Multivariable Calculus

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Contents

1 Sequence Theorems

1.1 Sunrise Lemma

The Sunrise Lemma states for any sequence $(a_n)_n^\infty = 1$ in $\mathbb{R}$, $\exists$ monotone subsequence $(a_{n_k})_{k=1}^\infty$, where $(n_k)_{k=1}^\infty$ is a strictly increasing sequence in $\mathbb{N}$ and $n_k \geq k$ for all $k \in \mathbb{N}$

Vistas are points in a sequence, a_n, where $N \in \mathbb{N}$, such that $a_N > a_n$ for all $n > N$

This means that for any sequence, there exists a subset of points within, such that within that sequence, the sequence is monotone

1.1.1 Well-Ordering Property

For any set $A \subseteq \mathbb{N}, A \neq \emptyset, min(A)$ exists

1.1.2 Proof

Case I: The set V of vistas, is infinite, such that $n_1 = min(v)$ and $n_k = min(V \cap n_{k-1}^\infty$, where $k \geq 2$, then a_{n_k} is strictly decreasing

Case II:

$$n_1 = \begin{cases} 1 & ifv = \emptyset \\ 1 + max(v) & ifv \neq \emptyset \end{cases}$$

$n_k = choice\{n > n_{k-1} | a_n \geq a_{n_{k-1}}$, thus $n_k \neq \emptyset$ because V is finite, thus a_{n_k} is increasing

1.2 Bolzano-Weierstrass Theorem

Every bounded sequence in $\mathbb{R}$ has at least one convergent subsequence

2 Extreme Value Theorem

For some function $f : [a, b]$