Geometry End of the Year Review Study Guide

Foundations of Geometry

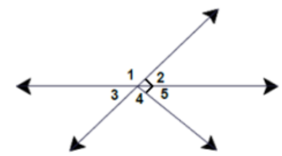
- **Inductive reasoning:** making a conjecture (guess) based on observation of patterns.
- **Deductive reasoning:** proving a statement based on facts (definitions, theorems, postulates, …)
- **Counterexample:** an example that disproves a statement.
- **Undefined terms:** point, line, plane.
- **Collinear:** on the same line.
- **Coplanar:** in the same plane.
- **Skew lines:** Non-coplanar and never intersect.
- **Postulate:** a statement that is assumed to be true (also called an "axiom").
- **Theorem:** a statement that must be proven true.

Reasoning and Proof

- **Hypothesis**: the "*if*" part of a conditional. (p)
- **Conclusion**: the "*then*" part of a conditional. (q)
- **Conditional statement:** an "if-then" statement.
- **Converse:** switch the "if" and the "then" parts of the conditional.
- **Inverse:** Negate both the "if" and the "then".
- **Contrapositive:** Switch and negate both.
- **Biconditional:** a conditional and its converse are both true and combined into one statement with "if and only if".
- **Counterexample**: a *specific* example where the *hypothesis* of a conditional is true but the *conclusion* is false.

Angle Relationships

- **Angle Bisector:** any figure that divides an angle into two congruent angles.
- **Midpoint of a Segment**: is a point that divides the segment into two congruent segments.
- **Segment Addition Postulate:** If B is between A and C, then AB + BC = AC.
- **Angle Addition Postulate:** If B is in the interior of $\angle AOC$, then m$\angle$AOB + m$\angle$BOC = m$\angle$AOC



- **Adjacent angles:** $\angle 3$ and $\angle 4$ (next to)
- **Vertical angles:** $\angle 2$ and $\angle 3$ ($\cong$)
- **Linear pair:** $\angle 1$ and $\angle 3$ (sum of 180)
- **Complementary Angles:** $\angle 2$ and $\angle 5$ (sum of 90)
- **Supplementary Angles:** $\angle 1$ and $\angle 3$ (sum of 180)

Properties of Equality and Congruence

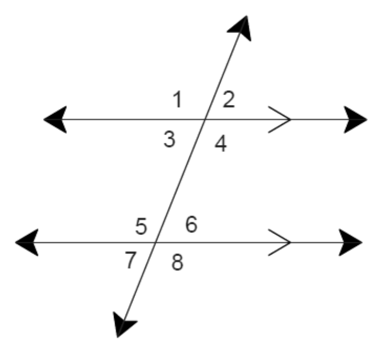
- **Reflexive Property of Equality:** a = a
- **Symmetric Property of Equality:** If a = b, then b = a.
- **Transitive Property of Equality:** If a = b and b = c, then a = c.
- **Substitution Property of Equality:** If a = b, then a can be substituted for b.

- **Reflexive Property of Congruence:** $\angle A \cong \angle A$
- **Symmetric Property of Equality:** If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.
- **Transitive Property of Equality:** If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$

Parallel and Perpendicular Lines

***Parallel Lines*:** If a **transversal** intersects parallel lines:

- **Corresponding Angles** are congruent. $\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$, $\angle 7 \cong \angle 3$, $\angle 8 \cong \angle 4$
- **Alternate Interior Angles** are congruent. $\angle 3 \cong \angle 6$, $\angle 5 \cong \angle 4$
- **Alternate Exterior Angles** are congruent. $\angle 1 \cong \angle 8$, $\angle 7 \cong \angle 2$
- **Consecutive Interior Angles** are supplementary (sum of 180). $\angle 5 + \angle 3 = 180$, $\angle 6 + \angle 4 = 180$
- **Consecutive Exterior Angles** are supplementary (sum of 180). $\angle 1 + \angle 7 = 180$, $\angle 2 + \angle 8 = 180$



◆ Use **Properties of Parallel Lines** to prove angle congruence.

◆ Use **Converses** to prove lines are parallel.

- If two lines are parallel to a third line, then they are parallel to each other.
- In a plane, if two lines are perpendicular to a third line, they are parallel to each other.

Triangle Angle Sum

Triangle angle sum: the angles in a triangle add up to 180 degrees.

Triangle exterior angles: each exterior angle is the sum of the two remote interior angles.

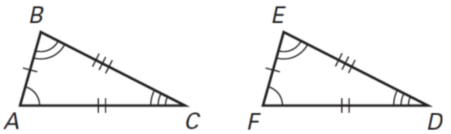
Polygon Angle Sum

Polygon angle sum for a polygon with *n* sides, the angles add up to $(n-2)180$

- Measure of a interior angle of a regular polygon is $\frac{(n-2)180}{n}$.
- Sum of the measures of exterior angles is 360.
- Measure of a single exterior angle is $\frac{360}{n}$.

Corresponding Parts

In two congruent figures, all the parts of one figure are congruent to the **corresponding parts** of the other figure.



$\triangle ABC \cong \triangle FED$

Corresponding angles: $\angle A \cong \angle F, \angle B \cong \angle E, \angle C \cong \angle D$

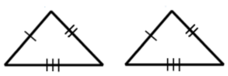
Corresponding sides: $\overline{AB} \cong \overline{FE}, \overline{BC} \cong \overline{ED}, \overline{AC} \cong \overline{FD}$

❖ *When you write a congruence statement always list the corresponding vertices in the same order.*

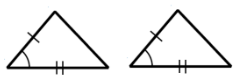
Congruent Triangles

Third Angle Theorem: If two angles of two triangles are congruent then the third angles are also congruent.

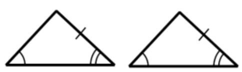
Triangle Congruence Postulates/Theorems:



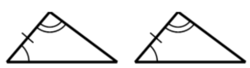
SSS



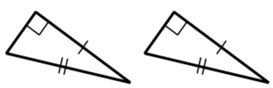
SAS



AAS



ASA

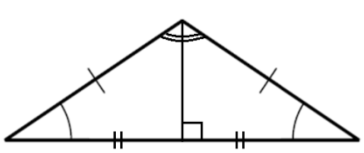


HL

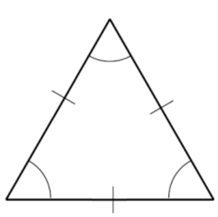
❖ Use **CPCTC** *after* proving triangles are congruent, to prove that parts of the triangles are congruent.

Isosceles and Equilateral Triangles

- **Isosceles Triangle Theorem:** If two sides of a triangle are congruent, then the angles opposite those two sides are congruent.
- In an **isosceles triangle,** the bisector of the vertex angle is the perpendicular bisector of the base.



- If a triangle is **equilateral**, then the triangle is **equiangular**.



Relationships Within Triangles

Point of concurrency: the point where 3 or more lines intersect.

Circumcenter (of a triangle): the point of concurrency of the perpendicular bisectors of a triangle.

- The circumcenter of a triangle is equidistant from the vertices.

Incenter: the point of concurrency of the angle bisectors.

- The incenter of a triangle is equidistant from the sides.

Centroid: the point of concurrency of the medians.

- The centroid is at a point on each median two-thirds of the distance from the vertex to the midpoint of the opposite side.

Orthocenter: the point of concurrency of the altitudes.

Midsegment: the segment that connects the midpoints of a two sides of a triangle.

- The midsegment is ½ the length of the 3rd side and is || to it.

Perpendicular Bisector: If a point lies on the perpendicular bisector of a segment, then it is equidistant from the endpoints.

Angle Bisector: If a point lies on the angle bisector of an angle, then it is equidistant from the sides of the angle.

Triangle Inequality

- The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
- The measure of the third side of a triangle must be less than the sum of the other two sides and greater than their difference.
- Longest side of a triangle is opposite the largest angle.
- Smallest side of a triangle is opposite the smallest angle.

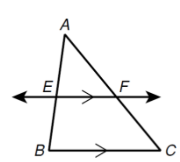
Similarity

Angle-Angle (AA) Similarity Postulate
If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.

Side-Side-Side (SSS) Similarity Theorem
If the corresponding side lengths of two triangles are proportional, then the triangles are similar.

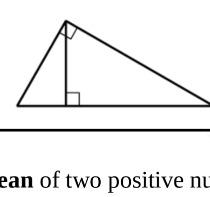
Side-Angle-Side (SAS) Similarity Theorem
If an angle of o ne triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.

- If a line parallel to a side of a triangle intersects the other two sides, then it divides those sides proportionally.



$$\frac{AE}{EB} = \frac{AF}{FC}$$

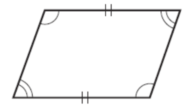
- The altitude to the hypotenuse of a right triangle forms two triangles that are similar to each other and to the original triangle.



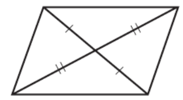
❖ The **geometric mean** of two positive numbers is the positive square root of their product.

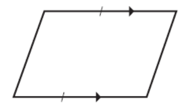
Quadrilaterals

Parallelograms:



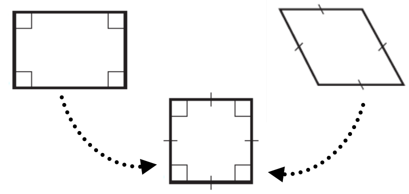
- If a quadrilateral is a parallelogram, then its opposite sides and its opposite angles are congruent.
- If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.
- If a quadrilateral is a parallelogram, then its diagonals bisect each other.
- If one pair of opposite sides of a quadrilateral is congruent and parallel, then the quadrilateral is a parallelogram.



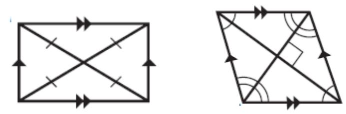


Special Parallelograms:

- A **quadrilateral is a rectangle** if and only if it has four right angles.
- A **quadrilateral is a rhombus** if and only if it has four congruent sides.
- A **quadrilateral is a square** if and only if it is a rhombus and a rectangle.

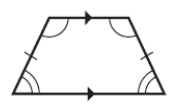


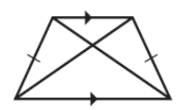
- A **parallelogram is a rectangle** if and only if its diagonals are congruent.
- A **parallelogram is a rhombus** if and only if its diagonals are perpendicular.
- A **parallelogram is a rhombus** if and only if each diagonal bisects a pair of opposite angles.



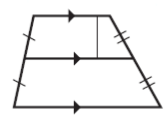
Trapezoids and Kites:

- If a **trapezoid is isosceles**, then each pair of base angles is congruent.
- A **trapezoid is isosceles** if and only if its diagonals are congruent.

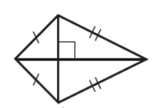


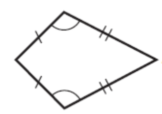


- The **midsegment of a trapezoid** is parallel to each base, and its length is one half the sum of the lengths of the bases.



- If a **quadrilateral is a kite**, then its diagonals are perpendicular.
- If a **quadrilateral is a kite**, then exactly one pair of opposite angles are congruent.





Right Triangle Trigonometry

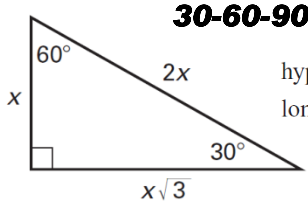
Pythagorean Theorem: $a^2 + b^2 = c^2$

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

- Converse of the Pythagorean Theorem: If $c^2 = a^2 + b^2$, then it is a right triangle.
- Acute triangle: If $c^2 < a^2 + b^2$, then it is an acute triangle.
- Obtuse triangle: If $c^2 > a^2 + b^2$, then it is an obtuse triangle.

Special right triangles:

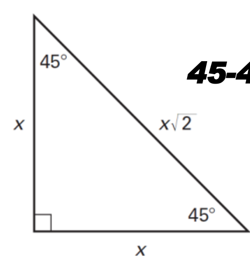
30-60-90 Triangle



hypotenuse = 2 • shorter leg

longer leg = shorter leg • $\sqrt{3}$

45-45-90 Triangle

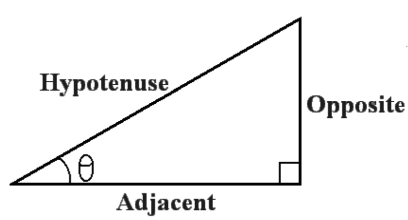


hypotenuse = leg • $\sqrt{2}$

Trigonometry:

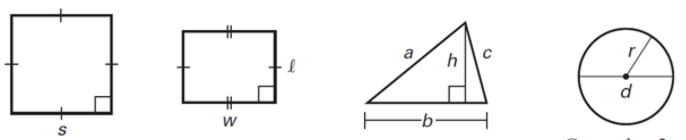
Is used to find the lengths of sides in a right triangle when *Pythagorean Theorem* or *Special Right Triangles* won't work.

- **Sine:** $\sin\theta = \dfrac{opposite}{hypotenuse}$ **(SOH)**
- **Cosine:** $\cos\theta = \dfrac{adjacent}{hypotenuse}$ **(CAH)**
- **Tangent:** $\tan\theta = \dfrac{opposite}{adjacent}$ **(TOA)**



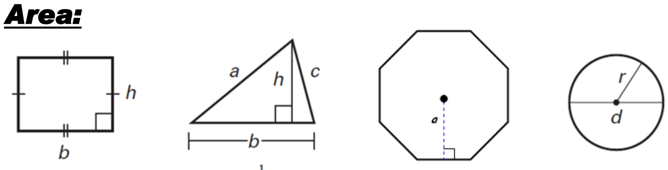
Perimeter and Area

Perimeter and Circumference:

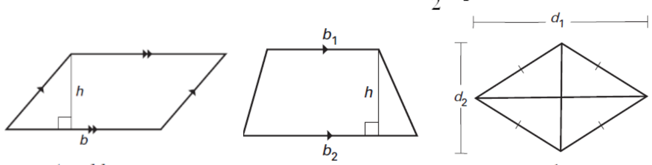


$P = 4s$ $\quad$ $P = 2\ell + 2w$ $\quad$ $P = a + b + c$ $\quad$ $C = \pi d = 2\pi r$

Area:



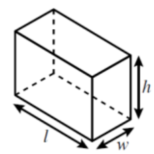
$A = bh$ $\quad$ $A = \frac{1}{2}bh$ $\quad$ $A = \frac{1}{2}a \cdot p$ $\quad$ $A = \pi r^2$



$A = bh$ $\quad$ $A = \frac{1}{2}h(b_1 + b_2)$ $\quad$ $A = \frac{1}{2}d_1 d_2$

Surface Area and Volume

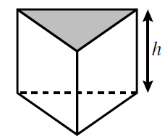
Prism



$L.A. = 2lw + 2lh$

$S.A. = 2lw + 2lh + 2wh$

$V = lwh$

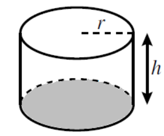


$L.A. = ph$

$S.A. = L.A. + 2B$

$V = Bh$

Cylinder

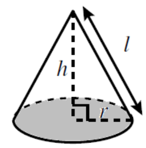


$L.A. = 2\pi rh$

$S.A. = 2\pi rh + 2B$

$V = \pi r^2 h$

Cone

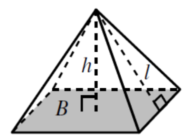


$L.A. = \pi rl$

$S.A. = \pi rl + B$

$V = \frac{1}{3}\pi r^2 h$

Pyramid

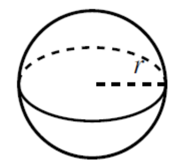


$L.A. = \frac{1}{2}pl$

$S.A. = L.A. + B$

$V = \frac{1}{3}Bh$

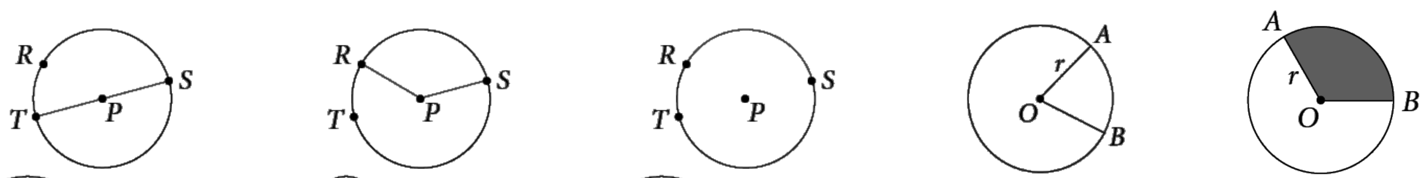
Sphere



$S.A. = 4\pi r^2$

$V = \frac{4}{3}\pi r^3$

Circles



$\widehat{TRS}$ is a semicircle. $m\widehat{TRS} = 180$

$\widehat{RS}$ is a minor arc. $m\widehat{RS} = m\angle RPS$

$\widehat{RTS}$ is a major arc. $m\widehat{RTS} = 360 - m\widehat{RS}$

length of $\widehat{AB} = \frac{m\widehat{AB}}{360} \cdot 2\pi r$

$A = \frac{m\widehat{AB}}{360} \cdot \pi r^2$

A **chord** is a segment whose endpoints are on a circle.

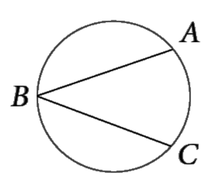
A **diameter** is a chord that contains the center of the circle.

A **secant** is a line that intersects a circle in two points.

A **tangent** is a line in the plane of a circle that intersects the circle in exactly one point, the *point of tangency*.

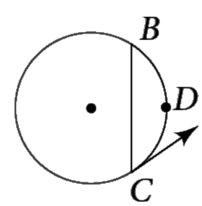
An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle.

Inscribed Angle



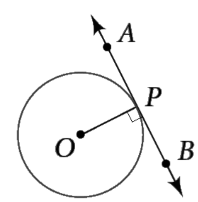
$m\angle B = \frac{1}{2}m\widehat{AC}$

Tangent-Chord



$m\angle C = \frac{1}{2}m\widehat{BDC}$

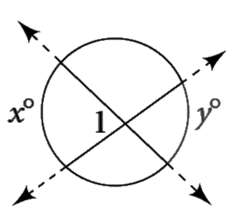
Tangent-Radius



$\overleftrightarrow{AB} \perp \overline{OP}$

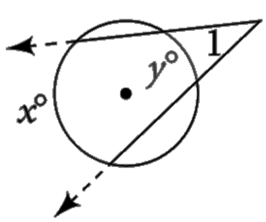
Intersecting Lines

Inside the Circle:



$m\angle 1 = \frac{1}{2}(x + y)$

Outside the Circle:

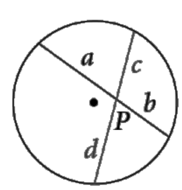


$m\angle 1 = \frac{1}{2}(x - y)$

Segment Lengths

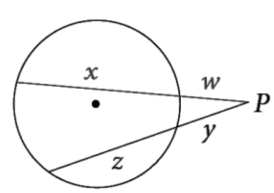
For a given point and circle, the product of the lengths of the two segments from the point to the circle is constant along any line through the point and circle.

Segments of Chords



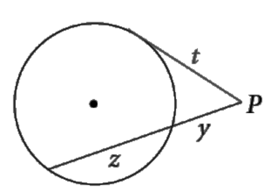
$a \cdot b = c \cdot d$

Segments of Secants



$(w + x)w = (y + z)y$

Segments of Tangent-Secant



$(y + z)y = t^2$

Coordinate Geometry

Slope formula : $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

- ***Parallel lines*** have equal slopes.
- ***Perpendicular lines*** have negative reciprocal slopes, and the product of their slopes equals -1. $\left(m_1 \cdot m_2 = -1\right)$

Distance Formula

- **Distance**: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint Formula

- **Midpoint:** $m = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

Transformations

A **transformation** is an operation that moves or changes a geometric figure in some way to produce a new figure. The new figure is called the **image**. Another name for the original image is the **preimage**.

A **translation** moves every point of a figure the same distance in the same direction. **(x, y)→(x+a, y+b)**

A **reflection** uses a line like a mirror to reflect an image. The mirror line is called the **line of reflection**.

Reflections

(x, y)→(x, -y)	Reflect over x-axis
(x, y)→(-x, y)	Reflect over y-axis
(x, y)→(-x, -y)	Reflect over both axes
(x, y)→(y, x)	Reflect over line $y = x$
(x, y)→(-y, -x)	Reflect over line $y = -x$

A **rotation** turns a figure about a fixed point, called the **center of rotation.**

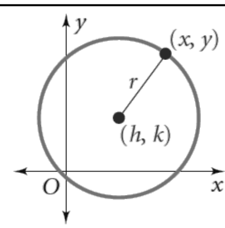
Rotations

(x, y)→(-y, x)	**90° rotation** (counter-clockwise)
(x, y)→(-x, -y)	**180° rotation**
(x, y)→(y, -x)	**270° rotation** (counter-clockwise)
(x, y)→(x, y)	**360° rotation**

Equation of a Circle

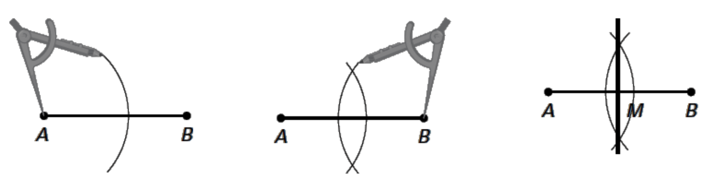
The **standard equation of a circle** with Center (h, k) and radius r is:

$$(x - h)^2 + (y - k)^2 = r^2.$$

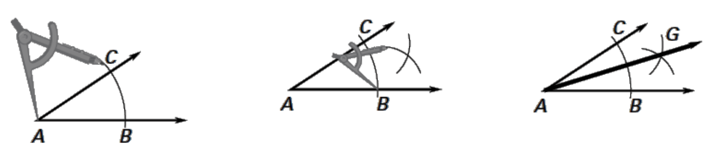


Constructions

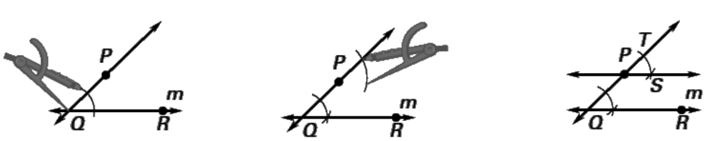
Construct a Perpendicular Bisector



Construct an Angle Bisector



Construct a Line Parallel to a Given Line



Copy an Angle