BECA / Huson / 11.1 IB Math SL Name:
15 September 2017

Homework: Chapter review for test on Wednesday September 20th
Write solutions on loose leaf paper. Spread your work out, working down the page. Write clearly.

Review exercise

1 **a** If $g(a) = 4a - 5$, find $g(a-2)$.
b If $h(x) = \frac{1+x}{1-x}$, find $h(1-x)$.

2 **a** Evaluate $f(x-3)$ when $f(x) = 2x^2 - 3x + 1$.
b For $f(x) = 2x + 7$ and $g(x) = 1 - x^2$, find the composite function defined by $(f \circ g)(x)$.

Function notation

- $f(x)$ is read as 'f of x' and means 'the value of function f at x'.

Composite functions

- The composition of the function f with the function g is written as $f(g(x))$, which is read as 'f of g of x', or $(f \circ g)(x)$, which is read as 'f composed with g of x'.
- A **composite function** applies one function to the result of another and is defined by $(f \circ g)(x) = f(g(x))$.

Inverse functions

- The **inverse** of a function $f(x)$ is $f^{-1}(x)$. It reverses the action of the function.
- Functions $f(x)$ and $g(x)$ are inverses of one another if:
  $(f \circ g)(x) = x$ for all of the x-values in the domain of g and
  $(g \circ f)(x) = x$ for all of the x-values in the domain of f.
- You can use the **horizontal line test** to identify inverse functions. If a horizontal line crosses a function more than once, there is no inverse function.

The graphs of inverse functions

- The graph of the inverse of a function is a reflection of that function in the line $y = x$.
- To find the inverse function algebraically, replace $f(x)$ with y and solve for y.
- The function $I(x) = x$ is called the identity function. It leaves x unchanged. So $f \circ f^{-1} = I$.

Transformations of functions

- $f(x) + k$ translates $f(x)$ vertically a distance of k units upward.
- $f(x) - k$ translates $f(x)$ vertically a distance of k units downward.
- $f(x + k)$ translates $f(x)$ horizontally k units to the left, where $k > 0$.
- $f(x - k)$ translates $f(x)$ horizontally k units to the right, where $k > 0$.
- $-f(x)$ reflects $f(x)$ in the x-axis.
- $f(-x)$ reflects $f(x)$ in the y-axis.
- $f(qx)$ stretches $f(x)$ horizontally with scale factor $\frac{1}{q}$.
- $pf(x)$ stretches $f(x)$ vertically with scale factor p.

3 Find the inverses of these functions.

a $f(x) = \frac{3x+17}{2}$

b $g(x) = 5x^3 - 4$

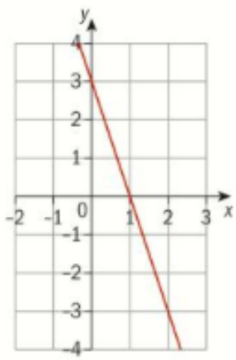
4 Find the inverse of $f(x) = -\frac{1}{5}x - 1$. Then graph the function and its inverse.

5 Find the inverse functions for

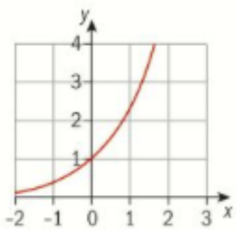
a $f(x) = 3x + 5$ **b** $f(x) = \sqrt[3]{x+2}$

6 Copy each graph and draw the inverse of each function.

a

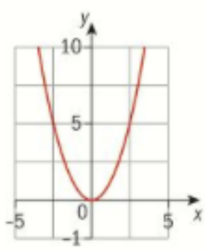


b



7 Find the domain and range for each of these graphs.

a



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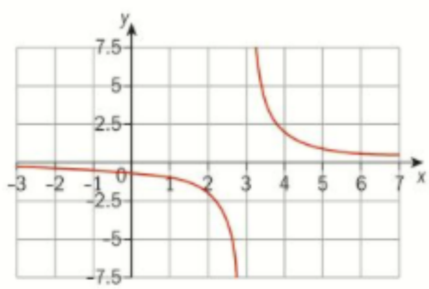


EXAM-STYLE QUESTION

8 For each function, write a single equation to represent the given combination of transformations.

a $f(x) = x$, reflected in the y-axis, stretched vertically by a factor of 2, horizontally by a factor of $\frac{1}{3}$ and translated 3 units left and 2 units up.

b $f(x) = x^2$, reflected in the x-axis, stretched vertically by a factor of $\frac{1}{4}$, horizontally by a factor of 3, translated 5 units right and 1 unit down.

9 **a** Explain how to draw the inverse of a function from its graph.

b Graph the inverse of $f(x) = 2x + 3$.

