

# Introduction to Physics III: Thermodynamics, Waves, and Relativity

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Fall 2016

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Primary Textbook: Heat and Thermodynamics by Zemansky and Dittman, 7th Edition

Secondary Textbook: Introduction to the Physics of Waves by Freegarde

Secondary Textbook: Special Relativity by Helliwell

Teacher: Dr. Johnson

# 1 Chapter 1 - Temperature and the Zeroth Law of Thermodynamics

## 1.1 Definition of Thermodynamics

1. The study of a natural system requires the creation of a boundary, separating a section of space and matter, or the system, from its surroundings, called closed if no matter is able to cross the boundary, open if there is an exchange of matter
2. Systems are then studied on either a macroscopic/human scale or a microscopic/molecular scale
  - (a) Systems are described on a macroscopic scale by composition, mass, volume, and temperature, and other aggregate properties, acting as macroscopic coordinates for the macroscopic description
    - i. Macroscopic coordinates have the properties of assuming nothing special about the matter structure, fields, or radiation, require few to describe a system, suggested by sensory observation, and can be directly measured
    - ii. While mechanics describes the external properties of a system and its energy, thermodynamics describes the internal properties
  - (b) Systems are described on a microscopic scale by statistical mechanics, describing the populations/number of particles in each energy state at equilibrium, and the interactions of particles with each other by collisions and fields, and with other systems in an ensemble
    - i. The equilibrium state is the state of the highest probability, and the system is assumed to have some high number of particles
    - ii. Microscopic descriptions have the properties of making assumptions about the structure of matter, fields, and radiation, requires many quantities, is based mathematical models rather than observation, and must be calculated rather than measured
  - (c) The scales must reconcile to the same conclusion, with the macroscopic view as the average over some amount of time of the microscopic characteristics
    - i. Since the microscopic view requires assumptions and models rather than observations, it is able to change as the result of increased data, unlike the macroscopic view which is used to test the assumptions
3. Thermodynamics is the study of the macroscopic properties of nature, especially the temperature of the system, and the identification of thermodynamic relations based on the fundamental laws
  - (a) Mechanical coordinates used within classical mechanics are from the mechanical/external energy of the system to measure the movement of the system overall, while thermodynamics uses macroscopic coordinates to deal with the internal energy of the system, called thermodynamic coordinates
  - (b) Systems described by thermodynamic laws are thermodynamic systems, found within each discipline
4. Thermodynamics is the branch of physics which describes the relationships between all forms of energy, conserved in an isolated system by the Law of Conservation of Energy
  - (a) Heat is energy in transit from one body at higher temperature to another with lower temperature, with 1 calorie as the heat to raise the temperature of 1 g of water by  $1^{\circ}\text{C}$  at 1 atm, equal to 4.184 J

## 1.2 Thermal Equilibrium and Temperature

1. For some thermodynamic system of constant mass and composition, where to describe the system requires two coordinates,  $X$  and  $Y$ 
  - (a) The system is at equilibrium state if the coordinates would remain constant if the external conditions are unchanged
  - (b) Equilibrium states depend on the proximity to other systems and the boundary within systems (adiabatic if any equilibrium state is present on both, such as wood, while diathermic if there must be specific combinations of equilibrium states, such as metal)
    - i. Thermal equilibrium is achieved by multiple systems separated by a diathermic wall when they have reached a combined system equilibrium
    - ii. Diathermic walls are thus boundaries that allow heat transfers, while adiabatic do not allow heat transfers, both not permitting matter transfers (assuming it does not break due to stress from the systems)
  - (c) The Zeroth Law of Thermodynamics states that if two systems are each in thermal equilibrium with another system simultaneously, then they are at thermal equilibrium with each other as well
    - i. It is called this due to the next two relying on this definition, found experimentally as a fundamental law of the discipline, creating the concept of temperature as a universal quantity
2. Temperature is designed as the macroscopic measure of hotness from an object observed, and microscopically as the average kinetic energy of particles, defined as some number  $\geq 0$  in science (Kelvin), ignoring the terms of coldness
  - (a) For any given state of a system, there exists a curve, or isotherm, of the locus of all states of another system which would be in thermal equilibrium with the first system, experimentally found to generally be continuous over some region of the curve
    - i. The same can be found for the other system at each set of coordinates, such that by the zeroth law, each isotherm has a corresponding isotherm in the other system, such that all points on those lines are at equilibrium with each other
    - ii. The property which determines if a system is at thermal equilibrium is called temperature, since all that is needed is a correspondence of equilibrium between two systems and a reference, such that it is a scalar, rather than a vector if there was not a transitive property
  - (b) Thus, for systems at the same temperature, they are on corresponding isotherms and thus at thermal equilibrium, such that temperature is a measure of thermal equilibrium
    - i. The converse of this is also true, such that thermal equilibrium implies same temperature
3. Temperature equality can be determined by a capillary of mercury, equally filled and sized, measuring the equality of the height the mercury rises, experimentally found to signify equilibrium, called a thermoscope
  - (a) Thermometers measure the temperature on an empirical scale, using some rule to assign a value to each isotherm, finding some path on the isotherm plane, with the varying variable as the thermometric property, the thermometric function,  $\theta(X)$  to get the temperature
    - i. The traditional mercury volume thermometer relies on the asymmetrical oscillation as energy increases, such that it is more likely to be at the larger side of oscillation, creating an average larger volume

- A. This is defined by  $U(r) = 4\epsilon((\frac{\omega}{r})^{12} - (\frac{\omega}{r})^6)$ , such that  $\epsilon$  is the minimum energy,  $r$  is the distance between molecules, and  $\omega^{\frac{2}{3}} = r_0$ , where  $r_0$  is the minimum distance between molecules
- B. All objects expand outward as a result, such that there would be increased space in a cavity as well, expanding only outward
- (b) The historic scale mercury pressure thermometer (constant volume) is based on a linear relationship,  $\theta(X) = aX + c$ , without a constant defining an absolute temperature scale (absolute zero at  $0^\circ$ )
  - i. The same relation when applied to other thermometers or other thermometer systems of the same type (the international standard being hydrogen pressure) produces other scales, with some defined coefficient for each
- (c) Thus, when a thermometer is placed in contact with a chosen standard system in a reproducible state, called the fixed point temperature
  - i. Until 1954, the world standard was Celsius, but which was too difficult to measure accurately due to water surrounding ice and sensitivity to minor pressure fluctuations
  - ii. After, the Kelvin system is based on the triple point of water (three state equilibrium, called the standard fixed point), able to measure accurately, as an absolute system, assigned 273.16 K, equal to  $0.01^\circ\text{C}$
  - A. This is measured by putting water under such pressure in a sealed tube that it begins to boil, using a freezing mixture to form a layer of ice on the edge, after which the mixture is removed and water forms between the ice and the edge, still evaporating, at the triple point

### 1.3 Thermometer Types

1. For each type of thermometer, a test using the same equation would produce drastically different results from those of the standard at all values except the reference
  - (a) Hydrogen gas pressure thermometers tend to be negligibly different at standard pressures, acting as the reference for normal temperature and pressure
2. Gas thermometers are made up of a measuring gas containing bulb, attached to a mercury column through a capillary, keeping the volume of the gas constant by adjusting the height of the mercury column
  - (a) The mercury column has two columns, one attached to the capillary, the other to the dead space, with a column of the measuring gas reaching from underneath the columns to the reservoir
    - i. If high enough pressure, the mercury can rise into the dead space/nuisance volume at the top of the capillary, adjusting the mercury in the column to the point that the mercury fills the capillary column fully to keep volume constant
  - (b) Thus, at this point, the pressure of the main column is the atmospheric pressure added to the difference in heights between the columns
    - i. It can then be measured when the bulb is surrounded by the triple point water and when surrounded by the measured system
  - (c) The pressures must be corrected for errors due to the gas being a different temperature than the bulb, the capillary not being uniform temperature, volume changes in any part as the temperature and pressure change

- i. It can also have error due to gradient in the capillary due to the diameter being close to the mean free path of the gas, gas adsorbed by the device, especially at low temperatures, and temperature and compressibility of the mercury creating error
- (d) As a result, once corrected, the measuring gas behavior appears close to that of an ideal gas
  - i. The basis of gas thermometers was the ideal-gas law, stating  $PV = nRT$ , where  $n$  is the number of moles of gas,  $T$  is the Kelvin temperature, and  $R$  is the molar gas constant
  - ii. This is able to be used for the reference, to derive the same relation between triple point pressure/temperature and measured pressure/temperature ( $\frac{P}{P_{TP}} = \frac{\theta}{273.16K}$ ), such that the temperature definition thus assumes an ideal gas for a Kelvin thermometer
- (e) Helium is the best measuring gas, not diffusing through platinum at high temperatures, but not becoming a liquid until extremely low temperatures
- 3. Vapor pressure thermometry depends only on the vapor pressure of the measuring gas at some temperature, thus easy to produce, and is easy to measure
- 4. Platinum resistance thermometry is secondary, due to being fundamentally defined in terms of temperature, but with a broader range than gas thermometers
  - (a) This is done by winding a long, thin wire around a frame, loose enough to avoid strain when compressing, able to also be wrapped around the object being measured, drilled into the object, or bonded onto the surface
  - (b) Potentiometric resistance thermometers have zero direct current into the leads, while bridge circuits are balanced such that there is no alternating current through
    - i. Bridge circuits are made possible as thermometers due to inductive voltage-dividers/ratio transformers and high sensitivity/signal-to-noise lock-in amplifiers
  - (c) Within the ideal measurement range,  $R'(T) = R'_{TP}(1 + aT + bT^2)$ , where  $a$  and  $b$  are constants, generally measured as  $W(T) = 1 + aT + bT^2$ , measuring the ratio of resistivity, which is less prone to material tension/composition error
- 5. Radiation thermometry, including optical, radiation, infrared, and total radiation, are used to measure thermal/blackbody radiation, using Planck's radiation law relating temperature to radiance, depending only on the walls of a closed body, not the shape
  - (a) The dimensions must be larger than the wavelengths themselves, and there must be a small hole to allow radiation to escape to allow measurement, negligible enough to not disturb equilibrium
  - (b) Pyrometers were made to measure high temperatures without contact, comparing visible radiation over a small band with that of a standard, using a photoelectric detector, correcting for emissivity of the source at that band
  - (c) Total-radiation pyrometers have a larger range able to measure, but are less precise, measuring all radiation from the object, rather than just blackbody
- 6. Thermocouples are an outdated method of measurement due to high inaccuracy of 0.2 K, made of two wires connected to copper wires in a melting point reference junction, connecting the two wires and measuring the EMF at various values
  - (a)  $\epsilon = c_0 + c_1\theta + c_2\theta^2 + c_3\theta^3$ , where  $c_n$  are constants based on the materials of the thermocouple, though for a small range of temperature,  $c_3$  approximates to 0
  - (b) The temperature range of the thermocouple depends on the materials used, advantageous because of the small, conductive mass allowing rapid temperature equilibrium with the

material being measured

- (c) The copper wires can be connected to electrical circuits for monitoring of electrical equipment temperatures or to a simple voltmeter for measurement

## 1.4 Temperature Scales

1. As the triple point pressure of a gas (due to lessening volume), approaches 0, the resulting temperature calculated is the ideal-gas temperature
  - (a) While thermometers depend on the specific measuring gas properties to provide the temperature, as it approaches the ideal gas temperature, it becomes independent of the measuring gas, though based on the properties of gases overall, rather than an individual gas, behaving ideally
    - i. Due to this ideal specific gas-independent temperature, it is an ideal state of matter for measuring of temperature
  - (b) For the temperature region in which a gas thermometer may be used, the measured Kelvin temperature scale and ideal gas temperature scales are identical
  - (c) It is noted that the idea of lack of atomic motion at absolute zero cannot be assumed, due to relying on the complete equivalency of temperature and atomic motion on different scales
    - i. In addition, at absolute zero, there is some residual energy expected based on quantum mechanics, resulting in the zero-point energy quantity
2. The Celsius scale was used prior to the Kelvin temperature scale until 1954, which slightly shifted the base from the ice point to the triple point of water (273.16 K) for accuracy and modifying to an absolute scale, with the same degree of magnitude between values
  - (a) Thus, the Celsius and Kelvin scales simply have a constant difference of the measured ice point, equal to 273.15 K
  - (b) As a result, the modern Celsius scale no longer has a fixed ice/steam point for water, but rather purely the triple point, such that Kelvin is the standard
3. Fahrenheit and Rankine are based on a scale 5/9ths that of the Celsius and Kelvin, based on the triple point of water, such that the ice point is found to be  $32^{\circ}F$ , providing a relationship to Celsius
4. The International Temperature Scale of 1990 was the creation of a practical scale for routine measurements and calibration of instruments, to ease measurement from time-consuming calibration by the single reference point, providing a series of fixed points for comparison
  - (a) It also provides close measurement interpolation for the remainder, providing a close approximation to the Kelvin scale, by other thermometer types
    - i. This uses polynomial approximations of constants to approximate the scale of another thermometer for that region
  - (b) At the minimum of 0.65 K to 3.2 K, it is defined by the  $^3He$  vapor pressure-temperature relation, from 1.25 K to 2.1768 K to 5 K by  $^4He$  of the same type
    - i.  $^3He$  fails below 0.3 K due to being too small to measure, and fails above 3.32 K due to the critical point (where only gas exists), while  $^4He$  fails below 1 K due to superfluid behavior causing less variation with temperature and errors, and above 5.2 K due to the critical point
  - (c) From 3 K to 24.5561 K by either He gases as a constant-volume gas thermometer
  - (d) From 13.8033 K to 1234.93 K, it is defined by platinum resistance thermometers with



fixed points within and deviation functions between, creating 11 sub-ranges

- (e) Above that, it is measured by an optical pyrometer, using blackbody spectral radiance concentration ratios by Planck's radiation law, using only a single reference of the freezing point of gold, silver, or copper, replacing the thermocouple

## 2 Chapter 2 - Simple Thermodynamic Systems

### 2.1 Thermodynamic Equilibrium

1. Change of state of a system is when the coordinates change either spontaneously or due to outside influence, isolated if the former, such that there is no outside influence, rarely found practically
  - (a) Systems not in mechanical equilibrium (with no unbalanced force or torque in the system interior or the surroundings/no unequal pressure) will have a change of state until it is corrected
  - (b) Systems not in chemical equilibrium (unequal chemical potential) result in a transfer of matter through or out/in the system, or a spontaneous change of internal structure as a reaction to restore it
  - (c) Systems in thermal equilibrium are those with no change in the system's thermodynamic coordinates when separated diathermically, such that there is no heat transfer (meaning temperature is equal with surroundings and within the system)
    - i. Otherwise, change of state will occur until thermal equilibrium is restored
  - (d) It is noted that systems not at thermodynamic equilibrium, but static over time due to some outside influence, can be broken into small increments, where it can be considered in equilibrium
2. Thermodynamic equilibrium is when thermal, mechanical, and chemical equilibrium are found simultaneously, such that it can be described by thermodynamic macroscopic coordinates at a particular instant, though not over time
  - (a) Thus, when not at mechanical, chemical, and thermal equilibrium, coordinates like composition, temperature, and pressure cannot be used to apply to the system as a whole, as it moves through nonequilibrium states
3. Equations of state relate the coordinates of a thermodynamic equilibrium system to each other, based on the individual specifics of a system, such that it is determined by molecular theory or experiment, not general theory, but can be assumed to exist for all thermodynamic equilibrium systems since all coordinates are not independent for a system
  - (a) At low pressures, the state equation of ideal gas is  $PV = nRT$ , or  $Pv = RT$ , where  $v$  is the molar volume ( $\frac{V}{n}$ )
  - (b) At high pressures, it is represented by the van der Waals equation, based on particle interactions and sizes, such that  $(P + \frac{a}{v^2})(v - b) = RT$ , where  $v$  is the molar volume
    - i.  $a$  and  $b$  are gas-specific constants, since equations of state are defined by existence, rather than any particular form without constants
    - ii.  $a$  is the molecular attraction while  $b$  is the atomic short range repulsion of the system, such that it accounts for higher pressures in which gases behave less ideally
  - (c) Nonequilibrium states cannot be described purely by coordinates as a whole, such that there is no equation of state for the system

## 2.2 Simple Systems

1. Simple systems are those which are described by three thermodynamic coordinates in an equation of state, with the special case of PVT systems called hydrostatic systems
2. Hydrostatic systems are also defined as an isotopic (uniform) system of constant mass and composition that exerts uniform hydrostatic pressure on the surroundings if there is no gravitational or electromagnetic effects
  - (a) They are divided into three categories, pure substances (a single compound of any mixture of states), homogeneous mixture of different compounds (mixture of different compounds of the same state or a solution), and heterogeneous mixtures (mixtures of multiple homogeneous multiple compound mixtures of different states)
    - i. It has been found experimentally that systems of a single state can be described by three coordinate equations of state, PVT
  - (b) These equations can describe some macroscopically small change in a measurement by a differential, though since they are macroscopic coordinates, it describes an amount greater than a few particles changing, rather than an infinitesimal amount
  - (c) Thus, for some function  $V = V(T, P)$ ,  $dV = \frac{\partial V}{\partial T}(T, P)dT + \frac{\partial V}{\partial P}(T, P)dP$ , abbreviated as  $dV = \frac{\partial V}{\partial T}dT + \frac{\partial V}{\partial P}dP$ 
    - i. The average coefficient of volume expansion is defined as the change in volume per unit volume divided by the change in temperature at constant pressure
      - A. As the change in temperature, and thus volume, approaches 0, the differential coefficient of volume expansion, or the volume expansivity,  $\beta = \frac{1}{V}(\frac{\partial V}{\partial T})$
      - B. It is generally a positive number, though there are exceptions such as water for  $0 - 4^\circ C$  melting point at which point water reaches maximum density
      - C. This is due to hydrogen bonding of ice and surface tension of water and is generally a function of P and T
      - D. It often varies negligibly with respect to P and T often within a small temperature and pressure range, such that it can be considered a constant over the range
      - E. It included division by V to signify the relative/percentage change in volume, rather the absolute change
    - ii. The average bulk modulus is defined as  $\frac{-V\Delta P}{\Delta V}$ , due to the pressure increasing causing a decrease in volume, such that it is a positive value
      - A. As the change in volume, and thus pressure, approaches 0 as temperature is constant, it is called the isothermal bulk modulus,  $B = -V(\frac{\partial P}{\partial V})$
      - B. Isothermal compressibility, or the reciprocal of isothermal bulk modulus,  $\kappa = \frac{-1}{V}(\frac{\partial V}{\partial P})$ , and is similarly often constant for a material over some PT range
  - (d) This can be similarly found for each equation of state of a different explicit variable, all of which are differentiated to determine an explicit exact differential equation, describing a slight change in state of equilibrium for that explicit differential in terms of the other variables
    - i. Exact differentials are contrasted as the differential of an actual function to that of an inexact differential, denoted by  $\delta$
    - ii. By combining two of exact differential equations equations, it follows that if  $y(x, z)$  and  $dz = 0$ ,  $dx \neq 0$ , then  $\frac{\partial x}{\partial y} \frac{\partial y}{\partial x} = 1$ 
      - A. Further, if  $dz \neq 0$  and  $dx = 0$ , then by extension,  $\frac{\partial x}{\partial y} \frac{\partial y}{\partial z} \frac{\partial z}{\partial x} = -1$

