

Principles of Physics IV: Modern Physics

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Spring 2017

Contents

1	Course Introduction	3
2	Chapter 3 - Quantization of Charge, Light, and Energy	4
3	Chapter 4 - The Nuclear Atom	7
4	Chapter 5 - Wavelike Properties of Particles	10
5	Chapter 6 - The Schrodinger Equation	12
6	Notes	15

Primary Textbook: Modern Physics by Tipler and Llewellyn

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1 Course Introduction

1. Kinematic assumptions originally stated that all items have a well-defined position that can be measured to any precision, and all observers agree on the time and position for a measurement
2. Newtonian mechanics were based on the idea of the deist clockwork universe
 - (a) Conservation of momentum follows from Newton's third law, conservation of energy follows from the work energy theorem
 - (b) Conservative forces have potential energy defined such that $F = -\nabla V$ (or U , depending on notation)
 - (c) Newton's Laws combined with the fact that all known major forces act on the center of an object, implies conservation of angular momentum
3. Relativistic mechanics has $p = \gamma mv$ and $E^2 = c^2 p^2 + m_0^2 c^4$, and defined rigid bodies as impossible, often using the center of mass
4. Waves are travelling disturbances in a medium, averaged over space to consider bulk properties, carrying energy and momentum
 - (a) Wave equations often relate curvature (second derivative w/ respect to x) to acceleration, $f_{xx} = (1/u^2)f_{tt}$ (u can be replaced with v , acting as an alternate velocity notation)
 - (b) Waves are travelling, such that $f(x, t) = g(x - ut)$
 - (c) $y = \exp(x - ct)$ is travelling, while $\sin(x)\cos(ct)$ is a sum of two travelling, but not a travelling on its own, such that any $g(x - ut)$ works
5. Waves but not particles can diffract, refract, and superposition, while both have velocity, generally localized position, momentum, and energy
 - (a) Superposition creates nodes of no displacement, antinodes of max displacement in a standing wave
 - (b) Energy is proportional to the square of the amplitude
6. Maxwell's equations show that since changing magnetic fields produce electric fields and vice versa, waves can be created by oscillating or accelerating charges, travelling forever in vacuum
7. Statistical mechanics are used to solve systems with too many objects to calculate, instead using statistics to predict average properties
 - (a) For some complicated system with total conserved energy and many ways to distribute the energy, the probability of any entity having a particular energy is independent of others, such that it depends only on others, but limits the possibilities of remaining energies
 - i. Thus, $P(E_1)P(E_2) = f(E_1 + E_2)$, since the probability of the joint energy depends only on the sum, found to be true if $P(E) = Ab^{\pm cE} = Ae^{-\beta E}$, where $\beta = \frac{1}{k_B T}$, $k_b = 1.38 * 10^{-23} J/K$
 - ii. $n(E)$ is defined as the number of distinct ways an object could have energy E , such that the sum of the energies must be 1, either a sum or an integral if discrete or continuous, such that $P(E) = \frac{e^{-\beta E}}{\sum_j n(E_j)e^{-\beta E_j}}$
 - (b) Thus, the average value is the sum of the value multiplied by the probability for all possible energy values, $g(\bar{E}) = \frac{\sum_j n(E_j)g(E_j)e^{-\beta E_j}}{\sum_j n(E_j)e^{-\beta E_j}}$
 - (c) The Boltzmann Distribution assumes $n(E) = 1$ for all E , and the energy distribution is continuous, such that $P(E) = \frac{e^{-\beta E}}{k_B T}$, $\bar{E} = k_B T$
8. Single photon cannot pair produce an electron and a positron, due to the center of mass frame proving lack of conservation of momentum, though a single photon can collide into a mass

and pair produce

9. Existence of atoms based on the macroscopic shape of crystals, the inability to convert elements to another type, stoichiometry chemistry rules, and periodic table able to predict properties of atoms
10. de Broglie later stated that massive particles could have wavelength by the same formula, $p\lambda = \lambda\sqrt{2mK} = h$, where K is kinetic energy
11. For two spheres on a collision course, they will collide if the center-center distance is at most $r_1 + r_2$, such that effective area/cross-section, $\sigma = \pi(r_1 + r_2)^2$
 - (a) Thus, the probability of an electron passing through a box with sides of area A/L^2 is $P = \frac{M\sigma}{A} = nL\sigma$, where M is the mass of the box and n is the density of the box
 - (b) For some surface, the number colliding per area, $\Delta N = -PN$ where N is the number incident on the surface per second, and the number through the box of thickness x is thus, $\frac{\Delta N}{\Delta x} = -nN\sigma$
 - (c) Thus, $N(x) = N(0)e^{-n\sigma x} = N(0)e^{-x/l}$ where l is the mean scattering length

