Partial Differential Equations

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Contents

Primary Textbook:
Teacher:

1 Introduction

Needed - Partial Derivatives - Ordinary Differential Equations - Green Theorem, Divergence, Etc - Complex Numbers - $\bar{z+w} = \bar{z} + \bar{w}, \bar{zw} = \bar{z} * \bar{w}$ - z*bar(z) = —z—^{2} $= a^2 + b^2 - z^(-1) = bar(z)/(|z|^2), z! = 0 - C is a field(closure under +, *, assocative for both, distributive, identity for both, inverses excep$ $|zw| = |z||w|, s.t. product of unit vectors is a unit vector - Conservation Laws and Flows, for some body bound by by \partial R,$ flows have a flux - $M_r = \int\int\int_R = \rho(\vec{v})dV, E_R(t) = \int\int\int_R e(\vec{v},t)dV, Q_R(t) = \int\int\int_R Q(\vec{v},t)dV$

- For f(x(t), y(t, s)), $\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t}$ - $\int_a^b \frac{\partial f}{\partial x}dx = f(b,y) - f(a,y) + c(y)$

2 Chapter 1 - Heat Equation

1. The analysis of a physical problem requires three stages, formulation, solution, and interpretation
2. For some one dimensional rod of constant cross-section and length L, the thermal energy density is defined by $e(x,t)$, assumed to be constant across a cross-section, such that for some cross-section, the heat energy $E = e(x,t)A\delta x$
   (a) It is assumed that heat energy change with respect to time $(\frac{\partial}{\partial t}(e(x,t)A\delta x)$ is equal to the energy flowing across boundaries combined with the energy generated inside
   (b) Heat flux is defined as the energy flowing to the right per unit time per unit surface area, $\phi(x,t)$, such that $\phi < 0$ means it is flowing to the left
   (c) Heat energy generated per unit volume per unit time is denoted as $Q(x,t)$, such that the conservation of heat energy can be written as $\frac{\partial e}{\partial t} = -\frac{\partial \phi}{\partial}x + Q$ for some slice
      i. Alternatively, it can be written not approximating for a small slice then taking the limit, such that $\frac{d}{dt}\int_a^b edx = \phi(a,t) - \phi(b,t) + \int_a^b Qdx$
      ii. This is found to also be equal to $\int_a^b \frac{\partial e}{\partial t}dx$ if a, b are constants and e is continuous
      iii. It is also noted that $\phi(a,t) - \phi(b,t) = -\int_a^b \frac{\partial \phi}{\partial x}dx$ if ϕ is continuous differentiable