- B. By extension of that,  $\frac{\partial y}{\partial z} \frac{\partial z}{\partial x} = -\frac{\partial y}{\partial x}$
- iii. Thus, by those theorems,  $\frac{\partial P}{\partial T} = \frac{\beta}{\kappa}$ , such that each partial derivative has a physical representation by the theorems
- A. As a result, a form  $dP = \frac{\beta}{\kappa} dT - \frac{1}{\kappa V} dV$  of the state equation is found, which can be integrated for each in terms of the others
- iv. In addition, it must be noted that by the chain rule, for some  $z(w(x, y), x)$ , it is found that  $\frac{\partial z}{\partial w}|_x \frac{\partial w}{\partial y}|_x = \frac{\partial z}{\partial y}|_x$
3. Wires are a one-dimensional system, usually assumed that pressure and volume are constant, such that the coordinates are tension,  $\tau$ , length,  $L$ , and temperature,  $T$ ,
- (a) While it cannot be expressed universally by a single equation, for a constant temperature in the portion of elasticity such that Hooke's law is true,  $\tau = -k(L - L_0)$ , where  $L_0$  is the length at no tension
- i. Since equations of state assume thermodynamic equilibrium, the tension must be balanced out by some external force
- (b) For some infinitesimal equilibrium state change for  $L = L(T, \tau)$ ,  $dL = \frac{\partial L}{\partial T}(T, \tau) dT + \frac{\partial L}{\partial \tau}(T, \tau) d\tau$
- i. By the limit of the average linear coefficient of expansion, linear expansivity,  $\alpha = \frac{1}{L} \frac{\partial L}{\partial T}$
- A. Metals tend to have a positive linear expansivity, but other materials such as rubber may be negative
- B. Linear expansivity is mainly constant for torque change, only changing for temperature, but over a small range, can be considered constant
- C. It is noted that the linear expansivity is derived to be related to the volume expansivity, such that it is  $\frac{1}{3}\beta$ , assuming the material is isotropic (same in each direction)
- D. Old thermostats were based on this, with two attached materials of different  $\alpha$ , such that it would curl as they expanded at different rates
- ii. By the limit of average Young's modulus (Stress \* Y = Strain), isothermal Young's modulus,  $Y = \frac{L}{A} \frac{\partial \tau}{\partial L}$
- A. Isothermal Young's modulus is always positive, only changing for temperature, but can be considered constant until some maximum strain ( $\frac{\Delta L}{L}$ ) as well
4. Surfaces are a 2D system, with different properties from those of the underlying material, acting as a stretched membrane, with each point exerting a force opposite and perpendicular to the opposite side of the surface
- (a) Surfaces are thus described by the area,  $A$ , the temperature,  $T$ , and the surface tension,  $\gamma$ , which is the force per unit length of the surface, at any point in any direction on the surface, with an equal and opposite force on the surface
- (b) While the substance within the surface is relevant, the volume and pressure of it can be ignored, due to remaining fairly constant for small changes in the surface coordinates
- (c) For pure liquids at equilibrium with their vapor phase, it is described by the Guggenheim equation of state,  $\gamma = \gamma_0(1 - \frac{T}{T_c})^n$ , where  $T_c$  is critical temperature,  $\gamma_0$  is surface tension at standard temperature, generally  $20^\circ C$ , and  $n$  is a constant, such that  $1 \leq c \leq 2$
- i. As a result, as  $T$  approaches  $T_c$ , the surface tension approaches 0, after which there is unable to be any liquid form
- (d) For oil films on liquid water, it is described by the equation  $(\gamma - \gamma_w)A = aT$ , where  $\gamma - \gamma_w$  is called the surface pressure, acting as a 2D version of the ideal gas law, and

where  $a$  is some constant

- (e) For soap bubbles, it functions as two surfaces trapping a small amount of liquid between the two, while another common surface is a single layer of gas on a solid surface

## 3 Chapter 3 - Work

### 3.1 Definitions

1. External work is work done on or by the system as a whole, while internal work is work done by part of the system on another part, such as interaction of molecules, only the former able to be done by macroscopic thermodynamics
  - (a) External work can be viewed as the changing of macroscopic coordinates by the modification of the configuration/position of some external mechanical device or system, such as thermodynamic electrochemical cell systems powering a mechanical device
    - i. Thus, assuming the system is in thermodynamic equilibrium and there is no change in the configuration of external systems, no external or internal work is done
  - (b) Work done on a system is denoted positively, while work done by the system is denoted negatively within thermodynamics and mechanics
2. Mechanical external work leads to forces and torques within the system, causing external acceleration, internal turbulence, waves, or other forms of internal motion
  - (a) This can lead to a nonuniform temperature distribution as well as a lack of thermal equilibrium with external systems
    - i. The lack of thermal and mechanical equilibrium can lead to spontaneous chemical reactions or gradients forming within the system
  - (b) As a result, mechanical external work can generally not be described thermodynamically, such that only an infinitesimal applied force, called a quasi-static process, such that it is infinitesimally close to equilibrium during the process
    - i. Thus, it can be thought to be described by an equation of state, being an idealization of work on any thermodynamic system, since the time to move to a new equilibrium state is negligible
    - ii. This is used to be able to ignore minor forces acting on a system, giving the ideas of ideal devices without friction and resistance, and to be able to allow change of state equations to apply
    - iii. As a result, all processes are reversible, due to no dissipative forces affecting the system, such that energy is not lost from the system
  - (c) Quasi-static process theory only applies to systems with proper boundaries, such that thermodynamic equilibrium is able to be reached
  - (d) It is notated that since the unbalanced force is infinitesimal, the equation of state variables and the force are considered to be equal for the system and external systems
3. While internal work does take place on a microscopic scale within the system, it is not the purview of thermodynamics, but rather within statistical mechanics, such that only external work is viewed on the macroscopic scale

### 3.2 Hydrostatic System Work

1. For some hydrostatic system in a closed cylinder with a piston to lower the height of the system, the pressure exerted by the system on the piston is  $P$ , the area of the piston  $A$ , such that  $F_i = PA$ , with some external force,  $F_e$ , only infinitesimally different
  - (a) For work done, lowering the volume of the system,  $dW = F_e dx = PAdx$ , since the difference between the forces is infinitesimal
    - i. This is under the assumption of a quasi-static system, such that dissipative processes or chemical movement/reactions would not have a large enough effect to render this invalid
    - ii. On the other hand, lack of mechanical equilibrium, such that there is not a standard pressure definition, would render this invalid
    - iii. This can be expanded to other hydrostatic systems of different shape, such that the underlying ideas are still valid
  - (b) As a result, since  $Adx = -dV$  by the decreasing volume, such that  $dW = -PdV$ , often with work measured in liter-atm for convenience, such that  $1 \text{ L-atm} = 100 \text{ J}$ 
    - i. This can be integrated for the work done by the external system, assuming the difference between the calculated work done and the actual is negligible, since the process is quasi-static, pressure is a function of temperature and volume by the equation of state
    - ii. Once the temperature pathway is found for the particular quasi-static process, it can be integrated purely in terms of  $V$
2. For the system, a PV diagram can be drawn for the relationship between pressure and volume as the piston moves, traversing the curve in opposite directions for expansion and contraction
  - (a) As a result, for some closed figure, it moves back to the original equilibrium state through other equilibrium states, called a cycle, such that the net work of the cycle is the difference between the work of the expansion and contraction temperature pathways
  - (b) It is noted that changing the direction of the cycle would change the sign of the line integral, due to depending on which path direction is traversed for expansion and contraction ( $\oint PdV$  for a clockwise curve due to each pathway integral being negative)
  - (c) Due to the infinitesimal force differences and possible chemical and thermal movement which is ignored, the path does not have to reverse directly, but rather use a different path such that net work is done, as well as the rate of the multiple controlled changing conditions
    - i. Thus, the path itself matters, such that for an isobaric process, in which pressure is constant, followed by an isochoric process in which volume is constant, it would be different in area from a direct path, such that it is path dependent
    - ii. Since the integral is not path-independent, due to infinite possible  $P(V)$  functions changing the outcome,  $dW = PdV$  is not valid, due to infinite possible values, but rather only valid for a specific path of  $P(V)$ , denoted as an inexact differential,  $\delta W = PdV$ 
      - A. This means that  $\delta W$  is not a small change in the function of work, but rather simply a small amount of work performed by some small change in  $dV$ , such that  $W$  is defined only in terms of the integral
      - B. Work as a result, is not some function of thermodynamic coordinates, or a state function of the system, but rather a path function depending on the specific change in the system

- C. For an isothermal change as a result, the path is the isotherm on the PV plane, such that the work is positional rather than path based, where  $\frac{\partial P}{\partial V}$  determines the relative work needed for a change in volume or pressure
- D. For a derivative of an inexact differential, it is noted that the path must still be determined for the differential, even with a designated change, assuming there are multiple paths for the change itself
- (d) This is all explained by the fact that work for a non-conservative force is a multi-variable path function, rather than a single variable state function, only state if conditions are imposed
  - i. This can also be written as the curl of the force being 0 for conservative forces, as a conservative vector field, which are path-independent on multiple variables as well
- 3. For a length of wire system,  $\delta W = \tau dL$ , while for a surface film (two layers containing some liquid between),  $\delta W = \gamma dA$ , proved similarly for each system
  - (a) The equation is derived for a film, but is equally applicable for a single surface, though the area of the surface is doubled for a film, due to repeating

## 4 Chapter 4 - Heat and the First Law of Thermodynamics

### 4.1 Internal Energy

1. Closed systems can have a change of state both by external work done on the system, and by heat transfer from a hotter substance within the system, rather than mechanically
  - (a) Heat is thus defined as something transferred between a system and surroundings by temperature difference only, such that adiabatic walls are also called heat insulators and diathermic walls are heat conductor
  - (b) Heat transfer and work can often be viewed as synonymously occurring, such that it depends on what is defined as the system and surroundings to define which is taking effect
2. Adiabatic work is work done within an adiabatic boundary performed by a coupled system acting on the adiabatically closed system, such that there is no transfer of heat into the system, but rather purely a mechanical energy transfer
  - (a) This can still result in a change in temperature, based on which parameters are allowed to vary, such as increasing the temperature of water through a resistor
  - (b) As a result, the first law of thermodynamics states that for any adiabatic change of state, the work done is equal for all adiabatic pathways connecting the states
    - i. Similar to a conservative force system, a thermodynamic system can thus be described in terms of a potential energy function, such that  $W_{adiabatic,i \rightarrow f} = U_f - U_i$ , called the internal energy function, where W is the work done on the system
    - ii. It is noted that not all states can be reached by purely adiabatic work due to entropy
    - iii. This as a result defines the law of the conservation of energy, and states that for any system, there exists an energy function describing the difference between two states as the energy change
      - A. It was noted that it was thought beta decay and the microscopic scale did not follow conservation of energy, due to less calculated after the decay, found to be due to the antineutrino for  $\beta^-$ , neutrino for  $\beta^+$
      - B. On an astronomical scale, it was found that dark energy and dark matter was

- necessary for conservation of energy to exist, but within general relativity in a nonconstant gravitational field, it was found to not apply
- (c) Since internal energy is a state function,  $dU$  can be expressed as an exact differential in terms of the coordinates necessary to describe a state

## 4.2 First Law of Thermodynamics

1. For nonadiabatic work, to allow conservation of energy to be consistent since  $\Delta U \neq W_{adi}$ , energy must be transferred by another means, heat transfer
  - (a) Thus, heat is defined thermodynamically as energy transferred by non-mechanical means into/from a closed system is called heat, such that  $\Delta U = Q + W$ , where  $W$  is adiabatic work and  $Q$  is diathermal heat transfer, positive for both if entering the system, as the full first law
    - i. This is true for all thermodynamic system changes, rather than just quasi-static process systems, due to not requiring an equation of state
    - ii. For an infinitesimal process, involving only infinitesimal changes in thermodynamic coordinates, it is written as  $dU = \delta Q + \delta W$ , or for quasi-static hydrostatic system,  $dU = \delta Q + PdV$
    - iii. For a composite system with multiple components,  $\delta W$  can be written as the sum of work done on each system within the overall system
  - (b) Thus, it can be seen from this the experimental idea that heat is energy transferred by a temperature gradient
  - (c) Temperature gradient is not the only relevant factor for transferred energy though, such that for an isochoric (constant volume) system due to a fixed diathermic boundary, it is internal energy transferred
    - i. On the other hand, isobaric heat (constant pressure) due to a movable boundary, it is enthalpy, rather than internal energy transferred as heat
    - ii. Thus, heat is either internal energy or enthalpy based on the system conditions being transferred, though it is only applicable as a path quantity, rather than a state,  $\delta Q$ , similar to work
  - (d) Further, by this equation, it is evident that for an adiabatic system, the loss of heat from one section of the system is equal to the gain by another part of the system
2. Heat capacity, more accurately called internal energy capacity, is the ability of a system to store energy, measured by the change in temperature at a point other than state changes
  - (a) It is measured with heat due to being easier to detect than work, such that  $C = \frac{\delta Q}{dT}$
  - (b) The specific quantity of the heat capacity, or the unit mass quantity, called specific heat capacity, is a property of the material itself, rather than the object
    - i. The molar heat capacity,  $c = \frac{C}{n}$ , where  $n$  is the number of moles, such that it is equal to the mass divided by the molecular mass multiplied by the molecules per mole (Avogadro's number,  $6.022 \times 10^{23}$ )
  - (c) Heat capacity depends on the process pathway on the P-V graph, such that heat capacity at constant pressure is denoted  $C_P(P, T)$ , while heat capacity at constant volume is denoted,  $C_V(V, T)$ 
    - i. For some small range of variation in  $T$  and either  $P$  or  $V$ , the heat capacity is virtually constant within the range
  - (d) Heat capacity is measured by a resistance wire around a cylindrical sample, such that

if both are the system, the electrical energy is doing work on the system, whereas if just the sample is, the electrical energy does work on the wire, moving as heat into the system

- i. The work required to produce the heat added to the system was originally called the mechanical equivalent of heat, later found just to be an alternate concept of work
- ii. This is called a heating coil, used to test the statistical mechanics results of a system
- iii. For some current in the wire,  $\delta Q = \epsilon I dt$ , building the calorimeter, coil, and connected thermometer based on the desired temperature range and material
  - A. For low temperature solids, it is suspended by a thin insulator, using a resistance thermometer placed in a drilled hole in the sample, using thin wires to avoid error
- iv. The temperature as a function of time graph is drawn, measuring temperature without activating the circuit as the foreperiod, closing the switch, then after  $\Delta t$ , opening it and graphing the afterperiod, though often measuring resistance instead of temperature by the thermometer
  - A. The foreperiod and afterperiod curves are extended to give the value in the center of the activated region, measuring the difference between as  $\Delta T$
  - B. Thus,  $c_P = \frac{\epsilon I \Delta t}{n \Delta T}$ , such that pressure is the constant variable due to atmospheric pressure keeping it constant
- (e) Originally, the calorie was the amount of heat to raise the temperature of 1g of water by  $1^\circ C$ , though later specified as  $14.5^\circ C$  to  $15.5^\circ C$ , measured as 4.186 J
3. The first law combined with the fact that for  $U(T, V)$ ,  $dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV$ , such that  $\frac{\delta Q}{dT} = \left(\frac{\partial U}{\partial T}\right)_V + \left(\frac{\partial U}{\partial V}\right)_T + P \frac{dV}{dT}$ 
  - (a) By extension, if  $V$  is constant, the second term cancels out ( $C_V = \left(\frac{\delta Q}{dT}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V$ )
    - i. The left side of the equation is able to be used to test a theoretical internal energy function experimentally
    - ii. It is noted that it takes high amounts of pressure to hold volume constant, such that this is not an easy method of measurement though
  - (b) On the other hand, if pressure is constant, it becomes  $C_P = \left(\frac{\partial U}{\partial T}\right)_V + \left(\frac{\partial U}{\partial V}\right)_T + P \frac{dV}{dT}$ 
    - i. This is due to the fact that since  $\frac{\delta Q}{dT}$  is path-dependent, due to  $T$  relying on two other parameters, some path as a function of  $P$  and  $V$  must be chosen
      - A. The only sections of the right hand side that are modified as a result of the path are  $P(T, V)$  and  $\frac{dV(T, P)}{dT}$ , the former held constant in this form, the latter becoming a partial
      - B. While the pathway by which  $P$  and  $T$  vary for within different parts of the equation rely on separate changes/parameters, it only determines the bounds of the end result, while the pathway chosen determines for both
    - ii. This is further modified to show that  $C_P = C_V + \left(\frac{\partial U}{\partial V}\right)_T + P \beta$ , providing another function of internal energy found theoretically, in terms of measurable state functions

### 4.3 Heat Flow

1. Temperature difference between a system and surroundings creates a lack of thermodynamic equilibrium, with a nonuniform temperature, though quasi-static process of heat flow behaves similarly to that of quasi-static work
  - (a) For some body of relatively large mass to the main system, it functions as a heat reservoir,



- such that virtually any amount of heat flow will not measurably change thermodynamic coordinates
- (b) Quasi-static heat reservoir contact would produce an isothermal change of state, due to the infinitesimal temperature gradient, adding energy to the system to produce other coordinate changes
    - i. On the other hand, a series of infinitesimally different heat reservoirs would produce an overall non-isothermal change of state, able to be performed for a high amount of time, using heat reservoirs such that their temperature remains constant
    - ii. As a result,  $\delta Q = CdT$ , where the path of the heat capacity determines the path of  $Q$ , allowing it to be integrated
2. Two parts of an adiabatically closed substance maintained at different temperatures at the edges have a continuous distribution of temperature in between, due to transporting energy between by heat conduction
- (a) Thus, it is found experimentally that  $\frac{\delta Q}{dt} = -KA\frac{dT}{dx}$ , where  $\frac{dT}{dx}$  is the temperature gradient, showing the rate of heat flow between the edges
    - i.  $K$  is the thermal conductivity, such that a large valued object is a thermal conductor, a small value is a thermal insulator
  - (b) Thermal conductivity varies as a function of temperature, such that for a large gradient, it varies throughout the object, though if small enough, it can be considered constant
    - i. For some small temperature difference,  $K = \frac{-L}{A\Delta T} \frac{\delta Q}{dt}$ , due to the temperature gradient and length being
  - (c) It has been found that thermal conductivity of metal is reduced by a large amount by any impurities, as well as changes in pressure of gases below a certain level and composition
    - i. Liquids have lower thermal conductivity, increasing as the temperature is raised, while metals tend to remain fairly constant until some minimum temperature where it decreases
    - ii. Nonmetals are poor conductors, similar to liquids, decreasing as temperature is raised, rising drastically at low temperatures
    - iii. Gases are especially bad conductors, independent of pressure after some maximum pressure, generally below 1 atm, increasing as the temperature rises
  - (d) Thermal conductivity in series is equal to the reciprocal of the sum of reciprocals, while in parallel is the sum, such that it is used in double pane glass with air between to lower the conductivity, due to air having a lower value
3. Convection currents are flows of liquid or gas that absorb heat and mix with another fluid of a lower temperature, releasing the heat, or vice verse
- (a) Convection currents by an external force/system is a forced convection, while if it is due to a combined density difference causing the flow along with a temperature difference, it is a natural convection
  - (b) For fluid in motion against some surface, there is some thin film of stationary fluid between, thinner the stronger the motion is
    - i. Heat is thus able to be transferred both by conduction through the film and convection
  - (c) The convection coefficient  $h$ , is defined such that  $\frac{\delta Q}{dt} = hA(T_{wall} - T_{fluid})$ , where  $h$  takes into account both the surface conduction through the stationary fluid and the convection
    - i. The coefficient depends on if the wall is flat/curved, vertical/horizontal, gas/liquid, density, viscosity, specific heat, and thermal conductivity of the fluid, laminar (low

