2. The law is only valid in inertial frames of reference and when mass is constant
3. m is the inertial mass of body, defined as the constant by which force is directly proportional to acceleration, measured typically in Kilograms (kg)
   (a) Mass of bodies added together is summed

(b) Mass is determined by the number of subatomic particles in the matter
(c) Within the CGS metric system, 1 dyne = 1 $g * cm/s^2 = 10^{-5}$ N, and in the British Imperial system, 1 pound = 4.4 N
4. Acceleration can thus be gained, use to calculate velocity, displacement, or time, but the equations should be put in terms of the constants given, not time

3.4 Weight

1. Weight is the gravitational force of the planet on a body
2. $F_w = mg$, such that g is the gravitational acceleration of the planet
   (a) Gravitational acceleration on the Earth is 9.8 m/s^2, though it varies throughout the Earth slightly due to rotation and revolution
   (b) Mass measured through a scale, using weight to measure it is called gravitational mass, and ideally, is equal to inertial mass
3. Apparent weight is the normal force of an object when the surface has additional upward or downward forces acting on it, changing the scale measurement, since scales measure by the normal force they exert
   (a) Apparent weightlessness is when in free fall, such that apparent weight is zero

3.5 Newton's Third Law

1. If one body exerts a force on another, then the other body exerts an equal but opposite force on the first body ($F_{aonb} = -F_{bona}$)
   (a) This law applies to both long range, and contact forces
   (b) The two forces are called an action-reaction pair

4 Chapter 5 - Applicatons of Newton's Laws

4.1 Friction Force

1. Friction force opposes movement, parallel to the surface, opposite the contact force of an object being pushed or pulled on a surface
   (a) Friction forces result from the microscopic ridges in two surfaces coming together and apart as surfaces come into contact, changing slightly throughout movement
   (b) Oil is used to prevent friction by producing a film over the ridges, preventing them from touching
   (c) $\mu_s \geq \mu_k \geq \mu_r$
2. The friction force as an object slides over a surface is kinetic friction force (f_k)
   (a) $f_k = \mu_k F_n$, where F_n is the normal force and μ_k is the coefficient of kinetic friction, determined by the object and surface material
3. The friction force when there is no relative motion between an object and a surface is static friction force (f_s)
   (a) $f_s \leq \mu_s F_n$, where μ_k is the coefficient of static friction, determined by the object and surface material

(b) As the contact force attempts to move the object, static friction rises to counter it, until it reaches the maximum, at which point it drops to kinetic
(c) Vehicles are able to move due to the wheels rotating, pushing backward, such that static friction pushes it forward, such that they can move in uniform circular motion using the same friction as the centripetal force
4. Rolling friction or tractive friction (μ_r) is the coefficient of friction used to calculate the kinetic friction with wheels
5. Fluid resistance is the force a gas or liquid exerts on a body moving through it, opposite the movement of the body itself
   (a) Drag (F_d) $= bv^2$ for larger bodies (tennis balls or larger), $F_d = bv$ for smaller bodies where b is the drag constant depending on size/cross-section, speed and shape of the body, and the viscosity fluid
   (b) Terminal speed is the speed where the contact force equals the drag resistance, at which point it cannot increase any further
   (c) $v = \frac{mg(1-e^{-bt/m})}{b}$ for a dropped body, such that as $b \to 0, e^x = 1 + x + x^2 + ... \approx 1 + x$, such that v = -gt, while as $t \to \infty, v \to v_{terminal} = \frac{mg}{b}$
6. Centripetal force is the net force which allows circular motion, toward the center of the circle, but is not a force in its own right

4.2 Fundamental Forces

1. All forces are a combination of or fall within the category of four fundamental forces of particle interaction
2. Gravitational interactions are those resulting from different bodies interacting on each other through gravitational force, such as weight, affecting larger bodies vastly more than smaller ones
3. Electromagnetic interactions include electric and magnetic forces
   (a) Electric forces result from the positive and negative charge of atoms, exerting forces on each other
   (b) Magnetic forces result from the movement of electric charges
   (c) These forces tend to cancel out as the bodies get larger, affecting smaller particles more
4. Strong interactions include the strong nuclear force which holds nuclei together, overcoming the electromagnetic repulsion, working only under vastly minute distances
5. Weak interactions include the weak nuclear force, which allows beta decay through ejection of an antineutrino and an electron, letting supernovae occur
6. Electroweak interactions were developed to encompass electromagnetic and weak interactions, leading to the grand unified theory containing electroweak and strong interactions
   (a) This has led to attempts to produce a theory of everything, containing all four types of interactions

5 Chapter 6 - Work and Kinetic Energy

5.1 Work

1. Work is the scalar product of displacement and the force in the direction of displacement, such that W = Fx and $W_{tot} = F_{net}x$ when force is constant
   (a) $W = \int_{x_1}^{x_2} F_{||}(x) \cdot dx$ for non-constant force, where $F_{||}$ is the force function tangential/parallel to the movement at that point, called a line integral
   (b) Work is measured in Joules (J), or N*m
   (c) Measured in the British Imperial System with ft*lb, since lb is the measurement for force
2. Work is zero if the force is perpendicular to the displacement, and negative if the force is opposite the displacement
3. Stable equilibrium is the point where a shift in any direction would cause movement back to equilibrium, unstable is where it would cause a shift from, and neutral equilibrium causes a lack of shift after slight movement
4. $W_{net} = \int_{x_1}^{x_2} F_{net}(x) \cdot dx = \sum_n W_n$

5.2 Kinetic Energy

1. $K = \frac{1}{2}mv^2$
2. Increase in speed means positive acceleration in the direction of displacement, work is positive, showing the relationship between work and velocity
3. The Work Energy Theorem states that $W_{total} = \Delta K$ for any single particle or composite system within any inertial frame of reference
   (a) This is true for any forces, conservative or nonconservative

5.3 Power

1. Power is the rate at which work is done, measured in Watts (W), or J/s
   (a) Can also be measured in ft*lb/s, or horsepower (hp), such that 1 hp = 500 ft*lb/s
2. $P_{av} = \frac{\Delta W}{\Delta t}$
3. $P = \frac{dW}{dt} = F \cdot v$ (if F is constant)

6 Chapter 7 - Potential Energy and Conservation of Energy

6.1 Gravitational Potential

1. Potential energy is the energy associated with position, which can be converted through work into kinetic energy, such that the change in potential energy is useful, rather than the actual potential energy
   (a) Thus, any point can be chosen as the zero point within convention, such that it is measured from that
2. Gravitational Potential Energy (U_{grav}) = $F_{weight}\Delta h$
   (a) Thus, $W_{grav} = -\Delta U_{grav}$

(b) If F_g is the only force acting on a body, then $\Delta K = -\Delta U_{grav}$

(c) Gravitational potential energy is considered a shared property of both bodies the force exists between, considered part of the same system

(d) Gravitational potential of a single body $= -\frac{GmM}{r}$, where r is the distance between the centers, m is the body, and M is the attracting body

6.2 Elastic Potential Energy

1. Elastic potential energy is the energy stored in a stretched elastic body, or a body which returns to its original shape and size after being changed
   (a) $U_{el} = \frac{1}{2}kx^2$, where x is the distance stretched from the relaxed position
   (b) $W_{el} = -\Delta U_{el}$

6.3 Conservative Forces

1. Conservative Forces are forces where the work is reversible, can be expressed as ΔU, if $\Delta x = 0, W = 0$, and depends only on Δx, not on the path taken
   (a) $W_{conservative} = -\Delta U$, where W is the work done by conservative forces and U is the total potential energy of the system
   (b) Total Potential Energy of the System (U) is the sum of all forms of potential energy within the system
   (c) E is constant if only conservative forces act on the body
   (d) Conservative forces include elastic, gravitational, and electric force
2. Nonconservative forces include friction or fluid resistance
   (a) Nonconservative forces which cause a loss in mechanical energy are called dissipative forces
3. Internal Energy (U_{int}) is the energy of the molecular state of the body, such as the body temperature, increasing when mechanical energy is lost
   (a) $W_{nonconservative} = -\Delta U_{int}$
   (b) Work on a closed path is not equal to 0, due to energy lost based on path
   (c) The increase in internal energy due to friction is generally considered to be the rotational kinetic energy for a rolling body
4. The Law of Conservation of Energy states $\Delta K + \Delta U + \Delta U_{int} = 0$
   (a) Thus, energy is never created or destroyed, but its form can be changed
   (b) Total Mechanical Energy of the System (E) = K + U_{grav}, such that $\Delta E + \Delta U_{int} = 0$, or $\Delta E = W_{nonconservative}$
5. Energy diagrams can be used to understand motion, graphing U(x), such that if there are only mechanical forces acting on it, y = E can be drawn to show the limits of motion
   (a) The relative minimums of the curve are stable equilibrium, which the force always attempts to restore
   (b) The relative maximums of the curve are unstable equilibrium, such that any force tends to push it away from the equilibrium
   (c) Since it cannot go beyond the bounds of the E line, the region between is the potential well, where the endpoints are the turning points where the particle is forced to change direction

6.4 Force and Potential Energy

1. $F = -\frac{dU}{dx}$, when U is a one dimensional potential energy
2. $\vec{F} = -(\frac{\partial U \vec{i}}{\partial x} + \frac{\partial U \vec{j}}{\partial y} + \frac{\partial U \vec{k}}{\partial z}) = -\vec{\nabla} U$, for 3 dimensional potential energy, able to be extended to other dimensions