

Multivariable Calculus

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1 Sequence Theorems

1.1 Sunrise Lemma

The Sunrise Lemma states for any sequence $(a_n)_n^\infty = 1$ in \mathbb{R} , \exists monotone subsequence $(a_{(n_k)})_{k=1}^\infty = 1$, where $(n_k)_{k=1}^\infty$ is a strictly increasing sequence in \mathbb{N} and $n_k \geq k$ for all $k \in \mathbb{N}$

Vistas are points in a sequence, a_n , where $N \in \mathbb{N}$, such that $a_N > a_n$ for all $n > N$

This means that for any sequence, there exists a subset of points within, such that within that sequence, the sequence is monotone

1.1.1 Well-Ordering Property

For any set $A \subseteq \mathbb{N}$, $A \neq \emptyset$, $\min(A)$ exists

1.1.2 Proof

Case I: The set V of vistas, is infinite, such that $n_1 = \min(v)$ and $n_k = \min(V \cap (n_{k-1}, \infty))$, where $k \geq 2$, then $a_{(n_k)}$ is strictly decreasing

Case II:

$$n_1 = \begin{cases} 1 & \text{if } v = \emptyset \\ 1 + \max(v) & \text{if } v \neq \emptyset \end{cases}$$

$n_k = \text{choice}\{n > n_{k-1} \mid a_n \geq a_{n_{k-1}}\}$, thus $n_k \neq \emptyset$ because V is finite, thus $a_{(n_k)}$ is increasing

1.2 Bolzano-Weierstrass Theorem

Every bounded sequence in \mathbb{R} has at least one convergent subsequence

2 Extreme Value Theorem

For some function $f : [a, b]$