- speed)/turbulent flow (high speed), and if evaporation, condensation, or formation of scale happens
4. Heat can be transmitted through empty space through radiation, most relevantly thermal (IR and visible), which is transmitted due to the high temperature of the origin solid/liquid (gases behave separately)
    - (a) At higher temperatures, greater frequencies of visible light are emitted, and more total energy is emitted
    - (b) The radiant exitance,  $R$ , of the body is the radiant power per infinitesimal area, depending on the nature of the surface of the body and the temperature just as absorptivity and emissivity does
    - (c) Isotropic radiation is thermal radiation incident on a body equally from all directions, where the fraction of incident radiant power absorbed is absorptivity
      - i. Total emissivity,  $\epsilon$ , is the fraction of power incident on the body that is emitted as thermal radiation, such that for thermal equilibrium, it is equal to absorptivity, depending on temperature and the material itself
      - ii. Ideal substances are those which absorbing and emit all radiation falling on it when at thermal equilibrium, called a blackbody, independent of the substance made from, approximated by a cavity with a small entrance hole and uniform high-temperature opaque walls
        - A. This is due to the radiation reflecting around the cavity until fully absorbed, such that it is almost entirely absorbed and is isotropic
    - (d) Irradiance is the radiant power incident per infinitesimal area on the surface of the cavity, corresponding to radiant exitance
      - i. Thus, for a blackbody with irradiance of  $H$  in thermal equilibrium with the cavity is placed within, such that radiant power absorbed per unit area of the blackbody is  $H\epsilon = H$
      - ii. Thus, radiant exitance of the blackbody,  $R$ , is equal to  $H$ , since the radiant power absorbed is equal to that emitted per unit area for the body, projected back to and from the cavity
    - (e) It has been found that the amount of radiation within the cavity is a function of the temperature produced by the thermal equilibrium and independent of the materials, detected by allowing a small amount of radiation to escape for measurement
    - (f) Radiant exitance of a non-blackbody depends on the surface material itself, as well as temperature, such that  $R = \epsilon H = \epsilon R_{bb}$ , called Kirchhoff's Law, used to measure thermal radiation emissivity of materials at some temperature
    - (g) There will only be a difference in the heat absorbed and emitted if there is a temperature difference between a body and its surroundings as a result
      - i. Thus,  $\frac{\delta Q}{dt} = A(\alpha(T)H - R) = A(\epsilon(T)R_{bb}(T_w) - \epsilon(T)R_{bb}(T)) = A\epsilon(T)(R_{bb}(T_w) - R_{bb}(T))$  for some body in a cavity, such that it is isotropic, where the cavity is large enough for the change in temperature to be negligible,  $T$  is the temperature of the body,  $T_w$  of the cavity, and  $A$  is surface area
    - (h) It was derived later thermodynamically that  $R_{bb}(T) = \sigma T^4$ , where  $\sigma$  is the Stefan-Boltzmann constant,  $5.67051 \times 10^{-8} \frac{W}{m^2 \cdot K^4}$ 
      - i. It follows that  $\frac{\delta Q}{dt} = A\epsilon\sigma(T_w^4 - T^4)$
      - ii. The Stefan-Boltzmann constant can be determined by the nonequilibrium method, placing a shielded silver disk in a blackened copper hemisphere until there is steam,

measuring the temperature  $T_W$ , uncovering the disk and measuring the temperature over time

- A.  $\frac{dT}{dt}$  is then found, such that since pressure is constant,  $\delta Q = C_P dT$  for the silver disk, where as a result,  $\sigma = \frac{C_P}{A(T_W^4 - T^4)} \frac{dT}{dt}$
- iii. It can also be found by the equilibrium method, placing a hollow blackened sphere suspended in a vessel of temperature  $T_W$ , adding electricity to the sphere at a constant power
  - A. This is done until it reaches equilibrium where radiation emission is equal to the power supplied, such that assuming it acts as a blackbody,  $\epsilon I = A\sigma(T^4 - T_W^4)$

## 5 Chapter 5 - Ideal Gases

### 5.1 Ideal Gas Laws

1. It has been found experimentally that  $\frac{PV}{n} = Pv = A(1 + BP + CP^2 \dots)$ , such that it is purely a function of pressure, called the virial expansion, where A, B, C are virial coefficients
  - (a) Until 40 atm, it is generally linear, such that only pressure is necessary, requiring more as pressure rises, though as pressure approaches 0, it is equal to A, only dependent on the temperature, rather than the gas itself
    - i. Virial coefficients are generally small, except at extremely low temperatures
  - (b) Since the ideal gas temperature assumes constant volume and the same amount of moles of gas for each measurement,  $T = 273.16K \lim_{P \rightarrow 0} \frac{Pv}{Pv_{TP}}$ 
    - i. As a result, R, the molar gas constant, is defined as  $\frac{\lim(Pv)_{TP}}{273.16K}$ , equal to 2.27102 kJ/mol\*K, difficult to measure due to absorption of gas on walls
    - ii. Thus, the ideal gas law is derived such that  $\lim(PV) = nRT$  and  $A = RT$ , such that  $\frac{Pv}{RT} = 1 + BP + CP^2 + \dots$
2. Adiabatic free/Joule expansion is the expansion in volume of a gas as there becomes a larger space to move into, such that no Q or W is done, and internal energy is constant
  - (a) Free expansion is noted not to be quasi-static, due to the fast modification from one state to another forcing it through nonequilibrium states
  - (b) The Joule coefficient,  $\frac{\partial T}{\partial V}|_U$ , testing with two diathermic vessels in a adiabatic water bath, measuring the change in gas temperature, found that the heat capacity of water was too high, and thus the coefficient was too difficult to measure directly
    - i. It was tested with water due to too much difficulty in making the temperature of gas
  - (c) Since for  $U(T, V)$ ,  $dU = \frac{\partial U}{\partial T}|_V dT + \frac{\partial U}{\partial V}|_T dV$ , such that if no temperature change or internal energy change takes place,  $dT = dU = 0$ , then  $\frac{\partial U}{\partial V}|_T = 0$ 
    - i. Similarly, for  $U(T, P)$ ,  $\frac{\partial U}{\partial P}|_T = 0$  in this case as well
      - A. Thus, as the gas approaches an ideal gas, it is equal to 0 as pressure approaches 0, such that there is no change in temperature in that case
    - ii. As a result, if free expansion causes no change in temperature, internal energy of the gas is a function of temperature only for free expansion, used to measure this fact
    - iii. It is also done by measuring the  $\frac{\partial u}{\partial P}|_T$ , where u is the molar internal energy, measured by a container with n moles of gas at some pressure, connected to the atmosphere

- by a long coil, surrounding the container with liquid at atmospheric temperature
- A. The valve is opened to allow small amounts of gas to flow out, using a heating coil to keep all temperatures constant with the atmosphere as the gas expands
  - B. As  $Q$  heat is absorbed by the gas from the coil, the work done by the gas is  $W = -P_0(nv_0 - V)$ , where  $P_0$  is atmospheric pressure,  $V$  is container volume, and  $v_0$  is molar volume at atmospheric temperature and pressure
  - C. It is noted that  $nv_0$  must be larger than the volume of the container, such that it expands when provided to the air
  - D.  $\Delta u(P, T)|_T = \frac{Q+W}{n}$ , after accounting for contraction of container walls, putting it in terms of change in pressure instead of volume, plotting for different pressures
- iv. On the other hand, it was noted that the experiment did not reach the ideal gas value of 0, and had the same issue of larger heat capacity of water, requiring precision in water temperature
3. Ideal gases are thus those which have low enough pressure for the ideal gas law, and as well that internal energy is a function of temperature, rather than temperature and pressure
- (a) Gases can be assumed to be ideal with increasing error as the pressure increases, though with only a few percent error below 2 atm
  - (b) For some quasi-static process, the ideal gas law can be written as  $PdV + VdP = nRdT$  by the product rule, and as a result of the second part, it can be stated that  $\frac{dU}{dT}|_V = C_V$ , such that  $\delta Q = C_V dT + PdV$ 
    - i. By combination of these, it is found that  $\frac{\delta Q}{dT} = C_V + nR - V \frac{dP}{dT}$ , such that if pressure is set constant,  $C_P = C_V + nR$  for an ideal gas
    - ii. Thus,  $C_V, C_P$  are functions of temperature only for ideal gases, and  $C_P \geq C_V$ , due to the gas expanding work to expand against the equal, external pressure when pressure is constant, whereas for constant volume, there is no work done ( $PdV = W$ )
    - iii. This is combined with the first law of thermodynamics to find that internal energy of an ideal gas is a function purely of temperature
  - (c) It is also derived following that  $\delta Q = C_P dT - VdP$  for an ideal gas, giving another form of the 1st law for an ideal gas

## 5.2 Heat Capacity Measurement

1. Heat capacities are measured by a gas in a thin-walled steel container, surrounded by a wire to add heat, measuring the rise in temperature at constant volume to get  $C_V$ , while  $C_P$  is through a calorimeter allowing volume to vary rather than pressure
  - (a) The inlet/initial and outlet/final temperatures are measured, found that for all ideal gases,  $C_V(T) < C_P(T)$ ,  $c_P - c_V = R$ , and  $\frac{C_P}{C_V} = \gamma$
  - (b) Monatomic gases, such as noble gases and most metallic vapors,  $C_V = c \approx \frac{3R}{2}$ , and  $C_P = c' \approx \frac{5R}{2}$
  - (c) Permanent diatomic gases, such as  $H_2, O_2, N_2, NO, CO, F_2, Cl_2$ , have constant values of  $C_V \approx \frac{5}{2}R, C_P \approx \frac{7}{2}R$ 
    - i. On the other hand though, at low temperatures,  $H_2$  uniquely acts as a monatomic gas
    - ii. For all temperatures, it may be written as  $\frac{C_P}{R} = \frac{7}{2} + f(T)$ , where  $f(T) = (\frac{b}{T})^2 \frac{e^{\frac{b}{T}}}{(e^{\frac{b}{T}} - 1)^2}$

generally for some constant  $b$ , generally too difficult to calculate, such that it is measured experimentally

- (d) Polyatomic gases and chemically active gases are those with values for  $C_P, C_V$  varying with temperature
- 2. For some quasi-static ideal gas adiabatic process,  $\delta Q = 0$ , such that since  $\delta Q = C_V dT + PdV = C_P dT - VdP$ , then  $\frac{dP}{P} = -\gamma \frac{dV}{V}$ , such that for some small temperature change,  $\gamma$  is constant, such that  $PV^\gamma = c$ 
  - (a) As a result, for some adiabatic process on a P-V graph,  $\frac{\partial P}{\partial V}|_Q = -\gamma \frac{P}{V}$ , such that since for a quasi-static isothermal process,  $\frac{\partial P}{\partial V}|_T = \frac{P}{V}$  by the ideal gas law, the curve for the former is steeper than the latter
- 3. Ruchhardt measured  $\gamma$  by placing gas in a jar of some volume, with a glass tube attached with a metal ball and piston within, such that  $P_{eq} = P_{atm} + \frac{mg}{A}$ , by  $F = PA$  where  $A$  is the area of the tube, assuming no friction
  - (a) With displacement, it will oscillate, such that  $dV = A dy$  and  $dP = \frac{dF}{A}$ , acting as a restoring force, causing oscillation, moving quickly enough to prevent heat transfer, such that it is adiabatic and minor enough to be quasi-static
  - (b) As a result, by the product rule and the ideal gas quasi-static adiabatic formula,  $dF = -\frac{\gamma P A^2}{V} dy$ , such that it is simple harmonic motion, where  $\tau = 2\pi \sqrt{\frac{m}{-\frac{F}{y}}} = 2\pi \sqrt{\frac{mV}{\gamma P A^2}}$ , measuring the period to get  $\gamma$
  - (c) A majority of the error originates from the assumption of the lack of friction within the glass tube
    - i. Error from assumptions was later corrected, using a steel piston in the middle of the gas container, dividing it in two and displacing it, using an electromagnet to hold the piston in place against weight, to prevent friction, using a current to move it slightly
    - ii. It also used the real equation of state, measuring the frequency by AC and amplitude by microscope, with a non-ideal non-adiabatic equation to create more precision
  - (d) For some small displacement of the piston in the divided container at constant velocity,  $\vec{v}_0$ , the pressure wave front due to the concentrated density near moves at velocity  $\vec{v}$ , such that it moves faster than the piston
    - i. Thus, the compressed portion of the gas at some time,  $t$ , had an original volume of  $V$ , mass of  $m = \rho_0 V = \rho_0 A \vec{v} t$
    - ii. Since the portion of the original volume that was compressed increases as the compressed wave front moves faster than the piston, such that the pressure lessens as it moves, but the compressed mass increases toward the total mass ( $\frac{dm}{dt} = \rho A \vec{v}$ )
    - iii. On the other hand, the additional velocity is due to the equalization of pressure, added to the velocity of the piston, such that the change in momentum is purely from the change in mass ( $\frac{dp}{dt} = \frac{dm}{dt} \vec{v}_0 = \rho A \vec{v} \vec{v}_0$ )
    - iv. The pressure from beyond the wave front is the normal pressure of the system, while the pressure from the piston is that added to the pressure exerted by the piston
    - v. It follows that  $F = A \Delta P = \rho A \vec{v} \vec{v}_0$ , and  $\frac{\Delta V}{V} = \frac{A \vec{v}_0 t}{A \vec{v} t}$ , able to be related such that  $\vec{v}^2 = \frac{-1}{\frac{P}{V} \frac{\Delta V}{\Delta P}}$ 
      - A.  $\frac{\Delta V}{V \Delta P}$  in this case is reversible adiabatic compressibility ( $\kappa_S$ ), rather than isothermal, proven by the above system, measuring heat transfer from the middle of the compressed and rarefacted/uncompressed portions,  $\frac{\lambda}{2}$

- B. It is found that the heat traveling by conduction in the time for the wave to travel  $\frac{\lambda}{2}$ ,  $Q = KA \frac{\Delta T \lambda}{\frac{\lambda}{2} 2\bar{v}} = KA \frac{\Delta T}{\bar{v}}$
- C. Since the heat to raise the temperature by  $\Delta T$  is  $Q = \rho A \frac{\lambda}{2} c_V \Delta T$ , it is only adiabatic if the heat transferred in that amount of time is insignificant in comparison to correcting the temperature difference, such that it is corrected as one system, rather than by heat
- D. Thus, it is found that this is true only if  $\lambda \gg \frac{2K}{\bar{v}\rho c_V}$ , such that the distance the wave is traveling must be drastically smaller than realistic, such that it is adiabatic
- vi. As a result, it is found that  $\kappa_S = \frac{1}{\gamma P}$ , such that  $\bar{v}^2 = \frac{\gamma P v}{M} = \frac{\gamma R T}{M}$ , where v is the molar volume, M is the molar mass
- vii. This can be used to measure the speed of sound in a gas, set with a movable piston with powder inside, vibrating the piston after placing it in such a position to create a standing wave
  - A. After the vibration begins, powder piles at the nodes, allowing measurement of half a wavelength between, multiplied by the frequency to find the velocity
  - B. This can be made more accurate by an acoustic interferometer instead of a tube, with a piezoelectric crystal, and a receiver, varying the frequency with a constant distance, measuring the resonance to find the nodes, accounting for viscosity, heat conduction, and boundary layer absorption
  - C. The speed of sound,  $\bar{v}(P)$  can thus be used to find R, as pressure approaches 0 such that the ideal gas law holds

### 5.3 Kinetic Theory of Ideal Gases

1. The theory was determined by Waterston and Kronig in 1859, finding that temperature is a function of particle motion for a monatomic gas (due to solid spherical atoms), assuming there is some high number of identical, chemically inert particles
  - (a) It is stated that the number of particles per mole is  $6.0221 \times 10^{23}$ , Avogadro's number, where the atomic mass is used to calculate the total mass, with a mole as  $22.4 \times 10^3 mL$  for an ideal gas
  - (b) Ideal gases are assumed to be in perpetual random motion with no preferred direction of motion, with a inter-atomic distance far larger than the atomic radius, with no forces of attraction between collisions, with velocities ranging from 0 to the speed of light
    - i. This can be integrated to infinity without error, due to such low density at higher velocities, and is assumed that the distribution at any particular velocity band is approximately constant
  - (c) The atoms are uniformly distributed ( $\frac{N}{V}$  is constant), such that  $dN = \frac{N}{V} dV$ , when fulfilling thermodynamic macroscopic qualifications
  - (d) Atomic collisions are assumed to be elastic against a smooth wall of the container, where only the perpendicular velocity with respect to the wall is changed, negated as a result without different speed
2. For some velocity vector from the origin,  $\vec{v}$ , to some elementary area,  $dA$ , on the surface of the sphere, such that for the sphere around the origin,  $dA = (r d\theta)(r \sin\theta d\phi)$ 
  - (a) The solid angle of a circle, as a 3D extension of angle,  $d\Omega = \frac{dA}{r^2} = \sin\theta d\theta d\phi$ , the maximum being that of the entire sphere,  $4\pi$  steradians (sr), such that it is the surface

area for a circle of radius 1

3. It is stated that there is no preferred atomic direction, it is written that  $dN|_{\vec{v},\theta,\phi} = dN|_{\vec{v}} \frac{d\Omega}{4\pi}$ , where N is the total number of atoms, such that the atoms going in any direction in a specific solid angle is equal to the fraction of the total atoms going in that direction
4. It is stated that atoms have no preferred location, such that for some portion of the wall, dA, some cylinder can be constructed of length  $\vec{v}dt$ , such that it is a short enough time that atoms won't collide, moving in the direction of  $\vec{v}$ 
  - (a) Due to the cylinder being non perpendicular to the base,  $dV = \vec{v}dt\cos\theta dA$ , such that it is stated that  $dC|_{\vec{v},\theta,dV,\phi} = dN|_{\vec{v},\theta,\phi} \frac{dV}{V}$ , such that the total atoms in that location moving in that direction divided by the total amount moving in that direction is equal to the percentage of volume
5. Since collisions are perfectly elastic,  $\Delta p$  per collision  $= 2m\vec{v}\cos\theta$ , such that per dt, the total change in momentum for collisions against dA  $= m\vec{v}^2 \frac{dN|_{\vec{v}}}{V} (\frac{1}{2\pi} \int_0^{2\pi} d\phi \int_0^{\frac{\pi}{2}} \cos^2\theta \sin\theta d\theta$ 
  - (a) Since the change in momentum due to collisions against some area is the definition of pressure, this is equal to  $dP|_{\vec{v}} = \frac{m}{3V} dN|_{\vec{v}}$ , integrated from 0 to  $\infty$  by the earlier assumption
  - (b) By definition, the average square of atomic velocity,  $\langle \vec{v}^2 \rangle = \frac{1}{N} \int \vec{v}^2 dN|_w$ , such that  $PV = \frac{Nm\langle \vec{v}^2 \rangle}{3}$
  - (c) Combining with the ideal gas law, produces the fact that  $T = \frac{2N}{3nR} \frac{1}{2} m \langle \vec{v}^2 \rangle = \frac{2\langle KE \rangle}{3nR}$ , where since for monatomic ideal gas atoms there are no rotational or vibrational kinetic energies, and due to lack of effect on other atoms, no potential,  $\langle K \rangle N = \sum KE = U$ 
    - i. Thus, it is shown that for a monatomic ideal gas, the internal energy and average kinetic energy are proportional purely to temperature
    - ii.  $C_V$  and  $C_P$  can then be calculated based on this function of internal energy, such that  $C_V = \frac{3}{2}nR$ ,  $C_P = \frac{5}{2}nR$ , such that it is independent of temperature
    - iii. The ratio  $k = 1.3807 * 10^{-23} J/K = \frac{R}{N_A}$ , where  $N_A$  is Avogadro's number, is the Boltzmann constant, providing the kinetic theory ideal gas law,  $PV = NkT$
6. The assumptions of non-interacting point-masses create error for real gases, such that van der Waals equation of state corrects for it, with a accounting for cohesive forces, b for volume occupied by being non-point masses

## 6 Chapter 6 - The Second Law of Thermodynamics

1. Work done in contact in a system to produce a higher temperature, but in contact with a heat reservoir of lower temperature, produces an unlimited amount of heat while work is done, without changing the state of the system itself, acting as a work engine
  - (a) Since the temperature of the system returns to the original due to losing heat to the sink, the work done is equal to the heat transmitted, such that there is 100% efficiency
2. The corresponding method to produce unlimited heat without a change of state is a heat engine, made up of a high temperature reservoir and a low temperature one, each connected to the system performing the work
  - (a) It then undergoes a cycle, or series of processes, to return it to the original state, measuring thermal efficiency,  $\eta = \frac{|W|}{|Q_H|}$ , which by the first law,  $\eta = 1 - \frac{|Q_L|}{|Q_H|}$ , such that it is only 100 if no heat goes to the lower reservoir