2 Chapter 3 - Quantization of Charge, Light, and Energy

1. In the 1800s, Faraday proved that a specific quantity of electricity could decompose one gram-ionic weight of monovalent ions, equal to a Faraday, or a mole of electrons, such that $Q = N_A e$, called Faraday's Law of Electrolysis, displaying discrete electric charges
 - (a) Zeeman later discovered that discrete spectral lines emitted by an atom in a magnetic field separate into three spaced lines of different frequencies, caused by the slightly different charge to mass ratios
 - i. This proved that the particles producing the light were negative, and found the charge to mass ratio of the electrons
 - (b) Thomson's cathode ray experiment later measured the same ratio as Zeeman, proving the existence of the electron, with the same ratio, as being the atomic negative component
 - i. This combined with Faraday's charge allowed the mass of the electron to be determined
 - ii. He also used a uniform magnetic field creating a circular path to measure the same ratio, found to be the same for all materials, showing it was universal for atoms
 - (c) Millikan attempted to use a cloud of water droplets with a charge, such that $Q = Ne$, using the mass of the cloud and the radius of the drop to find e, found to be difficult because of the evaporation
 - i. On the other hand, he found a single drop could be balanced in the air by an electric field, eventually picking up an ion causing a movement in some direction
 - ii. This resulted in the oil drop experiment, giving each charge and preserving it in midair, measuring the force on it, to confirm the electron charge
2. Absorbed radiation increases the kinetic energy of oscillating atoms, increasing the temperature, but resulting in increased radiation emission by electrons, reducing kinetic energy, called thermal radiation
 - (a) At thermal equilibrium, the rate of absorption and emission are equal, such that higher frequencies are present at higher temperatures, due to higher energy
 - (b) Ideal blackbodies emit and absorb all incident radiation by $R = P/A = \omega T^4$, where Stefan's constant (ω) is $5.67 * 10^8 W/m^2 K^4$

- i. Non-blackbodies emit multiplied by some emissivity constant, based on factors such as color, temperature, and composition
- (c) Spectral distribution of the radiation of a blackbody also only depends on temperature, where the maximum emitted wavelength, $\lambda_{max}T = 2.898 * 10^{-3}m * K$, called Wien's displacement law
- (d) Blackbodies are approximated by a cavity with a small hole to let radiation in, found that the power radiated out, $R = \frac{1}{4}cu$, where U is the total radiation energy density in the cavity
 - i. As a result, both are proportional to the wavelength, such that the energy density distribution can be found by the number of modes of oscillation
 - ii. It is found that the number of modes of standing wave oscillation per unit volume, $n = 8\pi\lambda^{-4}$, and the Rayleigh-Jeans equation states that $u = kTn$, such that R can be calculated
 - iii. As a result, while at higher wavelengths, it fits experimentally, at low wavelengths, it appears $R \rightarrow \infty$, called the ultraviolet catastrophe, such that total energy density over the spectrum from 0 to ∞ would be infinite as well
- (e) Planck's Law corrects for this, stating that since as λ approaches 0, n approaches infinity by classical formulas, u must be a function of wavelength, such that it approaches 0
 - i. Classically, electrons oscillating produce waves with equal frequency, where the average energy is found by the energy distribution function, $n(E) = Ae^{-E/kT}$ based on the Maxwell-Boltzmann distribution function, such that average energy, $\bar{E} = \int_0^\infty E f(E) dE = kT$
 - A. In this equation, n is the fraction of oscillators with energy E
 - B. This is based on the fact that for some standing wave, in space a, $\lambda = \frac{2a}{n}$ must be true for soem integer n, such that the number of modes, $\Delta n = \int_{f_1}^{f_2} \frac{4a}{c} df$, the additional multiple of 2 to account for two polarizations
 - C. In higher dimensions, $f = \frac{c}{2a} \sqrt{n_x^2 + n_y^2 + n_z^2}$, such that $N(f)df = 2(\frac{2a}{c})^3 (\frac{1}{8} 4\pi f^2 df)$, when the space is modeled as a sphere of radius a, only in a single octant
 - D. The higher dimensional mode equation combined with Boltzmann distribution gives the Rayleigh-Jeans formula
 - ii. Planck found it agreed with expermental data if oscillator energy is a multiple of a discrete value, such that $E = nhf$, where h is Planck's constant
 - A. The sum can be taken similarly such that $\bar{E} = \frac{hf}{e^{hf/kT} - 1}$
 - B. It follows that $u = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1}$, called Planck's Law, found to be a generalization of all known laws
- (f) Blackbody radiation has been used as proof of the Big Bang Theory, due to the universe predicted to act as a perfect black body in terms of energy distribution
- 3. In Hertz's spark gap experiment to generate EM waves and detect them, proving Maxwell's Theories, finding that light hitting a surface produced an electron current
 - (a) Lenard later proved that it was electrons, and observed the current was proportional to the intensity ($P = IA$), but found that there was no minimum intensity needed as would be classically expected, due to requiring enough energy to
 - i. Flux of photons are the photons per second per unit area, related to intensity
 - ii. Since the kinetic energy had to be great enough to avoid being pulled back to the metal surface cathode if there was a negative voltage, it required a voltage produced

