

### Homework: Chapter review for test on Wednesday September 20<sup>th</sup>

Write solutions on loose leaf paper. Spread your work out, working down the page. Write clearly.



## Review exercise

- 1 a If  $g(a) = 4a - 5$ , find  $g(a - 2)$ .  
b If  $h(x) = \frac{1+x}{1-x}$ , find  $h(1 - x)$ .
- 2 a Evaluate  $f(x - 3)$  when  $f(x) = 2x^2 - 3x + 1$ .  
b For  $f(x) = 2x + 7$  and  $g(x) = 1 - x^2$ , find the composite function defined by  $(f \circ g)(x)$ .

### Function notation

- $f(x)$  is read as 'f of x' and means 'the value of function f at x'.

### Composite functions

- The composition of the function  $f$  with the function  $g$  is written as  $f(g(x))$ , which is read as 'f of g of x', or  $(f \circ g)(x)$ , which is read as 'f composed with g of x'.
- A **composite function** applies one function to the result of another and is defined by  $(f \circ g)(x) = f(g(x))$ .

### Inverse functions

- The **inverse** of a function  $f(x)$  is  $f^{-1}(x)$ . It reverses the action of the function.
- Functions  $f(x)$  and  $g(x)$  are inverses of one another if:  
 $(f \circ g)(x) = x$  for all of the x-values in the domain of  $g$  and  
 $(g \circ f)(x) = x$  for all of the x-values in the domain of  $f$ .
- You can use the **horizontal line test** to identify inverse functions. If a horizontal line crosses a function more than once, there is no inverse function.

### The graphs of inverse functions

- The graph of the inverse of a function is a reflection of that function in the line  $y = x$ .
- To find the inverse function algebraically, replace  $f(x)$  with  $y$  and solve for  $y$ .
- The function  $I(x) = x$  is called the identity function. It leaves  $x$  unchanged.  
So  $f \circ f^{-1} = I$ .

### Transformations of functions

- $f(x) + k$  translates  $f(x)$  vertically a distance of  $k$  units upward.
- $f(x) - k$  translates  $f(x)$  vertically a distance of  $k$  units downward.
- $f(x + k)$  translates  $f(x)$  horizontally  $k$  units to the left, where  $k > 0$ .
- $f(x - k)$  translates  $f(x)$  horizontally  $k$  units to the right, where  $k > 0$ .
- $-f(x)$  reflects  $f(x)$  in the  $x$ -axis.
- $f(-x)$  reflects  $f(x)$  in the  $y$ -axis.
- $f(qx)$  stretches  $f(x)$  horizontally with scale factor  $\frac{1}{q}$ .
- $pf(x)$  stretches  $f(x)$  vertically with scale factor  $p$ .

3 Find the inverses of these functions.

a  $f(x) = \frac{3x+17}{2}$

b  $g(x) = 5x^3 - 4$

4 Find the inverse of  $f(x) = -\frac{1}{5}x - 1$ . Then graph the function and its inverse.

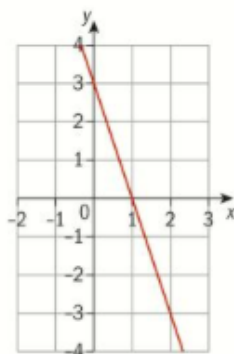
5 Find the inverse functions for

a  $f(x) = 3x + 5$

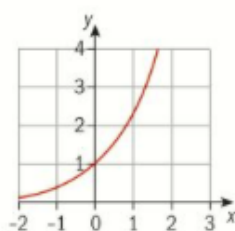
b  $f(x) = \sqrt[3]{x+2}$

6 Copy each graph and draw the inverse of each function.

a

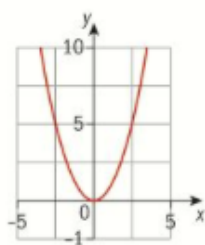


b

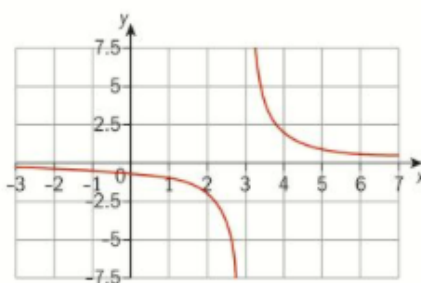


7 Find the domain and range for each of these graphs.

a



b



### EXAM-STYLE QUESTION

8 For each function, write a single equation to represent the given combination of transformations.

a  $f(x) = x$ , reflected in the  $y$ -axis, stretched vertically by a factor of 2, horizontally by a factor of  $\frac{1}{3}$  and translated 3 units left and 2 units up.

b  $f(x) = x^2$ , reflected in the  $x$ -axis, stretched vertically by a factor of  $\frac{1}{4}$ , horizontally by a factor of 3, translated 5 units right and 1 unit down.

9 a Explain how to draw the inverse of a function from its graph.

b Graph the inverse of  $f(x) = 2x + 3$ .

