12.1 IB Math - Unit 5: Integration
Bronx Early College Academy

Christopher J. Huson PhD

4 January 2019

BECA / Dr. Huson / 12.1 IB Math

GQ: How do we find the antiderivative of a function?

CCSS: F.IF.B.6 Calculate & interpret the rate of change of a function   5.1 Friday 4 January

Do Now. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$

1. Given $y = x^3 + x^2 + 17$.
2. Given $y = \frac{1}{4}x^4 + \frac{1}{2}x^2 + 9 - \frac{1}{x}$.
3. If $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + x$, find y.
4. Skills check #1 p. 290

Problem sets from January 2,3; Sigma notation, p 290

Lesson: Antiderivatives pp. 291-2

Exam review

Homework: Exercises 9A p. 293; test corrections due Monday

GQ: How do we find the indefinite integral of a function?

CCSS: F.IF.B.6 Calculate & interpret the rate of change of a function   5.2 Monday 7 January

Do Now. Find the antiderivative, $F(x)$, of each function, $f(x)$, such that $F'(x) = f(x)$

1. $f(x) = 4x^3 + 3x^2 + 1$.
2. $f(x) = x^4 + x^2 + 5$.
3. $f(x) = \sqrt{x}$

Test corrections due, review. (take home test tomorrow)

Lesson: Indefinite integral pp. 293-4

Homework: Exercises 9B p. 294

GQ: How do we integrate functions?

CCSS: F.IF.B.6 Calculate & interpret the rate of change of a function   5.3 Tuesday 8 January

Do Now. Handout-test review

Deltamath Integration practice (taking antiderivatives)

Homework: Deltamath project through Thursday

GQ: How do we apply boundary conditions to an integral?
CCSS: F.IF.B.6 Calculate & interpret the rate of change of a function   5.4 Wednesday 9 January

Do Now. Find each indefinite integral.

1. $\int (3x^2 + 2x + 1)\, dx$.
2. $\int (x^4 + 6x^2 + 1)\, dx$.
3. $\int \sqrt{x}\, dx$.
4. $\displaystyle\int \frac{\pi}{4}\sqrt[3]{x^2}\, dx$
5. $\int x^{-1}\, dx$

Lesson: Finding the constant C given boundary conditions. pp. 295-6

Homework: Exercises 9C p. 296-7

GQ: How do we integrate functions?
CCSS: F.IF.B.6 Calculate & interpret the rate of change of a function   5.5 Thursday 10 January

Do Now. Handout-test review

Deltamath Integration practice (taking antiderivatives)

Homework: Complete Deltamath calculus project

GQ: How do we integrate compositions of linear functions?

CCSS: F.IF.B.6 Calculate & interpret the rate of change of a function   5.6 Friday 11 January

Do Now. Derivatives and antiderivatives. (Use the chain rule)

1. $f(x) = e^{3x}$. Find $f'(x)$.
2. $f(x) = \ln(5x + 3)$. Find $f'(x)$.
3. $f(x) = (2x - 5)^3$. Find $f'(x)$.
4. Find $\int 4(x^2 + x + 1) \, dx$.
5. $y' = 2x^3 - 1$ and $y = 3$ when $x = 1$. Find y in terms of x.
6. Given $f'(x) = \sqrt[3]{x}$ and $f(0) = 1$, find $f(x)$.

Lesson: Antiderivatives of form $\int f(ax + b) \, dx$. pp. 297-9

Homework: Exercises 9D, 9E (odds) p. 298, 300

GQ: How do we integrate compositions of linear functions?

CCSS: F.IF.B.6 Calculate & interpret the rate of change of a function   5.7 Monday 14 January

Do Now. Derivatives and antiderivatives. (Use the chain rule)

1. $f(x) = e^{2x-3}$. Find $f'(x)$.
2. $f(x) = \ln(4x)$. Find $f'(x)$.
3. $f(x) = (3x + 2)^4$. Find $f'(x)$.
4. Find $\int (5x^3 + x^2 + 1)\, dx$.
5. $y' = 2x^2 - 1$ and $y = 3$ when $x = 3$. Find y in terms of x.
6. Given $f'(x) = \sqrt[3]{x^2}$ and $f(1) = 1$, find $f(x)$.

Lesson: Antiderivatives of form $\int f(ax + b)\, dx$. pp. 297-9

Homework: Exercises 9D, 9E (evens) p. 298, 300

GQ: How do we integrate functions?

CCSS: F.IF.B.6 Calculate & interpret the rate of change of a function   5.8 Tuesday 15 January

Do Now. Straight to DeltaMath

Deltamath Integration practice (taking antiderivatives)

Homework: Complete Deltamath

GQ: How do we use integration to solve for position?

CCSS: F.IF.B.6 Calculate & interpret the rate of change of a function 5.9 Wednesday 16 January

Do Now. Derivatives and antiderivatives.

1. $f(x) = \sin x$. Find $f'(x)$.

2. Find $\int \sin x \, dx$.

3. $s(t) = 4.9t^2$. Find $v(t) = s'(t)$ and $a(t) = s''(t)$.
   What might these functions represent?

4. Given $v(t) = \cos t$, find $s(t) = \int v(t) \, dt$. Assume $s(0) = 0$.

Lesson: Position problems (handout)

Homework: Calculus problem set

GQ: How do we calculate area with integration?
CCSS: F.IF.B.6 Calculate & interpret the rate of change of a function

Do Now

1. Find $\int (4x^3 - 3x + 1)\mathrm{d}x$.
2. Find $\int e^{5x}\mathrm{d}x$.
3. Find $\displaystyle\int \frac{1}{3x+1}\mathrm{d}x$.

Homework review #1, 5, 6 p. 302

Lesson: Reimann sums and the definite integral

Task: Example 8, page 304

Assessment: Calculator integration

Homework: Exercises 9H evens p. 308

GQ: How do we calculate area with definite integrals?
CCSS: F.IF.B.6 Calculate & interpret the rate of change of a function

Do Now

1. Use a calculator to find $\displaystyle\int_0^{\frac{\pi}{2}} \cos x \ \mathrm{d}x$
2. Differentiate $y = \sqrt{3x^3 - x}$
3. Find $\int (6x^2 - 2x - 5)\mathrm{d}x$.
4. Differentiate $y = (3x^2 - 5x)^5$
5. Find $\int 5(x^2 + 1)^4(2x)\mathrm{d}x$.
6. Find $\displaystyle\int \frac{3}{x} \ \mathrm{d}x$.

Lesson: Properties of definite integrals p. 307
Task: Review homework problems
Assessment: Problem #11 p. 300
Homework: Exercises 9H evens p. 308

GQ: How do we calculate area with definite integrals?
CCSS: F.IF.B.6 Calculate & interpret the rate of change of a function

Do Now

1. Use a calculator to find $\displaystyle\int_{-1}^{1} \frac{1}{x+2} \ dx$
2. Differentiate $y = \sqrt[3]{5x^2 - 2x}$
3. Find $\int (6x^2 - 2)^4 (12x) \ dx$.
4. Differentiate $y = (3x^2 - 5x)(\ln x)$
5. Find $\displaystyle\int_{1}^{2} \frac{3}{x^2} \ dx$. (check your result with a calculator)

Lesson: The fundamental theorem of calculus p. 309
Task: Practice Examples 11, 12 p. 310-1
Assessment: Problem 9J #1 p. 312
Homework: Exercises 9I, 9J p. 310-12

GQ: How do we calculate the area between two curves?
CCSS: F.IF.B.6 Calculate & interpret the rate of change of a function

Do Now: Consider the function $f(x) = -x^2 + 2x + 3$

1. Factor f and state its zeros.
2. Restate f in vertex form. Write down the vertex as an ordered pair.
3. Differentiate f. Show that the zero of $f'(x)$ is the vertex of f.
4. If $f(x)$ represents the height of a diver over the domain $0 \leq x \leq 3$, interpret $f(0)$ and $f'(0)$
5. What is the size of the area bounded by f, $x = 0$, and $y = 0$?