- greater than $-V_0$, called the stopping potential
- iii. Thus, it was found that $KE_{max} = eV_0$, such that KE_{max} was independent of the light intensity as well, rather than increasing the electron kinetic energy
- (b) Einstein postulated as a result that Planck's quantization was universal, such that $E = hf$ for all light quanta, such that $eV_0 = hf - \phi$, where ϕ is the work function, characteristic of the metal, to remove an electron
- i. This is equal to the maximum kinetic energy by energy conservation, though some electrons lose energy when leaving the metal further
 - ii. As a result, the threshold wavelength is equal to the work function divided by h
 - iii. This also explains the lack of a time lag for the production of photoelectrons, instead of the calculated time for enough energy if it is spread evenly over the surface, as assumed classically
4. Xrays originally were discovered by Roentgen with a cathode ray tube, noticing rays from the collision of electrons and the glass tube could activate flurescent photographic film and pass through opaque materials
- (a) He later observed no material was opaque, though less rays could pass through with higher densities
 - (b) He stated that their apparent lack of magnetic deflection, refraction, or interference, was due to a very short wavelength, finding they were defracted by a crystal lattice, also proving a regular crystal array
 - (c) He found they were produced by eletrons when deflected then stopped by the atoms of a target
 - (d) He then thought to use Bragg places, or face-centered cubic molecular structures of NaCl crystals, analyzing the scattered waves from each atom to view xray diffraction
 - i. The waves are in phase regardless of wavelength if the scattering angle is equal to the incident angle for two waves hitting atoms in a plane
 - ii. This condition is called the Bragg condition, true if $2d\sin(\theta) = m\lambda$, where d is the distance between atoms and θ is the angle from the surface, for constructive interference, found in soap bubbles
 - iii. The amount of ionization from xrays can then be measured to get the intensity of each wavelength, after it has been corrected by the Bragg condition to get the correct angle
 - (e) This produces a spectra for the xrays produced, based on the anode of the material, able to be used to determine the atomic spacing
 - i. This produces both the characteristic spectrum of sharp lines and the continuous spectrum, the former which is specific to the material
 - A. Maxwell had previously predicted the continuous spectrum was due to the electron bombardment/deacceleration in the strong electric field (called bremsstrahlung), though the cause of the characteristic was unknown
 - ii. There is also a cutoff wavelength, independent of the material, based on the energy of the bombarding electrons, by $\lambda = \frac{1.24 \cdot 10^3 nm}{V}$, called the Duane-Hunt Rule
 - A. This is explained as the opposite of the photoelectric effect, such that all the kinetic energy is converted ($\lambda = \frac{hc}{eV}$)
5. Compton later measured the scattering of xrays by free electrons, proving further both the photon and special relativity
- (a) Classical EM would have predicted a dipole oscillation due to the light field of the

particle, with the maximum at the original wavelength, based on $1 + \cos^2(\theta)$ for the angle of oscillation, rather than a shifted maximum

- (b) He observed that the scattered xrays were more easily absorbed, considering that the collision allowed an electron to absorb some of the photon energy, such that the wavelength became longer
- (c) As a result, he derived the Compton equation mechanically through relativity and quantization stating $\Delta\lambda = \frac{h}{mc}(1 - \cos(\theta))$, where $\frac{h}{mc}$ is called the Compton wavelength of the electron
 - i. This was observed for xrays due to the percent change in wavelength only being noticable for very short original wavelengths
 - ii. The unshifted, but scattered, portion is due to electrons tightly bound to the atom, such that the entire atom recoils

3 Chapter 4 - The Nuclear Atom

1. Newton's dispersion of white light was the first spectroscopy, later shown to have hundreds of numbers of dark lines inside, and bright light spectra formed by flames, forming the study of spectroscopy
 - (a) Continuous spectra are emitted by incandescent solids, showing no specific lines in any spectroscope
 - (b) Band spectra are closely packed groups of lines apparently continuous in low resolution produced by fire, while line spectra are those produced by unbound chemical elements, both characteristic of material
2. In 1885, Balmer found that visible and near-UV spectrum of H could be represented by $\lambda_n = 364.6 \frac{n^2}{n^2-4} nm$, where $n = 3, 4, \dots$, called the Balmer series
 - (a) The general form for other elements was later found as the Rydberg-Ritz formula, such that $\frac{1}{\lambda_{mn}} = R(\frac{1}{m^2} - \frac{1}{n^2}) \forall n > m$
 - i. R is the rydberg constant, varying slightly for elements, but negligably, such that $R_H = 1.096776 * 10^7 m^{-1}$ and $R_\infty = 1.097373 * 10^7 m^{-1}$
 - ii. Thus, for the Balmer series, $m = 2$, such that the maximum wavelength is as n approaches infinity
 - (b) The other series from higher energy to lower are Lyman, then Balmer, Paschen, Brackett, Pfund, and then Humphreys
3. Thomson's plum pudding model was the most popular atomic model after Balmer and Rydberg-Ritz formulas, trying to find stable configurations with normal modes of vibration of spectral lines
 - (a) On the other hand, the lack of continual emission due to electrostatic radiation and inability to make a model hindered this theory
4. Rutherford, while studying radioactivity, finding that alpha particles were doubly ionized helium, checking the spectral lines to prove it, used the particles to study atomic interiors
 - (a) He sent the radiation into gold foil, measuring the scattering angles, mainly undeflected or barely detected, though a few with right angles or more, showing the positively charged sphere could not have created such high angle scattering
 - i. Thomson's model did not have enough force at any point for a large enough deflection
 - (b) Rather there must be a dense center, such that Rutherford calculated angular distribu-