### EXAM-STYLE QUESTION

10 Let  $f(x) = 2x^3 + 3$  and  $g(x) = 3x - 2$ .

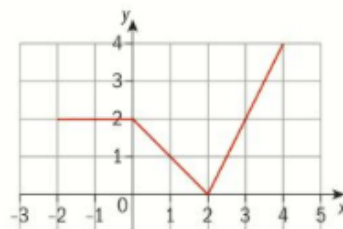
a Find  $g(0)$ .

b Find  $(f \circ g)(0)$ .

c Find  $f^{-1}(x)$ .

### EXAM-STYLE QUESTIONS

- 11** The graph shows the function  $f(x)$ , for  $-2 \leq x \leq 4$ .
- Let  $h(x) = f(-x)$ . Sketch the graph of  $h(x)$ .
  - Let  $g(x) = \frac{1}{2}f(x-1)$ . The point  $A(3, 2)$  on the graph of  $f$  is transformed to the point  $P$  on the graph of  $g$ . Find the coordinates of  $P$ .
- 12** The functions  $f$  and  $g$  are defined as  $f(x) = 3x$  and  $g(x) = x + 2$ .
- Find an expression for  $(f \circ g)(x)$ .
  - Show that  $f^{-1}(12) + g^{-1}(12) = 14$ .
- 13** Let  $g(x) = 2x - 1$ ,  $h(x) = \frac{3x}{x-2}$ ,  $x \neq 2$
- Find an expression for  $(h \circ g)(x)$ . Simplify your answer.
  - Solve the equation  $(h \circ g)(x) = 0$ .



The instruction 'Show that...' means 'Obtain the required result (possibly using information given) without the formality of proof'. For 'Show that' questions you do not usually need to use a calculator. A good method is to cover up the right-hand side of the equation and then work out the left-hand side until your answer is the same as the right-hand side.



## Review exercise

- Use your GDC to sketch the function and state the domain and range of  $f(x) = \sqrt{x+2}$ .
- Sketch the function  $y = (x+1)(x-3)$  and state its domain and range.
- Sketch the function  $y = \frac{1}{x+2}$  and state its domain and range.

### EXAM-STYLE QUESTIONS

- 4** The function  $f(x)$  is defined as  $f(x) = 2 + \frac{1}{x+1}$ ,  $x \neq -1$ .
- Sketch the curve  $f(x)$  for  $-3 \leq x \leq 2$ .
  - Use your GDC to help you write down the value of the  $x$ -intercept and the  $y$ -intercept.
- 5**
- Sketch the graph of  $f(x) = \frac{1}{x^2}$
  - For what value of  $x$  is  $f(x)$  undefined?
  - State the domain and range of  $f(x)$ .
- 6** Given the function  $f(x) = \frac{2x-5}{x+2}$
- write down the equations of the asymptotes
  - sketch the function
  - write down the coordinates of the intercepts with both axes.
- 7** Let  $f(x) = 2 - x^2$  and  $g(x) = x^2 - 2$ .
- Sketch both functions on one graph with  $-3 \leq x \leq 3$ .
  - Solve  $f(x) = g(x)$ .

### EXAM-STYLE QUESTIONS

- 8 Let  $f(x) = x^3 - 3$ .
- Find the inverse function  $f^{-1}(x)$ .
  - Sketch both  $f(x)$  and  $f^{-1}(x)$  on the same axes.
  - Solve  $f(x) = f^{-1}(x)$ .
- 9  $f(x) = e^{2x-1} + \frac{2}{x+1}$ ,  $x \neq -1$ .
- Sketch the curve of  $f(x)$  for  $-5 \leq x \leq 2$ , including any asymptotes.
- 10 Consider the functions  $f$  and  $g$  where  $f(x) = 3x - 2$  and  $g(x) = x - 3$ .
- Find the inverse function,  $f^{-1}$ .
  - Given that  $g^{-1}(x) = x + 3$ , find  $(g^{-1} \circ f)(x)$ .
  - Show that  $(f^{-1} \circ g)(x) = \frac{x-1}{3}$ .
  - Solve  $(f^{-1} \circ g)(x) = (g^{-1} \circ f)(x)$
- Let  $h(x) = \frac{f(x)}{g(x)}$ ,  $x \neq 3$ .
- Sketch** the graph of  $h$  for  $-6 \leq x \leq 10$  and  $-4 \leq y \leq 10$ , including any asymptotes.
  - Write down the **equations** of the asymptotes.

## CHAPTER 1 SUMMARY

### Introducing functions

- A **relation** is a set of ordered pairs.
- The **domain** is the set of all the first numbers ( $x$ -values) of the ordered pairs.
- The **range** is the set of the second numbers ( $y$ -values) in each pair.
- A **function** is a relation where every  $x$ -value is related to a unique  $y$ -value.
- A relation is a function if any vertical line drawn will not intersect the graph more than once. This is called the **vertical line test**.

### The domain and range of a relation on a Cartesian plane

#### Interval notation:

Use round brackets ( , ) if the value is not included in the graph or when the graph is undefined at that point (a hole or **asymptote**, or a jump).

Use square brackets [ , ] if the value is included in the graph.

- Set notation:

