BECA / Huson / 12.1 IB Math SL Name:
20 April 2018

Homework: Mixed review, short answer (57 marks). ***Paper 1, no calculator***

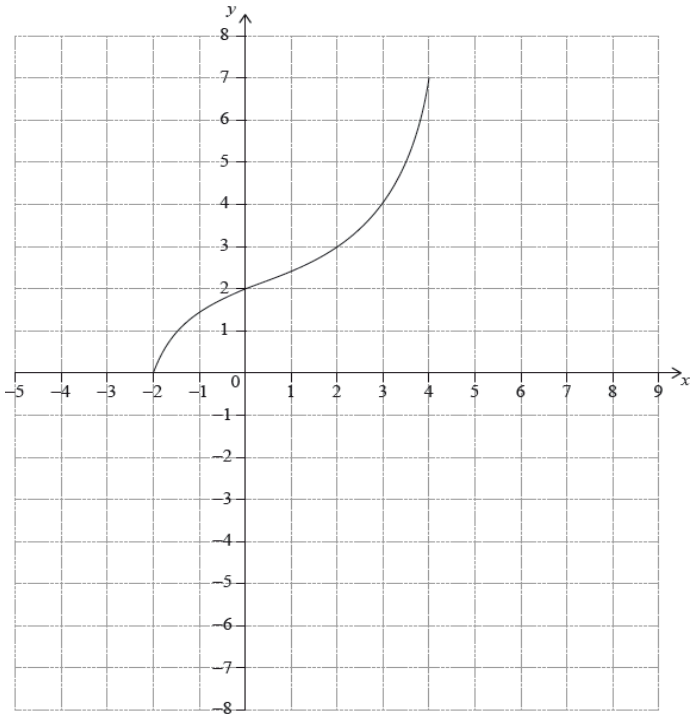
1a. In an arithmetic sequence, the first term is 8 and the second term is 5.

Find the common difference. *[2 marks]*

1b. Find the tenth term. *[2 marks]*

1c. Find the sum of the first ten terms. *[2 marks]*

2a. The following diagram shows the graph of a function f, with domain $-2 \leqslant x \leqslant 4$.



The points $(-2,\ 0)$ and $(4,\ 7)$ lie on the graph of f.

Write down the range of f. *[1 mark]*

2b. Write down $f(2)$; *[1 mark]*

2c. Write down $f^{-1}(2)$. *[1 mark]*

2d. On the grid, sketch the graph of f^{-1}. *[3 marks]*

3a. *[2 marks]*

Let $f(x) = 1 + \mathrm{e}^{-x}$ and $g(x) = 2x + b$, for $x \in \mathbb{R}$, where b is a constant.

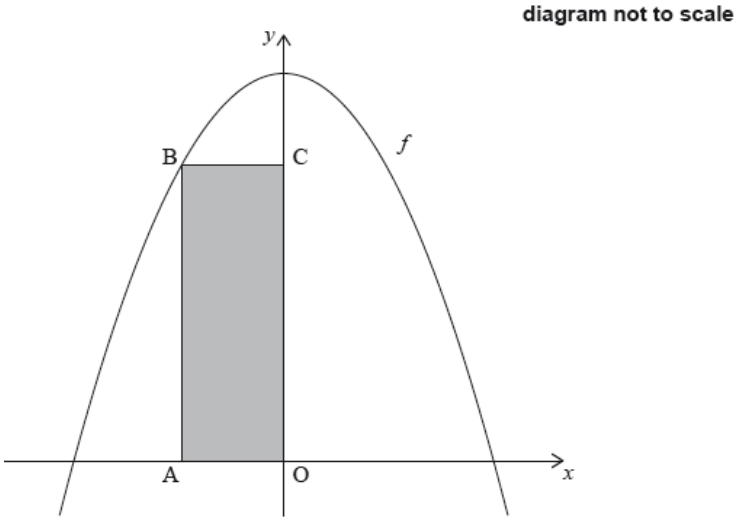
Find $(g \circ f)(x)$.

3b. *[4 marks]*

Given that $\lim_{x \to +\infty} (g \circ f)(x) = -3$, find the value of b.

4. *[7 marks]*

Let $f(x) = 15 - x^2$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f and the rectangle OABC, where A is on the negative x-axis, B is on the graph of f, and C is on the y-axis.



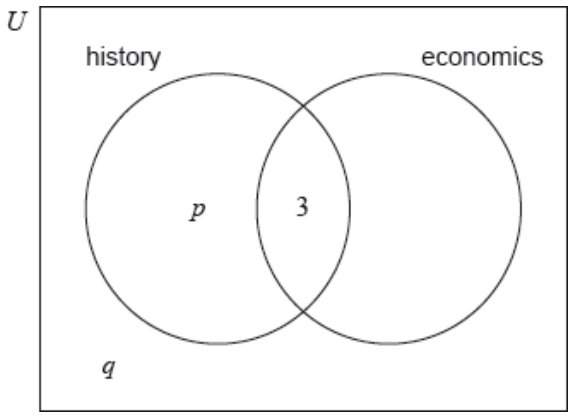
Find the x-coordinate of A that gives the maximum area of OABC.

5. *[7 marks]*

Consider $f(x) = \log k(6x - 3x^2)$, for $0 < x < 2$, where $k > 0$.

The equation $f(x) = 2$ has exactly one solution. Find the value of k.

6a. In a group of 20 girls, 13 take history and 8 take economics. Three girls take both history and economics, as shown in the following Venn diagram. The values p and q represent numbers of girls.



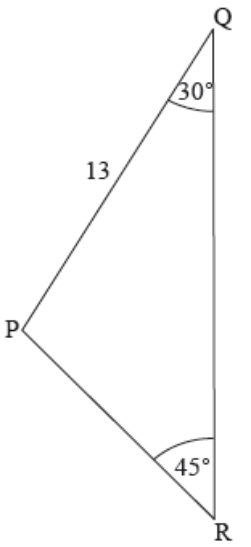
Find the value of p; *[2 marks]*

6b. Find the value of q. *[2 marks]*

6c. A girl is selected at random. Find the probability that she takes economics but not history. *[2 marks]*

7. The following diagram shows triangle PQR.

diagram not to scale



$\hat{PQR} = 30°$, $\hat{QRP} = 45°$ and $PQ = 13\,\text{cm}$.

Find PR. *[6 marks]*

8a. *[1 mark]*

Jim heated a liquid until it boiled. He measured the temperature of the liquid as it cooled. The following table shows its temperature, d degrees Celsius, t minutes after it boiled.

t (min)	0	4	8	12	16	20
d (°C)	105	98.4	85.4	74.8	68.7	62.1

Write down the independent variable.

8b. Write down the boiling temperature of the liquid. *[1 mark]*

8c. *[2 marks]*

Jim believes that the relationship between d and t can be modelled by a linear regression equation.

Jim describes the correlation as **very strong**. Circle the value below which best represents the correlation coefficient.

0.992 0.251 0 -0.251 -0.992

8d. *[2 marks]*

Jim's model is $d = -2.24t + 105$, for $0 \leqslant t \leqslant 20$. Use his model to predict the decrease in temperature for any 2 minute interval.

9a. *[4 marks]*

Find $\int x\mathrm{e}^{x^2-1}\mathrm{d}x$.

9b. *[3 marks]*

Find $f(x)$, given that $f'(x) = x\mathrm{e}^{x^2-1}$ and $f(-1) = 3$.