Multivariable Calculus

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1 Sequence Theorems

1.1 Sunrise Lemma

The Sunrise Lemma states for any sequence $(a_n)_n^{\infty} = 1$ in $\mathbb{R}$, $\exists$ monotone subsequence $(a_(n_k))_(^{\infty}k = 1)$, where $(n_k)_(^{\infty}k = 1)$ is a strictly increasing sequence in $\mathbb{N}$ and $n_k \geq k$ for all $k \in \mathbb{N}$

Vistas are points in a sequence, a_n, where $N \in \mathbb{N}$, such that $a_N > a_n$ for all $n > N$

This means that for any sequence, there exists a subset of points within, such that within that sequence, the sequence is monotone

1.1.1 Well-Ordering Property

For any set $A \subseteq \mathbb{N}, A \neq \emptyset, min(A)$ exists

1.1.2 Proof

Case I: The set V of vistas, is infinite, such that $n_1 = min(v)$ and $n_k = min(V \cap (n_(k - 1)^{\infty})), wherek \geq 2$, then $a_(n_k)$ is strictly decreasing
Case II:

$$n_1 = \begin{cases} 1 & ifv = \emptyset \\ 1 + max(v) & ifv \neq \emptyset \end{cases}$$

$n_k = choice\{n > n_(k-1)|a_n \geq a_(n_(k-1))$, thus $n_k \neq \emptyset$ because V is finite, thus $a_(n_k)$ is increasing

1.2 Bolzano-Weierstrass Theorem

Every bounded sequence in $\mathbb{R}$ has at least one convergent subsequence

2 Extreme Value Theorem

For some function $f : [a, b]$