

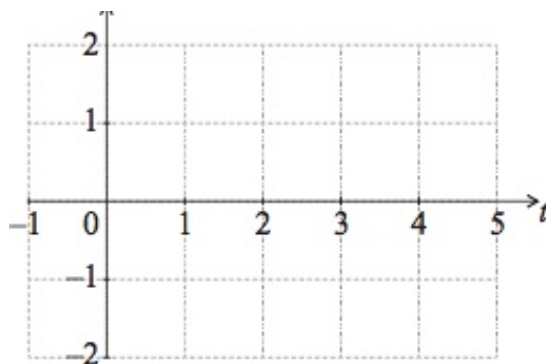
Test: Differential calculus

Show working for all problems. State answers exactly or to three significant figures.

Take the derivative of each function

1. $f(x) = x^2 - 2x + 11$.
2. $f(x) = \sqrt{x}$
3. $f(x) = x^2 e^x$
4. $f(x) = (x^2 - 5) \ln x$
5. $f(x) = \frac{\sin x}{x^3}$
6. $f(x) = \cos(1 - x^2)$
7. Let $f(x) = a(x - h)^2 + k$. The vertex of the graph of f is at $(2, 3)$ and the graph passes through $(1, 7)$.
 - (a) Write down the value of h and k .
 - (b) Find the value of a .
8. A function is given as $y = x^2 + kx - 20$.
 - (a) Find $\frac{dy}{dx}$.
 - (b) If the gradient of this function is 2 when x is 3, show that $k = -1$.
 - (c) Find the equation of the line tangent to the function through the point $(4, -8)$.
9. An arithmetic sequence is given by 5, 8, 11,
 - (a) Write down the value of d .
 - (b) Find
 - i. u_{100}
 - ii. S_{100}
 - (c) Given that $u_n = 1502$, find the value of n .

10. Let $f(x) = px^3 + px^2 + qx$.
- Find $f'(x)$.
 - Given that $f'(x) \geq 0$, show that $p^2 \leq 3pq$.
11. Given the function $f(x) = \ln x^2 + kx + 5, x \neq 0$.
- Find $f'(x)$.
 - The function $f(x)$ has a local maximum at $x = 2$. Show that $k = -1$
12. Given the function $f(x) = \frac{1}{x^2 - x - 2} + 3$.
- For what values of x is the function undefined?
 - Hence and otherwise, write down the equations of the two vertical asymptotes and one horizontal asymptote.
 - Find $f'(x)$.
 - Show that there is a local maximum at $x = \frac{1}{2}$
 - Find the equation of the normal to the function when $x = 1$.
13. The position of an object is given by the function $s = e^{\sin t} - 1$, for $0 \leq t \leq 5$.
- On the grid below, sketch the graph of s . (set your calculator to radians)



- Write down the positive t -intercept.
- Find the velocity of the object, $v(t)$.