7.7 Exam: Similarity ratios, dilation, the tangent function, transformations, symmetry

1. Given the following two linear equations:

   $l_1 : y = \frac{5}{4}x - 3$ $l_2 : 5x + 4y = 8$

   Write down the slopes of the two lines.

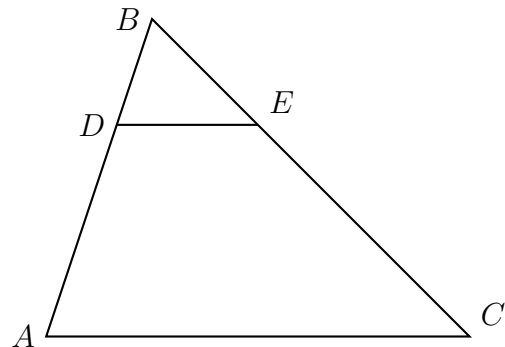 $m_1 =$ $m_2 =$

   Are the lines parallel, perpendicular, or neither? Justify your answer using the slopes.

2. Given $\triangle ABC \sim \triangle DEF$. $m\angle A = 88°$ and $m\angle F = 43°$. Find the measure of $\angle C$.

3. In the diagram below of $\triangle ABC$, D is a point on $\overline{BA}$, E is a point on $\overline{BC}$, and $\overline{DE}$ is drawn.
   If $BD = 6.5$, $DA = 13$, and $BE = 8$, what is the length of $\overline{BC}$ so that $\overline{AC} \parallel \overline{DE}$?
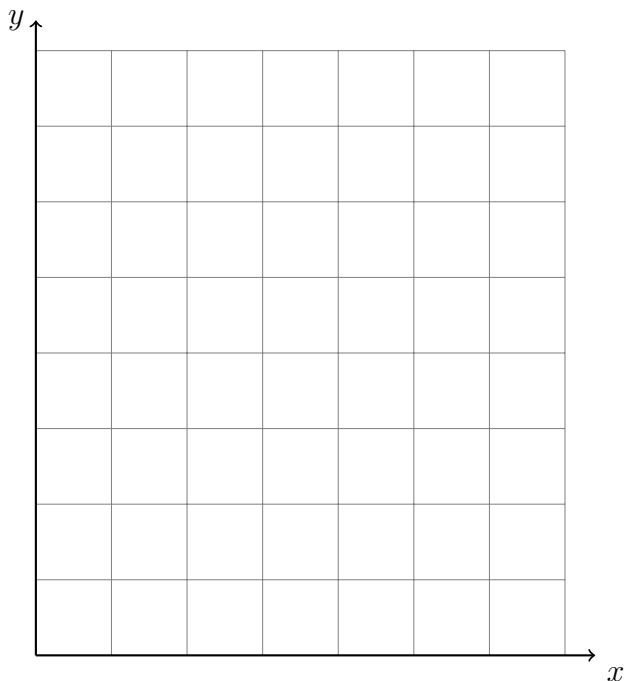

4. Find the image of $P(3, -5)$ after the translation $(x, y) \to (x - 5, y + 8)$.

5. (a) Graph and label $\triangle ABC$ with $A(0,0)$, $B(5,6)$, and $C(5,0)$. Calculate each length:

   i. $AC =$

   ii. $BC =$

   iii. $AB =$

(b) Write down the equation of the line $\overleftrightarrow{BC}$.

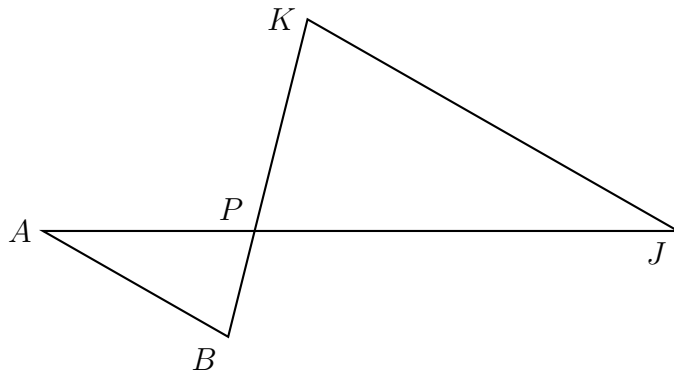(c) Write down the equation of the line $\overleftrightarrow{AB}$.

(d) The tangent of an angle is the ratio of the side lengths *opposite* over *adjacent* to the angle. Write down the value as a fraction.

   $\tan \angle BAC =$

(e) Find $m\angle A$ with a calculator's inverse tangent function, $m\angle BAC = \tan^{-1}(\frac{opp}{adj})$

6. Given $\triangle ABP \sim \triangle JKP$ as shown below. $AB = 13.5$, $AP = 10.0$, $BP = 9$, and $JP = 27.0$. Find JK.

7. The line l has the equation $y = \frac{3}{2}x + 5$. To each line below, circle whether l is parallel, perpendicular, or neither.
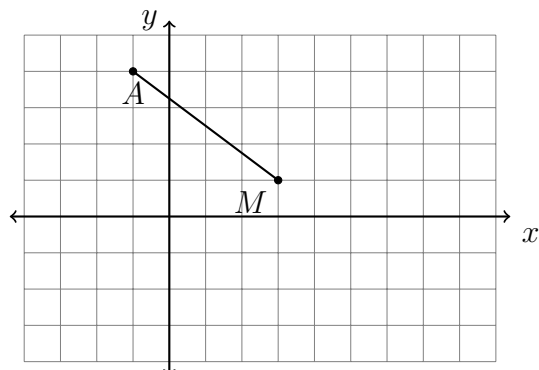
   (a) parallel   perpendicular   neither $y = \frac{3}{2}x - 2$

   (b) parallel   perpendicular   neither $y = \frac{2}{3}x + 7$

   (c) parallel   perpendicular   neither $3x - 2y = -6$

   (d) parallel   perpendicular   neither $2x + 3y = 9$

8. $A(-1, 4)$ is one endpoint of $\overline{AB}$. The segment's midpoint is $M(3, 1)$, as shown below. Find the coordinates of the other endpoint, B.

9. The vertices of $\triangle JKL$ have the coordinates $J(-4, -2)$, $K(4, 2)$, and $L(-2, 4)$, as shown.

Apply a dilation to $\triangle JKL \to \triangle J'K'L'$, centered on the origin and with a scale factor $k = 1.5$. Draw the image $\triangle J'K'L'$ on the set of axes below, labeling the vertices, and make a table showing the correspondence of both triangles' coordinate pairs.
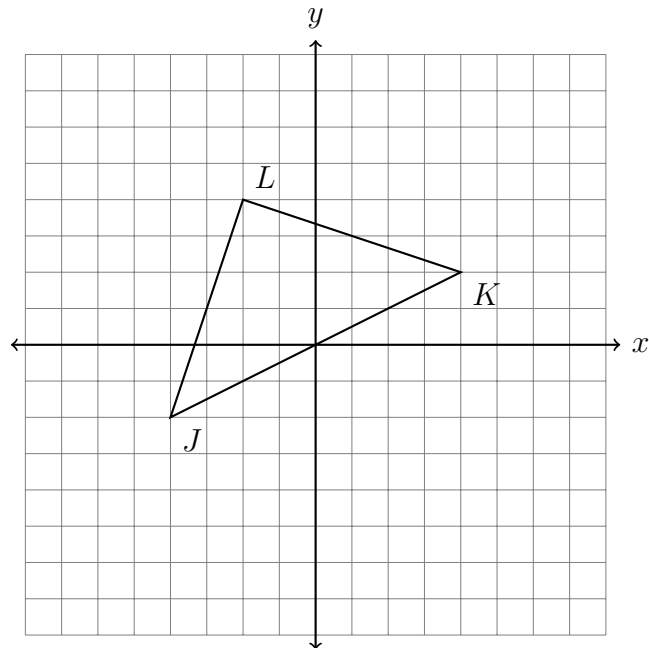
10. Given isosceles $\triangle ABC$ with $\overline{AB} \cong \overline{BC}$, $m\angle A = 53$. Mark and label the diagram, and then find $m\angle B$. (*the diagram is not to scale*)
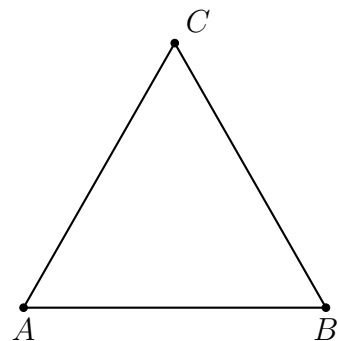
11. A translation maps $N(-3, 7) \to N'(-4, 1)$. What is the image of $M(0, -5)$ under the same translation?
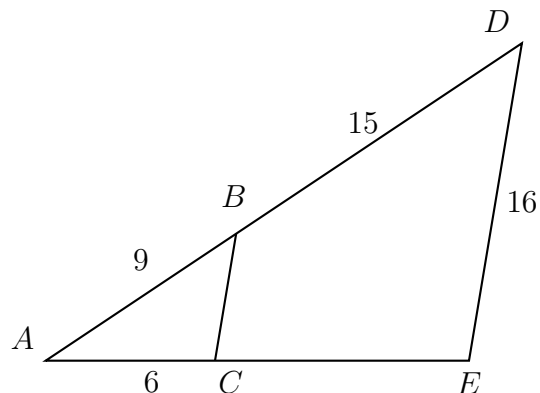
12. A dilation centered at A maps $\triangle ABC \rightarrow \triangle ADE$. Given $AB = 9$, $AC = 6$, $BD = 15$, and $DE = 16$. Find AD and the scale factor k. Then find AE and BC.
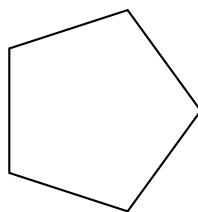
   (a) $AD =$
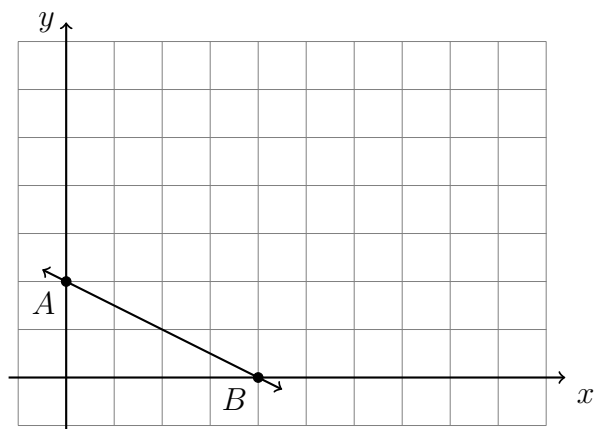
   (b) $k =$

   (c) $AE =$

   (d) $BC =$

13. What is the smallest non-zero angle of rotation about its center that would map the pentagon onto itself?

14. The line $\overleftrightarrow{AB}$ has the equation $y = -\frac{1}{2}x + 2$. Apply a dilation mapping $\overleftrightarrow{AB} \rightarrow \overleftrightarrow{A'B'}$ with a factor of $k = 2$ centered at the origin. Draw and label the image on the grid. Write the equation of the line $\overleftrightarrow{A'B'}$.
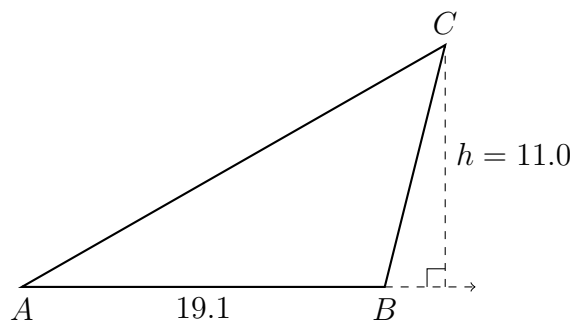
15. Given right $\triangle ABC$ with $m\angle C = 90°$, $AC = 13$, $m\angle A = 35°$. Find BC, rounded to the *nearest tenth*.
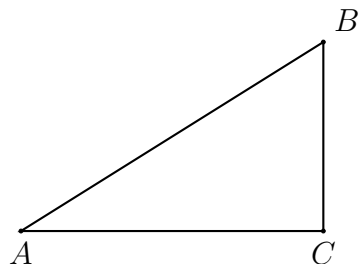
16. What transformation or series of transformations map $\triangle ABC$ onto $\triangle DEF$, shown below? Fully specify the transformation(s).
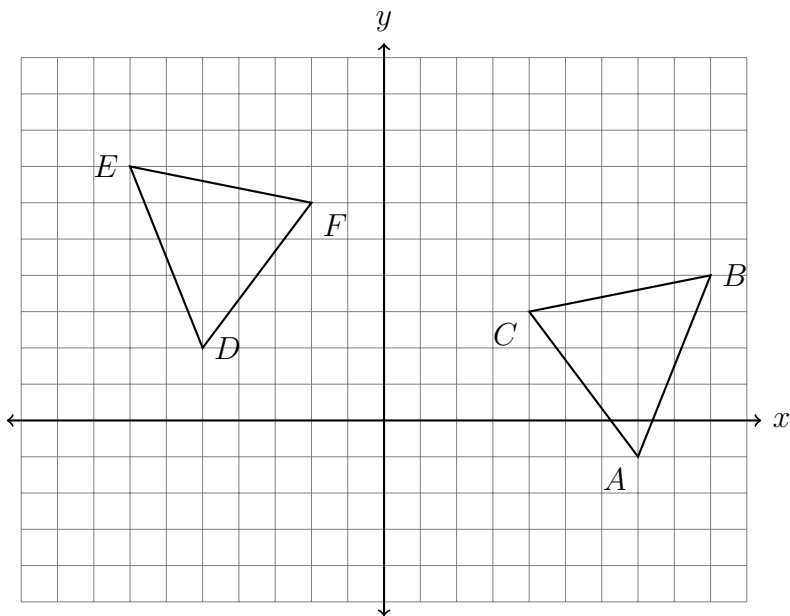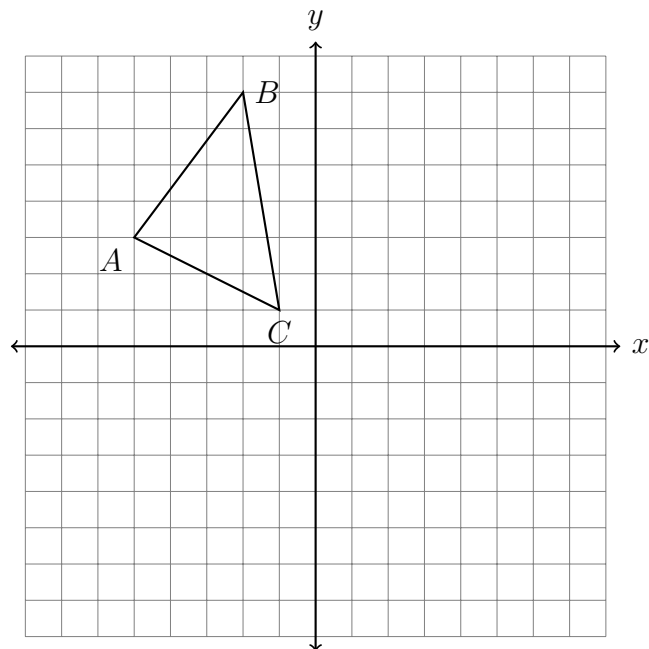

17. The side $\overline{AB}$ of triangle ABC is extended and an altitude to the vertex C is drawn, as shown below. The triangle's height is $h = 11.0$ and its base measures $AB = 19.1$. Find the area of the triangle.

18. Reflect $\triangle ABC$ over the y-axis. Make a table of the coordinates and plot and label the image on the axes.
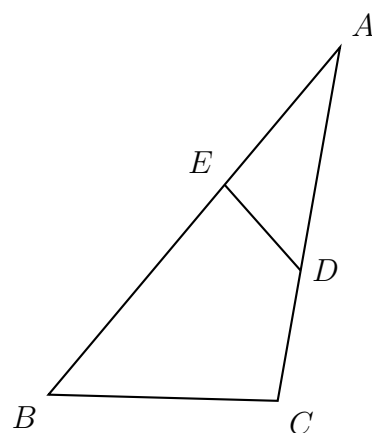
19. The diagram below shows $\triangle ABC$, with $\overline{AEB}$, $\overline{ADC}$, and $\angle ACB \cong \angle AED$. $AB = 18$, $AD = 12$, $AE = 9$, and $DE = 7$. Find the scale factor k, AC, and BC.
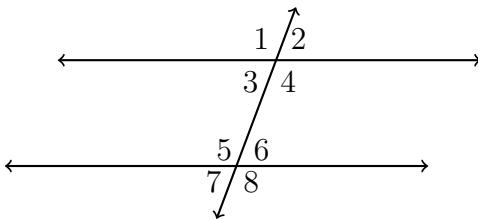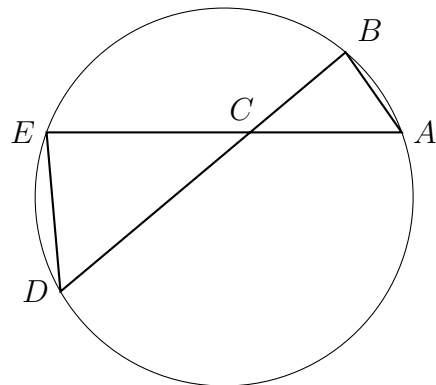
(a) $k =$

(b) $AC =$

(c) $BC =$

20. Find the midpoint M of $\overline{AB}$ with coordinates $A(-3, 1)$ and $B(7, 4)$.

21. Given two parallel lines and a transversal, as shown below. Given $m\angle 1 = 108°$.

(a) Find the measure $m\angle 2$.

(b) Find the measure $m\angle 8$.

(c) Given $m\angle 5 = (6x - 12)°$. Find x.

22. In the diagram below, the chords $\overline{AE}$ and $\overline{BD}$ intersect at C. Given $\triangle ABC \sim \triangle DEC$, $BC = 6$, $CD = 10$, and $CE = 8$. Determine the length of $\overline{CA}$.
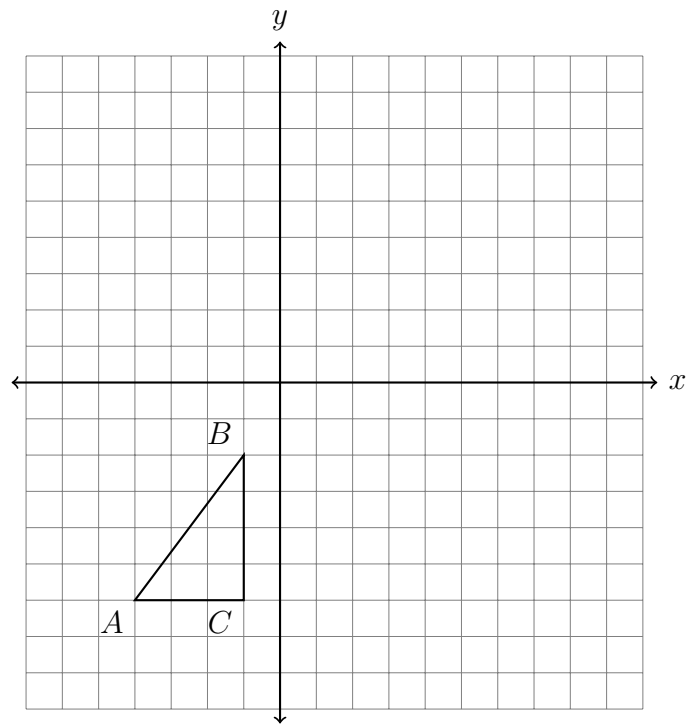

23. Given two points $A = -4.7$ and $B = 3.3$. Find the value of the midpoint M between A and B, and mark and label it on the numberline below.
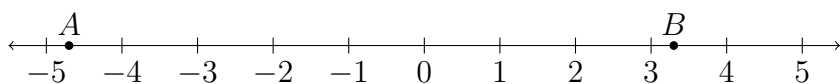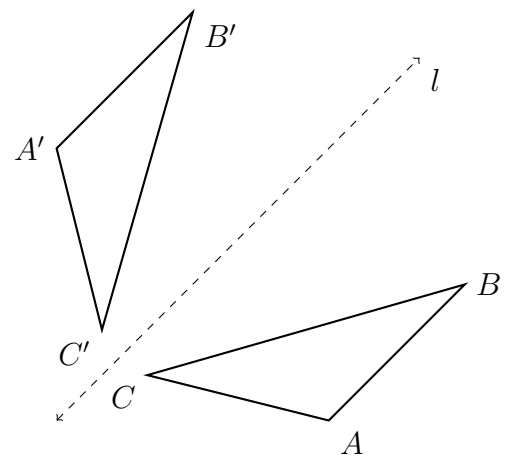
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24. Rotate $\triangle ABC$ 90° counterclockwise around the origin, yielding $\triangle A'B'C'$. Then translate it by $(x, y) \rightarrow (x + 2, y + 7)$. Make a table of the coordinates showing $\triangle ABC \rightarrow \triangle A'B'C' \rightarrow \triangle A''B''C''$ and plot and label the images on the axes.

25. The $\triangle ABC$ is reflected across l to yield $\triangle A'B'C'$. $AB = 4x + 4$, $A'B' = 7x - 8$, and $BC = 5x + 10$. Find the length $B'C'$.

Using the distance formula to prove an isosceles triangle

26. In this problem use the following theorem (copy it at the bottom of the page after your calculations):

    A triangle is isosceles if and only two of its sides are congruent.

    Shown below is triangle ABC, $A(-2, 2)$, $B(4, 5)$, and $C(1, -1)$.

    Prove it is an isosceles triangle by

    (a) finding the length of each of the three sides,
    (b) stating which sides are congruent,
    (c) copying the theorem as your conclusion, adding *therefore $\triangle ABC$ is isosceles.*