- (b) Heat engines are divided into internal-combustion, converting the fuel to high pressure and temperature by compressing, and external-combustion, using high temperature surroundings to convert the fuel to high pressure and temperature
- 3. The gasoline engine is made up of a piston connected to a rotating mechanism, rotating in one direction as it moves, using a six-process, four-stroke (piston-moving) cycle
  - (a) The intake stroke moves the piston outward, such that the volume moves from 0, pulling gasoline vapor and air from the open intake valve, due to being depressurized compared with atmospheric outside, rotating the mechanism manually
  - (b) The compression stroke moves the piston inward after closing the intake valve, increasing the pressure and temperature by decreasing the volume, after which combustion by a spark plug produces extremely high temperature and pressure (combustion), rotating manually
  - (c) The power stroke is caused by the combustion products expanding, pushing the piston, and rotates the mechanism by the pressure increase, after which the exhaust valve is opened to lower temperature and pressure and release some products (exhaust)
  - (d) The exhaust stroke then lowers the volume back to 0, pushing the remaining products back through the exhaust valve, rotating the mechanism manually
- 4. The Otto cycle is an idealized gasoline engine, in which there is no error introduced by conduction of heat through the engine, friction, turbulence, and chemical reaction between the vapor and air
  - (a) This engine is assumed to have air acting as an ideal gas, with constant heat capacities, and purely reversible quasi-static processes
  - (b) The intake stroke is an isobaric atmospheric pressure ( $P_0$ ) process, moving the volume from 0 to  $V$ , such that it is described by the ideal gas law,  $P_0V = nRT_1$ , where  $T_1$  is the temperature outside the engine, describing the state after intake
  - (c) The compression stroke is an adiabatic process, such that it moves from  $V_1, T_1 \rightarrow V_2, T_2$ , described by  $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ , where  $V_1 > V_2$
  - (d) The combustion is an isochoric increase of temperature and pressure due to external heat added from the hot reservoir, assumed to be a series of slightly different hot reservoirs, such that the change in temperature is quasi-static
    - i. This gives the equation that  $|Q_H| = \int_{T_2}^{T_3} C_V dT = C_V(T_3 - T_2)$
  - (e) The power stroke is an adiabatic process from some temperature,  $T_3$  after combustion to  $T_4$ , at a lower pressure and temperature, such that  $T_3V_2^{\gamma-1} = T_4V_1^{\gamma-1}$ , moving back to the volume before the compression
  - (f) The exhaust is the isochoric decrease in temperature and pressure due to external heat added to the sequence of cold reservoirs, where  $|Q_L| = C_V(T_4 - T_1)$
  - (g) The exhaust stroke is an isobaric process at atmospheric pressure again, the reversal of the intake stroke
  - (h) It is found as a result of these equations that  $\eta = 1 - \frac{T_1}{T_2} = \frac{T_4}{T_3}$ , such that it is based on the temperatures before and after each adiabatic process
    - i. In addition,  $\eta = 1 - \frac{1}{(\frac{V_1}{V_2})^{\gamma-1}}$ , where  $\frac{V_1}{V_2}$  is the compression ratio, such that complete efficiency would require no volume/gas
      - A. For real engines, it is found that a compression ratio greater than 10 would cause spontaneous combustion, interfering in the timing of the engine, though used in diesel
    - ii. Otto gasoline engines are assumed to go from 300 K to 580 K, such that there is



- 48% efficiency, though real gasoline engines tend to be about 25%
5. Diesel engines are internal-combustion engines as well, taking in only air during intake, spraying oil after compression, using the temperature of the air to trigger combustion, acting similarly to a gas engine otherwise
    - (a) The oil spraying rate is adjusted such that the combustion is slow enough that it is isobaric as the piston moves out to increase the volume for the power stroke
    - (b) Since air is used rather than gasoline, the exhaust stroke and intake strokes can be skipped, such that the power of the engine is doubled
      - i. This involves blowing fresh air into the cylinder to remove the remaining combustion products that didn't move out on their own
    - (c) An idealized diesel cycle, removing the same issues present during a real gasoline engine, acting similar to the Otto cycle, such that the only difference is during the combustion
      - i. Since the combustion is isobaric instead of isochoric as heat enters from the hot reservoir, such that  $|Q_H| = \int_{T_2}^{T_3} C_P dT = C_P(T_3 - T_2)$ , such that  $\eta = 1 - \frac{1}{\gamma} \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{1}{\gamma} \frac{r_E^\gamma - 1}{r_E - 1} \frac{T_1}{T_2}$ , where  $r_E$  is the expansion volume ratio,  $\frac{V_f}{V_i}$
      - ii. The expected ideal value is approximately 55%, while the realistic value is approximately 32%
  6. Steam engines are an external-combustion engine, used largely in nuclear powered maritime vehicles and submarines, functioning by a constant mass of water, using the feed-water pump to pump water up into the boiler, at which point a nearby hot reservoir heats and boils it
    - (a) Since the boiler has much higher pressure than the remainder, the pump is necessary, compressing it as it rises with only small changes in temperature and volume
      - i. The boiling takes place isobarically, raising the temperature, after which it is allowed to flow into a cylinder, expanding adiabatically against a piston to perform work
      - ii. Eventually, the temperature and pressure lower until that of the condenser as volume increases, after which it is allowed to flow into there near the cold sink, condensing into the original water temperature and pressure, below atmospheric levels
    - (b) Error is introduced to the engine by turbulence from the pressure differences causing flow, friction, or conduction of heat through the walls
    - (c) The idealized steam engine cycle is the Rankine cycle, starting from liquid water in the condenser, using the pump to compress and move it to the boiler adiabatically
      - i. After, the water is isobarically superheated and vaporized to above boiling point and vapor saturation at that pressure, expanding in the boiler
      - ii. After, it adiabatically expands from superheated steam to saturated/equilibrium steam, performing work on the piston, and finally the isobaric and isothermal condensation of steam back into water by loss of heat
    - (d) Since  $Q_L$  is nonzero, due to needing to take heat to condense the water back into liquid, the efficiency cannot be 100%, historically far lower than diesel, but currently actually about 35%
  7. Stirling engines are external combustion engines, invented to be less likely to explode than steam engines, used before the invention of internal combustion engines for low horsepower machines
    - (a) It has the overall advantage of being able to use any form of heat source, operates quietly, and does not produce any toxic byproducts, currently used for artificial heart power
    - (b) Ideal Stirling engines have two pistons on each side of a shaft with a fixed mass of helium in between, the left/expansion piston side in contact with high temperature reservoir,

- the right/compression piston with the low reservoir/atmosphere
- i. In the middle is the regenerator made of fine wire screens, allowing heat in and out into an internal reservoir by a fixed quantity reversibly
- (c) The Stirling cycle is made up of the heat loss stage, in which the left piston fully compressed, the right piston compressing halfway isothermally, triggering heat loss through the cold sink, performing work to compress
- i. After, the left piston expands, and the right fully compresses isochorically forced through the regenerator to the other side, raising the temperature and pressure without work or volume change through heat provided
  - ii. After, the right piston remains static, as the left piston is expands due to the pressure, to allow such that the volume increases, absorbing heat such that it is isothermal, performing work
  - iii. Finally, the left piston compresses fully as the right piston expands fully, giving the same heat absorbed by the regenerator as moving back through, isochorically
- (d) Error is introduced into the system by non-ideal gas, leakage of gas, heat transfer through walls, heat conducted from the regenerator to the device, or friction
- i. Realistic thermal efficiency of a Stirling engine is approximately 40%
- (e) The Ringbom Stirling engine was a modification using one piston instead of two, with a displacer/regenerator moving between the cylinder and piston
8. The Second Law of Thermodynamics Kelvin-Planck statement states that it is impossible to construct a cyclical heat engine that will do no effect other than extraction of heat from a reservoir and performing equal work
- (a) This is based on the fundamental cycles of a heat engine, relying on a process which takes heat from an high temperature reservoir, and a process which rejects heat to a lower temperature reservoir
    - i. This can be restated as that it is impossible for a hot object to utilize mechanical work from internal energy of matter, cooling below its surrounding objects (100% thermal efficiency)
  - (b) Theoretical machines which create energy, violating the first law, are perpetual motion machines of the first kind
  - (c) Those which use internal energy of a single heat reservoir only, converting heat into work alone, thus violating the second law, are perpetual motion machine of the second kind
9. Refrigerators are the opposite of a heat engine, taking some small amount of heat from a cold reservoir, returning a higher amount by work done on the system, the working substance of the system called refrigerant
- (a) It must be able to be cyclical, such that there is no change of state from a full cycle, able to convert an unlimited amount of work into heat
  - (b) The Clausius statement of the Second Law states that it is impossible to construct a refrigerator that produces no effect other than transferring heat from a low temperature reservoir to high temperature, such that there is no work needed
    - i. This is proven to be the Second Law, by the fact that combining an engine and a refrigerator, the latter violating this, would allow violation of the Kelvin statement
    - ii. Further, for an engine that violates the second law, feeding the work into the refrigerator, the refrigerator would thus violate the Clausius statement
10. Reversible processes are those which can be performed such that the system returns to its original state without any external changes

- (a) Any isothermal process involving a dissipation of heat, to be reversible, would require the complete conversion of heat from a reservoir into work, thus impossible
  - i. Any adiabatic process increasing the internal energy of a system from work, resulting in an increase in temperature, to be reversible, would require removing all added internal energy as heat, and converting it fully into work, also impossible
  - ii. These dissipative processes are considered to have external mechanical irreversibility, present in all mechanical devices, such that a machine which violates it is a perpetual motion machine of the third kind
- (b) Internal mechanical irreversibility processes are those which convert internal energy into mechanical energy, then back to internal energy without external heat or work, unable to be reversed without adding energy to the system, such as free expansion
  - i. This can be visualized as a process, which to be reversed, would require work, but also which all added energy would have to be removed fully as heat and fed back to the source of the work
  - ii. This can be viewed microscopically as being due to mechanical instability, causing the initial change, followed by dissipation causing the second change, making it irreversible
- (c) External and internal thermal irreversibility are those involving a transfer of heat to/from a reservoir, or within a body, by a finite temperature difference, impossible to reverse by the Clausius statement
- (d) Chemical irreversibility are those involving spontaneous chemical reactions, mixing of chemical substances, changes of phase, and transport of matter between contacting phases (solutions, osmosis)
  - i. Mixing of substances can be viewed as the free expansion of two substances, such that they are shown to be irreversible, while others are proved by chemical thermodynamics
- (e) By extension, it is shown that all natural processes are irreversible, featuring characteristics of lack of thermodynamic equilibrium during (not quasi-static) and dissipative effects
  - i. Since heat reservoirs are defined as a body such that any amount of heat transfer would produce a negligible effect, the transfer of heat can be considered reversible for a heat reservoir
    - A. This is due to acting quasi-statically and producing little enough work relative to internal energy of the body for dissipative effects to be negligible
  - ii. On the other hand, quasi-static processes do not imply lack of dissipation, in cases of heat being created from work, making it non-dissipative
  - iii. On the other hand, since none of the ideal cycles rely on this conversion of work into heat, it is found that they are reversible, assuming no friction or other dissipative processes

## 7 Chapter 7 - Carnot Cycle and Thermodynamic Temperature Scale

1. The Carnot cycle is a theoretical ideal, reversible heat engine, with the working substance beginning in thermal equilibrium with the low temperature,  $T_L$

- (a) Reversible adiabatic process is then preformed, such that the temperature rises to  $T_H$ , after which a reversible isothermal process is performed such that  $|Q_H|$  is absorbed
  - (b) After which a reversible adiabatic process is performed, such that the temperature drops back to the low temperature, and a reversible isothermal process is performed, rejecting  $|Q_L|$  to the cold reservoir
  - (c) This is distinct from a real heat engine, in that since it is isothermal rather than isochoric or isobaric, it doesn't require a series of reservoirs to be quasi-static, but rather a single
2. For a gas, this is drawn simply on a P-V graph, while for a reversible state change for added heat, which is isobaric and isothermal, with reversible adiabatic temperature change, it is written differently
- (a) In this case, it is noted that the change in volume for temperature change is drastically larger for gases than liquids, but the change in pressure is approximately the same
    - i. It is found that for an Carnot engine, any system can be graphed on some two coordinate plane, such that it is signified in a diagram by some rectangle, with R written inside to show reversibility
  - (b) The work done by the heat engine as a result is simply modified by changing the length of the isothermal process, and as a result, the amount of heat allowed to enter
3. Any reversible heat engine only requiring two reservoirs for operation and reversibility must be Carnot engines, unlike other cycles, which require a series of reservoirs to be reversible, providing a definition for Carnot engines
- (a) Carnot engines used in the reverse of the cycle are called Carnot refrigerators, distinct from other cycles, due to perfectly reversing the process formed by the engine, impossible due to dissipative effects
  - (b) Thus, Carnot's Theorem states that no heat engine operating between two given reservoirs can be more efficient than a Carnot engine operating between the same two reservoirs
    - i. This is proven by joining a Carnot refrigerator with an irreversible heat engine, such that by contradiction of the 2nd Law, it must be more efficient
    - ii. As a result of this, the Carnot cycle does not depend on the actual substances which make up the Carnot engine, but rather have a fixed efficiency between two reservoirs
    - iii. This provides a bound for the efficiency of a heat engine, as the maximum due to being able to absorb all heat at the high temperature, and release all heat at the low temperature, rather than by a series of reservoirs
4. The thermodynamic absolute temperature scale is the ideal, independent of the working substance, such as a constant-volume ideal gas thermometer, thus since a Carnot engine is independent, it makes a thermodynamic absolute scale
- (a) Efficiency of a Carnot engine is  $\mu_R = 1 - \frac{|Q_L|}{|Q_H|}$ , as some function of the two temperatures, such that  $f(T_H, T_L) = \frac{1}{1-\mu_R} = \frac{|Q_H|}{|Q_L|}$
  - (b) Since two Carnot engines, passing heat through some arbitrary heat sink, is equivalent to one bypassing it in terms of the initial and final heat, and total work, since the middle heat sink cancels out,  $\frac{|Q_H|}{|Q_L|} = \frac{\phi(T_H)}{\phi(T_L)}$
  - (c) As a result,  $\phi(T)$  is defined as the thermodynamic temperature function, independent of any substance, not even relying on gas properties like an ideal gas system, as the ratio of the heats
    - i. For a logarithmic  $\phi(T)$ , varying from  $-\infty(Q=0)$  to  $\infty(Q \rightarrow \infty)$ , advantageous due to moving absolute zero far from the freezing point of water

- A. The triple point of water was set at  $0^\circ$ , increasing linearly for each factor of 10, as a more symmetrical numerical system, denoted  $^\circ L$ , called the logarithmic thermodynamic scale
- ii. On the other hand, the common scale remains Kelvin or Rankine, applied to thermodynamic measurement systems
- (d) For an absolute linear scale,  $\phi(T)$  can just be written as  $T$  on the other hand, such that the triple point can be set as before, such that  $T = 273.16K \frac{|Q|}{|Q_{TP}|}$ , such that heat is a thermometric property for a Carnot thermodynamic scale

## 8 Chapter 8 - Entropy

### 8.1 Definition of Entropy

1. For some reversible process plotted on a smooth curve on a two-coordinate graph, versus a piecewise reversible continuous process, drawn made up of isothermal and adiabatic processes, with the same amount of work done from the same initial and final states
  - (a) Since the change in internal energy is equal, and the work done is equal, the heat transfer is equal for the process, able to ignore the adiabatic portions
  - (b) Since adiabatic and isothermal lines are unable to intersect due to being parallel relatively, any number of isothermal and adiabatic alternating processes can be written for the same process
  - (c) For two isothermal reactions between the same adiabatic curves, it forms a Carnot cycle, such that  $\frac{|Q_1|}{T_1} = \frac{|Q_2|}{T_2}$ , going in opposite directions, such that the heat signs are opposite
    - i. Thus, for reversible Carnot cycles, these reactions can be summed, such that  $\sum_n \frac{Q_n}{T_n} = \oint_R \frac{\delta Q}{T} = 0$ , such that any reversible cycle can be thought of as a series of Carnot cycles such that this equation is true, called the Claudius Theorem
2. By Claudius's Theorem, for any two reversible paths making a closed cycle,  $\int_{R1i}^f \frac{\delta Q}{T} + \int_{R2f}^i \frac{\delta Q}{T} = 0$ 
  - (a) Since each path is reversible,  $\int_{R2f}^i \frac{\delta Q}{T} = - \int_{R2i}^f \frac{\delta Q}{T}$ , such that the integral of  $\frac{\delta Q}{T}$  for each reversible path with the same initial and final state is equal
  - (b) Thus, the state function of this integral is called entropy,  $S$ , such that  $\delta Q_R = TdS$ , allowing an exact differential form of heat for a reversible process
    - i. T-S diagrams can thus be plotted similar to a P-V diagrams for a reversible process, such that for some  $T(S)$  curve, the region under the heat done, forming a rectangle for a Carnot cycle
    - ii. In addition, for some reversible adiabatic process, it must be isentropic, such that the entropy remains constant as well
3. Caratheodory proved the Second Law mathematically, based on the concept that around any point,  $P_0$ , there exists points not accessible along solution curves of  $A(x, y, z)dx + B(x, y, z)dy + C(x, y, z)dz = 0$  if the equation is able to be integrated
  - (a) This is able to be integrated if there exists  $\Lambda(x, y, z), F(x, y, z)$ , such that  $A dx + B dy + C dz = \Lambda dF$ , both facts holding for any amount of dimensions
  - (b) For some system of thermodynamic coordinates  $A, B, C$ ,  $\delta Q = A dx + B dy + C dz$ , as form of the first law, such that there are states of the system inaccessible by purely quasi-static adiabatic pathways, as a form of the second law ( $\delta Q = 0$ )

- i. Thus, the second law is only valid if there exists some functions,  $T$ ,  $S$ , such that  $\delta Q = TdS$  under this form, such that by the second law,  $\delta Q_R = TdS$  must be valid
4. For an ideal gas as a result, it is found that  $dS = \frac{\delta Q_R}{T} = C_P \frac{dT}{T} - \frac{V}{T} dP = C_P \frac{dT}{T} - nR \frac{dP}{P}$ 
  - (a) As a result, if  $C_P$  is constant over the temperature range,  $S - S_r = C_P \ln \frac{T}{T_r} - nR \ln \frac{P}{P_r}$ , or  $S = C_P \ln T - nR \ln P + S_0$ , where  $r$  is the reference point for a relative entropy scale, allowing an entropy table to be calculated
  - (b) The general function for entropy of an ideal gas as a result by this and the similar formula for volume, is  $\Delta S = \int_{T_r}^T C_P \frac{dT}{T} - nR \ln \frac{P}{P_r} = \int_{T_r}^T C_V \frac{dT}{T} - nR \ln \frac{V}{V_r}$ 
    - i. For a reversible isobaric process as a result,  $\frac{\partial T}{\partial S}|_P = \frac{T}{C_P}$  and for a reversible isochoric process  $\frac{\partial T}{\partial S}|_V = \frac{T}{C_V}$ , dependent only on the material and the temperature, rather than other coordinates
5. For some reversible process, the entropy change of the universe is both that due to the process and the reservoir, such that since it is quasi-static, the temperature of each is equal
  - (a) As a result, the entropy of the system and the reservoirs are inverses, such that for a reversible process, there is no change in entropy of the universe

## 8.2 Entropy Change

1. For some irreversible process between equilibrium states, since entropy is a state function, the change in entropy is equal to that of a reversible process with the same initial and final equilibrium states
  - (a) External mechanical irreversibility processes with isothermal transformation of work into internal energy of a reservoir has no change on the system due to work rather than heat and no temperature change, while since the reservoir gains heat, that is the universal entropy change
    - i. For a system with an adiabatic conversion of work into internal energy of a system, the entropy change is  $\Delta S = \int_{T_i}^{T_f} C_P \frac{dT}{T} > 0$ , such that if  $C_P$  is constant,  $\Delta S = C_P \ln \frac{T_f}{T_i}$
  - (b) Internal mechanical irreversibility is found by free expansion, such that  $\Delta S = \int_{V_i}^{V_f} nR \frac{dV}{V} = nR \ln \frac{V_f}{V_i}$ , such the entropy in the system increased
    - i. While in a free expansion, the heat exchange, and thus entropy, would be 0, it is approximated as an isothermal increase in volume, such that the entropy can be calculated by the entropy equation
  - (c) External thermal irreversibility, such that there is a finite temperature difference, have heat flow, where the same amount of heat is transferred from high temperature to low, such that entropy increases
  - (d) It is seen fundamentally thus, for some irreversible process, the change in entropy must be positive, such as chemical irreversibility of diffusion (acting as two free expansions)
  - (e) This is proven by a theoretical system of a reversible engine feeding heat into an irreversible engine with an internal temperature difference/cold reservoir, feeding from an external hot reservoir
    - i. In a cycle, such that there are no internal changes,  $W = \oint dQ_H \leq 0$ , since it cannot produce work without a cold reservoir, requiring dissipation to get back to the initial state, such that neither engine is able to produce work
    - ii. If the work done was 0, it would be moving back to the original state, returning all

