

26 March 2018

**Homework: Pre-Exam Sequences and series**

**1a.** In an arithmetic sequence, the first term is 3 and the second term is 7.

Find the common difference.

[2 marks]

**1b.** Find the tenth term.

[2 marks]

**1c.** Find the sum of the first ten terms of the sequence.

[2 marks]

**2a.** The first three terms of an arithmetic sequence are  $u_1 = 0.3$ ,  $u_2 = 1.5$ ,  $u_3 = 2.7$ .

Find the common difference.

[2 marks]

**2b.** Find the 30th term of the sequence.

[2 marks]

**2c.** Find the sum of the first 30 terms.

[2 marks]

**3a.** The first three terms of a geometric sequence are  $u_1 = 0.64$ ,  $u_2 = 1.6$ , and  $u_3 = 4$ .

Find the value of  $r$ .

[2 marks]

**3b.** Find the value of  $S_6$ .

[2 marks]

**3c.** Find the least value of  $n$  such that  $S_n > 75\,000$ .

[3 marks]

**4a.** The first three terms of a geometric sequence are  $\ln x^{16}$ ,  $\ln x^8$ ,  $\ln x^4$ , for  $x > 0$ .

Find the common ratio.

[3 marks]

**4b.** Solve  $\sum_{k=1}^{\infty} 2^{5-k} \ln x = 64$ .

[5 marks]

**5.** Consider a geometric sequence where the first term is 768 and the second term is 576.

Find the least value of  $n$  such that the  $n$ th term of the sequence is less than 7.

[6 marks]

26 March 2018

6a. Consider the following sequence of figures.

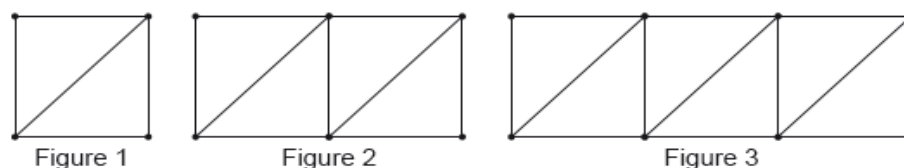


Figure 1 contains 5 line segments.

Given that Figure  $n$  contains 801 line segments, show that  $n = 200$ .

[3 marks]

6b. Find the total number of line segments in the first 200 figures.

[3 marks]

7. An arithmetic sequence has the first term  $\ln a$  and a common difference  $\ln 3$ .The 13th term in the sequence is  $8 \ln 9$ . Find the value of  $a$ .

[6 marks]

8a. The first two terms of an infinite geometric sequence, in order, are

$$2\log_2 x, \log_2 x, \text{ where } x > 0.$$

Find  $r$ .

[2 marks]

8b. Show that the sum of the infinite sequence is  $4\log_2 x$ .

[2 marks]

8c. The first three terms of an arithmetic sequence, in order, are

$$\log_2 x, \log_2 \left(\frac{x}{2}\right), \log_2 \left(\frac{x}{4}\right), \text{ where } x > 0.$$

Find  $d$ , giving your answer as an integer.

[4 marks]

8d. Let  $S_{12}$  be the sum of the first 12 terms of the arithmetic sequence.

$$\text{Show that } S_{12} = 12\log_2 x - 66.$$

[2 marks]

8e. Given that  $S_{12}$  is equal to half the sum of the infinite geometric sequence, find  $x$ , giving your answer in the form  $2^p$ , where  $p \in \mathbb{Q}$ .

[3 marks]