Mathematics Class Slides

Bronx Early College Academy

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August [was 23 May] 2018

How do we use and interpret trigonometric functions?

HSF.LB.B.5 Interpret the parameters in a trigonometric function in context 11.2

Do Now: Exam review problems

1. Handout practice of graphing and solving

Lesson: Using periodic functions for modeling
Task: Regents practice - Complete mastery grid cells
Assessment: Test tomorrow
Homework: Study pretest problems for test

How do we apply trig to solve problems?

HSG Define trigonometric ratios and solve problems involving right triangles 11.1

Do Now handout: Sine graph, complex number review

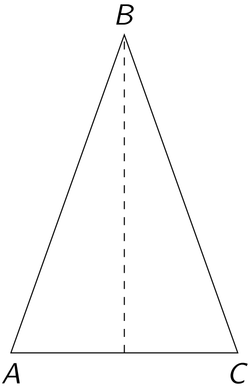
1. Confirm that your calculator is in radian mode
2. Use the Table → SET (F5) function
3. For complex number problems, first write down the four powers of i

Lesson: The Pythagorean identity p. 378. The sine rule p. 381
Assessment: Exercise 11G #1a p. 383
Homework: Read through p. 382; Exercises 11G #1-4 p. 383

Exercise 11C #1

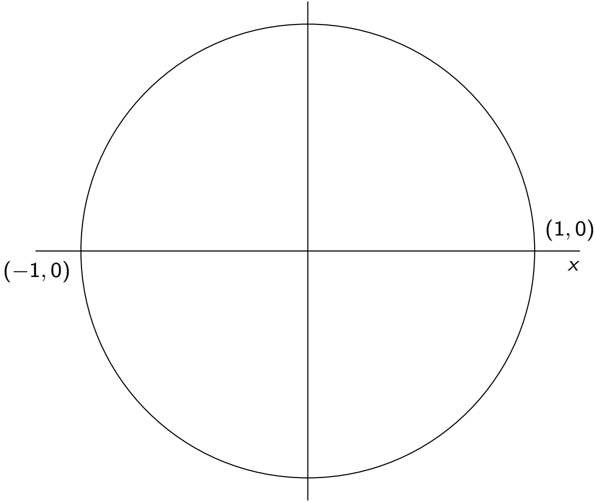
Isosceles triangle ABC has base $AC = 10$ cm and sides $AB = CB = 15$ cm.

- Find the height of the triangle
- Find the sizes of $B\hat{A}C$ and $A\hat{B}C$



Unit circle

Circle with radius of one centered on the origin.



Communication protocols

How to send information and common conventions to follow

- Mail ("post"), by messenger: formal, fancy, legally secure
- Text (cell phone): brief, informal, immediate, transitory
- Email: versatile, threaded, transitory or permanent (insecure)
  Subject line, salutations, handle@domain
- Attachments: .pdf is universally readable, can't be edited
  Microsoft Word, Excel, .ppt can be edited, commented
- Link: extended collaboration, commercial paywalls
  If you are not paying for it, you're not the customer; you're the product being sold.

Extra help

- Algebra 2 Regents prep
  7th period pullout to room 414 (sometimes 1st period)
  Twice a week
  Alesha, Elisabeth, Nicole, Emelyn, Stephen, Mivian, Joshua

- IB Math, exploration paper, general help
  Thursday lunch (usually Mondays & Fridays too)
  Thursday after school room 414 (Johnsen & Guarnaccia)
  Saturday (Guarnaccia)

Graphing on a calculator to solve equations

To solve an equation, separate the equality into two functions,

$$f(x) = g(x)$$

The intersection of their graphs is the solution.

- Solve for x: $|x-1|-3=-2x^2+x+3$.
- As a check, first make a quick sketch of the functions.

Notes:
Learn to resize the calculator window efficiently
Use the calculator's graph-solve function
Even simple equations can be solved this way. e.g. $e^{0.12x}=5.25$

Steps for writing technical papers

Practice writing mathematics according to IB requirements, as per IA criteria.

Proposal

1. Define an “aim,” including success criteria.
2. Outline paper, especially Method including data collection, graphs, formulas; list references
3. Draft introduction, including rationale and aim.
4. Structure data tables, sketch graphs, begin formula and algebra (all handwritten, perhaps spreadsheets or Desmos)
5. Draft Method section text

Method

1. Collect data (survey, search, simulation, etc.)
2. Work interactively with spreadsheets, graphing software, math
3. Refine Method section, draft results and discussion.

Complete mathematics and paper. Proofread carefully. Rewrite. Receive peer feedback. Rewrite. Submit final draft.

Standards for writing technical papers

Practice writing mathematics according to IB requirements, as per IA criteria.

Criterion C: Personal engagement (0-4 points)

1. Address a personal interest; "make it your own"
2. Think independently and/or creatively
3. Present mathematical ideas in your own way

Criterion D: Reflection (0-3 points)

1. Review, analyze, and evaluate the mathematics throughout the paper. Go beyond just describing results
2. Link to the aims, comment on what has been learned, consider limitations, and compare different mathematical approaches
3. Consider what's next, discuss the implications of results, strengths and weaknesses of approaches, and consider different perspectives

Technical writing

Write a short paper answering the query:
"How many subsets can be picked from a group of four students?"

1. Logical, step-by-step explanation, using an example
2. Precise terminology, succinct: combination, permutation, order (matters), event, sample space, set, subset, with /without replacement, factorial
3. Notation: algebra symbols, tables, trees, grids
4. Summary, big-picture, conceptual idea
5. Audience: student peers

Standard conventions for mathematical notation

Practice writing mathematics according to IB requirements, as per exam rubrics.

1. Use the formula sheet.
2. Chose the appropriate formula (M1).
   (you do not have to copy the formula)
3. Substitute values correctly (A1).
4. Solve, showing key steps (A1).
   (skip routine algebra if you like)
5. Write down the exact solution or copy the calculator display.
   An ellipsis (. . .) indicates more digits (A1).
6. Round to 3 significant digits (use $\approx$)(A1).

Standard conventions for mathematical notation

Practice writing mathematics according to IB requirements, as per exam rubrics.

Examples of key algebraic techniques

1. Setting a quadratic function $= 0$
2. Converting an exponent to a log
3. Reading a value from a graph
4. When writing lists, you may write only the first two and the last terms. For example,

$$\sum_{k=1}^{5} 3 \cdot 2.25^k = 3 + 6.75 + \ldots + 76.8867\ldots$$

$$= 135.99609\ldots \approx 136$$

Descriptive statistics terminology

Make a list of these terms, find their definitions in the textbook.

Univariate data, bivariate
Population, sample, random/biased sample, survey, census
Discrete/continuous data, quantitative/qualitative
Central tendency, mean ($\overline{x}, \mu$), median, mode; quartiles, percentiles
5-figure summary, box & whisker plots, range, interquartile range, outlier
Dispersion, standard deviation (σ), variance ($v = \sigma^2$)
Frequency distributions (tables/bar charts/histograms)
Grouped data, class, mid-interval value, boundaries, modal class
Cumulative frequency distributions

Bias and fairness, random variation, & combinations

When rolling two dice, why aren't all the possible totals equally likely?

Definition:
A fair (p. 67) or unbiased (p. 79) process

In mathematics we usually simplify and assume a random process follows exact, idealized probabilities. For example, we assume heads and tails are equally likely results of a coin toss.

Bias and fairness, random variation, & combinations

The limits of observational studies

Surveys and censuses are *observational* studies. Correlation shows an association between variables, but not cause and effect.

An experimental (or empirical results, p. 65) can show causation if there is random assignment and a *control* group.

In real life, results have a degree of random variation. The observed relative frequencies are estimates of the underlying theoretical probabilities, estimates which grow more accurate with additional trials. *Simulations* can quantify the uncertainty of findings.

Bias and fairness, random variation, & combinations

When rolling two dice, why aren't all the possible totals equally likely?