- heat taken in, such that it would be reversible, thus  $\oint dQ_H < 0$
- iii. Since the first engine is a Carnot engine, the thermodynamic temperature ratio is valid, such that  $\frac{dQ_H}{T_H} = \frac{-dQ_L}{T_M} = \frac{dQ_I}{T_M}$ , since the heat lost is taken by the irreversible engine
  - iv. Thus, it is found that  $\oint \frac{dQ_I}{T_M} < 0$  for an irreversible engine cycle, where  $T_M$  is the temperature it is entering the system, rather than the temperature of the system/internal reservoir
- (f) As a result, the Clausius statement of the second law of thermodynamics states that  $\oint \frac{\delta Q}{T} \leq 0$
- i. It is noted though that the temperature of the entropy statement change is that of the heat entering the irreversible engine, rather than the temperature within the irreversible engine, since the temperatures are not equal due to not being quasi-static/reversible
2. Since entropy is a state function, for some closed loop, reversible or irreversible,  $\oint dS = 0$ , while since  $\oint \frac{\delta Q}{T} < 0 = \oint dS$
- (a) As a result, it can be generally stated for all processes that  $dS \geq \frac{\delta Q}{T}$ , such that two out of isentropic, reversible, and adiabatic imply the third, but one does not imply the others
3. For some nonequilibrium state moving toward an equilibrium state, the nonequilibrium can be broken into incremental volume/mass elements, such that it can be viewed as infinitesimal equilibrium states
4. The Entropy Principle states that for any irreversible process, the entropy of the universe increases, already shown for irreversible nonadiabatic processes, acting as an alternate form of the Second Law
- (a) For some irreversible adiabatic process,  $\Delta S = S_f - S_i$ , it can be brought through a combination of a reversible isothermal and two reversible adiabatic processes to bring to the initial state
    - i. Since for the reversible adiabatic process, there is no change in entropy,  $\Delta S$  is equal to the negative of the change in entropy of the reversible isothermal process
    - ii. In addition,  $\Delta S$  is then equal to the negative of the heat gained then divided by the temperature during the process after the first reversible adiabatic process
    - iii. As a result, during a full cycle, the only action done is the absorption of heat during the isothermal process, followed by work done on the system during the irreversible adiabatic process, such that there cannot be heat absorbed, but rather  $\geq 0$
    - iv. Thus, the change in entropy during the irreversible process  $\geq 0$ , and cannot be 0 since that would imply reversibility
  - (b) For some nonhomogeneous composition, temperature, and pressure system, for an irreversible adiabatic process, the system is assumed to be able to be divided into infinitesimal parts, each with definite entropy and coordinates
    - i. Thus, each section can be viewed as a homogeneous system, such that the total entropy of the system increases for each
    - ii. This assumes it can be broken up into infinitesimal parts and reversible processes for chemical reactions can be found, both of which relying on experimental proof
  - (c) For some object being cooled from the surrounding temperature  $T_i$  to  $T_f$  by a Carnot refrigerator, the minimum work for a real refrigerator can be calculated
    - i.  $\Delta S_{body} + \Delta S_{reservoir} = S_f - S_i + \frac{Q+W}{T_i} = 0$ , such that it provides the minimum work

for a real process, using an entropy table to determine

5. It can be seen from the process of irreversible, isothermal state changes or by free expansion, that an increase in entropy in the system manifests as a decrease in the atomic order/structure, describing a state just as entropy does
  - (a) In the conversion of work to heat, this is viewed as the conversion of ordered kinetic energy to random microscopic kinetic energy
  - (b) Thus, entropy can also be viewed as a mechanism of time, such that entropy must always increase over time, used as a measurement
6. The Third Law of Thermodynamics states that at absolute 0 ( $0^\circ K$ ), the entropy of the system is set to 0

## 9 Miscellaneous Thermodynamics Notes

Boundary layer is what changes temperature, rather than the wall itself, such that it is what modifies? Convection and conduction simultaneously? Ice water == resistor in series - Homework ice problem Oscillating electric charge emits an electromagnetic waves/radiation, emitting many types in a sample due to random motion Opaque bodies have 0 transmission fraction, transparent have 1, white bodies have reflective fraction of 1 Planck radiation law solved the lack of classical theory infinite 5000K radiation, leading to photons and quantum mechanics Higher frequency peak as temperature increases

Air is a poor conductor, such that atmosphere has chemical change, radiation, and convection (particle flow) for heat change, not conduction, processes within it can be considered adiabatic, such that for problem 10,  $M\Delta ng = M\rho A\Delta yg = \frac{MPAg\Delta y}{RT}$

From above,  $F = P(y + \Delta y)A$ , from below  $F = P(y)A$ , gravity from above, such that at equilibrium, it can be solved for the density and pressure with height 29 g/mol - air average  $-dT/dy = -9.77$  K/km

Heat is conversion of ordered work into unordered entropy on a microscopic scale, such that the production of heat from non-internal energy is dissipative, even if it is due to prior internal energy Lack of nondissipative processes is fundamental law of nature, due to view as entropy, but not as a result of first two laws of thermodynamics, under which perpetual of third kind is allowed Engines converted ordered energy to heat to ordered energy and heat, but increase overall universal entropy, even if geothermal machine passign heat somehow

Carnot cycle is a theoretical, but practically createable, and reversible engine, due to including no inherently irreversible processes, unlike other engines

Irreversibility cannot be seen from a PV diagram, due to only paying attention to the system for dissipative, thus just changing the values, and for non-quasi-static, pressure and volume at any points except the extremes are meaningless

$dS = C_V dT/T + P/T dV$  by first law  $\frac{\partial T}{\partial S}|_V = \frac{T}{C_V}$ ,  $\frac{\partial T}{\partial S}|_P = \frac{T}{C_P}$ , since  $C_V < C_P$ , the slope of the former is greater on a TS diagram



## 10 Freegarde Chapter 1 - Wave Motion Essence

1. Physics can be viewed by Lagrangian Particle Theory or Euler Field Theory, as a duality of perspectives, most apparent on a quantum scale
  - (a) As a result, when considering a dynamic/time-dependent system, it becomes Kinetic Theory and Wave Theory respectively
2. Waves are defined microscopically as a collective bulk disturbance, created at a point as a delayed response to the disturbance at adjacent points
  - (a) The disturbance progression to adjacent points is called the propagation, going through the medium
  - (b) The medium does not need to be linear, homogeneous, with sinusoidal or periodic propagation to classify as a wave
  - (c) Positions in the medium are defined by coordinates, each position requiring at least one additional variable to describe the disturbance
    - i. The coordinates and disturbance variables are each wavefunctions, defined with respect to time
3. Waves must be viewed in terms of both cause and effect, such that it is defined macroscopically as a time-dependent field effect due to finite speed of propagation of a causal effect
  - (a) Thus, it can be viewed as the solution to systems in dynamic equilibrium, more generalized than static equilibrium theory
  - (b) Electric Coulomb waves can thus be viewed as a result of a rotating dipole acting on a single charge at some distance
    - i. If the dipole radius is assumed to be far less than the distance from the charge, it can be simplified such that  $E(t) = \frac{2kq}{r_0^3}a(t - \frac{r_0}{c})$ 
      - A.  $a(t)$  is the perpendicular height of each side of the dipole, while  $r_0$  is the distance from the center of the dipole to the particle
      - B. The time delay due to the relativistic limitations on which the change in force is received create the time lag/retardation, allowing the wave
      - C. The magnetic component of electromagnetic waves are found by a Lorentz transformation taking into account the motion of the charges and dipoles themselves
    - ii. Gravitational waves have a similar wave effect for a gravitational single body, such that  $g = \frac{Gm}{r_0^3}a(t - \frac{r_0}{c})$ 
      - A. On the other hand, since the rotation of the causal body requires a third body to rotate with it similar to a dipole, it results for similar bodies in a higher order (inverse fourth root law), canceling out the effect to some degree
      - B. Thus, it is found that  $g = \frac{2Gma_0^2}{r_0^4}\sin(2\omega t)$  assuming the distance between the two causal bodies is vastly smaller
  - (c) As a result, waves can be viewed as some function dependent on an action at a prior time at some other object
4. It was believed until the 1900s that objects required a medium to influence each other in a vacuum, leading to the idea of the aether
  - (a) As a result of relativity and electromagnetism, it became apparent that there is no ether, such that forces can be sent through a vacuum
  - (b) On the other hand, for waves, there remains the concern of diffraction in a vacuum, with apparently nothing deflecting, and the concept of radiation, sending energy through a

vacuum alone

- i. Current theories create no issues with these concepts, though they remain difficult to conceptualize
5. Transverse wave motions, such as surface tension, electromagnetic, or gravitational, where the disturbance is perpendicular to the direction of wave propagation
  - (a) Longitudinal waves are those where the disturbance is parallel to the direction of propagation, such as sound, generally due to pressure differences on each side
    - i. On the other hand, gravitational or electromagnetic waves can be created longitudinal rather than transverse
  - (b) Spin waves are those with both transverse and longitudinal components, such as seismology
  - (c) Waves can also be scalar quantity displacement, such as thermal waves (based on heat transfer), quantum wavefunction, or chemical composition waves (based on a reaction-diffusion system)
  - (d) Waves are also not required to be moving through a continuous medium, as long as they fulfill the definition

## 11 Freegarde Chapter 2 - Basic Wave Equations

### 11.1 General Concepts

1. Wave equations are partial differential equations which relate the wavefunction (derivatives of wave displacement) to time and position, derived first from the general physical situation
2. It is found that for traveling waves, the shape remains the same though the location of the disturbance changes with time
  - (a) This is due to the lack of loss of energy during propagation, such that there is no dissipation/damping of the wave
  - (b) As a result, it is found that  $\psi(x, t) = \psi(x - \delta x, t - \delta t) = \psi(x - v\delta t, t - \delta t)$ , since  $\delta x = v\delta t$ 
    - i. Thus, if  $\delta t = t$ ,  $\psi(x, t) = \psi(x - vt, 0) = \psi(u)$ , such that the variables must appear in that combination for a forward traveling wave
    - ii. Thus, since the wavefunction is the same value over a specific curve, this single variable function is found for a traveling wave
    - iii. This all follows from the method of viewing waves as the motions of individual particles, collectivized, such that x determines which particle is being viewed, while t is at which time
  - (c) In addition, both sides can be subtracted by  $\phi(x, t - \delta t)$ , by the x-t relation, such that  $\frac{\partial \phi}{\partial t} = -v \frac{\partial \phi}{\partial x}$ 
    - i. This assumes that  $\phi$  is continuous, or at least can be approximated as a continuous function
    - ii. Further, it is found similarly that  $\frac{\partial^n \phi}{\partial t^n} = (-v)^n \frac{\partial^n \phi}{\partial x^n}$ , though the equation would have to be modified due to the sign of n changing
3. Generally, wave motion may be done relating the derivative with respect to time at some point to that relating the derivative with respect to location at that time
  - (a) This is then used to determine the general form of the wave equation, followed by inputting specific parameters for the solution

## 11.2 Long String Waves

1. For some string with tension  $T$ , assuming gravity effect is negligible, such that for some unit length mass  $M$  with length  $\delta x$ , tension is acting on both sides
  - (a) The angle is assumed to be slightly varying on the sides of the length, with small enough total displacement that  $\cos\theta_1 \approx \cos\theta_2 \approx 1$ 
    - i. Since the tension on the side in the direction of the wave is toward the positive wave function, while it is toward the negative for the other at some point,  $F_{\text{perpendicular}} = T(\sin\theta_2 - \sin\theta_1)$
    - ii. When this distance is infinitesimal, the change in angle for each side is infinitesimal, moving the wave toward the slightly steeper direction
  - (b) By definition of the angle,  $\tan\theta = \frac{\partial\psi}{\partial x}$  for each end of  $\delta x$ 
    - i. Since the angles are small enough that  $\cos\theta \approx 1$ , it can be stated that  $\sin\theta \approx \tan\theta$
    - ii. As a result by this and Newton's Law,  $F_n = T(\frac{\partial\psi}{\partial x}_{x_2} - \frac{\partial\psi}{\partial x}_{x_1}) = M\delta x(\frac{\partial^2\psi}{\partial t^2}|_x)$ 
      - A. This is due to the fact that the acceleration at any one point is equal to the wave function second derivative with respect to time
    - iii. Thus, as divided by the length and taken to the limit as the length approaches 0, it is found that  $\frac{\partial^2\psi}{\partial t^2} = \frac{T}{M} \frac{\partial^2\psi}{\partial x^2}$
2. Using the formula for traveling waves in terms of one variable combined with the chain rule and the above formula, it is found that  $v^2 \frac{d^2\psi}{du^2} = \frac{T}{M} \frac{d^2\psi}{du^2}$ 
  - (a) Thus, for a traveling string wave,  $v = \pm\sqrt{\frac{T}{M}}$ , such that the velocity depends on the tension and density
3. Since the wave equation has linearity, such that superposition applies to it, the general solution is equal to the linear combination of individual solutions, spanning the solution set
  - (a) Thus, the two respective solutions are for a positive and negative velocity traveling waves, such that it is equal to the solution to a wave with positive and negative velocity
    - i. Thus, this forms the idea that since the velocity can be in either direction when assuming a standing wave, all waves on the string are made up of two standing wave directions
  - (b) Thus, it is found that  $\psi(x, t) = \psi(x \pm vt) = \psi_+(x - |v|t) + \psi_-(x + |v|t)$ , to provide the forward and backward wave components, taking the specific solution at  $t = 0$ 
    - i. The constant multipliers of the linear multiplier are contained within the functions themselves
    - ii. This forms  $\psi(x, 0) = \psi_+(x) + \psi_-(x)$  and by the derivative with respect to  $t$ ,  $\frac{\partial\psi}{\partial t}(x, 0) = \frac{\partial\psi_+(x-|v|t)}{\partial t}(x, 0) + \frac{\partial\psi_-(x+|v|t)}{\partial t}(x, 0)$ 
      - A. The first equation serves to provide the initial condition for a unique solution
    - iii. By the chain rule, the second equation becomes  $\frac{\partial\psi}{\partial t}(x, 0) = |v|(\frac{d\psi_-(x)}{dx} - \frac{d\psi_+(x)}{dx})$
  - (c) For some guitar string connected at two ends,  $[0, l]$ , pulled at some point  $x_0$  to initial height  $a_0$  at  $t = 0$ , with initial velocity of 0
    - i. By integrating the velocity formula for  $t = 0$ ,  $\psi_-(x) = \psi_+(x) + c$ , such that at  $t = 0$ ,  $\psi(x, 0) = 2\psi_+(x) + c = 2\psi_-(x) - c$
4. By these formulas for  $\psi_{\pm}$  in terms of the initial wavefunction of some point,  $\psi(x, t) = \frac{1}{2}\psi(x - |v|t, 0) + \frac{1}{2}\psi(x + |v|t, 0)$ 
  - (a) It follows from this and the general formula that the separate wave components are equal to half of the initial wavefunction, moving in each respective direction

- (b) This assumes that  $x$  is not at the endpoints at some nonzero time, since it is undefined for a negative displacement or a displacement greater than the length
- (c) This equations can be supplemented by the fact that  $\psi(0, t) = \psi(l, t) = 0$  for any time  $t$ , acting as a boundary condition
- (d) This is used to find the restriction that for any  $x, t$ ,  $\psi_+(x - |v|t) = -\psi_-(2l - x + |v|t) = \psi_+(x - |v|t - 2l)$ , such that the component waves are periodic triangular/sawtooth
  - i. This is found by the fact that  $u_+ = 2x - u_i$ , giving the relationship between the component functions if the overall function is 0, such that  $u_+ = -u_i$  and  $u_+ = 2l - u_-$  must be true for all  $u$  by the boundary conditions
- 5. The wave solution must be valid for the entire domain of  $x, t$  values which are possible, to be a solution

### 11.3 Light Properties

1. By Maxwell's equations, the wave equation for light is derived, such that  $\frac{\partial B_z}{\partial x} = \mu_0 \epsilon_0 \frac{\partial E_y}{\partial t}$ , such that it is solved to find that  $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ 
  - (a) This results also in the fields oscillating sinusoidally, perpendicularly, and transversely in phase with each other
  - (b) In addition, by Maxwell's equations, it is found that  $E = cB$  at any point in the wave, giving a relationship between the amplitudes
  - (c) Similarly, within other mediums,  $v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\kappa\epsilon_0 k_m \mu_0}$ , where  $\kappa$  is the dielectric constant and  $k_m$  is the relative permeability
    - i.  $k_m$  is generally equal to 1 for dielectrics, such that it can be ignored
    - ii. This provides a relationship between these constants and the index of refraction, such that  $n = \sqrt{\kappa k_m} = \sqrt{\kappa}$ , such that  $\kappa$  is a function of
2. By Maxwell's equations, the energy density for each field is found such that  $u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2 = \epsilon_0 E^2$ , such that the magnetic and electric field energies are equal at all points
  - (a)  $S = \epsilon_0 E^2 c = \frac{EB}{\mu_0}$ , such that the Poynting vector is the vector instantaneous energy flow per unit area in the direction of propagation,  $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$ , with the magnitude as the power per unit area
  - (b) The intensity is equal to the average of the Poynting vector as a result, such that  $I = \langle S \rangle = \frac{E_{max} B_{max}}{2\mu_0} = \frac{1}{2}\epsilon_0 c E_{max}^2$ , with total power as  $P = \int I A$  assuming the waves are isotropic
    - i. In addition, the root-mean-square field value is defined as  $E_{rms}\sqrt{2} = E_{max}$ , and cooresponding for magnetic fields, such that  $I = \frac{E_{rms}^2}{c\mu_0}$

## 12 Freegarde Chapter 4 - Sinusoidal Waveforms

1. Sinusoidal wave solutions are often used to approximate a wave, such that while they are not the only form of motion, can be used to form any linear wavemotion
  - (a) Pseudo-sinusoidal equations are also used for dissipative systems, able to be described generally by a main sinusoidal component
2. The main sinusoidal wave form, fulfilling the traveling wave criteria, is  $\psi(x, t) = a \sin(k(x - vt) + \phi)$ , where  $\phi$  is the initial phase/offset (relative to an origin) and  $a$  is the amplitude

- (a) As a result, the wave angular frequency,  $\omega = vk = kf\lambda$ , where  $k$  is the constant wavenumber, continuing infinitely
- (b) This can be rewritten by several variables, oscillation period ( $\tau f = 1$ ), frequency ( $\omega = 2\pi f$ ), wavelength ( $k\lambda = 2\pi$ ), or the spectroscopists' wavenumber ( $\bar{\nu}\lambda = 1$ )
- 3. Wavefronts are defined as a continuous function with constant sinusoidal phase, such that for the main form,  $k(x - vt) + \phi = c$  for some constant  $c$ 
  - (a) As a result, since it describes the movement of a point of constant wavefunction over the function over time ( $x = vt + (c - \phi)/k$ )
  - (b) This includes the crest of a wave, or a point where the wavefunction remains 0, or a point that moves with the wave, keeping constant displacement
  - (c) The velocity of the wavefront, moving with constant phase, is called the phase velocity, denoted  $v_p$ , commonly just as  $v$
- 4. For some traveling string wave ( $u = x - vt$ ), assuming amplitude is far smaller than wavelength and tension is constant, kinetic energy is by the transverse motion ( $v_t = \frac{\partial \psi}{\partial t}$ )
  - (a) Thus,  $K = \frac{1}{2}\delta m v_t^2 = \frac{M\delta x}{2}(\frac{\partial \psi}{\partial t})^2$ , by chain rule,  $K = \frac{v_p^2 M\delta x}{2}(\frac{\partial \psi}{\partial u})^2$ 
    - i. Due to lack of longitudinal motion, each mass item is constant per unit length, and kinetic energy is purely transverse
  - (b) Potential energy of a string element is the work done stretching the string to displace it, creating the tension
    - i. As a result,  $U = \frac{T\delta x}{2}(\frac{\partial \psi}{\partial x})^2$ , by the chain rule,  $U = \frac{T\delta x}{2}(\frac{\partial \psi}{\partial u})^2$
    - ii. Thus, potential and kinetic energy of any length element are equal, such that  $E = K + U = T(\frac{\partial \psi}{\partial x})^2\delta x = T(\frac{\partial \psi}{\partial u})^2\delta x$
  - (c) The energy density,  $\epsilon$ , is the energy per unit length
  - (d) This is an example of the virial theorem, stating that potential and kinetic energy are related for any dynamic system
    - i. In addition, this shows that both forms of energy are related to the square of the wave amplitude
  - (e) As a result, the intensity is the energy moving through some point with the wave at some moment, or the power at some point such that  $I = Tv_p(\frac{d\psi}{du})^2 = \epsilon v_p$ 
    - i. Thus, intensity is defined as the energy propagating through some point per unit time, such that total power through the region is  $P = \int IA$
  - (f) It is noted that for superpositioned waves, the intensity and energy must take into account the sums of various directions of each
- 5. Sinusoidal waves are generally used as an ideal solution class to the wave equations, due to commonality, but relying on a single frequency to be able to define phase velocity
  - (a) On the other hand, it is ideal due to acting as solutions to linear, dispersive systems with varying phase velocity as well, and due to standing waves and orthogonality
- 6. Standing waves are a class of solutions to a non-dispersive, linear string system, in which the motion is standard, differing only in magnitude throughout the string
  - (a) As a result, it can be written in the format of  $\psi(x, t) = X(x)T(t)$ , placed within the string wave function to get two second order ODEs, forming sinusoidal solutions if  $X$ ,  $T$ , and  $k$  are real
  - (b) In this case,  $X(x) = X_0 \sin(kx + \phi)$ ,  $T(t) = T_0 \sin(\omega t + \theta)$ , where  $\omega/v_p$  are constant based on string characteristics
    - i. Thus,  $\psi(x, t) = \psi_0 \sin(kx + \phi) \sin(\omega t + \theta)$ , where  $\psi_0 = T_0 X_0$