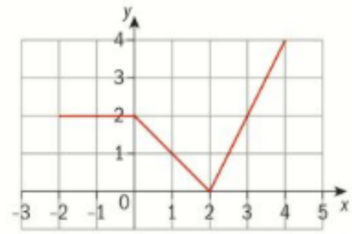
EXAM-STYLE QUESTION

10 Let $f(x) = 2x^3 + 3$ and $g(x) = 3x - 2$.

a Find g(0). **b** Find $(f \circ g)(0)$. **c** Find $f^{-1}(x)$.

EXAM-STYLE QUESTIONS

11 The graph shows the function $f(x)$, for $-2 \le x \le 4$.



a Let $h(x) = f(-x)$. Sketch the graph of $h(x)$.

b Let $g(x) = \frac{1}{2}f(x-1)$. The point $A(3, 2)$ on the graph of f is transformed to the point P on the graph of g. Find the coordinates of P.

12 The functions f and g are defined as $f(x) = 3x$ and $g(x) = x + 2$.

a Find an expression for $(f \circ g)(x)$.

b Show that $f^{-1}(12) + g^{-1}(12) = 14$.

13 Let $g(x) = 2x - 1$, $h(x) = \frac{3x}{x-2}, x \neq 2$

a Find an expression for $(h \circ g)(x)$. Simplify your answer.

b Solve the equation $(h \circ g)(x) = 0$.

The instruction 'Show that…' means 'Obtain the required result (possibly using information given) without the formality of proof'.
For 'Show that' questions you do not usually need to use a calculator.
A good method is to cover up the right-hand side of the equation and then work out the left-hand side until your answer is the same as the right-hand side.

Review exercise

1 Use your GDC to sketch the function and state the domain and range of $f(x) = \sqrt{x+2}$.

2 Sketch the function $y = (x + 1)(x - 3)$ and state its domain and range.

3 Sketch the function $y = \frac{1}{x+2}$ and state its domain and range.

EXAM-STYLE QUESTIONS

4 The function $f(x)$ is defined as $f(x) = 2 + \frac{1}{x+1}, x \neq -1$.

a Sketch the curve $f(x)$ for $-3 \le x \le 2$.

b Use your GDC to help you write down the value of the x-intercept and the y-intercept.

5 **a** Sketch the graph of $f(x) = \frac{1}{x^2}$

b For what value of x is $f(x)$ undefined?

c State the domain and range of $f(x)$.

6 Given the function $f(x) = \frac{2x-5}{x+2}$

a write down the equations of the asymptotes

b sketch the function

c write down the coordinates of the intercepts with both axes.

7 Let $f(x) = 2 - x^2$ and $g(x) = x^2 - 2$.

a Sketch both functions on one graph with $-3 \le x \le 3$.

b Solve $f(x) = g(x)$.

EXAM-STYLE QUESTIONS

8 Let $f(x) = x^3 - 3$.

a Find the inverse function $f^{-1}(x)$.

b Sketch both $f(x)$ and $f^{-1}(x)$ on the same axes.

c Solve $f(x) = f^{-1}(x)$.

9 $f(x) = e^{2x-1} + \frac{2}{x+1}$, $x \neq -1$.

Sketch the curve of $f(x)$ for $-5 \leq x \leq 2$, including any asymptotes.

10 Consider the functions f and g where $f(x) = 3x - 2$ and $g(x) = x - 3$.

a Find the inverse function, f^{-1}.

b Given that $g^{-1}(x) = x + 3$, find $(g^{-1} \circ f)(x)$.

c Show that $(f^{-1} \circ g)(x) = \frac{x-1}{3}$.

d Solve $(f^{-1} \circ g)(x) = (g^{-1} \circ f)(x)$

Let $h(x) = \frac{f(x)}{g(x)}$, $x \neq 3$.

e **Sketch** the graph of h for $-6 \leq x \leq 10$ and $-4 \leq y \leq 10$, including any asymptotes.

f Write down the **equations** of the asymptotes.

CHAPTER 1 SUMMARY

Introducing functions

- A **relation** is a set of ordered pairs.
- The **domain** is the set of all the first numbers (x-values) of the ordered pairs.
- The **range** is the set of the second numbers (y-values) in each pair.
- A **function** is a relation where every x-value is related to a unique y-value.
- A relation is a function if any vertical line drawn will not intersect the graph more than once. This is called the **vertical line test**.

The domain and range of a relation on a Cartesian plane

Interval notation:
Use round brackets (,) if the value is not included in the graph or when the graph is undefined at that point (a hole or **asymptote**, or a jump).
Use square brackets [,] if the value is included in the graph.

- Set notation: