

3 January 2018

Classwork & homework review. Due Monday.**1a. [2 marks]**

Let $f(x) = 3x - 2$ and $g(x) = \frac{5}{3x}$, for $x \neq 0$.

Find $f^{-1}(x)$.

1b. [2 marks]

Show that $(g \circ f^{-1})(x) = \frac{5}{x+2}$.

1c. [2 marks]

Let $h(x) = \frac{5}{x+2}$, for $x \geq 0$. The graph of h has a horizontal asymptote at $y = 0$.

Find the y -intercept of the graph of h .

1d. [3 marks]

Hence, sketch the graph of h .

1e. [1 mark]

For the graph of h^{-1} , write down the x -intercept;

1f. [1 mark]

For the graph of h^{-1} , write down the equation of the vertical asymptote.

1g. [3 marks]

Given that $h^{-1}(a) = 3$, find the value of a .

2a. [3 marks]

Let $f(x) = \sqrt{x-5}$, for $x \geq 5$.

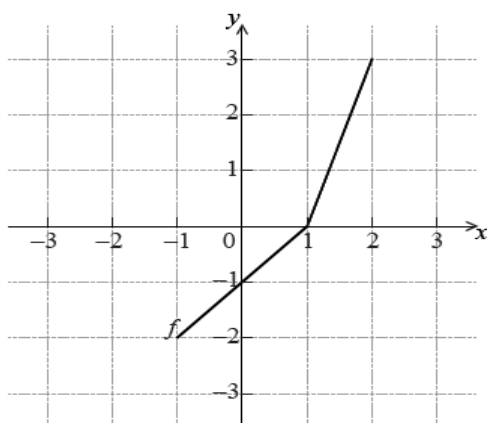
Find $f^{-1}(2)$.

2b. [3 marks]

Let g be a function such that g^{-1} exists for all real numbers. Given that $g(30) = 3$, find $(f \circ g^{-1})(3)$.

3a. [3 marks]Let $f(x) = 4x - 2$ and $g(x) = -2x^2 + 8$.Find $f^{-1}(x)$.**3b.** [3 marks]Find $(f \circ g)(1)$.**4a.** The diagram below shows the graph of a function f , for $-1 \leq x \leq 2$.

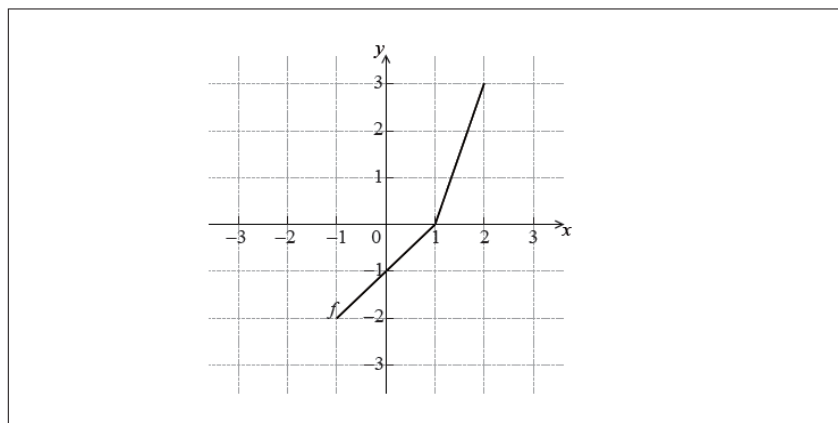
[1 mark]

Write down the value of $f(2)$.**4b.** Write down the value of $f^{-1}(-1)$.

[2 marks]

4c. Sketch the graph of f^{-1} on the grid below.

[3 marks]



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5a. [6 marks]

Let f and g be functions such that $g(x) = 2f(x + 1) + 5$.

(a) The graph of f is mapped to the graph of g under the following transformations:

vertical stretch by a factor of k , followed by a translation $\begin{pmatrix} p \\ q \end{pmatrix}$.

Write down the value of

(i) k ;

(ii) p ;

(iii) q .

(b) Let $h(x) = -g(3x)$. The point A(6, 5) on the graph of g is mapped to the point A' on the graph of h . Find A' .

5b. [3 marks]

The graph of f is mapped to the graph of g under the following transformations:

vertical stretch by a factor of k , followed by a translation $\begin{pmatrix} p \\ q \end{pmatrix}$.

Write down the value of

(i) k ;

(ii) p ;

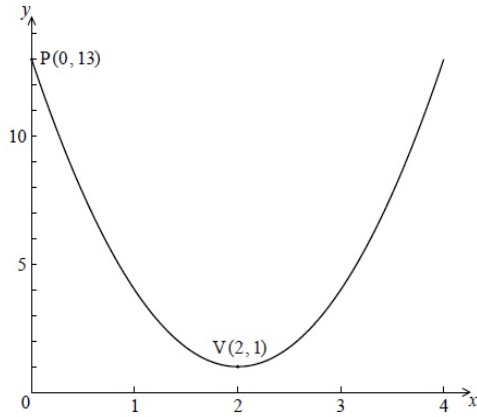
(iii) q .

5c. Let $h(x) = -g(3x)$. The point A(6, 5) on the graph of g is mapped to the point A' on the graph of h . Find A' . [3 marks]

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6a. [4 marks]

The following diagram shows the graph of a quadratic function f , for $0 \leq x \leq 4$.



The graph passes through the point $P(0, 13)$, and its vertex is the point $V(2, 1)$.

The function can be written in the form $f(x) = a(x - h)^2 + k$.

(i) Write down the value of h and of k .

(ii) Show that $a = 3$.

6b. [3 marks]

Find $f(x)$, giving your answer in the form $Ax^2 + Bx + C$.

6c. [8 marks]

Calculate the area enclosed by the graph of f , the x -axis, and the lines $x = 2$ and $x = 4$.

7a. [5 marks]

Consider $f(x) = 2kx^2 - 4kx + 1$, for $k \neq 0$. The equation $f(x) = 0$ has two equal roots.

Find the value of k .

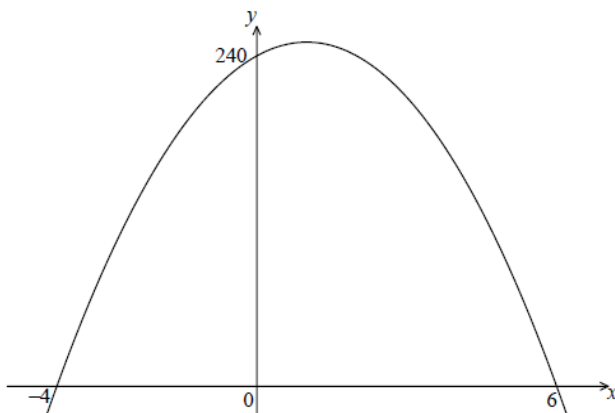
7b. [2 marks]

The line $y = p$ intersects the graph of f . Find all possible values of p .

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8a. [2 marks]

The following diagram shows part of the graph of a quadratic function f .



The x -intercepts are at $(-4, 0)$ and $(6, 0)$, and the y -intercept is at $(0, 240)$.

Write down $f(x)$ in the form $f(x) = -10(x - p)(x - q)$.

8b. [4 marks]

Find another expression for $f(x)$ in the form $f(x) = -10(x - h)^2 + k$.

8c. [2 marks]

Show that $f(x)$ can also be written in the form $f(x) = 240 + 20x - 10x^2$.

8d. [7 marks]

A particle moves along a straight line so that its velocity, $v \text{ ms}^{-1}$, at time t seconds is given by

$$v = 240 + 20t - 10t^2, \text{ for } 0 \leq t \leq 6.$$

(i) Find the value of t when the speed of the particle is greatest.

(ii) Find the acceleration of the particle when its speed is zero.

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9a. Let $f(x) = \log_3 \frac{x}{2} + \log_3 16 - \log_3 4$, for $x > 0$.

Show that $f(x) = \log_3 2x$.

[2 marks]

9b. Find the value of $f(0.5)$ and of $f(4.5)$.

[3 marks]

9c. [6 marks]

The function f can also be written in the form $f(x) = \frac{\ln ax}{\ln b}$.

(i) Write down the value of a and of b .

(ii) Hence on graph paper, **sketch** the graph of f , for $-5 \leq x \leq 5$, $-5 \leq y \leq 5$, using a scale of 1 cm to 1 unit on each axis.

(iii) Write down the equation of the asymptote.

9d. [1 mark]

Write down the value of $f^{-1}(0)$.

9e. [4 marks]

The point A lies on the graph of f . At A, $x = 4.5$.

On your diagram, sketch the graph of f^{-1} , noting clearly the image of point A.

10a. [2 marks]

Let $f(x) = 3x^2$. The graph of f is translated 1 unit to the right and 2 units down. The graph of g is the image of the graph of f after this translation.

Write down the coordinates of the vertex of the graph of g .

10b. Express g in the form $g(x) = 3(x - p)^2 + q$.

[2 marks]

10c. The graph of h is the reflection of the graph of g in the x -axis.

Write down the coordinates of the vertex of the graph of h .

[2 marks]

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11a. [2 marks]

Let $f(x) = \log_3 \sqrt{x}$, for $x > 0$.

Show that $f^{-1}(x) = 3^{2x}$.

11b. [1 mark]

Write down the range of f^{-1} .

11c. [4 marks]

Let $g(x) = \log_3 x$, for $x > 0$.

Find the value of $(f^{-1} \circ g)(2)$, giving your answer as an integer.

12. [7 marks]

Solve $\log_2 x + \log_2(x - 2) = 3$, for $x > 2$.

13a. [3 marks]

Let $f(x) = k \log_2 x$.

Given that $f^{-1}(1) = 8$, find the value of k .

13b. [4 marks]

Find $f^{-1}\left(\frac{2}{3}\right)$.

14a. [3 marks]

Let $f(x) = e^{x+3}$.

(i) Show that $f^{-1}(x) = \ln x - 3$.

(ii) Write down the domain of f^{-1} .

14b. [4 marks]

Solve the equation $f^{-1}(x) = \ln \frac{1}{x}$.

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15a. [1 mark]Find $\log_2 32$.**15b.** [4 marks]

Given that $\log_2 \left(\frac{32^x}{8^y} \right)$ can be written as $px + qy$, find the value of p and of q .

16a. Expand $(x - 2)^4$ and simplify your result. [3 marks]

16b. Find the term in x^3 in $(3x + 4)(x - 2)^4$. [3 marks]

17a. Let $f(x) = 3(x + 1)^2 - 12$.

Show that $f(x) = 3x^2 + 6x - 9$. [2 marks]

17b. [8 marks]For the graph of f

(i) write down the coordinates of the vertex;

(ii) write down the **equation** of the axis of symmetry;(iii) write down the y -intercept;(iv) find both x -intercepts.

17c. Hence sketch the graph of f . [2 marks]

17d. [3 marks]

Let $g(x) = x^2$. The graph of f may be obtained from the graph of g by the two transformations:

a stretch of scale factor t in the y -direction

followed by a translation of $\begin{pmatrix} p \\ q \end{pmatrix}$.

Find $\begin{pmatrix} p \\ q \end{pmatrix}$ and the value of t .