- (c) These appear as non-moving waves, though by the trigonometric identities, can be converted to the sum of sinusoidal travelling waves, creating an alternate wave format
- (d) Nodes of a standing wave are points where motion is always 0, such that  $kx_m + \phi = m\pi$ , where  $m$  is an integer, able to be fixed at those points without changing the wave
  - i. Thus, for some fixed string of length  $l$ , assuming no phase shift,  $k = \frac{n\pi}{l}$ , where  $n$  is some integer, or  $l = \frac{n\lambda}{2}$
  - ii. As a result, the allowed modes of vibration of a standing wave are all lengths of integer  $n$  for some wavelength
    - A. For  $n = 1$ , it is the fundamental mode of vibration/first harmonic, while higher modes are harmonics/overtones ( $n = 2$ , the second harmonic, first overtone)
  - iii. Nodes can also be determined by the superposition of forward and backward travelling waves, such that if  $\psi_+(u) = \psi_0 \cos(ku)$ , for each node, it must be true by boundary conditions that  $\psi_+(u - 2x)$ 
    - A. As a result, it must be true that  $kx = n\pi$  where  $n$  is some integer
    - B. This is able to be taken as well, such that  $\psi_-(u) = -\psi_0 \cos(k(x + |v|t))$ , the standing wave equation can be derived
- (e) The energy over an entire string,  $E = T(\int (a\frac{\partial\psi_1}{\partial x} + b\frac{\partial\psi_2}{\partial x})^2)$ , expanded to find that  $E = a^2 E_1 + b^2 E_2 + 2Tab \int_0^l \frac{\partial\psi_1}{\partial x} \frac{\partial\psi_2}{\partial x} dx$ 
  - i. As a result, it is found that the energy is greater than the sum of the component energies
  - ii. On the other hand, it is found that sinusoidal functions,  $\sin(k_1x), \sin(k_2x)$  have the integrated product of 0 if the functions are linearly independent, called orthogonal, making them ideal for energy analysis

## 13 Freegarde Chapter 5 - Complex Wavefunctions

1. Sinusoidal wave forms, due to the cycle of derivatives, are those with wavefunctions of the form  $\sum_{n_i, n_j, \dots} a_{n_i, n_j, \dots} \frac{\partial^{n_i + n_j + \dots} \psi}{\partial x_i^{n_i} \partial x_j^{n_j} \dots} = 0$ 
  - (a) This assumes that the order of the sum of partial derivatives are all even or all odd ( $n_i + n_j + \dots$ )
  - (b) The main wave types not with sinusoidal wave forms, are dissipative waves, thermal diffusion equation, and the Schrodinger quantum equation, which are not all odd or even, and thus not fully sinusoidal
    - i. In other cases, the complex harmonic waveform is used, such that it is a well-defined sinusoidal function, but with possibly imaginary amplitudes
  - (c) For some complex wavefunction,  $\psi(x, t) = a \cos(kx - \omega t) + i a \sin(kx - \omega t) = a e^{i(kx - \omega t)}$ , such that it is a travelling wave where  $u = k(x - v_p t)$ , and can be broken separably
    - i. This can be proven to be the solution to any wavefunction, with any partial derivative sum parity
    - ii. It can also thus be broken up into a separable standing wave function, breaking the exponents up
    - iii. It can be superimposed with other complex exponentials to produce a real sinusoidal wave, allowing for easier calculation
  - (d) For some dissipative system, such as a string wave with a drag force, such that  $F = -\mu v_t$ ,

solved to find  $k = \pm \sqrt{\frac{M}{T}}\omega \sqrt{1 + \frac{\mu i}{M\omega}} = \pm k_0(p + iq)$

- i. This can then be solved for the real and imaginary parts, such that  $p^2 = \frac{1}{2}(1 + \sqrt{1 + (\frac{\mu}{M\omega})^2})$ ,  $q = \frac{\mu}{2M\omega}$
- ii. Weak damping is if  $p \approx 1$ , such that  $\mu \ll M\omega$  and  $\psi = ae^{i(k_0x - \omega t)}e^{-k_0qx}$ , where  $k_0q$  is the decay constant
- iii. This provides the spatial decay wave equation, though it can be rewritten to create a complex  $\omega$  and real  $k$  for the temporally decaying form
- iv. These equations can also be solved by complex standing wave form of each standing wave part, separating the variables, creating a single variable differential equation for the same function
- v. These display dispersion as a result, changing the shape during propagation, rather than being a constant travelling wave, such that phase velocity and angular frequency are dependent on wave number
  - A. As a result, dispersive waves are only sinusoidal travelling waves if the wavenumber is constant
2. The displacement due to sinusoidal wave motion at some point is drawn with a phasor arrow, proportional in length to amplitude
  - (a) The phasor is rotated to show the phase of the wave propagation, able to be represented as a complex exponential, providing the time difference between each neighbor effect
  - (b) The phasors are thus able to be added as vectors, providing the overall wave displacement at some point
    - i. For phasors of constant phase difference, it becomes an infinite geometric series, decreasing in magnitude from the original amplitude, such as light reflecting on oil
    - ii. The light is then reflected partially at the bottom of the oil to the top, repeating infinitely, summing for the net effect

## 14 Freearde Chapter 6 - Huygens Wave Propagation

### 14.1 Huygens Description

1. Huygen's principle states that a wavefront of light propagates from an initial disturbance at the speed of light, equally in all directions, to form a spherical wavefront
  - (a) This has since been expanded to apply to all forward-traveling waves, rather than just light waves
  - (b) It also states that as the wavefront moves, it creates secondary sources, forming secondary wavefronts, being created at all positions continuously
  - (c) As a result of interference, adding the amplitudes as superposition of waves from different paths to enforce or cancel out others
    - i. This results in the lack of backwards travelling waves, averaging to cancel each other out in that direction, superposed purely on the advancing planar wavefront
      - A. This creates a wavefront tangent to the secondary wavelets, where the set of wavelets is the envelope of the wavelets
    - ii. This can be shown by phasars for each wavefront at some point, the central waves at the lowest angle due to being the first, and so forth

- A. This can be mathematically analyzed by finding the exponential function,  $a(x, y) = \frac{e^{ikr}}{r}$ ,  $r = \sqrt{x^2 + y^2}$ , such that the amplitude is the inverse square dependent on the distance
- B. This can then be taken for all points at the same x value, integrated, to get the overall amplitude at that point
- C. It is found to be proportional to the Fresnel integral,  $\int_{-\infty}^{\infty} e^{iks^2} ds$ , where  $r - x = s^2$  if  $kx \ll 1$
- D. The Cornu/Euler Spiral/Clothoid is the graph on the complex plane of the result of  $a_0(s_0) = \int_0^{s_0} e^{iks^2} ds$  for all  $s_0$ , graphing the line from the ends of the curve for the Fresnel integral vector
- E. This as a result approximates to a resultant with a phase of  $\frac{\pi}{4}$
- iii. Huygen's description implies a backward-travelling wavefront similarly, acting as a snapshot mechanism, such that each wavefront is replaced by the new wavefront of subsequent sources
  - A. This is due to the velocity for the wave being of equal importance to the displacement at some point, for determining the wave, providing the initial conditions for a specific solution
  - B. Thus, the lack of a specific mechanism or velocity of propagation prevents a specific waveform from being determined
  - C. For electromagnetic waves, the electric and magnetic fields provide the displacement and velocity, the cross product being the Poynting vector of the direction of energy propagation
  - D. It can also be explained by the forward wave producing secondary waves in phase with each other, while the backwards secondary waves don't produce in phase, but rather at any point after
- iv. The Fresnel integral vector is found to have a  $\frac{\pi}{4}$  angle/phase, doubled for a 2D wave, as the basis of Fresnel-Kirchhoff diffraction theory
  - A. It could also be attributed to some degree to an overly literally derived mathematical model of Huygen's description, but can also be derived by Maxwell's equations
  - B. In addition, if the assumption of  $kx \ll 1$  is false, the value collapses towards the real value in the direction of the wavefront
- 2. Huygen's principle is not independent, but rather a means of calculating wave phenomena, able to be separately derived for individual wave types

## 14.2 Geometric Properties

1. Reflection is explained as creating secondary wavefronts at the collision with the surface, spreading outward, the waves hitting the mirror moving further, creating a new horizontal wavefront
  - (a) It can be then found geometrically that the wavefronts are at equal angles to the normal of the surface of the mirror, creating the Law of Reflection
    - i. The rays are considered to be propagating in the plane of incidence, perpendicular to the material boundary
  - (b) This prevents the backward-travelling waves from being canceled out and it blocks/disrupts the forward travelling waves



- (c) Reflection at a definite angle of a smooth surface is specular reflection, while a rough surface causes diffuse reflection
- (d) Reflection takes place in all types of materials, allowing them to be seen by the colors reflected
- 2. Refraction is described similarly, though it only partially disrupts the forward travelling wave
  - (a) It must take into account the relative speeds within each medium, such that the distances are based on time, such that the wave travels at a different velocity within the new medium, causing the angle change
  - (b) It is thus partially reflected, but forms a wave within the border, such that the angles must be specifically to allow the same time with different velocity
- 3. The relationship between the angles and velocity are embodied by Snell's law of refraction, commonly written in terms of a refractive index,  $\eta$  of the medium, such that  $\eta = \frac{c}{v_p} \geq 1$ , such that if  $\theta$  is the angle from the normal line,  $\eta_1 \sin(\theta_1) = \eta_2 \sin(\theta_2)$ 
  - (a) Gradual changes in the velocity, rather than at an interface, causes refraction, though not reflection since there is no sudden medium shift to trigger reflection, changing due to the density shift, such as in the atmosphere, creating mirages
  - (b) The refractive index is distinct for different materials and since all real waves are dispersive, since the velocity depends on the wavelength/wavenumber, the index of refraction does as well
    - i. It follows that since  $v = \lambda f$ , if  $\lambda_0$  is the wavelength in a vacuum,  $\lambda = \frac{\lambda_0}{\eta}$
  - (c) As a result of this law, the path of a ray is reversible, moving by the same path in the other direction
  - (d) It is possible for all light to be reflected from a non-opaque medium shift if the resultant angle is  $90^\circ$ , called the critical angle, such that above that, none can be refracted
    - i. This is called total internal reflection, only able to occur if the index of refraction of the original medium is larger than that of the other medium, such that there is no refraction
    - ii. This allows a hollow Porro prism (45-90-45) to internally reflect all light within to each of the legs of the prism, due to air/glass having a lower critical angle
    - iii. This also allows a light pipe, such as a fiber optic cable of glass or plastic, to transmit an image, built so the angles are all greater than the critical angle, acting as a thinner and more insulator than normal wires
- 4. Fermat's Principle of Least Time is distinct from, but consistent with, the Huygens construction, stating that a wave follows the path that takes the least time when travelling between two points
  - (a) The path is defined as the ray path, either of a narrow beam from a slit blocking the light wave, larger than a wavelength, smaller than the main components, such that there are enough secondary sources for the wavefront, but a narrow wavefront
    - i. This can also be viewed as the direction of energy flow or as the normal to the wavefront
  - (b) This is found to correspond to Snell's Law if applied to a medium transition, such that refraction produces the path of least time
    - i. This can be thought of in terms of phasors, such that phasors of longer time vary more in direction, cancelling each other out, such that only the shortest time remain
  - (c) This law is akin to the Principle of Least Action in mechanics, such that the same law applies to particles as well as waves to an extent

5. Dispersion is the dependence of wave speed and index of refraction on wavelength, not taking place in a vacuum, allowing light to be dispersed/separated into a spectrum when diffracted
  - (a) The amount of dispersion depends on the difference between the refractive indices, causing diamonds to have large dispersion
6. Rainbows are created partially from light being partially refracted, refracted, and dispersed, most reflected into some angle less than  $\theta$ , the majority at  $\theta$ 
  - (a) The down-sun point is the point at which for some drop of water, the horizontal axis intersects the angle, generally outside the drop, appearing as the bright line of the rainbow
    - i. The angle depends on the refractive index for each wavelength, such that it is the angle light at the top of the drop reflects to for the primary rainbow, bottom for secondary
    - ii. For purple light, refracted the most, the angle is approximately  $42^\circ$
  - (b) The primary rainbow has one reflection and two refractions, while the secondary rainbow has two reflections and two refractions, with larger angle for each wavelength ( $52^\circ$  for red)
  - (c) The secondary rainbow as a result has reversed colors, with the areas between the primary and secondary and above the secondary darker than the rest, due to far fewer light waves
    - i. The second rainbow is fainter due to less light remaining after two reflections causing some diffracted
7. Polarization is a characteristic of all transverse waves, linearly polarized if there is only displacement on one axis, performed by polarizing filters
  - (a) Polarizing filters can be thin slits to stop strings in any other direction, or metal/long-chain-molecule slits to absorb all electric fields in those directions by free electrons canceling it out
    - i. Polaroid filters are dichroismic materials, such that it transmits 80% of the waves on the polarizing axis, and only 1% otherwise
    - ii. Ideal polarizing filters are those which transmit all light on the polarizing axis
  - (b) Radio waves are linearly polarized parallel to the antenna, but other emitted light is due to random molecule orientation, such that it is unpolarized/natural light
  - (c) Malus's Law states that since for two polarizers (the second called an analyzer) with some angle  $\theta$  between the axes,  $\vec{E} = \vec{E}_{max} \cos\theta$ , such that  $I = I_{max} \cos^2\theta$
  - (d) Light is also able to be partially polarized by reflection, due to light with electric fields parallel to the surface being reflected more
    - i. There is also some angle, the polarizing angle, such that all light is refracted, rather than reflected, except those parallel to the surface, which has some reflected
      - A. In addition, refracted light from this angle is partially polarized by that mainly perpendicular to the surface in the same manner
    - ii. Brewster's law states that the reflected and refracted rays are perpendicular at the polarizing angle, such that  $\tan(\theta_p) = \frac{n_b}{n_a}$
    - iii. This allows sunglasses to work better, polarizing to remove light that is reflected off the horizontal ground to reduce the light
  - (e) Circular/elliptical polarization is caused by polarized transverse waves perpendicular to each other, out of phase, such that each point has circular motion, appearing as a spiral
    - i. For a quarter phase difference, it is circular, while otherwise, it is elliptical, while

- as the wave moves towards the viewer, the direction of rotation is right if clockwise, left otherwise
- ii. This can be done by an antenna by adding a delay for one of two perpendicular antennas from the transmitter
  - iii. Materials with different indices of refraction for different polarizations are birefringent, dependent on the material and the wavelength of light
    - A. This is able to produce the phase difference for perpendicularly polarized light, due to the different velocity within for each polarization
    - B. Crystals such as calcite which are thick enough to produce a quarter phase difference are called quarter-wave plates
    - C. Photoelasticity is the property of materials to become birefringent when under stress, allowing light to move through perpendicular polarizers if passing through a stressed object between
8. Scattering is the process of atoms absorbing and reradiating light, such that blue is scattered downward more, leading to the red sunset of light not scattered, but the blue sky
- (a) This collision polarizes the light, due to the molecule oscillating like an antenna, propagating in every other direction than the original wave, due to no electric field in that direction
    - i. It then sends waves in the direction of oscillation, such that the waves in those directions are polarized
  - (b) It is found that  $I_{scattered} = kf^4$ , such that higher frequencies are scattered more
  - (c) Higher concentrations generally appear white, due to all wavelengths eventually scattering within

## 15 Freegarde Chapter 7 - Geometric Optics

1. Geometric optics are the movement of a ray/wavefront moving in a straight line, and the effects of a change in medium, predating physical optics which requires wave behavior
  - (a) Optics are generally the study of electromagnetic waves/light
  - (b) Until Newton, light was believed to be a string of particles emitted, though as it displayed properties of waves, the consensus changed until Maxwell proved it was an electromagnetic wave
    - i. It was later found that light was in discrete quanta, or photons, displaying wave-particle duality, leading to the study of quantum electrodynamics
    - ii. As a result, propagation is generally studied as a wave, while absorption and emission are studied as particles
  - (c) Wavefronts, while realistically somewhat curved, can be viewed on some small area relative to the total size of the wavefront, as a planar wave
2. For some glass sphere, the paraxial approximation states that the angle made by any ray to the sphere is small, and the thin-lens approximation states that the distance from the horizontal axis is smaller than the radius
  - (a) Situations where the approximations do not apply are considered optical aberrations, preventing the height from being ignored
  - (b) The sphere is made up of a sphere connected to a continuing medium, found geometrically that  $\frac{\eta_1}{d_1} + \frac{\eta_2}{d_2} = \frac{\eta_2 - \eta_1}{R}$

- i.  $d_1$  is the distance from the ray origin on the axis to the point of refraction  $d_2$  is the distance from when the diffracted ray hits the axis to the image, and  $R$  is the radius
  - ii. It is noted that it is independent of the height from the horizontal axis at which the ray hits the sphere
  - iii. The focus of the rays is the point at which the rays are refracted to, reversible, such that the points are images of each other
  - iv. It is also found that the same point is valid for all rays from a particular initial axis point
- (c) For some point at the same distance from the sphere, but either above or below the axis, it is mapped as if the axis was rotated
  - i. This creates a correspondence between each point, such that it creates image formation in the sphere, flipped in orientation
- (d) The existence of a focal point at which all the rays converge produces a real image, allowing it to be recorded
  - i. For some critical distance, the rays are parallel, such that they intersect at infinity, and no image is produced
    - A. This distance is called the focal length,  $f_n = \frac{\eta_n R}{\eta_2 - \eta_1}$ , where  $n$  is the medium,  $\eta_2$  is the sphere,  $\eta_1$  is the other medium
  - ii. For a greater distance, they diverge from some point beyond the original source, called a virtual image
  - iii. Due to the images being reversible, the sources/objects can be real or virtual, either from a physical object or an image object
  - iv. Negative sources and images are denoted with a negative distance, positive if real, with the radius positive for a source outside the interface, negative for within
- 3. Two spheres of different curvatures/radii are often combined to send the image into the original medium on the opposite side, called a lens
  - (a) Lenses can have each side calculated for diffraction, combined with the thin lens approximation, to give the lensmaker's formula, such that  $\frac{1}{s_1} + \frac{1}{p_2} = \frac{\eta_2 - \eta_1}{\eta_1} (\frac{1}{R_1} - \frac{1}{R_2})$ 
    - i.  $s_1$  is the distance from the object to the lens, while  $p_2$  is the distance from the lens to the image, defined purely by the lens properties
    - ii. For either an infinitely distant image or source, one term can be removed, such that  $\frac{1}{f} = \frac{\eta_2 - \eta_1}{\eta_1} (\frac{1}{R_1} - \frac{1}{R_2})$
    - iii. As a result, it can also be written as  $\frac{1}{s} + \frac{1}{p} = \frac{1}{f}$
  - (b) Converging lenses are those with a positive focal length, while those with a negative focal length are diverging lenses
  - (c) This can be extended to a spherical mirror, such that  $\eta_1 = -\eta_2$  to describe reflection, with  $R_1 = R_2$ 
    - i. As a result,  $\frac{1}{f} = \frac{-2}{R} = \frac{1}{s} + \frac{1}{p}$ , such that the focal length is half the curvature radius
    - ii. This applies to both forms of mirrors, such that  $R$  is positive if the image is outside the radius interface, negative otherwise
- 4. The focal plane is the plane that passes perpendicular to the lens axis, such that the angle of the parallel rays to the lens axis is based on the initial height of the object in the plane
  - (a) Thus, it is determined by the Gaussian lens equation that  $y = f\theta$
- 5. Fermat's principle states that while there is no preferred direction, rays travel the path of least time, such that all ray paths take the same time to reach the image point from some source