- tion and dependence on nuclear charge, angle, and kinetic energy, validated experimentally
- (c) This can be calculated mechanically, where b is the impact parameter, or the distance from the line through the nucleus, such that $b = \frac{kq_\alpha Q}{m_\alpha v^2} \cot(\frac{\theta}{2})$
 - i. Thus, it is found that for intensity I_0 (in particles per second), the number scattered by one nucleus through angles greater than θ is the number with impact parameters less than $b(\theta)$, equal to $\pi b^2 I_0$
 - ii. πb^2 is called the cross section ω for scattering angles greater than θ , such that it is multiplied by intensity and number of nuclei for the number of scatterings above that value
 - A. The number of nuclei is calculated such that $n = \frac{\rho N_A}{M}$, where M is the molar mass
 - B. As a result, the fraction of scatters above some angle θ is $\pi b^2 n t$, where t is the thickness of the surface
 - (d) Rutherford later derived an equation for the number of particles scattered at some angle, $\Delta N = (\frac{I_0 A_{sc} n t k Z e^2}{2 r^2 E_k})^2 \frac{1}{\sin^4(\frac{\theta}{2})}$, where A_{sc} is the detector area and r is the distance from the foil to the detector
 - i. In this case, Z is the atomic number and t is the thickness
 - ii. This was later verified experimentally, showing the legitimacy of atomic theory
 - (e) Rutherford's model does not assume a point charge, but rather just a ball, and assumes the alpha particle does not penetrate it, such that for each angle, the closest distance from the nucleus can be calculated
 - i. Thus, for the largest angle (180°), the collision is almost head on, providing an upper bound for the atomic radius
 - ii. For some nucleus, by conservation of energy, $r = \frac{kq_\alpha Q}{\frac{1}{2} m_\alpha v^2}$, using the point when the number of collisions at some angle changes as the radius based on the initial kinetic energy
5. Bohr later specified the charge and mass of electrons, stating they orbited by Coulomb force, but was unstable due to accelerating toward the center, with EM predicting that light of $f = \frac{v}{2\pi r}$, proportional to $\frac{1}{r^2}$
- (a) For the electrons, $E = \frac{1}{2} m v^2 - \frac{k Z e^2}{r} = -\frac{k Z e^2}{2 r_n}$, proportional to $-\frac{1}{r}$, such that it will be losing energy, with decreasing radius
 - i. As the energy is lost due to the emission of radiation, emitting a continuous spectrum as the radius changes, the atom will decay, rendering it invalid
 - (b) Bohr solved this by stating electrons has certain stationary states they could orbit without emission, and that the radiation emitted was due to the change in state, $h f = E_i - E_f$, called the Bohr frequency condition
 - i. He also made the correspondence principle, stating that the limit of large orbits and energies (or as quantum numbers reach higher, rather than \hbar reaches 0, as was thought during the ultraviolet catastrophe), all quantum must align with classical
 - (c) This allowed him to determine that angular momentum is quantied, assuming values of $\frac{n\hbar}{2\pi}$, determining the spectra for any atom with -e charge
 - i. By the centripetal Coulomb force, $v = (\frac{k Z e^2}{m r})^{1/2}$, with $L = m v r = n \frac{\hbar}{2\pi} = n\hbar$, where n is a quantum number, such that $r_n = \frac{n^2 a_0}{Z}$, where a_0 is the Bohr electron radius
 - ii. Thus, $E_n = -E_0 \frac{Z^2}{n^2}$, where $E_0 = \frac{m k^2 e^4}{2 \hbar^2} = 13.6 eV$, such that the energy is quantized,

with $n = 1$ as the ground state for the single electron

- A. The energy of the ground state is the ionization/binding energy to fully remove the electron from the atom, with the differences between states able to be drawn by an energy level diagram
 - iii. This is able to be converted to the Rydberg-Ritz equation, finding the Rydberg constant (R) as $\frac{E_0}{hc}$
 - (d) The Rydberg constant assumes a stationary nucleus due to infinite mass, such that if it has mass M , the nucleus and the electron have total kinetic energy of $E = \frac{p^2}{2\mu}$ where $\mu = \frac{m}{1+\frac{m}{M}}$
 - i. As a result, μ is the reduced electron mass, substituted for the electron mass to account for regular nuclei, such that $R = R_\infty\mu$
 - ii. μ is taken from classical physics, assuming central forces, acting similarly on electrons as on astronomical bodies
 - (e) The correspondence principle as a result implies that as energy levels are closely enough spaced, quantization should not matter, such that it is true as $n \rightarrow \infty$
 - i. As a result, the frequency of an single electron jump emission is equal to the frequency of the revolution of the electron
 - ii. While it appears that multiple jumps would be greater, this is remedied by allowing elliptical orbits, such that the energy depends on the major axis, rather than the eccentricity
 - (f) Relativistic corrections implied that a highly eccentric orbit would need a larger correction as the electron moves closer, approximated for $n = 1$ in Hydrogen to $\frac{v}{c} = \frac{ke^2}{hc} = \alpha$, called the fine-structure constant
 - i. As a result, there are many ellipses for a single energy level, and since the velocity depends on the orbit, the energy radiated is slightly different, approximately ranging α^2
 - ii. α is also able to be used to simplify equations acting as a universal constant
 - (g) Large atoms are called Rydberg atoms, as the electron energy approaches the ionization energy, the atomic radius is able to increase, used to study the correspondence principle and electromagnetic fields
6. X-ray spectra were found by Moseley to change from element to element regularly, due to being caused by inner electron transitions, depending only on the less complex lower orbits and not affected by interatomic binding forces
- (a) The Mosley plot is based on the fact that energy levels are proportional to Z^2 , such that characteristic X-ray lines are found by $f^{1/2} = A_n(Z - b)$, where A_n, b are constants for the line
 - i. The K series has $b = 1$ while the L series has $b = 7.4$
 - ii. These occur when the bombarding electron cause the ionization of an inner electron, resulting in another electron falling down into the $n = 1/K$ shell, and so forth
 - (b) Bohr's equation can provide a similar relation, using $n_f = 1$ with $Z = Z - 1$ to find the frequency, finding $f = cR_\infty(Z - b)^2(\frac{1}{n_f^2} - \frac{1}{n^2})$
 - i. Thus, for the K-series, $A_n^2 = cR_\infty(1 - \frac{1}{n^2})$
 - ii. The $(Z - b)$ factor is due to the shielding of nuclear charge by other electrons in the lower shells, allowing corrections to the periodic table from weight to Z , solving property discrepancies

