

18 December 2017

**Homework: Binomial distribution**

1a. A discrete random variable  $X$  has the following probability distribution.

$x$	0	1	2	3
$P(X=x)$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{2}{10}$	$p$

Find  $p$ .

[3 marks]

1b. Find  $E(X)$ .

[3 marks]

2a. The following table shows the probability distribution of a discrete random variable  $X$ .

$x$	0	2	5	9
$P(X=x)$	0.3	$k$	$2k$	0.1

Find the value of  $k$ .

[3 marks]

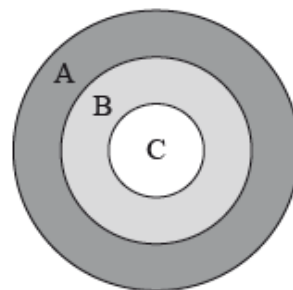
2b. Find  $E(X)$ .

[3 marks]

3a. The following diagram shows a board which is divided into three regions  $A$ ,  $B$  and  $C$ .

A game consists of a contestant throwing one dart at the board. The probability of hitting each region is given in the following table.

Region	A	B	C
Probability	$\frac{5}{20}$	$\frac{4}{20}$	$\frac{1}{20}$

Find the probability that the dart does **not** hit the board.

[3 marks]

3b. The contestant scores points as shown in the following table.

Region	A	B	C	Does not hit the board
Points	0	$q$	10	-3

Given that the game is fair, find the value of  $q$ .

[4 marks]

**4a.** In a large university, the probability that a student is left handed is 0.08. A sample of 150 students is randomly selected from the university. Let  $k$  be the expected number of left-handed students in this sample.

Find  $k$ . [2 marks]

**4b.** Hence, find the probability that exactly  $k$  students are left handed; [2 marks]

**4c.** Hence, find the probability that fewer than  $k$  students are left handed. [2 marks]

**5a.** A box holds 240 eggs. The probability that an egg is brown is 0.05.

Find the expected number of brown eggs in the box. [2 marks]

**5b.** Find the probability that there are 15 brown eggs in the box. [2 marks]

**5c.** Find the probability that there are at least 10 brown eggs in the box. [3 marks]

**6a.** The probability of obtaining “tails” when a biased coin is tossed is 0.57. The coin is tossed ten times. Find the probability of obtaining **at least** four tails. [4 marks]

**6b.** The probability of obtaining “tails” when a biased coin is tossed is 0.57. The coin is tossed ten times. Find the probability of obtaining the fourth tail on the tenth toss. [3 marks]

**7a.** A factory makes lamps. The probability that a lamp is defective is 0.05. A random sample of 30 lamps is tested.

Find the probability that there is at least one defective lamp in the sample. [4 marks]

**7b.** A factory makes lamps. The probability that a lamp is defective is 0.05. A random sample of 30 lamps is tested.

Given that there is at least one defective lamp in the sample, find the probability that there are at most two defective lamps. [4 marks]

**8.** Two lines with equations  $\mathbf{r}_1 = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + s \begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix}$  and  $\mathbf{r}_2 = \begin{pmatrix} 9 \\ 2 \\ 2 \end{pmatrix} + t \begin{pmatrix} -3 \\ 5 \\ -1 \end{pmatrix}$  intersect at the point P. Find the coordinates of P. [6 marks]

**9a.** Consider the points A (1, 5, 4), B (3, 1, 2) and D (3,  $k$ , 2), with (AD) perpendicular to (AB).

Find

(i)  $\overrightarrow{AB}$ ;

(ii)  $\overrightarrow{AD}$  giving your answer in terms of  $k$ .

[3 marks]

**9b.** Show that  $k = 7$ .

[3 marks]

**9c.** The point O has coordinates (0, 0, 0), point A has coordinates (1, -2, 3) and point B has coordinates (-3, 4, 2).

The point C is such that  $\overrightarrow{BC} = \frac{1}{2}\overrightarrow{AD}$ .

Find the position vector of C.

[4 marks]

**9d.** Find  $\cos \widehat{ABC}$ .

[3 marks]

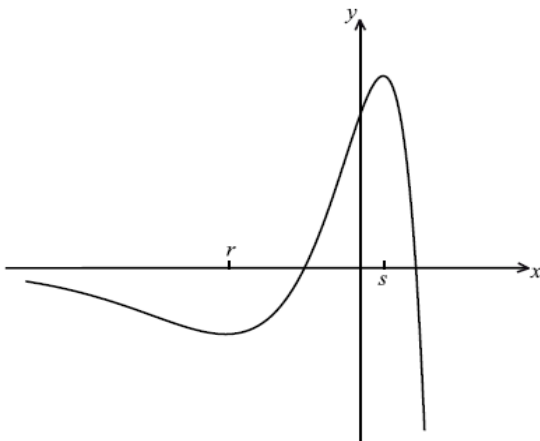
**10a.** Let  $f(x) = e^x(1 - x^2)$ .

Show that  $f'(x) = e^x(1 - 2x - x^2)$ .

[3 marks]

**10b.** Part of the graph of  $y = f(x)$ , for  $-6 \leq x \leq 2$ , is shown below. The  $x$ -coordinates of the local minimum and maximum points are  $r$  and  $s$  respectively.

[1 mark]



Write down the **equation** of the horizontal asymptote.

**10c.** Write down the value of  $r$  and of  $s$ .

[4 marks]

**10d.** Let  $L$  be the normal to the curve of  $f$  at  $P(0, 1)$ . Show that  $L$  has equation  $x + y = 1$ .

[4 marks]