

Spiral Review: 6-1 P1 (No Calculator) Calculus Tangents

1. 10N.1.sl.TZ0.2

Let $g(x) = 2x \sin x$.

(a) Find $g'(x)$ [4 marks]

(b) Find the gradient of the graph of g at $x = \pi$. [3 marks]

2. 12M.1.sl.TZ1.3

Let $f(x) = e^{6x}$.

(a) Write down $f'(x)$ [1 mark]

(b) The tangent to the graph of f at the point $P(0, b)$ has gradient m . [4 marks]

i. Show that $m = 6$.

ii. Find b .

(c) Hence, write down the equation of this tangent. [1 mark]

3. 09M.1.sl.TZ1.3

Let $f(x) = e^x \cos x$. Find the gradient of the normal to the curve of f at $x = \pi$.

4. 13M.1.sl.TZ1.3

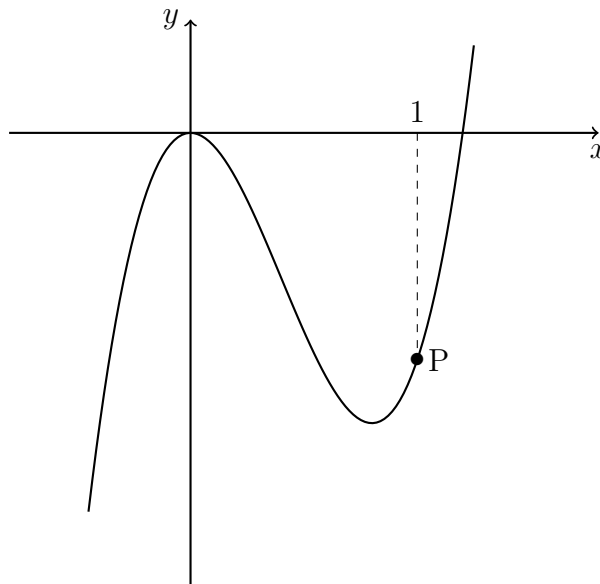
Consider $f(x) = x^2 \sin x$.

(a) Find $f'(x)$. [4 marks]

(b) Find the gradient of the curve of f at $x = \frac{\pi}{2}$. [3 marks]

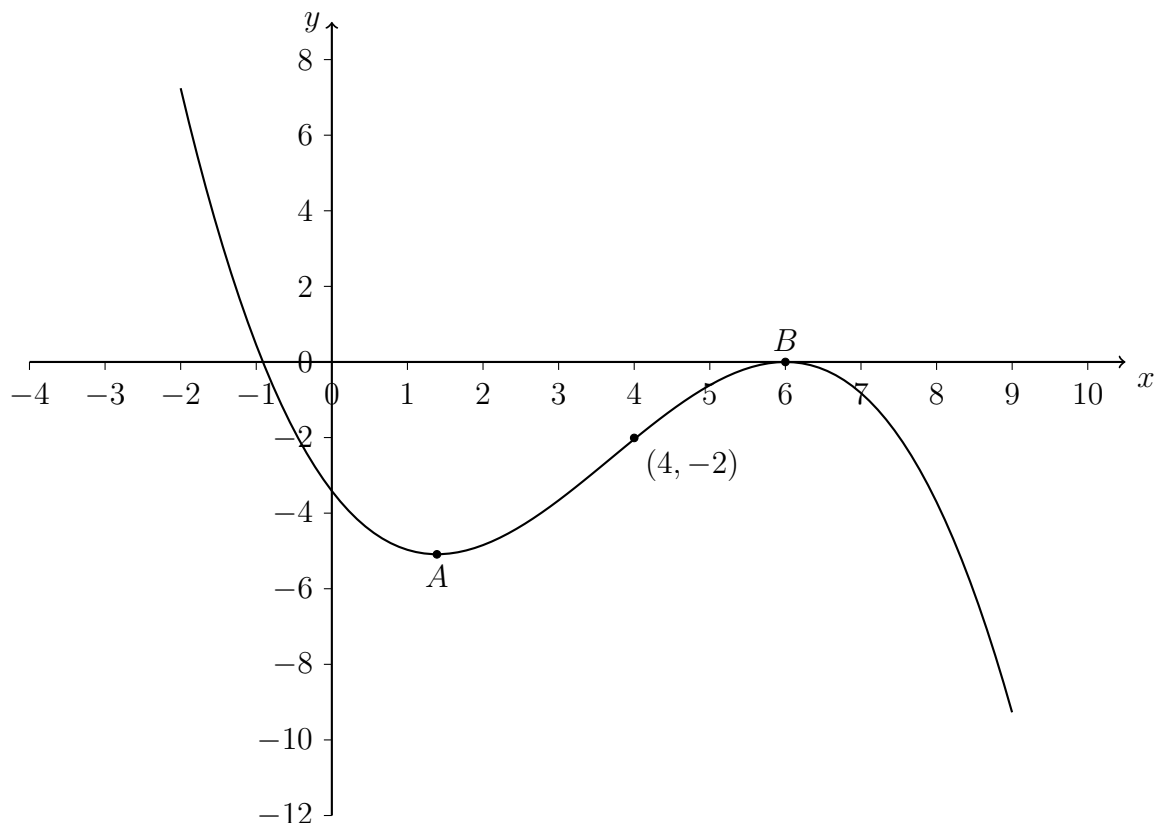
5. 12N.1.sl.TZ0.4

Part of the graph of $f(x) = ax^3 - 6x^2$ is shown below.



The point P lies on the graph of f . At P , $x = 1$.

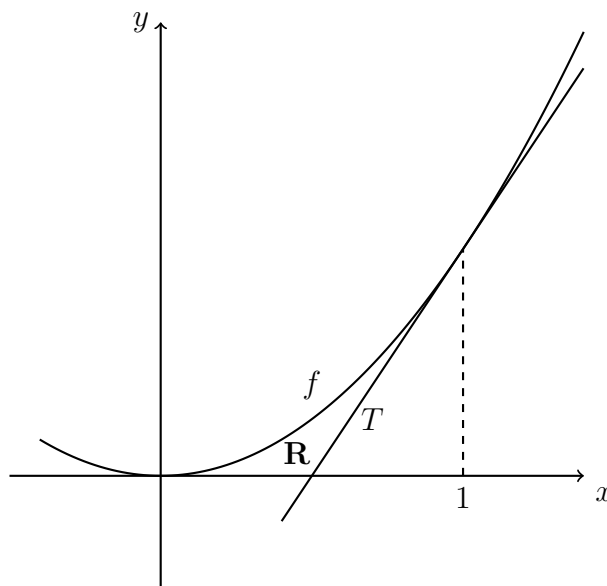
- (a) Find $f'(x)$. [2 marks]
- (b) The graph of f has a gradient of 3 at the point P . Find the value of a . [4 marks]
6. 17N.1.sl.TZ0.5
Let $f(x) = 1 + e^{-x}$ and $g(x) = 2x + b$, for $x \in \mathbb{R}$, where b is a constant.
- (a) Find $(f \circ g)(x)$. [2 marks]
- (b) Given that $\lim_{n \rightarrow \infty} (f \circ g)(x) = -3$, find the value of b . [4 marks]
7. 10M.1.sl.TZ2.5 [6 marks]
Let $f(x) = kx^4$. The point $P(1, k)$ lies on the curve of f . At P , the normal to the curve is parallel to $y = -\frac{1}{8}x$. Find the value of k .
8. 13N.1.sl.TZ0.6 [6 marks]
Let $f(x) = e^{2x}$. The line L is the tangent to the curve of f at $(1, e^2)$.
Find the equation of L in the form $y = ax + b$.
9. 17M.1.sl.TZ1.6
The following diagram shows the graph of f' , the derivative of f .



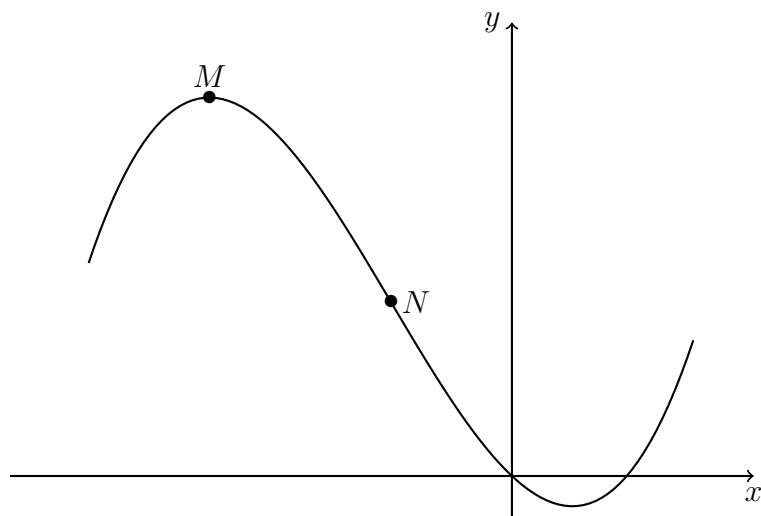
The graph of f' has a local minimum at A , a local maximum at B and passes through $(4, 2)$. The point $P(4, 3)$ lies on the graph of the function, f .

Name:

- (a) Write down the gradient of the curve of f at P . [1 mark]
- (b) Find the equation of the normal to the curve of f at P . [3 marks]
- (c) Determine the concavity of the graph of f when $4 < x < 5$ **and** justify your answer. [2 marks]
10. 18M.1.sl.TZ1.7 [7 marks]
Consider $f(x)$, $g(x)$ and $h(x)$, for $x \in \mathbb{R}$ where $h(x) = (f \circ g)(x)$.
Given that $g(x)$, $g'(3) = 4$, and $f'(7) = -5$, find the gradient of the normal to the curve of h at $x = 3$.
11. 11M.1.sl.TZ2.8
The following diagram shows part of the graph of the function $f(x) = 2x^2$.



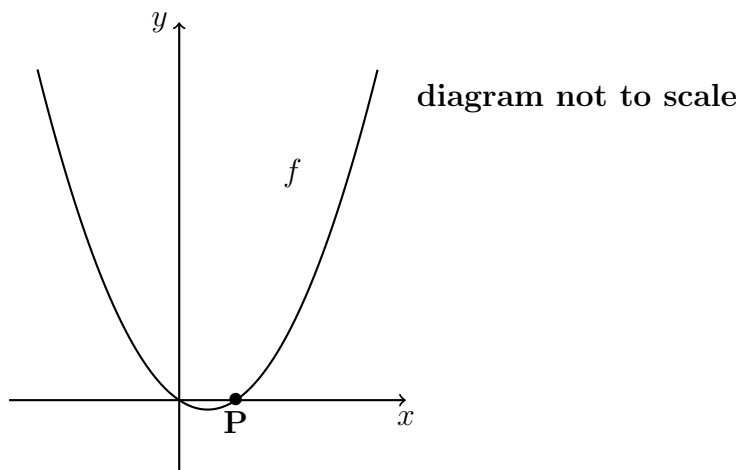
- The line T is the tangent to the graph of f at $x = 1$.
- (a) Show that the equation of T is $y = 4x - 2$. [5 marks]
- (b) Find the x -intercept of T . [2 marks]
- (c) The shaded region R is enclosed by the graph of f , the line T , and the x -axis. [9 marks]
- Write down an expression for the area of R .
 - Find the area of R .
12. 08M.1.sl.TZ1.8
Consider $f(x) = \frac{1}{3}x^3 + 2x^2 - 5x$. Part of the graph of f is shown below. There is a maximum point at M , and a point of inflexion at N .



- (a) Find $f'(x)$. [3 marks]
- (b) Find the x-coordinate of M . [4 marks]
- (c) Find the x-coordinate of N . [3 marks]
- (d) The line L is the tangent to the curve of f at $(3, 12)$. Find the equation of L in the form $y = ax + b$. [4 marks]

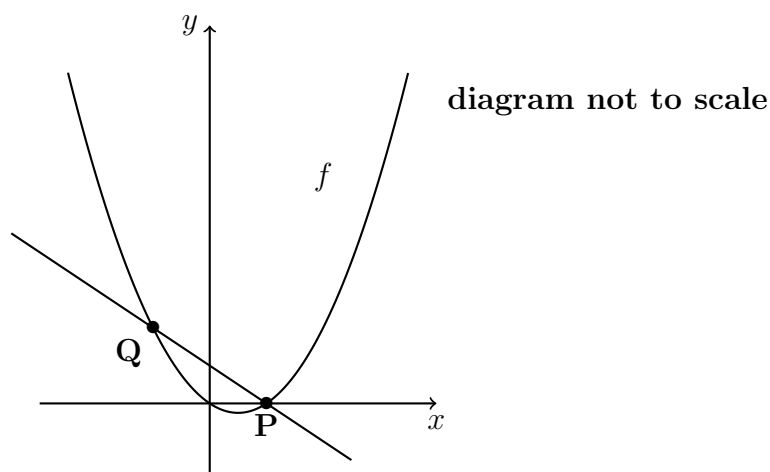
13. 13

Let $f(x) = x^2 - x$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f .



The graph of f crosses the x -axis at the origin and at the point $P(1, 0)$.

- (a) Show that $f'(1) = 1$. [3 marks]
- (b) The line L is the normal to the graph of f at P .
Find the equation of L in the form $y = ax + b$. [3 marks]
- (c) The line L intersects the graph of f at another point Q , as shown in the following diagram.



Find the x -coordinate of Q . [4 marks]

(d) Find the area of the region enclosed by the graph of f and the line L . [6 marks]

14. 14

A quadratic function f can be written in the form $f(x) = a(x - p)(x - 3)$. The graph of f has an axis of symmetry $x = 2.5$ and y -intercept at $(0, -6)$.

(a) Find the value of p . [3 marks]

(b) Find the value of a . [3 marks]

(c) The line $y = kx - 5$ is a tangent to the curve of f . Find the values of k . [8 marks]

15. 15

A function f has its derivative given by $f'(x) = 3x^2 - 2kx - 9$, where k is a constant.

(a) Find $f''(x)$. [2 marks]

(b) The graph of f has a point of inflexion when $x = 1$.
Show that $k = 3$. [3 marks]

(c) Find $f'(2)$. [2 marks]

(d) Find the equation of the tangent to the curve of f at $(-2, 1)$, giving your answer in the form $y = ax - b$. [4 marks]

(e) Given that $f'(-1) = 0$, explain why the graph of f has a local maximum when $x = -1$. [3 marks]

16. 16

Let $f(x) = \sin x + \frac{1}{2}x^2 - 2x$, for $0 \leq x \leq \pi$.

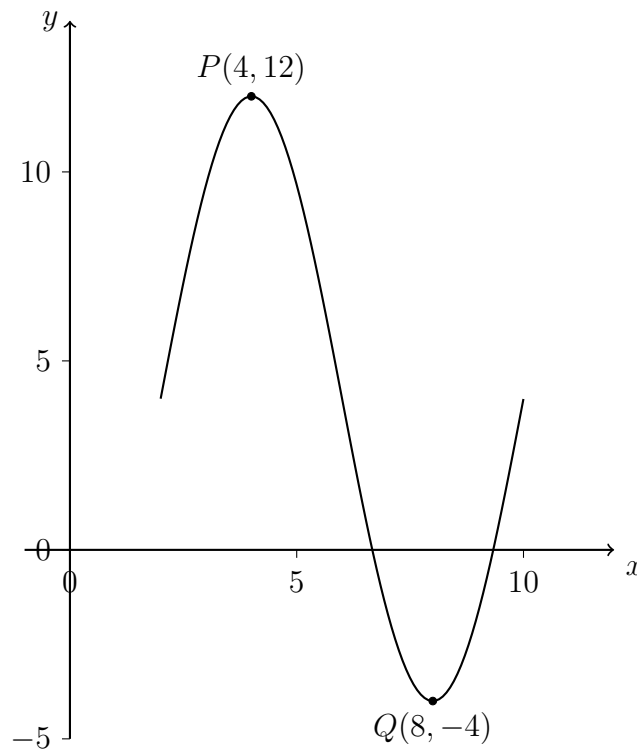
(a) Find $f'(x)$. [3 marks]

(b) Let g be a quadratic function such that $g(0) = 5$. The line $x = 2$ is the axis of symmetry of the graph of g .
Find $g(4)$. [3 marks]

- (c) The function g can be expressed in the form $g(x) = a(x - h)^2 + 3$.
- Write down the value of h .
 - Find the value of a .
- (d) Find the value of x for which the tangent to the graph of f is parallel to the tangent to the graph of g . [6 marks]

17. 17

The following diagram shows the graph of $f(x) = a \sin(b(x - c)) + d$, for $2 \leq x \leq 10$.



There is a maximum point at $P(4, 12)$ and a minimum point at $Q(8, -4)$.

- (a) Use the graph to write down the value of [3 marks]
- a ;
 - c ;
 - d .
- (b) Show that $b = \frac{\pi}{4}$. [2 marks]
- (c) Find $f'(x)$. [2 marks]
- (d) At a point R , the gradient is -2π . Find the x -coordinate of R .