- (c) The Auger effect is an alternative to x-ray emission, ejecting an extra electron from an outer shell, such that each element has a characteristic Auger electron kinetic energy spectrum
- 7. The Franck-Hertz experiment used a cathode ray tube to send electrons to atoms of some material by voltage, after which there is a small voltage in the opposite direction, measuring the current that reaches it
 - (a) It was noted that the current increased as the initial voltage did, until it suddenly dropped, repeating as a pattern
 - (b) This was due to a forced elastic collision with the atom until the energy was high enough to excite an electron, which was found to correspond to light emitted on the spectra
 - (c) This formed the basis of electron energy loss spectroscopy, measuring the locations of inelastic collisions to gain the energy structure of materials

4 Chapter 5 - Wavelike Properties of Particles

- 1. The De Broglie Hypothesis stated that the cause of the quantization of electrons was an effect of the standing wave condition for the wavelength of the electron, such that $E = \frac{hc}{\lambda} = pc$, such that nonrelativistically for electrically accelerated particles, $\lambda = \frac{hc}{(2mc^2eV_0)^2}$
 - (a) This had not been previously noticed due to the extremely small wavelength of larger mass objects
 - (b) It was determined that low energy electrons would have a low enough wavelength to detect diffraction in a Bragg plane, noticing the maxima and minima in the scattering by the Bragg condition
 - i. As the energy is varied, the diffraction maximum changes, such that ****ANGLE VOLTAGE ADD****
 - (c) The wave properties of neutral atoms and molecules were found using natural thermal energy as motion, only using the top plane of the Bragg's planes due to the lack of energy, measuring the scattering
- 2. The De Broglie equation for relativistic particles is found by $E^2 + (pc)^2 + E_0^2$, where $E = K + E_0$, such that $\frac{\lambda}{\lambda_c} = \frac{1}{(2(\frac{K}{E_0}) + (\frac{K}{E_0})^2)^{1/2}}$, where the Compton wavelength, $\lambda_c = \frac{h}{mc}$
- 3. Particle waves do not have a material or ether, but rather the wave is the probability of finding the particle, represented by superposition of a group of harmonic waves to form a time-space localized wave packet
 - (a) Wave packets have a group velocity ($v_g = \frac{d\omega}{dk}$), and as more waves with infinitesimally similar values of k are added, it becomes more localized
 - i. Phase velocities, $v_p = f\lambda = \frac{\omega}{k}$, are the velocities of the individual harmonic waves, such that $v_g = v_p + k \frac{dv_p}{dk}$
 - ii. Mediums in which the phase velocity is the same for all frequencies is nondispersive, such that it is the same as the group velocity, in which the wave shape is constant
 - (b) It is found classically that for wave packets, $\Delta k \Delta x \approx 1$ where Δk is the range of k values of the harmonic waves, Δx is the approximate length of the wave packet
 - i. It follows that $\Delta \omega \Delta t \approx 1$ as well, called the response time-bandwidth relation
 - ii. Both are approximate due to depending on how the range is defined as well as the shape of the packets, serving as a magnitude and defining the approximate wave characteristics

