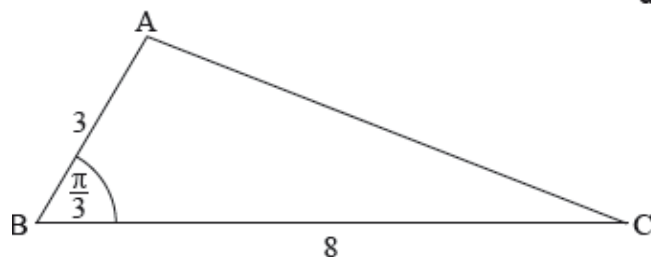


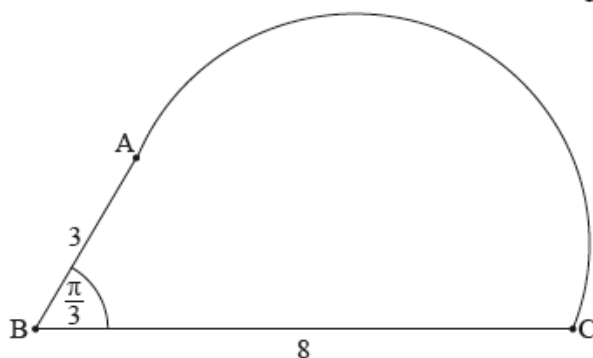
17 April 2018

Homework: Trig functions

- 1a. The following diagram shows triangle ABC, with  $AB = 3$  cm,  $BC = 8$  cm, and  $\hat{ABC} = \frac{\pi}{3}$ .

**diagram not to scale**Show that  $AC = 7$  cm.*[4 marks]*

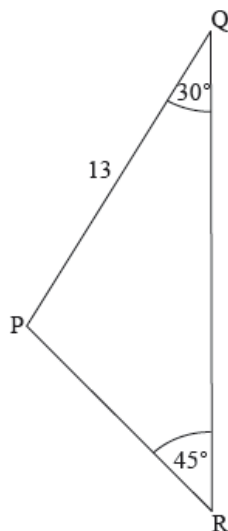
- 1b. The shape in the following diagram is formed by adding a semicircle with diameter  $[AC]$  to the triangle.

**diagram not to scale**

Find the exact perimeter of this shape.

*[3 marks]*

2. The following diagram shows triangle PQR.

**diagram not to scale** $\hat{PQR} = 30^\circ$ ,  $\hat{QRP} = 45^\circ$  and  $PQ = 13$  cm.

Find PR.

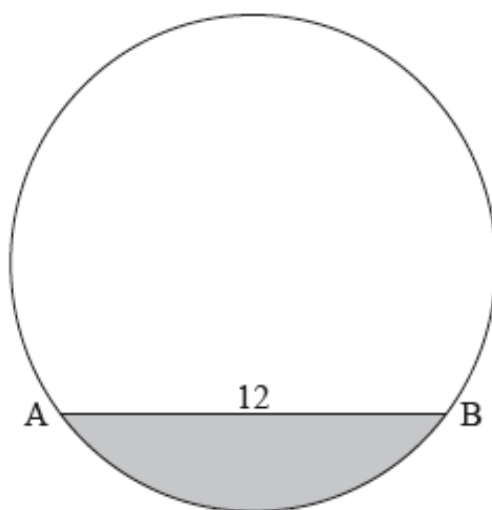
*[6 marks]*

17 April 2018

3. [7 marks]

The following diagram shows the chord  $[AB]$  in a circle of radius 8 cm, where  $AB = 12$  cm,

**diagram not to scale**

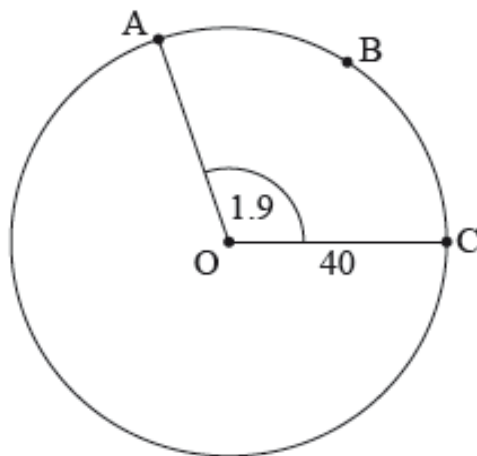


Find the area of the shaded segment.

4a. [2 marks]

The following diagram shows a circle with centre  $O$  and radius 40 cm.

**diagram not to scale**



The points  $A$ ,  $B$  and  $C$  are on the circumference of the circle and  $\hat{AOC} = 1.9$  radians.

Find the length of arc  $ABC$ .

4b. Find the perimeter of sector  $OABC$ .

[2 marks]

4c. Find the area of sector  $OABC$ .

[2 marks]

**5a.** The depth of water in a port is modelled by the function  $d(t) = p \cos qt + 7.5$ , for  $0 \leq t \leq 12$ , where  $t$  is the number of hours after high tide.

At high tide, the depth is 9.7 metres.

At low tide, which is 7 hours later, the depth is 5.3 metres.

Find the value of  $p$ .

[2 marks]

**5b.** Find the value of  $q$ .

[2 marks]

**5c.** Use the model to find the depth of the water 10 hours after high tide.

[2 marks]

**6a.** [3 marks]

Let  $\sin \theta = \frac{\sqrt{5}}{3}$ , where  $\theta$  is acute.

Find  $\cos \theta$ .

**6b.** [2 marks]

Find  $\cos 2\theta$ .

**7a.** Let  $f(x) = 3 \sin\left(\frac{\pi}{2}x\right)$ , for  $0 \leq x \leq 4$ .

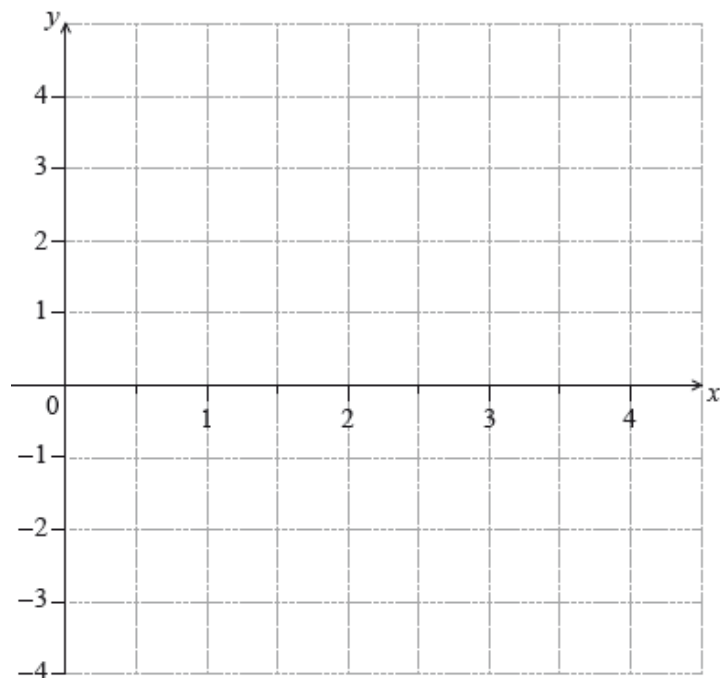
(i) Write down the amplitude of  $f$ .

(ii) Find the period of  $f$ .

[3 marks]

**7b.** [4 marks]

On the following grid sketch the graph of  $f$ .



17 April 2018

**8a.** [5 marks]

Let  $f(x) = 6x\sqrt{1-x^2}$ , for  $-1 \leq x \leq 1$ , and  $g(x) = \cos(x)$ , for  $0 \leq x \leq \pi$ .

Let  $h(x) = (f \circ g)(x)$ .

Write  $h(x)$  in the form  $a \sin(bx)$ , where  $a, b \in \mathbb{Z}$ .

**8b.** [2 marks]

Hence find the range of  $h$ .

**9a.** The height,  $h$  metres, of a seat on a Ferris wheel after  $t$  minutes is given by

$$h(t) = -15 \cos 1.2t + 17, \text{ for } t \geq 0.$$

Find the height of the seat when  $t = 0$ .

[2 marks]

**9b.** The seat first reaches a height of 20 m after  $k$  minutes. Find  $k$ .

[3 marks]

**9c.** Calculate the time needed for the seat to complete a full rotation, giving your answer correct to one decimal place.

[3 marks]

**10a.** Let  $f(x) = 3 \sin(\pi x)$ .

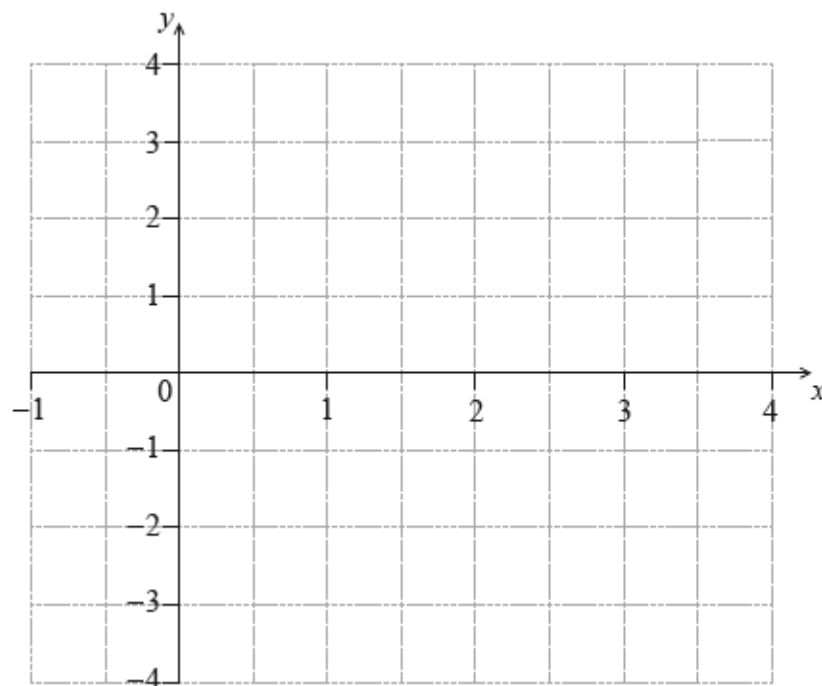
Write down the amplitude of  $f$ .

[1 mark]

**10b.** Find the period of  $f$ .

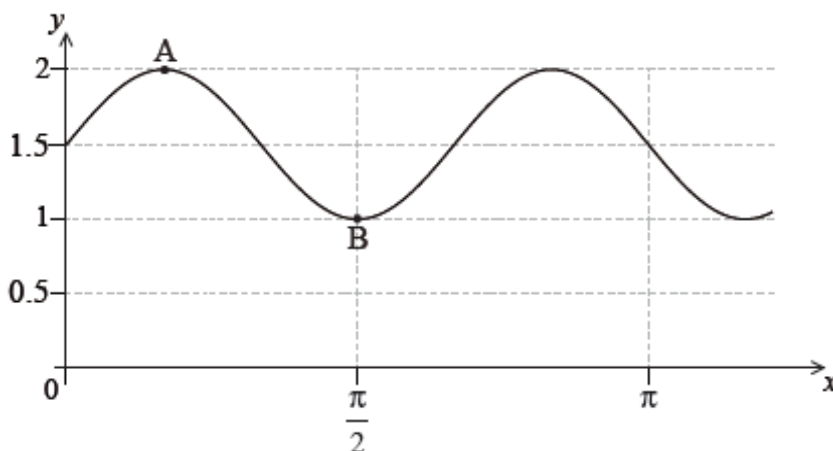
[2 marks]

**10c.** On the following grid, sketch the graph of  $y = f(x)$ , for  $0 \leq x \leq 3$ .



[4 marks]

**11a.** The following diagram shows part of the graph of  $y = p \sin(qx) + r$ .



The point  $A \left( \frac{\pi}{6}, 2 \right)$  is a maximum point and the point  $B \left( \frac{\pi}{6}, 1 \right)$  is a minimum point.

Find the value of  $p$ ;

[2 marks]

**11b.**  $r$ ;

[2 marks]

**11c.**  $q$ .

[2 marks]

**12a.** [3 marks]

Let  $f(x) = \cos\left(\frac{\pi}{4}x\right) + \sin\left(\frac{\pi}{4}x\right)$ , for  $-4 \leq x \leq 4$ .

Sketch the graph of  $f$ .

**12b.** [5 marks] Find the values of  $x$  where the function is decreasing.

**12c.** [3 marks]

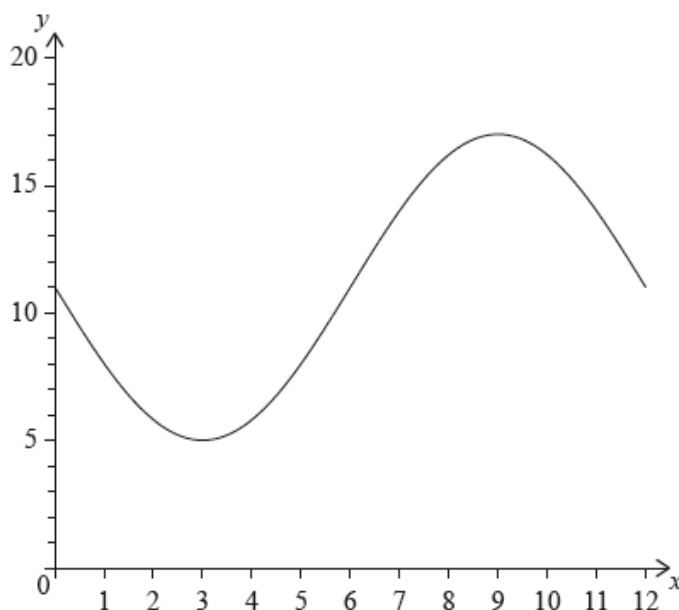
The function  $f$  can also be written in the form  $f(x) = a \sin\left(\frac{\pi}{4}(x + c)\right)$ , where  $a \in \mathbb{R}$ , and  $0 \leq c \leq 2$ . Find the value of  $a$ ;

**12d.** [4 marks]

The function  $f$  can also be written in the form  $f(x) = a \sin\left(\frac{\pi}{4}(x + c)\right)$ , where  $a \in \mathbb{R}$ , and  $0 \leq c \leq 2$ . Find the value of  $c$ .

**13a.** [6 marks]

The following diagram shows the graph of  $f(x) = a \sin bx + c$ , for  $0 \leq x \leq 12$ .



The graph of  $f$  has a minimum point at  $(3, 5)$  and a maximum point at  $(9, 17)$ .

(i) Find the value of  $c$ .

(ii) Show that  $b = \frac{\pi}{6}$ .

(iii) Find the value of  $a$ .

**13b.** [3 marks]

The graph of  $g$  is obtained from the graph of  $f$  by a translation of  $\begin{pmatrix} k \\ 0 \end{pmatrix}$ . The maximum point on the graph of  $g$  has coordinates  $(11.5, 17)$ .

(i) Write down the value of  $k$ .

(ii) Find  $g(x)$ .

**13c.** [6 marks]

The graph of  $g$  changes from concave-up to concave-down when  $x = w$ .

(i) Find  $w$ .

(ii) Hence or otherwise, find the maximum positive rate of change of  $g$ .