12.1 IB Math - Unit 5: Integration

Bronx Early College Academy

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BECA / Dr. Huson / 12.1 IB Math - Unit 5 Integration

5.1 Drui: Antiderivatives. Friday 4 January

5.2 Drui: Indefinite Integral. Monday 7 January

5.3 Drui: Deltamath Integration Tuesday 8 January

5.4 Drui: Boundary conditions. Wednesday 9 January

5.5 Drui: Deltamath Integration Thursday 10 January

5.6 Drui: Compositions of linear functions. Friday 11 January

5.7 Drui: Compositions of linear functions. Monday 14 January

5.8 Drui: Deltamath Integration Tuesday 15 January

5.9 Drui: Integration - position problems. Wednesday 16 January

5.10 Drui: Definite integrals and area. Thursday 17 January

5.11 Definite integrals and area. Friday 18 January

5.12 Deltamath: definite integrals and area. Tuesday 29 January

5.13 Integration by substitution. Wednesday 30 January

Cold-day 6-1 P1 Tangents A spiral review. Thursday 31 January

5.14 Deltamath: Area between two curves. Friday 1 February

5.15 Area between two curves. Monday 4 February

5.16 Test review. Wednesday 6 February

5.17 Unit test: Integration. Thursday 7 February

5.18 Calculus overview review. Friday 8 February

GQ: How do we find the antiderivative of a function?

CCSS: F.IF.B.6 Calculate & interpret rate of change 5.1 Friday 4 January

Do Now. Find $\frac{dy}{dx}$

1. Given $y = x^3 + x^2 + 17$.
2. Given $y = \frac{1}{4}x^4 + \frac{1}{2}x^2 + 9 - \frac{1}{x}$.
3. If $\frac{dy}{dx} = 3x^2 + x$, find y.
4. Skills check #1 p. 290

Problem sets from January 2,3; Sigma notation, p 290

Lesson: Antiderivatives pp. 291-2

Exam review

Homework: Exercises 9A p. 293; test corrections due Monday

GQ: How do we find the indefinite integral of a function?

CCSS: F.IF.B.6 Calculate & interpret rate of change 5.2 Monday 7 January

Do Now. Find the antiderivative, $F(x)$, of each function, $f(x)$, such that $F'(x) = f(x)$

1. $f(x) = 4x^3 + 3x^2 + 1$.
2. $f(x) = x^4 + x^2 + 5$.
3. $f(x) = \sqrt{x}$

Test corrections due, review. (take home test tomorrow)

Lesson: Indefinite integral pp. 293-4

Homework: Exercises 9B p. 294

GQ: How do we integrate functions?

CCSS: F.IF.B.6 Calculate & interpret rate of change 5.3 Tuesday 8 January

Do Now. Handout-test review

Deltamath Integration practice (taking antiderivatives)

Homework: Deltamath project through Thursday

GQ: How do we apply boundary conditions to an integral?

CCSS: F.IF.B.6 Calculate & interpret rate of change 5.4 Wednesday 9 January

Do Now. Find each indefinite integral.

1. $\int (3x^2 + 2x + 1)\,\mathrm{d}x.$
2. $\int (x^4 + 6x^2 + 1)\,\mathrm{d}x.$
3. $\int \sqrt{x}\,\mathrm{d}x.$
4. $\int \frac{\pi}{4}\sqrt[3]{x^2}\,\mathrm{d}x$
5. $\int x^{-1}\,\mathrm{d}x$

Lesson: Finding the constant C given boundary conditions. pp. 295-6

Homework: Exercises 9C p. 296-7

GQ: How do we integrate functions?

CCSS: F.IF.B.6 Calculate & interpret rate of change 5.5 Thursday 10 January

Do Now. Handout-test review

Deltamath Integration practice (taking antiderivatives)

Homework: Complete Deltamath calculus project

GQ: How do we integrate compositions of linear functions?

CCSS: F.IF.B.6 Calculate & interpret rate of change 5.6 Friday 11 January

Do Now. Derivatives and antiderivatives. (Use the chain rule)

1. $f(x) = e^{3x}$. Find $f'(x)$.
2. $f(x) = \ln(5x + 3)$. Find $f'(x)$.
3. $f(x) = (2x - 5)^3$. Find $f'(x)$.
4. Find $\int 4(x^2 + x + 1)\,\mathrm{d}x$.
5. $y' = 2x^3 - 1$ and $y = 3$ when $x = 1$. Find y in terms of x.
6. Given $f'(x) = \sqrt[3]{x}$ and $f(0) = 1$, find $f(x)$.