- iii. The derivative of the equation relating k and λ gives $\Delta k = \frac{2\pi\Delta\lambda}{\lambda^2}$, such that $\Delta x \Delta \lambda \approx \frac{\lambda^2}{2\pi}$
- iv. This represents the minimum uncertainty/error of the measurement, though human error in the measurement would increase the actual error
- (c) The particle wavefunction is denoted $\psi(x, t)$, such that for each harmonic wave, $v_p = \frac{E}{p} = \frac{v}{2}$, where v is the velocity of the particle itself
 - i. The group velocity, on the other hand, is equal to the velocity of the particle itself, acting as a nondispersive wave packet, found nonrelativistically by $v_g = \frac{d\omega}{dk} = \frac{dE}{dp} = v$
- (d) Wave behavior of photon particles are seen in low intensity diffraction, where it is the probability of hitting some location, where $E = hf$ per photon and E_{tot} is proportional to \vec{E}^2
 - i. The electric field, \vec{E} , is the wavefunction for light, coresponding to Schrodinger's equation for electrons
 - ii. For matter waves, $P(x)dx = |\psi|^2 dx$, where P is the probability at that location, acting as complex wavefunctions
- 4. Since the wavefunction is nonzero for multiple values, there is uncertainty to the exact position of the electron, such that by the classical uncertainty relations, $\Delta x \Delta p \approx \Delta E \Delta t \approx \hbar$, called the Uncertainty Principle
 - (a) For Gaussian distribution functions (total probability of 1), $\sigma_x \sigma_k = \frac{1}{2}$, where σ is the standard deviation, such that uncertainty is defined as the standard deviation, such that $\Delta x \Delta p \geq \frac{\hbar}{2}$
 - (b) As a result, particles cannot have 0 average kinetic energy, since for some box of length L , $\Delta x \leq L$, such that $\Delta p \geq \frac{\hbar}{L}$, ignoring the Gaussian $\frac{1}{2}$ term due to generally not being Gaussian and not affecting the order of magnitude
 - i. As a result, $(\Delta p)^2 = (p - \bar{p})_{av}^2 = \bar{p}^2 \geq (\frac{\hbar}{L})^2$, since \bar{p} is equal to 0 for a symmetric box
 - ii. Thus, $\bar{E} \geq \frac{\hbar^2}{2mL^2}$, called the zero point energy of the box, since the average energy must be ≥ 0
 - (c) For an electron with momentum p and distance r from the proton, $E = \frac{p^2}{2m} - \frac{ke^2}{r}$, $\Delta x = r$ such that $(\Delta p)^2 = \bar{p}^2 \geq \frac{\hbar^2}{r^2}$, providing a radius for the minimum energy and a minimum electron energy at that radius
 - (d) Since a precise measurement of the energy of a system requires infinite time, such that the mean decay time, or the lifetime (τ), used as the time of measurement
 - i. As a result, the measurement of the wavelength of spectral lines has a degree of uncertainty due to the uncertain in measurement, with the natural line width being $\Delta E = \frac{\hbar}{\tau}$, denoted Γ_0
 - ii. By the formula that $E - E_0 = \frac{hc}{\lambda}$, the derivative is taken such that $dE = -hc \frac{d\lambda}{\lambda^2}$, the range of λ can be found for ΔE
- 5. It is seen that wave-particle duality is seen in matter and light, such that emission and absorption are particle-based, propagation is wave-based
 - (a) This is also able to be thought of as interactions and observations of matter and light are particle-based, while predictions of observations are described by waves
 - i. Interactions cause the changing of the wavefunction, changing the propagation after
 - (b) For wavelengths smaller than any objects, particle theory can describe propagation as well as wave theory, giving the same results, due to wavelike behavior being too small to be observed, while for time averages of energy/momentum exchange, wave theory works

as well

5 Chapter 6 - The Schrodinger Equation

1. The Schrodinger Equation is a fundamental law, unable to be derived, and is non-relativistic, replaced by Dirac's Relativistic Wave Equation later
 - (a) This wave equation is similar to that of light, $\frac{\partial^2 \vec{E}}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}$, with solution $\vec{E} = \vec{E}_0 \cos(kx - \omega t)$ to give the equation, $\omega = kc$, which is equivalent to $E = pc$
 - i. Similarly for electrons, since $E = \frac{p^2}{2m} + V$ is the energy of the electron where V is the potential energy of the electron, $\hbar\omega = \frac{\hbar^2 k^2}{2m} + V$, such that k and ω are not linearly related
 - ii. This implies that for a harmonic electron wavefunction, the first time derivative is related to the second spacial derivative, and will involve potential energy
 - iii. The wave equation must be consistent with the De Broglie relations, classical conservation of energy, squared as the probability of finding the particle at that location, and if potential is constant, energy and momentum is constant
 - A. As a result, an ideal form for $V = 0$ would be able to be reduced to $E = \frac{p^2}{2m}$, such that cos/sin fail, but a complex exponential works
 - iv. The wave equation must also be linear with respect to the wavefunction, due to allowing constructive and destructive interference, such that each term is linear with respect to ϕ or some derivative of ϕ
 - (b) The equation in 1D states that $\frac{-\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2} + V(x,t)\psi(x,t) = i\hbar \frac{\partial \psi(x,t)}{\partial t}$
 - i. For a free particle, such that $V(x,t) = V_0$ is constant, it is found that individual harmonics are not solutions, but a complex exponential is, such that $\psi = Ae^{i(kx - \omega t)}$ provides the previous equation
 - (c) By the probabilistic interpretation of the wavefunction, $P(x,t)dx = |\psi(x,t)|^2 dx = \psi^*(x,t)\psi(x,t)dx$, where $\psi^*(x,t)$ is the conjugate, such that P(x, t) is called the probability density, and $\psi(x,t)$ is called the probability density amplitude/probability amplitude
 - i. Thus, the normalization condition states that $\int_{-\infty}^{\infty} \psi^* \psi dx = 1$
 - (d) Schrodinger then showed that energy quantization can be explained in terms of standing waves, called the stationary states, or eigenstates, of the particles, in which the potential energy is independent of time
 - i. For these situations, the solution is separable, such that $\psi(x,t) = \gamma(x)\phi(t)$, producing ordinary derivatives instead of partial, with each side as a function of a single variable
 - ii. Since they are single variable, they both must be equal to some separation constant, such that $\phi(t) = e^{\frac{-iEt}{\hbar}} = \cos(2\pi \frac{Et}{h}) - i\sin(2\pi \frac{Et}{h})$, such that $f = \frac{E}{h}$, or $E = hf$
 - A. With E in the spacial portion of the equation, it gives the time-independent Schrodinger equation, with the time dependent side replaced by $E\psi(x)$ whose probability density is found to be equal for this situation to the multivariable density
 - (e) Wavefunctions must fit with the type of potential energy function, which is allowed to be discontinuous, solved separately in each region, but require smooth joining at the point of discontinuity
 - i. The wavefunction itself must be continuous as well as the first derivative, such that