- (a) As a result, the initial direction by which a ray is emitted is independent of the time to reach the image
- (b) The image can thus be defined as the point in which all rays from a source are in phase together

## 16 Freegarde Chapter 8 - Interference

1. Wave interference and superposition state that the field can be added for all different wave paths at a specific point
  - (a) The theory of interference ignores the phase differences and use the paraxial approximation again to allow angle approximations, preventing the measurement at the interference directly
  - (b) Young's analysis of interference also assumes a specific frequency to allow use of phasors by sinusoidal components
2. Young observed that light shined into opaque cardboard with two slits within, showed periodic spacial intensity
  - (a) This was due to allowing only a single secondary wave source for each slit, such that the phasor sum of the secondary waves produced the periodic nature
  - (b) For some slits at  $x = 0, y_{1/2} = \pm \frac{d}{2}$ ,  $\psi(x, y) = \psi_1(x, y) + \psi_2(x, y)$ , where  $\psi_{1/2} = \frac{\psi_0}{r_{1/2}} e^{ikr_{1/2}}$ 
    - i. This is due to the phasors acting as the components for superpositioned waves, producing constructive and destructive interference, proving wave behavior
    - ii.  $r_{1/2} = r_0 \sqrt{1 + \frac{\mp d + \frac{d^2}{4}}{r_0^2}}$  where  $r_0$  is the distance from the center of the slit sheet
      - A. If  $d \ll r_0$ , this approximates that  $r_{1/2} = r_0(1 \mp \frac{dy}{2r_0^2})$ , and  $\psi(x, y) = 2 \frac{\psi_0}{r_0} e^{ikr_0} \cos(k \frac{d}{2r_0} y)$
      - B. By the prior formulas for intensity of light, the intensity at any point is approximated as  $I(x, y) \approx I_0 \cos^2(\frac{kdy}{2r_0}) = I_0 \cos^2(kd \sin(\frac{\theta}{2}))$ , where  $\theta$  is the angle from the perpendicular axis to the slits to the point
3. Speakers use sound meant to go to each ear through headphones, made to produce the proper intensities and depth, in a wide area creating cross-talk
  - (a) As a result, in theory, the cross-talk sound can be cancelled out by the same negated sound from the other speaker, adding a phase shift to cancel it out, used for surround sound
  - (b) On the other hand, this can create a faint disturbance, and relies on being placed precidely between

## 17 Miscellaneous Wave Notes

All waves can be viewed as the sum of a sinusoidal traveling wave, such that  $\psi(x, t) = A \cos(k(x - vt) + \phi) = A \cos(kx - \omega t + \phi)$  by forier series of normal modes

$\frac{\partial^2 \psi}{\partial t^2} = v^2 \frac{\partial^2 \psi}{\partial x^2}$  is the general wave equation

Since  $\sin kx \sin \omega t = \frac{1}{2}(\cos(kx - \omega t) - \cos(kx + \omega t))$ , it can be written as travelling waves For a string attached at the ends, since  $\psi(0, t) = A \sin \phi \sin(\omega t + \theta) = 0$  for a standing wave, and  $\psi(L, t) = A \sin(kL + \phi) \sin(\omega t + \theta) = 0$ ,  $kL$  is limited to specific values for a standing wave to occur

v independent for a string, such that  $f\lambda$  is constant,  $f_n = \frac{nv}{2L} = nf_1$ , where  $f_1$  is the fundamental frequency on the string. Fundamental frequency is the frequency of a standing wave equal to half a wavelength.

$\psi(x, t) = \sum a_n \sin(\frac{n\pi x}{L}) \cos(\frac{n\pi vt}{L})$ , represented as a Fourier series, equal to just the sine component at  $t = 0$ , from  $n = a_1$  to  $a_\infty$ .

In addition, it proceeds to get more rounded as it goes along, due to damping causing the higher frequencies to fade?? or vice versa.

Each stationary standing wave point is a normal node, while normal string waves are triangular, they can be approximated by a standing wave curve, getting closer as the Fourier series is allowed to extend to infinity.

Three terms is generally enough to approximate a string properly. Superposition of electronic states leads to electrons in many distinct states simultaneously until observed.

Energy in Wave Motion -  $K = \frac{1}{2} \Delta m v^2 = \frac{1}{2} \mu \Delta x (\frac{\partial \psi}{\partial t})^2$

For string,  $\Delta L = \sqrt{(\Delta x)^2 + (\Delta \psi)^2} - \Delta x = \Delta x (\sqrt{1 + (\frac{\partial \psi}{\partial x})^2} - 1)$

$\sqrt{1 + x^2} = 1 + \frac{1}{2}x^2$   $U = T \Delta L$ ,  $E = T (\frac{\partial \psi}{\partial x})^2 \Delta x$

Energy can be expressed in normal modes, electromagnetic waves in powerpoint.

Poynting vector in the direction of propagation of light, equal to the cross product of the electric and magnetic fields  $E = E_0 \cos(kx - \omega t)$ ,  $B = B_0 \cos(kx - \omega t)$   $E_0 = cB_0$ ,  $\vec{S} = \frac{1}{\mu_0} E \times B$ ,  $S =$  poynting,  $|S| = \frac{1}{\mu_0} EB = \frac{P}{A}$ ,  $I = \langle |S| \rangle = \frac{1}{\mu_0} \langle EB \rangle = \frac{1}{c\mu_0} \langle E^2 \rangle$   $v = c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$  At some angle, total internal reflection takes place, at which point there is no refraction, only reflection, only when moving from high index of refraction to low. Called critical angle, when the transmission angle is parallel to the medium boundary.

$c = \frac{1}{\sqrt{\epsilon \mu_0}} = \frac{1}{\sqrt{\kappa \epsilon_0 \mu_0}}$ ,  $n = \sqrt{\kappa} > 1$

Mirage is caused by hot air having a lower angle of refraction, such that light bends, allowing two images of the same object, one down, one straight. It is turned upside down due to the top part being bent such that the ray is below the unbent ray of the bottom.

Polarization in the direction of oscillation, using a filter barrier to remove all those not oscillating in the right direction. Can be done by a metal grating, the electric charges changing to cancel out the field with electric fields in that orientation. Can also be done by long stretched chain molecules, acting similar to a metal due to fairly mobile electrons.

Intensity =  $\frac{EB}{\mu} = \frac{E^2}{c\mu_0}$

45 degrees then 45 degrees again of a polarization field can shift the light into components, such that it can pass through the two perpendicular filters.

I unpolarized  $I_0$  -> Filter,  $I = \frac{\langle E^2 \rangle}{c\mu_0} = \frac{E_0^2 \langle \cos^2 \theta \rangle}{c\mu_0} = I_0 \langle \cos^2 \theta \rangle = \frac{I_0}{2}$

Scattering - Rayleigh (Main form,  $\lambda \gg D$ ), causing electron to oscillate, producing new wave as atom rotates,  $I = kf^4$ , such that more blue scattered than red, causing sky colors.

Oscillating dipoles produce the partially polarized light from reflection.

Wavelets caused by oscillating electrons at the same rate, such that frequency is the same, just velocity and wavelength different

Backtravelling waves are canceled out with an infinite sized plane wave with an infinite number of wavefronts

Virtual image produced by plane mirror due to diverging light

Magnification =  $y'/y = -S'/S$

$1/S + 1/S' = 2/R = 1/f$

Youngs constructive/destructive interference ( $r_2 - r_1 = n \cdot \lambda$  - Hyperbola shape for each  $\pm n$ )  
- Asymptotic far away from slits, forming parallel lines angled diagonally

Michelson interferometer in Morley-Michelson experiment measured speed of light perpendicular to earth versus parallel to measure the effect on light speed, finding constructive rather than expected destructive, disproving ether Inferometer splits light in different directions by wavelength, bouncing off mirrors before going back to source, giving different distances

Complete solutions need boundary + specific velocity/displacement

Normal mode of oscillation all at same frequency in standing wave

Energy of solutions  $\psi = a\psi + b\psi$  is  $E = a^2 E + b^2 E$  for superposition of normal modes, which all can be broken into fourier series of  $\psi(x, t) = \sum_{n=1}^{\infty} \sin(\frac{n\pi x}{L}) \cos(\frac{n\pi v}{L} t)$

$dB^2$ , not dB

Intensity = Instantaneous power/Area of Solid wave reached

Chapter 4 - Potential energy string derivation rely on not using small angle approximation, and using binomial approximation ( $\sqrt{1+x^2} - 1 = \frac{1}{2}x^2$ )' Chapter 8 - Approximation as  $d$  approaches 0

Focal length inside radius is negative for non-lenses For a lens, rays moving through the center are considered to be negligably refracted, continuing straight approximately Positive focal radius lens = converging/convex, negative = diverging/concave Erect = not upside down

## 18 Helliwell Chapter 1 - Inertial Frames and Classical Mechanics

1. Frames of reference are necessary to describe motion, creating a coordinate axis, easiest to measure motion when inertial frames
  - (a) Inertial frames of reference are those where Newton's First Law is true, found to be those not accelerating or rotating relative to any other inertial frame
    - i. This is compensated for on Earth by local reference frames only being negligably rotating and accelerating with the Earth
  - (b) This allows infinite inertial frames of reference, moving with different velocities
  - (c) Newton's other laws are only valid for describing motion by inertial observers
2. Events are defined as something that happens at an instant in time and in space, requiring both to measure position

- (a) The measurement of position of an event depends on the frame of reference
  - (b) For some frame F, and some frame F' moving with velocity v with respect to F, some event at (x, y, z, t),  $x' = x - vt$ ,  $y' = y$ ,  $z' = z$ ,  $t' = t$ , called the Galilean transformation of F to F'
    - i. Within a Galilean transformation, length and time intervals are the same between frames
    - ii. It is noted that to measure length of an object, the two event measurements must be simultaneous for the lengths to be measured equally, and clocks must be synchronized
  - (c) As a result of Galilean transformations, space and time are distinct and do not affect motion as a result
  - (d) It also follows that  $v'_x = v_x - v$  and  $a'_x = a_x$ , providing the relationship between velocities and accelerations in different frames
    - i. Since mass is assumed to also be a constant quantity, force is independent of frame of reference as a result within inertial frames
  - (e) Thus, within classical mechanics, it is assumed that there is no preferred frame of reference, also called the principle of relativity
3. The reference frame such that the center of mass remains fixed in the frame for system momentum of 0 is called the center of mass frame, giving infinite rotated center of mass frames for any system
  4. Classical physics assumes the validity of Newton's Laws, both in terms of force and momentum, as well as the universal conservation of energy

## 19 Helliwell Chapter 2 - Light and the Ether

1. In the 1800s, it was thought light required a medium to propagate in, called the ether, existing in all environments, assuming light moving at c for a frame of reference of the ether at rest
2. The aberration of light is the annual apparent circular motion of the stars in the sky, due to the telescope having to be pointed at an angle to view a star above, approximately  $\tan(\theta) = \theta = \frac{v}{c} = \frac{30 \cdot 10^3 \text{ m/s}}{c} = 10^{-4} \text{ rad}$ 
  - (a) This is due to the motion of the telescope due to the rotation of the earth in the time it takes the light to reach the telescope due to the rotation of the Earth
  - (b) As the star moves overhead twice per year, it requires angling the telescope in opposite directions due to the revolution of the Earth
  - (c) As a result, the ether cannot be moving at the same speed as the rotation of the Earth, or the telescope would not require being angled
3. The Michelson-Morley Experiment tried to measure the velocity of the ether, sending light through a separator in perpendicular directions towards a mirror, travelling separate distances to each mirror, interfering to the observer
  - (a) This is very sensitive to changes in distance due to the small wavelength of visible light
  - (b) As a result, the ether would change the time it takes the light to move back, changing the relative velocity, such that  $t_{\text{longitudinal}} = \frac{d}{c-v} + \frac{d}{c+v} = \frac{2d}{c(1-\frac{v^2}{c^2})}$ 
    - i. For the transverse movement, it would move in a triangle relative to the ether, such that  $t_{\text{transverse}} = \frac{2d}{c\sqrt{1-\frac{v^2}{c^2}}}$



- (c) This is then measured with the perpendicular setup with the same distances, to differentiate the effect of the different distances and the ether movement
- 4. It was found that the ether was not moving at the surface of the Earth at a velocity opposite the orbital speed of the Earth such that it is at rest relative to space, apparently showing the ether movement on Earth, contradicting the aberration effect

## 20 Helliwell Chapter 3 - Einstein's Postulates

1. Einstein's postulates of special relativity made up for the lack of an ether frame of reference for the speed of light, found to also not be relative to the emitting object,
  - (a) These stated that absolute uniform motion cannot be detected and that the velocity of light does not depend on the velocity of its source relative to the observer, both only applying in an inertial frame of reference
    - i. The first implied that there was no preferred inertial frame of reference by which all motion is compared, such that the laws of physics must be valid for all frames
    - ii. The second is generally not true for other phenomena, such as sound, in which it is only independent of the source when the observer is at rest with respect to the air/medium
  - (b) As a result, light does not depend on the source velocity, and by the first postulate, for any inertial frame of reference of the observer, it is the same, such that the velocity of the observer with respect to the source doesn't change the velocity of the light
2. For an explosion of light moving from the coinciding origins at  $t = 0$ , when  $F'$  moving at  $v$  with respect to  $F$ , it will appear to an observer in each frame to be moving in a circle at the same rate from the origin with respect to their frame
  - (a) On the other hand, an explosion of sound will only move out spherically from the origin in the frame with velocity 0 with respect to the air, such that in  $F'$ , the origin from which the sound moves will appear to be moving with velocity  $-v$

## 21 Helliwell Chapter 4 - Time Dilation

1. Light clocks are those that flash each time they detect the reflection of their previous pulse, such that  $\Delta t = \frac{2D}{c}$ , where  $D$  is the distance to the mirror
  - (a) For clocks on a moving vehicle, they function in the same manner by the first principle, moving at the same rate as each other, but must function at the same rate as other clocks of any type on the same vehicle
  - (b) The rate of the clock for those on the vehicle is by the previous formula, but for observers of the clock as the ship moves, stationed perpendicular to the motion
    - i. Thus, the light has to move both the distance the vehicle is moving in that time, while moving towards the mirror, such that  $\Delta t = \frac{2D}{\sqrt{c^2 - v^2}}$
    - ii. As a result, the time between each tick appears to be longer for clocks on the ship to a stationary observer than for a relative stationary clock
2. As a result, it is shown that  $t_{moving} = t_{stationary} \sqrt{1 - \frac{v^2}{c^2}}$ , such that since all clocks move slower as a result, including technological and biological, time moves slower on the moving vehicle, called dilation, such that time is not universal

- (a) This formula assumes that the velocity of the vehicle/the relative velocity of frames of reference is unable to be greater than the speed of light, acting as a maximum speed for the reasoning
- 3. It is difficult to measure time dilation due to the time required for the light from the clock to reach the viewer giving a delay, instead placing synchronized clocks at both endpoints
  - (a) A third moving clock is then synchronized with the first when it is at the same location, observing the time on the third and second clock as it passes it
  - (b) This has been observed by the decay of unstable particles from high energy colliders or cosmic rays, by the movement of galaxies to/from the Earth, and in satellites/airplanes
    - i. Particles such as muons decay in their own time frame within  $2.2\mu s$ , but are found to reach the planet in cosmic rays due to time dilation, lasting longer in the detectors reference frame, acting as a clock
    - ii. Muons are created due to pion decay, which in turns are produced by cosmic protons colliding with the air, testing the muons at high altitude versus low at  $0.995c$  approximately, finding they survive approximately  $8.8 \pm 0.8x$  longer, close to the calculated  $8.4 \pm 2x$

## 22 Helliwell Chapter 5 - Length Contraction

- 1. Time dilation assumed that transverse lengths were not changed by motion, found to be true by the first postulate, due to this causing a nonsymmetric effect depending on the frame of reference
  - (a) This is proven by the wooden stick knife experiment, one running toward the other such that if the length changed, it would either be sliced off or slice off the blade of the other from the stick, showing a preferred frame of reference
- 2. Longitudinal lengths, on the other hand, are affected, such that if measured for some clock moving with velocity  $v$  along some length  $D$ , the time it takes the clock to reach the end of the stick would then be  $\Delta t = \frac{D}{v\sqrt{1-\frac{v^2}{c^2}}}$ , due to time moving slower for the moving clock
  - (a) Within the clock's frame of reference, the clock will still state the same value when the stick has passed, such that either the velocity of the stick depends on the frame, or the length of the stick depends on the velocity
    - i. Since velocity is independent of frame of reference, due to the first postulate, the distance depends on the frame of reference
    - ii. Thus, within the clock's frame of reference, the time for the stick to pass is distance divided by velocity, while in the sticks frame, time is moving slower for the clock
  - (b) Thus, Lorentz-Fitzgerald contraction of longitudinal lengths is designated by  $d_{moving} = d_{rest}\sqrt{1-\frac{v^2}{c^2}}$  for some object
- 3. This explains the fact that for those moving at some velocity, since the clocks move slower, they appear to travel distances in shorter times, rather travelling shorter distances
- 4. For a light clock sending light in the direction of motion, sending a distance  $D$  when not moving,  $\Delta t = \frac{2D}{c\sqrt{1-\frac{v^2}{c^2}}}$  is the time between each tick, equal to that of a transverse sending clock
  - (a) This is due to the stationary observer viewing the length  $D$  as  $D\sqrt{1-\frac{v^2}{c^2}}$ , such that on the ship, less time is passing, but on the stationary frame, the light is travelling a shorter

distance

## 23 Helliwell Chapter 6 - Simultaneity

1. Relativity also causes the order of events, or simultaneity, to be relative, rather than definite, such as that seen when two ships move in opposite direction, triggering an explosion on each side as the stationary observer sees the head of one reach the tail of the other
  - (a) For an observer inside the ship, the light from the direction towards which they are moving would reach them before the light from the direction they are moving from, although the explosions occurred when they were halfway
  - (b) This can be explained from the perspective of an observer in the ship, as the other ship appears shorter, such that the explosion of reaching its tail must happen before vice versa
2. As a result of the lack of simultaneity between reference frames, clocks must be synchronized in each specific reference frame
  - (a) This cannot be done by a single observer, aligning the time of one with that of another at some other location due to the lag it takes the light to reach the observer causing a difference in times
  - (b) This can also not be done by moving clocks to a different location after synchronizing them all together, due to time dilation of transport causing them to run slightly slower based on the distance and velocity of transport, unless precisely synchronizing transport as well
  - (c) Clocks are properly synchronized by a light flash when  $t_1 = 0$ , setting the second clock when the light reaches to  $t_2 = \frac{d}{c}$ 
    - i. This can also be done similarly by a lightbulb in between, setting both to  $t = 0$  as the light reaches each of the clocks
  - (d) On the other hand, by this method for a moving observer with stationary clocks, the clocks appear to be moving in the opposite direction with a contracted length between them
    - i. Since they are moving, the flash of light hits one of the clocks before it reaches the other, due to one moving towards the wavefront, the other away
    - ii. This time difference can be calculated, such that the time until it reaches the first clock is  $t = \frac{D\sqrt{1-\frac{v^2}{c^2}}}{2(c+v)}$ , due to the contracted distance the light travels in the clock's frame of reference
      - A. On the other hand, the other clock has a denominator of  $2(c-v)$ , such that the time difference is  $\Delta t = \frac{Dv\sqrt{1-\frac{v^2}{c^2}}}{c^2-v^2}$
      - B. Since that is within the frame of reference of the clocks, it is multiplied by the Lorentz factor, such that when the clock moving from the light waves is at  $t = 0$ , the other is at  $t = \frac{vD}{c^2}$ , giving the time lag factor for clocks in the same frame
      - C. The clock in the front is moving from the light wave, called the leading clock, is the clock with a time lag
    - iii. As a result, distance and velocity create a time lag, such that simultaneity depends on frame of reference, worse by distance