Lesson: Antiderivatives of form $\int f(ax + b)\,\mathrm{d}x$. pp. 297-9

Homework: Exercises 9D, 9E (odds) p. 298, 300

GQ: How do we integrate compositions of linear functions?

CCSS: F.IF.B.6 Calculate & interpret rate of change 5.7 Monday 14 January

Do Now. Derivatives and antiderivatives. (Use the chain rule)

1. $f(x) = e^{2x-3}$. Find $f'(x)$.
2. $f(x) = \ln(4x)$. Find $f'(x)$.
3. $f(x) = (3x+2)^4$. Find $f'(x)$.
4. Find $\int (5x^3 + x^2 + 1)\,\mathrm{d}x$.
5. $y' = 2x^2 - 1$ and $y = 3$ when $x = 3$. Find y in terms of x.
6. Given $f'(x) = \sqrt[3]{x^2}$ and $f(1) = 1$, find $f(x)$.

Lesson: Antiderivatives of form $\int f(ax+b)\,\mathrm{d}x$. pp. 297-9

Homework: Exercises 9D, 9E (evens) p. 298, 300

GQ: How do we integrate functions?

CCSS: F.IF.B.6 Calculate & interpret rate of change 5.8 Tuesday 15 January

Do Now. Straight to DeltaMath

Deltamath Integration practice (taking antiderivatives)

Homework: Complete Deltamath

GQ: How do we use integration to solve for position?

CCSS: F.IF.B.6 Calculate & interpret rate of change 5.9 Wednesday 16 January

Do Now. Derivatives and antiderivatives.

1. $f(x) = \sin x$. Find $f'(x)$.
2. Find $\int \sin x \, \mathrm{d}x$.
3. $s(t) = 4.9t^2$. Find $v(t) = s'(t)$ and $a(t) = s''(t)$.
   What might these functions represent?
4. Given $v(t) = \cos t$, find $s(t) = \int v(t) \, \mathrm{d}t$. Assume $s(0) = 0$.

Lesson: Position problems (handout)

Homework: Calculus problem set

GQ: How do we calculate area with integration?

CCSS: F.IF.B.6 Calculate & interpret rate of change 5.10 Thursday 17 January

Do Now

1. Find $\int (4x^3 - 3x + 1)\mathrm{d}x$.
2. Find $\int e^{5x}\mathrm{d}x$.
3. Find $\displaystyle\int \frac{1}{3x+1}\mathrm{d}x$.

Homework review #1, 5, 6 p. 302

Lesson: Reimann sums and the definite integral

Task: Example 8, page 304

Assessment: Calculator integration

Homework: Exercises 9H evens p. 308

GQ: How do we calculate area with definite integrals?

CCSS: F.IF.B.6 Calculate & interpret rate of change 5.11 Friday 18 January

Do Now

1. Use a calculator to find $\int_0^{\frac{\pi}{2}} \cos x \, \mathrm{d}x$
2. Differentiate $y = \sqrt{3x^3 - x}$
3. Find $\int (6x^2 - 2x - 5)\mathrm{d}x$.
4. Differentiate $y = (3x^2 - 5x)^5$
5. Find $\int \frac{3}{x} \, \mathrm{d}x$.

Lesson: Properties of definite integrals p. 307
The fundamental theorem of calculus p. 309

Homework: Have a nice break!

GQ: How do we calculate area with definite integrals?

CCSS: F.IF.B.6 Calculate & interpret rate of change 5.12 Tuesday 29 January

Do Now: 6-1 Calculus Tangent lines spiral review handout

1. Select and solve one of the problems: mild, medium, or spicy. (early finishers, do another)
2. As a class, check work
3. Record result in personal tracker

Lesson: Definite integrals
Task: Deltamath practice
Homework: Exercises 9I p. 310-1

GQ: How do we anti-differentiate using the chain rule?

CCSS: F.IF.B.6 Calculate & interpret rate of change 5.13 Wednesday 30 January

Do Now

1. Differentiate $y = \sqrt[3]{5x^2 - 2x}$
2. Find $\int (6x - 2)^4(6)\ \mathrm{d}x$.
3. Differentiate $y = (3x^2 - 5x)(\ln x)$
4. Use a calculator to find $\displaystyle\int_{-1}^{1} \frac{1}{x+2}\ \mathrm{d}x$
5. Find $\displaystyle\int_{1}^{2} \frac{3}{x^2}\ \mathrm{d}x$. (check your result with a calculator)

Lesson: The substitution method of integration p. 300
Task: Practice Examples 7 p. 300
Homework: Exercises 9J p. 312-13

GQ: How do we find the tangent of a curve?

CCSS: F.IF.B.6 Calculate & interpret rate of change Cold-day, Thursday 31 January

Do Now

1. 6-1 P1 Tangents A spiral review

Homework: 6-1 P1 Tangents B spiral review

GQ: How do we calculate the area between two curves?

CCSS: F.IF.B.6 Calculate & interpret rate of change 5.14 Friday 1 February

Do Now: 6-1 Calculus Tangents spiral review handout
Early finishers - Take the derivative of each function

1. $f(x) = \sin x^3$
2. $g(x) = \sqrt{x^4 + 2}$
3. $h(x) = \ln(x^2 + 1)$

Lesson: The area between two functions p. 313
Assessment: Example #13 p. 314
Homework: Exercises 9K p. 316

GQ: How do we calculate the area between two curves?

CCSS: F.IF.B.6 Calculate & interpret rate of change 5.15 Monday 4 February

Do Now: Integration practice

1. $\int (x^2 + 2x + 7)\,\mathrm{d}x.$
2. $\int (\sin x + \frac{1}{x} - \sqrt{x})\,\mathrm{d}x.$
3. $\int e^{3x-4}\,\mathrm{d}x.$
4. $\displaystyle\int \frac{3x^2 + 5}{(x^3 + 5x)^3}\,\mathrm{d}x$
5. $\displaystyle\int_0^2 2\sqrt{2x - x^2}\,\mathrm{d}x$ (use a calculator)

Homework review #9K p. 316
Lesson: Determining definite integrals' boundaries p. 317
Assessment: Example #14 p. 317 (test Thursday)
Homework: Pre-test; 6-2 P1 Differentiation spiral review

GQ: How do we calculate the area between two curves?

CCSS: F.IF.B.6 Calculate & interpret the rate of change 5.16 Wednesday 6 February

Do Now: Integration practice

1. $\int_0^{\frac{\pi}{2}} \sin x \, \mathrm{d}x$ (Do this without a calculator. Sketch the unit circle.)

6-2 P1-A Differentiation spiral review
Lesson: Pretest review (test tomorrow)
Homework: Study for exam.
6-2 P1-B Differentiation spiral review (Friday)

GQ: How do we calculate the area between two curves?

CCSS: F.IF.B.6 Calculate & interpret rate of change 5.17 Thursday 7 February

Unit test: Antidifferentiation, definite integrals, finding areas

Homework: Exercises #9L 317

GQ: How do we calculate the area between two curves?

CCSS: F.IF.B.6 Calculate & interpret rate of change 5.18 Friday 8 February

Do Now: Integration practice

1. $\int_0^{\frac{\pi}{2}} \sin x \, \mathrm{d}x$

Homework review #9L p. 317
6-2 P1-B Differentiation spiral review
Lesson: Differentiation review, concavity
Homework: Integration practice; 6-2 P1-C Differentiation spiral review

GQ: How do we calculate a volume of rotation?

CCSS: F.IF.B.6 Calculate & interpret rate of change 5.15 Monday 11 February

Do Now: Sketch the functions $f(x) = 10x + x^2 - 3x^3$ and $g(x) = x^2 - 2x$

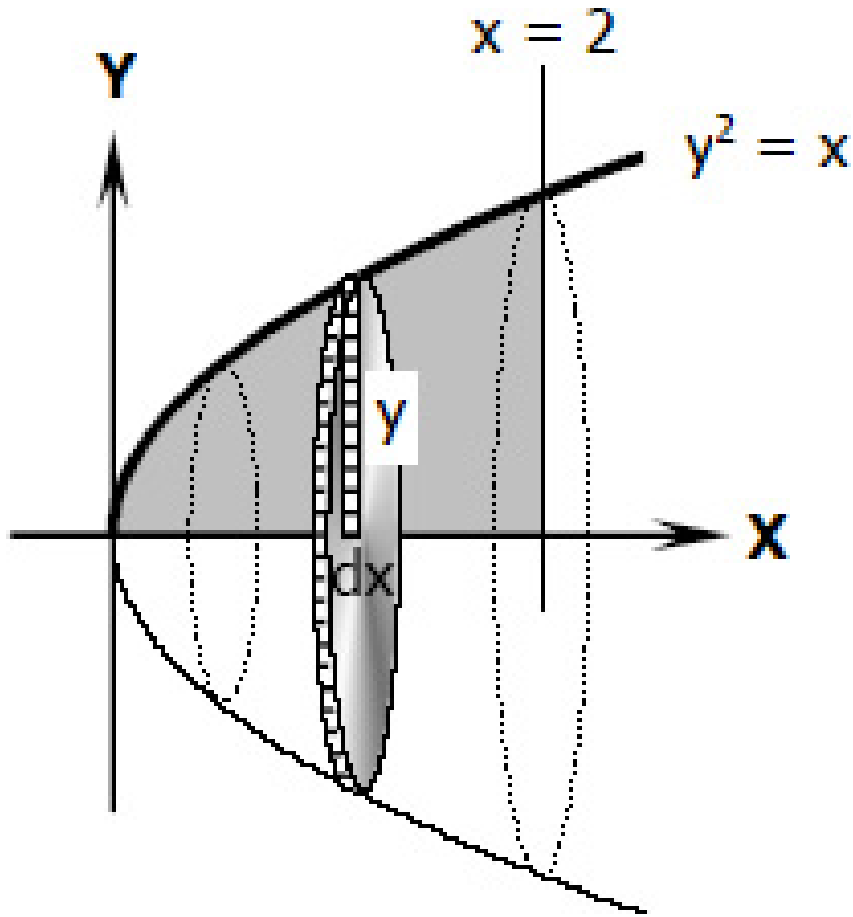
1. What are their intersections? (i.e. $f(x) = g(x)$)
2. What is the definite integral representing the area between the curves?
3. Using a calculator, what is the size of the area? (this may not be a trivial question)

Lesson: Integrating circle areas, modeling a solid p. 318
Assessment: Example #15 p. 319
Homework: Exercises 9L & 9M p. 317, 319

The volume of a function rotated around the x-axis

Differentiate over x, but use the area of a disk defined by $A = \pi r^2$

video



Credit: MATHalino.com - Pinoy Math Community Romel Verterra

GQ: How do we calculate a volume of rotation?

CCSS: F.IF.B.6 Calculate & interpret rate of change 5.21 Wednesday 13 February

Do Now: Volumes of rotation

1. Medium: $f(x) = -x^2 + 4$.
   - 1.1 What is the range of f? The roots?
   - 1.2 Find $f'(x)$.
   - 1.3 Find the volume generated when the curve of f for $-2 \leq x \leq 2$ is revolved through 2π about the x-axis.
2. Spicy: $g(x) = \sin^3 x$.
   - 2.1 What is the range of g?
   - 2.2 Find $g'(x)$.
   - 2.3 Find the volume generated when the curve of g for $0 \leq x \leq \frac{\pi}{2}$ is revolved through 2π about the x-axis.

Lesson: Kinematics p. 321

Homework: Textbook exercises (9N p. 320) 9O p. 324-5 test corrections due Friday (before field trip)

GQ: How do we calculate a volume of rotation?

CCSS: F.IF.B.6 Calculate & interpret rate of change 5.22 Thursday 14 February

Do Now: Volumes review

1. Volumes review

Lesson: Integrating circle areas, modeling a solid p. 318

Homework: Problem set test corrections due tomorrow

GQ: How do we calculate displacement from velocity?

CCSS: F.IF.B.6 Calculate & interpret the rate of change of a function 12.1

Do Now: Do the calculations below and read the handout

1. Lance Armstrong's average speed in his six Tour de France victories from 1999-2004 was about 24 miles per hour. Assuming that he pedals at his average speed and takes no breaks, how long would it take him to ride 38 miles to the top of a 10,000 ft. volcano?
2. People who are not Lance Armstrong can travel at about 12 miles per hour on a bike. At that speed, how long would it take to reach the top of the volcano?

Lesson: Integrating velocity over time, displacement p. 321
Assessment: Example #18 p. 323
Homework: Exercises 9N & 9O p. 320, 324

GQ: How do we calculate displacement from velocity?

CCSS: F.IF.B.6 Calculate & interpret the rate of change of a function 12.1

Do Now: continued, 38 mile ride in 10 hours

1. Using the velocity vs time graph from yesterday, integrate to show that the areas representing the distance covered by the three riders are equal ($v \times t = d$).
2. Show that a rider accelerating according to $v(t) = \frac{76}{100}t$ also arrives at $(10, 38)$.

Lesson: Integrating velocity over time, displacement p. 321
Task: Review 9F, 9M, probability
Assessment: Example #18 p. 323 (take home test Thursday)
Homework: Exercises 9P p. 326, any remaining problem sets