- the function is smooth, except possibly at the boundary (since if there is infinite potential energy, the wavefunction must be 0)
- ii. Both also must be finite and single-valued to have measurable quantities, and must follow the normalization condition
2. The infinite square well is a frictionless wire with increasing potential from points near the end, until the maximum at the border of the box, able to be made arbitrarily steep and large potentials, such that $V(x) = 0$ if $0 < x < L$, $V(x) = \infty$ otherwise
 - (a) This problem is related to the classical vibrating string problem and is a good approximation to the motion of free electrons in a metal
 - (b) As a result, the boundary conditions state that it must be 0 at both ends of the box, allowing quantization similar to the standing wave condition
 - i. For a standing wavelength of the particle, $E = n^2 \frac{\pi^2 \hbar^2}{2mL^2}$, able to be derived from the time-independent Schrodinger equation by providing $\psi(L) = A \sin(kx)$, such that $k_n = \frac{n\pi}{L}$, providing quantized energy values/energy eigenvalues
 - A. n is the quantum number of the system, allowing the specific mode of the system to be determined
 - ii. The normalization condition can then be used to get that $A_n = (\frac{2}{L})^{1/2}$, such that the eigenfunctions are $\psi_n(x) = \sqrt{\frac{2}{L}} \sin(\frac{n\pi x}{L})$
 - iii. This provides the probability of finding the particle at each location, depending on the total energy of the particle system
 - (c) Classically, since there is no force except an infinitely large force at the edges, any speed and energy is possible, but by the uncertainty principle the velocity and position can't be found simultaneously, the minimum energy is below the energy uncertainty
 - i. In addition, as quantum number increases, the peaks approach infinite, similar to classical prediction
 - (d) The complete wavefunction is found as $\psi_n(x, t) = \psi_n(x)e^{-i\omega t} = \psi_n(x)e^{-i(E_n/\hbar)t}$, such that by the identity, $\sin(k_n x) = \frac{(e^{ik_n x} - e^{-ik_n x})}{2i}$, multiplied by the time portion to show that the standing wave is equal to two equal and opposite travelling waves
 3. For a finite square well, such that the bounds of the region have potential V_0 , where the total energy is assumed to be less than V_0 , such that within the well, the equation is the same as the infinite well, $\psi''(x) = -k^2\psi(x)$, $k^2 = \frac{2mE}{\hbar^2}$, such that it is a sine/cosine within the well
 - (a) Outside the well, it requires the complete time-independent equation, but is not required to be 0, rather only needing to follow the normalization condition
 - (b) Since the second derivative/curvature and the derivative have the same sign, the function always curves away from the x-axis, rather than towards like a sine/cosine
 - i. This will appear in the form, $\psi(x) = ce^{-\alpha x} = ce^{-\frac{2m}{\hbar^2}(V_0-E)x}$
 - ii. As a result, for most energy values of the wavefunction, it goes towards $\pm\infty$ as x approaches $\pm\infty$, such that it is invalid, such that it must move towards 0 as $x \rightarrow \pm\infty$, restricting energy states
 - iii. It is found that there is a finite number of possible energies, decreasing in quantity as the voltage decreases, to 1 possible state at very low V_0
 - (c) Since it is possible for the particle to be outside the well, kinetic energy appears to be negative, but if the probability is assumed to be negligible at $\Delta x = \alpha^{-1}$, the momentum uncertainty gives a minimum energy equal to the negative kinetic energy to cancel it
 - (d) This applies to all functions with $E > V$ in the box, even if V varies such that it is not