## 24 Helliwell Chapter 7 - Paradoxes

1. Paradoxes are apparent impossibilities within relativity, aiding understanding of the concepts once the logical contradictions are resolved
2. The clock speed paradox is the idea that if a clock is apparently slow to one observer, and yet to an observer in the frame of reference of that clock, a clock in the first observers frame of reference appears slow to them
  - (a) This is resolved by the fact that the rate of the moving clock is measured by two synchronized clocks within the stationary frame, but from the moving frame, the two clocks are not synchronized
  - (b) This is shown by two clocks in frame A, apparently taking  $t = \frac{D}{V}$  for the clock in frame B to reach the second, reading that it took it less time by the Lorentz factor
    - i. Within frame B, it took the time read on the second clock, due to the distance travelled being contracted
    - ii. The time on each clock when the moving clock reaches it does not depend on the frame of reference, but the clocks do not appear synchronized within the B frame, but rather that the second clock is at  $t = \frac{VD}{c^2}$  when the first is at  $t = 0$
    - iii. As a result, the time that passes within the A frame according to the B frame is the apparent time in A multiplied by the Lorentz Factor squared, taking into account both the time dilation and length contraction
      - A. When adding this to the initial value of the second clock from the perspective of the B frame, it reads the same time as in A, such that time still slowed in the clocks in the A frame apparently
  - (c) The Twin Paradox is an extension of this, caused by a twin changing to a near-light speed, then slowing back, such that when in the same frame of reference, one will have to have had more time pass than the other, although both could argue their clock is faster
    - i. This is resolved by the fact that the moving twin is accelerating at certain points, such that the rules of special relativity do not apply
      - A. This is due to acceleration not being relative, but rather able to be measured by the force on an object, such that the accelerating frame does not have the formula apply
    - ii. On the other hand, within the nonaccelerating frame, the rules apply, such that the faster clock exists within this frame, due to the rules applying normally
3. The meterstick length paradox is similar to the clock speed paradox, instead about the idea that each meterstick appears longer to those in its frame of reference than others
  - (a) The length is measured by simultaneous measurement of both sides of the moving stick by stationary observers in frame A, but observers in frame B would see those as non-simultaneous
  - (b) Since the second clock within frame B lags behind by  $\frac{VD}{c^2}$  in terms of the A frame, the meterstick in B appears to an observer in B to have been measured shorter than it actually is
    - i. The same logic applies to a meterstick within the A frame, such that it will appear to an observer in A that B is measuring the stick shorter than it should be
4. Special relativity also causes a problem with rigid bodies, preserving the distance between any two points, such that it acts as a point mass, due to length contraction and by limiting

the speed of an effect to less than the speed of light, such that a force won't reach all points simultaneously

## 25 Helliwell Chapter 8 - Lorentz Transformation

- The Lorentz Transformation is a reformulation of the Galilean transformation applying the concepts of relativity to it, such that F' is moving with velocity  $v$  relative to F
  - By the Galilean transformation, for an observer in F,  $x' = x - vt$ , but the distance is contracted, such that an observer in F' observes distance  $x'$ , in F observes  $x' \sqrt{1 - \frac{v^2}{c^2}} = (x - vt) \sqrt{1 - \frac{v^2}{c^2}}$
  - Just as in a Galilean transformation,  $y' = y$  and  $z' = z$ , due to being perpendicular to the direction of motion
  - For the time transformation, it is assumed the clocks are synchronized at  $t = 0$  as they are at the same location, such that  $t = t' \sqrt{1 - \frac{v^2}{c^2}}$ 
    - For some following clock within F', it is leading by  $\frac{vx}{c^2}$ , since the distance from the origin initially is independent of frame, such that for some clock at  $(x, y, z)$  in F', the relationship between F and F' is  $t = t' \sqrt{1 - \frac{v^2}{c^2}} + \frac{vx}{c^2}$
  - This transformation is reversible, such that F can be viewed as moving with  $-v$  from the stationary F' for the same result
- Thus, the overall Lorentz transformation in terms of F is  $x' = \gamma(x - vt)$ ,  $y' = y$ ,  $z' = z$ ,  $t' = \gamma(t - \frac{vx}{c^2})$ , where  $\gamma$  is the Lorentz factor, or  $\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$
- This can be taken for  $t'$  where  $x = x'$  for all  $t$  in the F frame to get time dilation,  $x_1 - x_2$  at the same time within the F frame to get the formula for length contraction, and  $t'_1 - t'_2$  for simultaneous time measurements ( $t_1 = t_2$ ) in the same frame based on location
- It is found by differentiating that  $\frac{dx'}{dt'} = \frac{\frac{dx'}{dt}}{\frac{dt'}{dt}}$ , similar for  $y$  and  $z$ , such that  $v'_x = \frac{v_x - v}{1 - \frac{vv_x}{c^2}}$  and  $v'_{y/z} = \frac{v_{y/z} \sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{vv_{y/z}}{c^2}}$ 
  - As a result, as the speed of light approaches infinity, it becomes the Galilean transformation, but otherwise, it depends on both  $v_x$  and the velocity on the axis itself
  - It is also calculated from this that  $v_{F'} = c$  if  $v_F = c$  for all  $v$  between the frames of reference, such that it follows Einsteins second postulate
  - It also follows from these equations that for any velocity greater than the speed of light, the value is imaginary, such that it is not possible
- The transformation can be rewritten in terms of F, such that  $x = \gamma(x' + vt')$ ,  $y = y'$ ,  $z = z'$ ,  $t = \gamma(t' + \frac{vx'}{c^2})$

## 26 Helliwell Chapter 9 - Spacetime

- Within relativistic physics, distance and time depend on each other, warping together as the frame of reference changes, such that it can be viewed as a coordinate system, called Minkowski spacetime

- (a) Lorentz transformations are viewed as a rotation of the coordinate system, though not following the same rotation equations as a normal coordinate plane, using  $ct$  as the time axis such that it is the same units
  - (b) This allowed general relativity, based on the idea of curved spacetime, rather than Euclidean perpendicular axes, allowing non-inertial reference frames, such as a large/global scale in which gravity applies
  - (c) As a result, for some light ray on the 2D graph of time and space in one dimension, it appears as the lines  $y = x$  and  $y = -x$ , both moving forward in time, intersecting at the origin, forming the locus of light rays
    - i. With a second spacial dimension, it forms a light cone as the locus of light rays, all at the same angle to the time axis, and with a third spacial dimension, forming a sphere changing in size over time, still called the light cone
    - ii. For any particle, it moves within the cone, through the origin, staying inside due to moving less than at the speed of light, drawn over time called the worldline
2. Thus, the absolute past for the particle is defined as all events on or within the lower light cone, while the absolute future are those of the upper light cone, due to only those events being able to influence or be influenced by the particle
- (a) The relative past and future of the particle are those taking place outside the cone, depending on the frame of reference velocity relative to the particle, to determine the time of the event relative to the particle
  - (b) All observers will agree on an event within the absolute past, due to the effects being present in any frame of reference
3. For some standard displacement vector in 3D space, it is noted that even if the coordinate plane is rotated, the magnitude is still constant, called invariant
- (a) Similarly, the position vector in spacetime is denoted by  $(x_\mu)$ ,  $\mu = 0, 1, 2, 3 = (ct, x, y, z)$ , called a four-vector, defined such that the magnitude is constant in all frames
    - i. This is found to not be true for the Euclidean space magnitude vector formula, but rather with the magnitude defined for four-vectors as  $\sqrt{-c^2t^2 + x^2 + y^2 + z^2} = |(x_\mu)|$ , such that Minkowski space is called a pseudo-Euclidean space
      - A. Similarly, the magnitude is given the term, four-scalars, remaining constant under rotation
      - B. It is noted that  $ct$  is used such that the units are the same for all, and that the Lorentz transformation for  $ct$  results in the same equation as used for  $x$  ( $x'_0 = \gamma(x_0 - \frac{vx_1}{c})$ )
    - ii. This is due to the fact that longer time results in longer length, rather than shorter length as the alternate formula would imply
  - (b) Displacement within spacetime is called the spacetime interval, such that the magnitude is the same within all frames of reference
    - i. It is noted that it can be both positive or negative, and may be zero even if the location of the events are not the same
    - ii. For some events with spacetime interval magnitude greater than 0 is called spacelike, with spacial separation greater than  $c\Delta t$ 
      - A. As a result, there is some frame of reference, such that  $\Delta t = 0$ , such that the events are simultaneous, where the distance between in that frame is the proper length, only able to be found in a spacelike interval
    - iii. For some events with spacetime interval magnitude less than 0 is called timelike,

such that it is possible to find a frame of reference where  $\Delta x = \Delta y = \Delta z = 0$ , such that the proper time ( $\tau$ ) can be measured

- iv. For some events with a null spacetime interval ( $(\Delta s)^2 = 0$ ), it is possible for them to be at different coordinates, such that they must exist along some specific light ray
  - A. Thus,  $\Delta x = \pm c\Delta t$ , and there is no frame of reference in which they are simultaneous events, or in which they are at the same spacial location
- v. The magnitude of interval is constant, such that the proper time and length can be measured, the former of which for some stationary clock,  $\Delta t$ , it is found that
 
$$\Delta\tau = \Delta t \sqrt{1 - \frac{v^2}{c^2}}$$

## 27 Helliwell Chapter 10 - Momentum

1. Within classical mechanics, momentum is defined as  $\vec{p} = m\vec{v}$ , where if there is no net external forces acting on an object, it is conserved in all inertial frames of reference, found to not be valid under a Lorentz transformation
2. Momentum must be defined as a four-vector, denoted  $(p_\mu) = m(v_\mu)$ , where the four-vector velocity,  $v_\mu = \frac{dx_\mu}{d\tau}$ , where  $\tau$  is the proper time in the particles frame of reference, such that time is a four-scalar rather than a four-vector, and thus the velocity is a four-vector
  - (a) Four-scalars keep a constant value in a Lorentz tranformation, while four-vectors are those which change by the Lorentz formula formula for each component in a frame shift
  - (b) Since the time of a stationary clock,  $\Delta t = \frac{\Delta\tau}{\sqrt{1 - \frac{v^2}{c^2}}}$ , such that  $\lim_{\Delta\tau \rightarrow 0} \frac{\Delta t}{\Delta\tau} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$
3. As a result,  $(p_\mu) = (\frac{mc}{\sqrt{1 - \frac{v^2}{c^2}}}, m\frac{dt}{d\tau}\frac{dx}{dt}, \dots) = (\frac{mc}{\sqrt{1 - \frac{v^2}{c^2}}}, \frac{mv_x}{\sqrt{1 - \frac{v^2}{c^2}}}, \dots)$ , where  $v_x$  is the unprimed frame velocity
  - (a) The three spacial components of the four-vector momentum are denoted as the standard momentum within any frame of reference, such that  $\vec{p} = \frac{m\vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m\vec{v}$ , where  $\gamma$  uses the overall velocity magnitude of the particle in that frame of reference
  - (b) As a result, as the velocity is far less than c, the definition reduces to the standard definition of momentum
4. It is then calculated by the Lorentz transformation that for the four vector momentum,  $(p_\mu)$ , if it is conserved in one frame of reference, it is conserved in all, such that it is a proper four-vector
  - (a) Further, it has been determined experimentally, that the spacial momentum components are conserved as well

## 28 Helliwell Chapter 11 - Energy

1. In 1905, Einstein stated that the inertia of an object depends on its energy content, based on the Einstein box thought experiment, based on a cardboard tube of length L in empty space without gravity at rest in some inertial frame
  - (a) It has a metal cap of mass m at each end, with the cardboard tube as massless, from one of which a light pulse is emitted towards the right, with momentum  $\Delta E = \Delta pc$  by Maxwell's equations

- (b) By conservation of momentum, when the light is emitted, the tube must move towards the left with an equal amount of momentum,  $\Delta p = 2m\Delta v_{tube}$ , such that assuming the time difference is negligible based on the tube movement,  $\Delta x = \Delta v_{tube}t = \frac{\Delta EL}{2mc^2}$ 
  - i. After the light hits the other side, the momentum is restored, although the center of mass has changed, without any outside influence or apparent change in mass distribution
- (c) It is noted that this ignores the lack of rigidity in special relativity, in which case the box would stretch, still shifting the center of mass
- 2. This apparent change in center of mass could be explained by a change in mass caused by the movement of light, such that there is no change in center of mass, such that  $\Delta E = \Delta mc^2$ 
  - (a) It was observed that this would align with the first momentum four-vector term,  $p_0$ , such that  $E = p_0c = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$ , such that at rest, the total energy would be purely mass energy
  - (b) As the velocity changes, the total energy would increase due to kinetic energy by the same formula, such that  $KE = mc^2(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1)$ 
    - i. By the binomial approximation, this reduces to  $KE = \frac{1}{2}mv^2$ , the classical mechanics definition of kinetic energy
  - (c) In addition, since both the spacial and overall momentum four-vector components are conserved in all frames if in one, this component must be conserved in all events as well
  - (d) It is also noted that it is found that  $|(p_\mu)| = -m^2c^2$ , describing why it is constant under a Lorentz transformation
    - i. It follows from the magnitude,  $|(p_\mu)|$ , that  $E^2 = \vec{p}^2c^2 + m^2c^4$ , providing a relationship between the rest mass and momentum to energy
- 3. As a result, for some particle fission of initial mass  $M$  into two  $m$  masses,  $E_f = \frac{2mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$ , such that by conservation of energy,  $M = \frac{2m}{\sqrt{1-\frac{v^2}{c^2}}}$ , such that mass is not conserved
  - (a) Thus, the separate laws of Conservation of Mass and Energy have been modified into the single Conservation of Energy including mass energy law
  - (b) Due to the low energy in relativistic particle collisions, the electron volt ( $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ ) is used, equal to the energy gained by an electron accelerating through 1 V voltage
    - i. Thus, momentum can be measured in  $\frac{eV}{c}$  and mass can be measured in  $\frac{eV}{c^2}$  by the  $E^2 = \vec{p}^2c^2 + m^2c^4$  formula, sometimes leaving off the unit denominator as shorthand
- 4. In 1905, Einstein stated that light acts in some cases as discrete energy quanta, called photons, with energy  $E = hf$ , such that the momentum and energy equations are invalid due to 0 mass and infinite  $\gamma$ 
  - (a) On the other hand, the four-vector magnitude is valid, such that it is found  $E = pc$ , where  $p$  is the magnitude of the momentum

## 29 Helliwell Chapter 12 - Applications

- 1. The ionization energy of an atom is the energy required to separate, also called the binding energy of the electron and the atom, equal for a hydrogen atom to 13.6 eV
  - (a) While there is some kinetic energy of the two particles, it can be considered negligible



- due to being vastly smaller than ionization or particle energy, such that  $E_H + E_{ion} = E_p + E_e$ , such that the mass of the original atom is less than that of the particles
- (b) As a result, larger binding energy of two particles results in a smaller mass of the combination
  2. Conversely, during the joining of two oxygen atoms, such as during combustion, a photon is released, such that some of the mass energy is lost as kinetic energy of the photon
  3. The nuclear strong force is a strong, short-range force which acts on nucleons (neutrons/protons) close together, creating a far greater binding energy than that of ionization
    - (a) As atoms get heavier, the bonding energy increases drastically, peaking around Iron (56 protons), then slowly decreasing, due to the mass of protons repelling each other countering the strong force of all of the nucleons
    - (b) This allows low mass atoms to be fused easily into an atom with higher binding energy, while high mass atoms are fissioned easily into the same, causing a higher binding energy per nucleon and a high release of kinetic energy as a result
  4. Mass-energy/momentum equations combined with conservation of momentum and energy can be used to calculate the characteristics of particle decay purely with decay masses
    - (a) It was noted that the energies of electron beta decay from a Carbon-14 atom varied, such that a third neutral particle must be emitted, called a neutrino, such that the energies and velocities have infinite possibilities
  5. Stable particles are often studied by collisions of stable particles into an unstable particle, such as a photon and proton to produce either a neutral pion and proton or positive pion and neutron
    - (a) The collision into the stationary proton by the photon produces two particles moving in the same direction at different angles from the x-axis,  $\theta \geq 0, \phi \leq 0$ 
      - i. As a result, momentum is conserved by component, such that  $p_\gamma = p_\pi \cos(\theta) + p_p \cos(\phi)$  and  $0 = p_\pi \sin(\theta) - p_p \sin(\phi)$
      - ii. The momentum vector triangle can be drawn, such that by the law of cosines,  $p_p^2 = p_\pi^2 + p_\gamma^2 - 2p_\pi p_\gamma \cos(\theta)$
    - (b) By the energy-momentum equation, energy conservation can be written as  $p_\gamma c + m_p c^2 = \sqrt{p_\pi^2 c^2 + m_\pi^2 c^4} + \sqrt{p_p^2 c^2 + m_p^2 c^4}$ , such that if either  $\cos(\theta), p_\pi$ , or  $p_p$  are measured, the remaining two can be found by these equations
  6. The Compton experiment fired x-rays at graphite and measured the wavelength of the x-rays scattered by electrons as a function of the angle deflected, displaying particle behavior
    - (a) By the law of cosines,  $p_e^2 = (\frac{hf_i}{c})^2 + (\frac{hf_f}{c})^2 - 2\frac{hf_i hf_f \cos(\theta)}{c^2}$ , where  $p_e$  is the momentum of the recoil electron, related by  $p_e c^2 = E_e^2 - m_e^2 c^4$ , such that  $E_e$  is the energy of the electron after recoil
    - (b) This can then be solved, substituting wavelength into frequency, such that the scattered wavelength is longer, such that energy is lost
    - (c) Electrons that were not changed were those that reflected off an atom as a whole, rather than an electron, such that the high mass results in infinitesimal change in wavelength
  7. Conservation of momentum and energy also imply certain forbidden reactions, such as an electron and positron moving with equal but opposite momentum, colliding to produce a photon, which is unable to have zero momentum/energy
    - (a) Similarly, particles are unable to break into multiple particles with a greater total mass than the original particle by energy conservation
  8. Photon rockets are spaceships sending photons as fuel, making it more efficient due to the

high speed

- (a) It is found that  $\gamma = \frac{1 + \frac{M^2}{M_0^2}}{2 \frac{M}{M_0}}$ , such that the portion of the ship taken up by fuel would have to be high to achieve a high gamma factor
- (b) This could be done by lasers, though these would be heavy and inefficient, or by matter-antimatter annihilation, though this is difficult to produce and store

### 30 Relativity Miscellaneous Notes

Lorentz Factor =  $\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$  Blue shift when approaching, red shifted when leaving Binomial approximation -  $(1+x)^a \approx 1+ax$  Is simultaneous clock lag based on difference within moving or stationary frame? Relativistic mass follows from momentum, rest mass times the lorentz factor of the velocity Is there a greater gravitational effect - relativistic mass

$\Delta KE = \int_A^B \vec{F}_{net} \cdot d\vec{r} = \dots = \int_A^B \vec{v} \cdot d\vec{p} = \vec{v} \cdot \vec{p} - \int_A^B \vec{p} \cdot d\vec{v} = mv^2\gamma + \frac{mc^2}{\gamma} |_A^B$  If  $v_A = 0$ ,  $K = (\gamma - 1)mc^2$ , reduces to classical form by binomial expansion, and  $E = \gamma mc^2 = KE + mc^2$

$E^2 = p^2 c^2 + m^2 c^4$  - p is relativistic  $E = hn = pc, p = hn/c = h/\lambda$  for photon