BECA / Huson / 12.1 IB Math SL Name:
19 October 2017

Test: Differential calculus

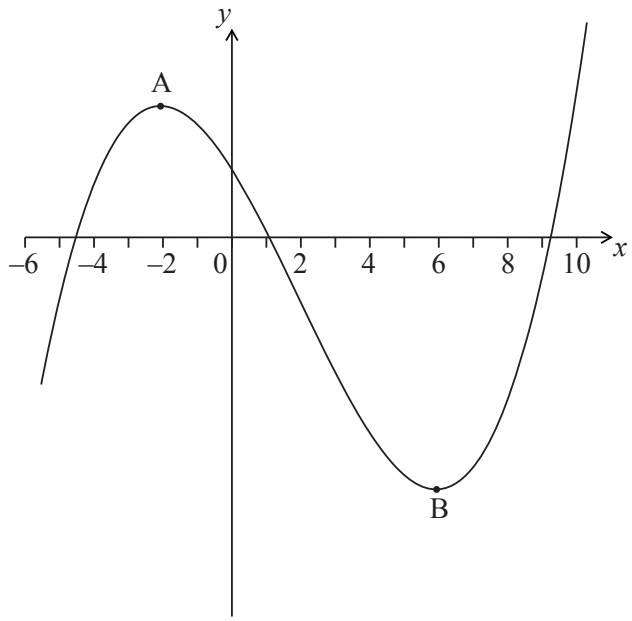
Show working for all problems. State answers exactly or to three significant figures.

Take the derivative of each function

1. $f(x) = x^2 - 2x + 11$.
2. $f(x) = \sqrt{x}$
3. $f(x) = x^2 e^x$
4. $f(x) = (x^2 - 5)\ln x$
5. $f(x) = \dfrac{\sin x}{x^3}$
6. $f(x) = \cos(1 - x^2)$
7. Let $f(x) = a(x-h)^2 + k$. The vertex of the graph of f is at $(2, 3)$ and the graph passes through $(1, 7)$.
   (a) Write down the value of h and k.
   (b) Find the value of a.
8. A function is given as $y = x^2 + kx - 8$.
   (a) Find $\dfrac{dy}{dx}$.
   (b) If the gradient of this function is 2 when x is 3, show that $k = -4$.
   (c) Find the equation of the line tangent to the function through the point $(4, -8)$.
9. An arithmetic sequence is given by 5, 8, 11, ...
   (a) Write down the value of d.
   (b) Find
      i. u_{100}
      ii. S_{100}
   (c) Given that $u_n = 1502$, find the value of n.

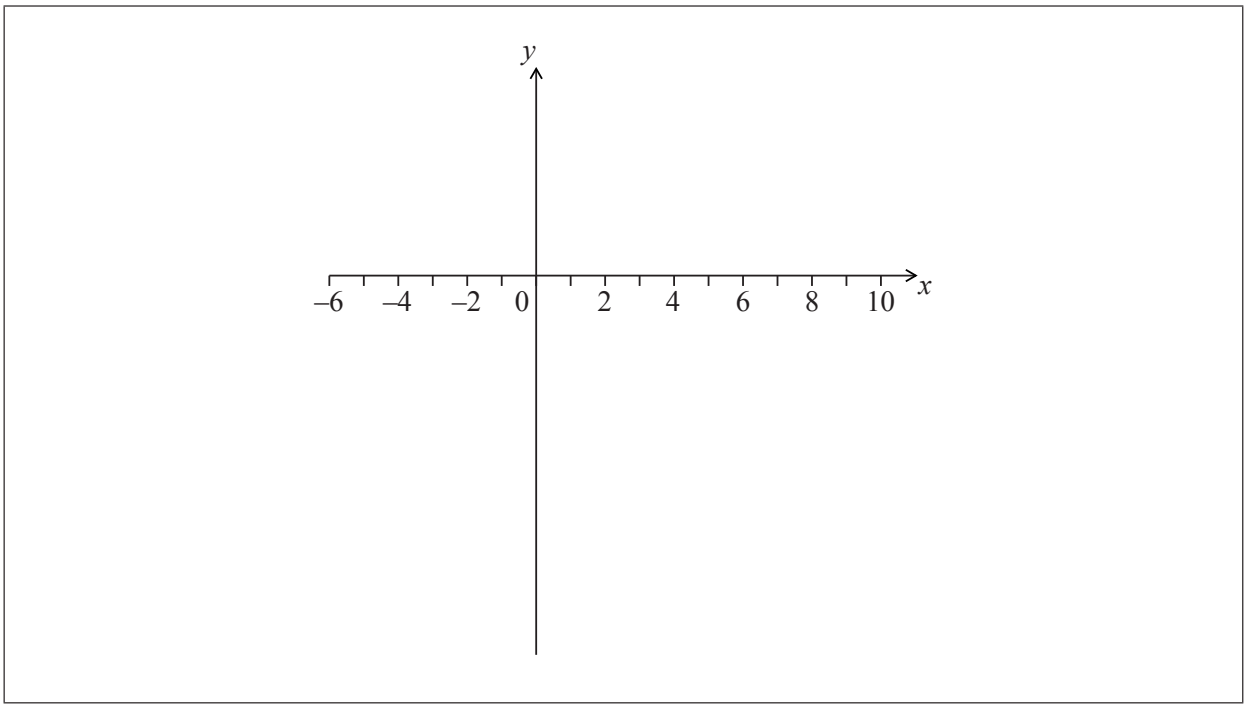
6. *[Maximum mark: 6]*

The following diagram shows part of the graph of $y = f(x)$.



The graph has a local maximum at A, where $x = -2$, and a local minimum at B, where $x = 6$.

(a) On the following axes, sketch the graph of $y = f'(x)$. *[4]*



(b) Write down the following in order from least to greatest: f (0), f '(6), f "(-2)

10. Let $f(x) = px^3 + px^2 + qx$.

    (a) Find $f'(x)$.

    (b) Given that $f'(x) \geq 0$, show that $p^2 \leq 3pq$.

11. Given the function $f(x) = \ln x^2 + kx + 5, x \neq 0$.

    (a) Find $f'(x)$.

    (b) The function $f(x)$ has a local maximum at $x = 2$. Show that $k = -1$

12. Given the function $f(x) = \dfrac{1}{x^2 - 4} + 3$.

    (a) For what values of x is the function undefined?

    (b) Hence and otherwise, write down the equations of the two vertical asymptotes and one horizontal asymptote.

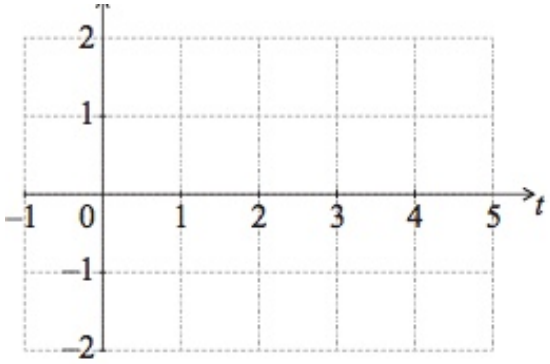
    (c) Find $f'(x)$.

    (d) Show that there is a local maximum or minimum at $x = 0$

    (e) Find the equation of the normal to the function when $x = 1$.

13. The position of an object is given by the function $s = e^{\sin t} - 1$, for $0 \leq t \leq 5$.

    (a) On the grid below, sketch the graph of s. (set your calculator to radians)



    (b) Write down the positive t-intercept.

    (c) Find the velocity of the object, $v(t)$.

6. *[Maximum mark: 6]*

Let f and g be functions such that $g(x) = 2f(x+1) + 5$.

(a) The graph of f is mapped to the graph of g under the following transformations:

vertical stretch by a factor of k, followed by a translation $\begin{pmatrix} p \\ q \end{pmatrix}$.

Write down the value of

(i) k;

(ii) p;

(iii) q. *[3 marks]*

(b) Let $h(x) = -g(3x)$. The point $\mathrm{A}(6, 5)$ on the graph of g is mapped to the point A' on the graph of h. Find A'. *[3 marks]*

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