- simple sinusoidal, with $E < V$ outside the box, restricting energy states
4. The expectation value of $f(x)$, $\langle f(x) \rangle = \int_{-\infty}^{\infty} \psi^* f(x) \psi dx$, is the average value of $f(x)$ expected to be obtained by the measurement of particles with the wavefunction (either the time-independent or time-dependent)
 - (a) Operators are the set of functions replacing $f(x)$, acting on the wavefunction to provide the weighted average, not always functions of x such as when the uncertainty principle prevents it, like momentum
 - i. The momentum x-component operator is $\frac{\hbar}{i} \frac{\partial}{\partial x}$, squared for the p^2 operator
 - ii. The Hamiltonian/total energy operator in classical mechanics is $H = \frac{p^2}{2m} + V$, such that the Hamiltonian operator is found by replacing p by the p operator
 - A. As a result, the time independent Schrodinger equation can be replaced by $H_{op}\psi = E\psi$
 - iii. The time dependent Hamiltonian operator is $i\hbar \frac{\partial}{\partial t}$, kinetic energy is $-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2}$, and the z component of angular momentum is $-i\hbar \frac{\partial}{\partial \phi}$
 5. The simple harmonic oscillator, such that $V(x) = \frac{1}{2}m\omega^2 x^2$, acting as the finite well, has a classical probability of $P(x)dx = c \frac{dx}{v}$, with any energy value possible
 - (a) Due to the symmetric potential function, the probability must be symmetric, such that the wave function must be either symmetric or anti-symmetric
 - i. Parity operations are those flipping the x-axis, such that $x \rightarrow -x$, true for the Hamiltonian operator such that $H_{op}\psi(-x) = E\psi(-x)$
 - ii. If there are multiple separate solutions for one energy state, it is called degenerate, doubly degenerate for two, though since it is found for the Schrodinger equation to be at most by a factor of -1 for odd parity ($\psi(-x) = -\psi(x)$)
 - (b) It is found that $E_n = (n + \frac{1}{2})\hbar\omega$, such that the minimum energy is $E_0 = \frac{1}{2}\hbar\omega$, due to the minimum uncertainty principle energy
 - i. Thus, it is found that $\psi_n(x) = C_n e^{-m\omega x^2/2\hbar} H_n(x)$, where $H_n(x)$ is the Hermite polynomial of order n , symmetric for even n , antisymmetric for odd n
 - (c) It is also found that $\int_{-\infty}^{\infty} \psi_n^* x \psi_m dx = 0$ unless $n = m \pm 1$, called the selection rule, such that radiation can only change the energy of the oscillator by one at a time
 6. For an unbound state problem, such that E is greater than $V(x)$ as $x \rightarrow \pm\infty$, the second derivative and wavefunction must have opposite signs to avoid moving towards infinity, curving to the x axis, such that any value of E is possible
 - (a) While it is not normalizable over the entire domain, it is bounded within a specific range, such that $\int_a^b |\psi(x)|^2 dx = \int_a^b \rho dx = \int_a^b dN = N$, where N is the number of particles in the interval
 - (b) For some step potential, such that for $x < 0, V(x) = 0$ and for $x > 0, V(x) = V_0$, such that by the Schrodinger equation, for $x < 0, \frac{d^2\psi(x)}{dx^2} = -\frac{\sqrt{2mE}}{\hbar}$, and $x > 0, \frac{d^2\psi(x)}{dx^2} = -\frac{\sqrt{2m(E-V_0)}}{\hbar}$
 - i. For solutions of beams of particles moving to the right multiplied by the time portion, each regions solution is the sum of the travelling waves in each direction
 - ii. There is assumed to be no leftward moving beam from the $x > 0$ side, such that the coefficient of that term is 0, with the functions for each side required to be continuous at $x = 0$
 - iii. The coefficients of reflection R and transmission T are the relative rates by which particles are reflected and transmitted, with $R = (\frac{k_1 - k_2}{k_1 + k_2})^2$, $T = \frac{4k_1 k_2}{(k_1 + k_2)^2}$, such that

$$T + R = 1$$

- (c) As a result of Schrodinger's wave nature, unlike the classical idea that none would be reflected due to the change in potential, a portion of the particles are, depending on the change in wavenumbers, but not in the sign of the change

6 Notes

1. Absorption is a subset of the emission spectra, due to multiple possible paths of emission for higher levels of hydrogen atoms
2. $\frac{1}{\lambda} = \frac{m_e \alpha^2 Z^2}{2} \left(\frac{1}{n^2} - \frac{1}{m^2} \right)$, $\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} \approx \frac{1}{137}$
3. Fractional change of n vs m = $\frac{n-m}{n}$
4. Hydrogen atom is $f = cZ^2 R \left(\frac{1}{n^2} - \frac{1}{m^2} \right)$, such that Moseley's Law applies this to xrays for analyzing wavelength
5. Regular as n goes to infinity, L'Hopital used
6. The area of a sphere is $4\pi r^2$
7. $\lambda = \frac{h}{\sqrt{2mK}}$
8. $L = mvr = I\omega = mr^2\omega$ (for single particle)
9. ****Pay attention to units****
10. Things maybe to know: Light frequencies of different types, eV vs J conversion, angular momentum mechanics
11. Powder scattering is the scattering of light by a collection of crystallites with random orientations, forming circles of peaks in a target formation
12. Electrons observed appear to behave with probabilities due to the light preventing the interference pattern, only going through one slit, changing the outcome, or low intensity makes not enough scatter and low momentum/high wavelength don't measure properly
 - (a) Complementary principle, only one type of property, wave or particle, can be measured, but not both can be
 - i. Since momentum is proportional inversely to wavelength, it is a wave property, such that it or position can be measured, but not both, leading to Heisenberg
13. Copenhagen interpretation states that initial conditions can only determine probability, not definite future conditions
14. Oscillating functions are often represented in complex exponential notation, keeping only the real portion for the actual equation
15. Useful integrals include $\int_{-\infty}^{\infty} e^{-(x/a)^2} dx = a\sqrt{\pi}$, $\int_0^{\infty} x e^{-(x/a)^2} dx = \frac{a^2}{2}$, $\int_{-\infty}^{\infty} x^2 e^{-(x/a)^2} dx = \frac{a^3\sqrt{\pi}}{2}$, $\int_{-\infty}^{\infty} e^{ikx-(x/a)^2} dx = a\sqrt{\pi} e^{-(a^2 k^2)/4}$
16. $R_{\infty} = 1.0973 * 10^7 m^{-1}$ (Rydberg Constant)