

19 December 2017

**Homework: Binomial distribution and review**

- 1a. The random variable  $X$  has the following probability distribution, with  $P(X > 1) = 0.5$ .

$x$	0	1	2	3
$P(X = x)$	$p$	$q$	$r$	0.2

Find the value of  $r$ .

[2 marks]

- 1b. Given that  $E(X) = 1.4$ , find the value of  $p$  and of  $q$ .

[6 marks]

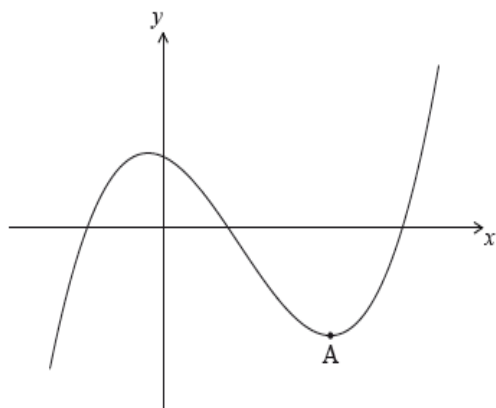
2. The random variable  $X$  has the following probability distribution.

$x$	1	2	3
$P(X = x)$	$s$	0.3	$q$

Given that  $E(X) = 1.7$ , find  $q$ .

[6 marks]

- 3a. The following diagram shows the graph of a function  $f$ . There is a local minimum point at  $A$ , where  $x > 0$ .

The derivative of  $f$  is given by  $f'(x) = 3x^2 - 8x - 3$ .Find the  $x$ -coordinate of  $A$ .

[5 marks]

- 3b. The  $y$ -intercept of the graph is at  $(0, 6)$ . Find an expression for  $f(x)$ .

The graph of a function  $g$  is obtained by reflecting the graph of  $f$  in the  $y$ -axis, followed by a translation of  $\begin{pmatrix} m \\ n \end{pmatrix}$ .

[6 marks]

**4a.** Let  $L_x$  be a family of lines with equation given by  $r = \begin{pmatrix} x \\ \frac{2}{x} \end{pmatrix} + t \begin{pmatrix} x^2 \\ -2 \end{pmatrix}$ , where  $x > 0$ .

Write down the equation of  $L_1$ .

[2 marks]

**4b.** A line  $L_a$  crosses the  $y$ -axis at a point  $P$ .

Show that  $P$  has coordinates  $\left(0, \frac{4}{a}\right)$ .

[6 marks]

**4c.** The line  $L_a$  crosses the  $x$ -axis at  $Q(2a, 0)$ . Let  $d = PQ^2$ .

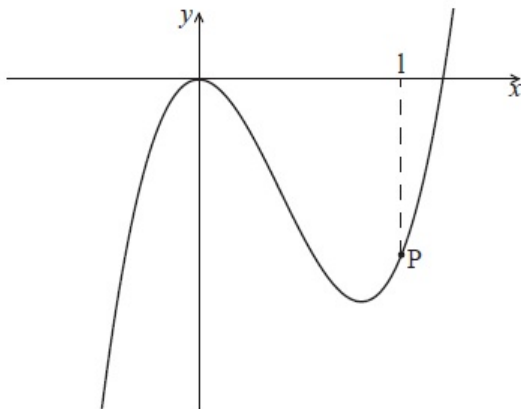
Show that  $d = 4a^2 + \frac{16}{a^2}$ .

[2 marks]

**4d.** There is a minimum value for  $d$ . Find the value of  $a$  that gives this minimum value.

[7 marks]

**5a.** Part of the graph of  $f(x) = ax^3 - 6x^2$  is shown below.



The point  $P$  lies on the graph of  $f$ . At  $P$ ,  $x = 1$ .

Find  $f'(x)$ .

[2 marks]

**5b.** The graph of  $f$  has a gradient of  $3$  at the point  $P$ . Find the value of  $a$ .

[4 marks]

6a. In this question, you are given that  $\cos \frac{\pi}{3} = \frac{1}{2}$ , and  $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ .

The displacement of an object from a fixed point, O is given by  $s(t) = t - \sin 2t$  for  $0 \leq t \leq \pi$ .

Find  $s'(t)$ . [3 marks]

6b. In this interval, there are only two values of  $t$  for which the object is not moving. One value is  $t = \frac{\pi}{6}$ .

Find the other value. [4 marks]

6c. Show that  $s'(t) > 0$  between these two values of  $t$ . [3 marks]

6d. Find the distance travelled between these two values of  $t$ . [5 marks]

7a. Consider the following sequence of figures.

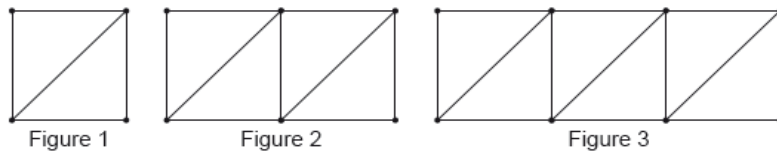


Figure 1 contains 5 line segments.

Given that Figure  $n$  contains 801 line segments, show that  $n = 200$ . [3 marks]

7b. Find the total number of line segments in the first 200 figures. [3 marks]

8a. Let  $x = \ln 3$  and  $y = \ln 5$ . Write the following expressions in terms of  $x$  and  $y$ .

$\ln\left(\frac{5}{3}\right)$ . [2 marks]

8b.  $\ln 45$ . [4 marks]

9. Three consecutive terms of a geometric sequence are  $x - 3$ , 6 and  $x + 2$ .

Find the possible values of  $x$ . [6 marks]