Lesson: The area between two functions p. 313
Assessment: Example #13 p. 314
Homework: Exercises 9K p. 316

GQ: How do we calculate a volume of rotation?
CCSS: F.IF.B.6 Calculate & interpret the rate of change of a function

Do Now: Sketch the functions $f(x) = 10x + x^2 - 3x^3$ and $g(x) = x^2 - 2x$

1. What are their intersections? (i.e. $f(x) = g(x)$)
2. What is the definite integral representing the area between the curves?
3. Using a calculator, what is the size of the area? (this may not be a trivial question)

Lesson: Integrating circle areas, modeling a solid p. 318
Assessment: Example #15 p. 319
Homework: Exercises 9L & 9M p. 317, 319; probability handout

GQ: How do we calculate displacement from velocity?

CCSS: F.IF.B.6 Calculate & interpret the rate of change of a function   12.1

Do Now: Do the calculations below and read the handout

1. Lance Armstrong's average speed in his six Tour de France victories from 1999-2004 was about 24 miles per hour. Assuming that he pedals at his average speed and takes no breaks, how long would it take him to ride 38 miles to the top of a 10,000 ft. volcano?

2. People who are not Lance Armstrong can travel at about 12 miles per hour on a bike. At that speed, how long would it take to reach the top of the volcano?

Lesson: Integrating velocity over time, displacement p. 321
Assessment: Example #18 p. 323
Homework: Exercises 9N & 9O p. 320, 324

GQ: How do we calculate displacement from velocity?

CCSS: F.IF.B.6 Calculate & interpret the rate of change of a function   12.1

Do Now: continued, 38 mile ride in 10 hours

1. Using the velocity vs time graph from yesterday, integrate to show that the areas representing the distance covered by the three riders are equal ($v \times t = d$).

2. Show that a rider accelerating according to $v(t) = \frac{76}{100}t$ also arrives at $(10, 38)$.

Lesson: Integrating velocity over time, displacement p. 321
Task: Review 9F, 9M, probability
Assessment: Example #18 p. 323 (take home test Thursday)
Homework: Exercises 9P p. 326, any remaining problem sets

GQ: How do we integrate a function?

CCSS: HSF.IF.B.6 Calculate and interpret the area under a function   12.1

Do Now: Chain rule - Take the derivative of each function

1. $f(x) = \sin x^3$
2. $g(x) = \sqrt{x^4 + 2}$
3. $h(x) = \ln (x^2 + 1)$

Lesson: Take home exam papers assessment & review
Task: Work problems on board
Assessment: Test corrections due
Homework: Integration exam problems

GQ: How do we calculate volume (of a rotated function)?
CCSS: HSF.IF.B.6 Calculate and interpret the area under a function   12.1

Do Now: Identifying problem types. On your homework, underline the "M1" points you earned. Examples:

1. $X \sim B(10, 0.5)$
2. bell curve sketch
3. $u = \&\quad u' =$

Lesson: $\displaystyle\int_a^b \pi r^2 \mathrm{d}x$
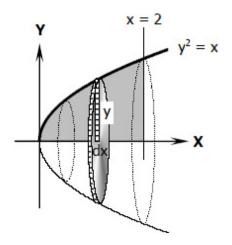
Task: Work homework problems on board

Assessment: self-reflection on mixed versus block problem sets

Homework: Solids of rotation & mixed exam problems

The volume of a function rotated around the x-axis

Differentiate over x, but use the area of a disk defined by $A = \pi r^2$

video

Credit: MATHalino.com - Pinoy Math Community Romel Verterra