Name: ____________________________________ Date: __________________

INTEGER EXPONENTS
COMMON CORE ALGEBRA II

We've finished our review of linear functions. Linear functions grow by equal differences for equal domain intervals (changes in *x*). In this unit we will concentrate on exponential functions which grow by equal **factors** for equal domain intervals. To understand exponential functions, we first need to understand exponents.

***Exercise* #1:** The following sequence shows powers of 3 by repeatedly multiplying by 3. Fill in the missing blanks.

3^{-3}	3^{-2}	3^{-1}	3^{0}	3^{1}	3^{2}	3^{3}
______	______	______	______	3	9	______

$\times 3$ $\times 3$ $\times 3$ $\times 3$ $\times 3$ $\times 3$

This pattern can be duplicated for any **base** raised to any **integer exponent**. Because of this we can now define positive, negative, and zero exponents in terms of multiplying the number 1 repeatedly or dividing the number 1 repeatedly.

INTEGER EXPONENT DEFINITIONS

If n is any positive integer then:

1. $b^n = 1\cdot\underbrace{b\cdot b\cdot b\cdot\ldots\cdot b\cdot b}_{n\text{-times}}$

2. $b^0 = 1$

3. $b^{-n} = \dfrac{1}{\underbrace{b\cdot b\cdot b\cdot\ldots\cdot b\cdot b}_{n\text{-times}}} = \dfrac{1}{b^n}$

***Exercise* #2:** Given the exponential function $f(x) = 20(2)^x$ evaluate each of the following without using your calculator. Show the calculations that lead to your final answer.

(a) $f(2)$ (b) $f(0)$ (c) $f(-2)$

(d) When x increases by 3, by what **factor** does y increase? Explain your answer.

There are many basic **exponent properties or laws** that are critically important and that can be investigated using integer exponent examples. Two of the very important ones we will see next.

***Exercise* #3:** For each of the following, write the product as a single exponential expression. Write (a) and (b) as extended products first (if necessary).

(a) $2^3 \cdot 2^4$

(b) $2^6 \cdot 2^2$

(c) $2^m \cdot 2^n$

It's clear why the exponent law that you generalized in part (c) works for positive integer exponents. But, does it also make sense within the context of our negative exponents?

***Exercise* #4:** Consider now the product $2^3 \cdot 2^{-1}$.

(a) Use the exponent law found in Exercise 3(c) to write this as a single exponential expression.

(b) Evaluate $2^3 \cdot 2^{-1}$ by first rewriting 2^3 and 2^{-1} and then simplifying.

(c) Do your answers from (a) and (b) support the extension of the **Addition Property of Exponents** to negative powers as well? Explain.

Let's look at another important exponent property.

***Exercise* #5:** For each of the following, write the exponential expression in the form 3^x. Write (a) and (b) as extended products first (if necessary).

(a) $\left(3^2\right)^3$

(b) $\left(3^4\right)^2$

(c) $\left(3^m\right)^n$

Again, let's look at how the **Product Property of Exponents** still holds for negative exponents.

***Exercise* #6:** Consider the expression $\left(3^{-2}\right)^4$. Show this expression is equivalent to 3^{-8} by first rewriting 3^{-2} in fraction form.

Name: ______________________________________ Date: ___________________

INTEGER EXPONENTS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Write each of the following exponential expressions without the use of exponents such as we did in lesson *Exercise* #1.

(a)

$\boxed{2^{-3}}$	________	
		$\times 2$
$\boxed{2^{-2}}$	________	
		$\times 2$
$\boxed{2^{-1}}$	________	
		$\times 2$
$\boxed{2^{0}}$	________	
		$\times 2$
$\boxed{2^{1}}$	2	
		$\times 2$
$\boxed{2^{2}}$	4	
		$\times 2$
$\boxed{2^{3}}$	________	

(b)

$\boxed{5^{-3}}$	________	
		$\times 5$
$\boxed{5^{-2}}$	________	
		$\times 5$
$\boxed{5^{-1}}$	________	
		$\times 5$
$\boxed{5^{0}}$	________	
		$\times 5$
$\boxed{5^{1}}$	5	
		$\times 5$
$\boxed{5^{2}}$	25	
		$\times 5$
$\boxed{5^{3}}$	________	

2. Now let's go the other way around. For each of the following, determine the integer value of n that satisfies the equation. The first is done for you.

(a) $2^n = \frac{1}{8}$

$2^n = \frac{1}{2^3}$

$2^n = 2^{-3}$

$n = -3$

(b) $4^n = 16$

(c) $3^n = \frac{1}{81}$

(d) $7^n = 1$

(e) $5^n = \frac{1}{25}$

(f) $10^n = \frac{1}{10{,}000}$

(g) $13^n = 1$

(h) $2^n = \frac{1}{32}$

3. Use the **Addition Property of Exponents** to simplify each expression. Then, find a final numerical answer *without* using your calculator.

(a) $2^{-5} \cdot 2^{3} \cdot 2^{4}$ (b) $5^{3} \cdot 5^{7} \cdot 5^{-10}$ (c) $10^{3} \cdot 10^{-7} \cdot 10^{2}$

4. Use the **Product Property of Exponents** to simplify each exponential expression. You do not need to find a final numerical answer.

(a) $\left(2^{3}\right)^{4}$ (b) $\left(3^{-2}\right)^{2}$ (c) $\left(\left(5^{2}\right)^{-4}\right)^{-2}$

5. The exponential expression $\left(\frac{1}{8}\right)^{4}$ is equivalent to which of the following? Explain your choice.

(1) 4^{-8} (3) 8^{-2}

(2) 2^{-12} (4) 32^{-1}

REASONING

6. How can you use the fact that $25^{2} = 625$ to show that $5^{-4} = \frac{1}{625}$? Explain your process of thinking.

7. We've extended the two fundamental exponent properties to negative as well as positive integers. What would happen if we extended the **Product Exponent Property** to a fractional exponent like $\frac{1}{2}$? Let's play around with that idea.

(a) Use the **Product Property of Exponents** to justify that $\left(9^{1/2}\right)^{2} = 9$.

(b) What other number can you square that results in 9? Hmm...