Counting events in a sample space (p. 78) or calculating combinations (p. 184)

The six possible results of rolling a single die are equally likely, $\mathrm{P}(x) = \frac{1}{6}$, if we assume the die is fair. Similarly, the probability of any of the 36 (6×6) possible results of rolling two dice are equally likely, $\mathrm{P}(x) = (\frac{1}{6})^2$. However, the probability of a particular total varies according to how many combinations lead to that total. Thus, for example, 7 can be rolled six different ways, so $\mathrm{P}(7) = \frac{6}{36}$, while 2 can only result one way, $\mathrm{P}(2) = \frac{1}{36}$.

Sets, subsets, & proper subsets

Definitions:
A set is an unordered collection of elements.
e.g. $\{$red, white, blue$\}$ (do not repeat elements)

Subset: Set A is a subset of set B if and only if all of the elements of A are elements of B.
Written: $A \subseteq B$

Proper subset: $A \subseteq B$ and A is not equal to B. Written: $A \subset B$

The empty set is a subset of all sets. $\{\}$ or $\emptyset$

GQ: Combinatorics problem

CCSS: F.IF.B.6 Calculate & interpret the rate of change of a function

Show the formula and then use your calculator function

1. You have a \$1 bill, a \$5 bill, a \$10 bill, a \$20 bill, a quarter, a dime, a nickel, and a penny. How many different total amounts can you make by choosing six bills and coins?

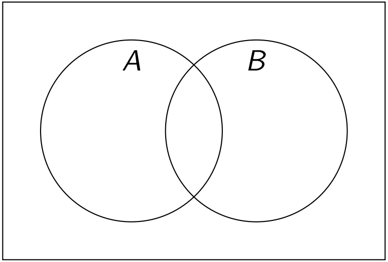
What is the number of the set you are choosing from?
How many are you picking?
Does their order matter?

Do Now #1: Phone preferences by gender

Given the frequency table, make a Venn diagram

	Android	iPhone
Boys	15	5
Girls	5	15

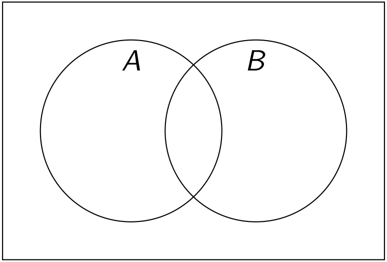
$A = \{\text{prefers Android}\}$ and $B = \{\text{is a boy}\}$



Do Now #2: Independence

Given the situation, make a Venn diagram, frequency table, and tree representing

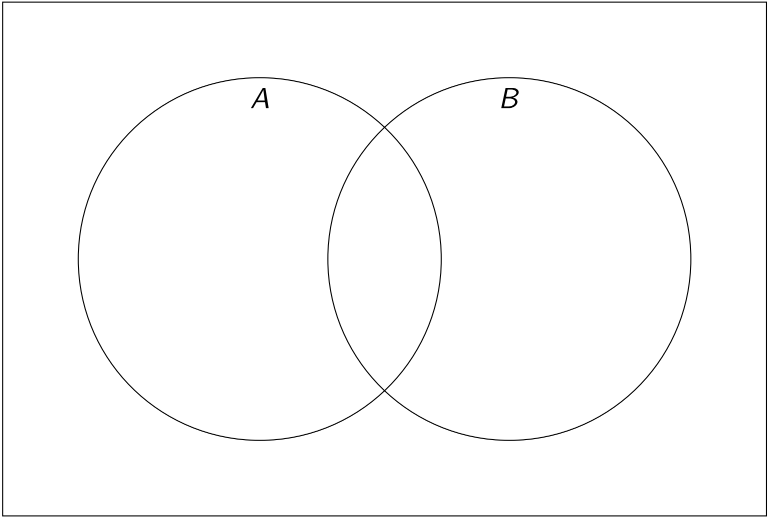
$$P(A) = 0.6,\ P(B) = 0.5,\ P(A \cap B) = 0.3$$



	A	A'
B		
B'		

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

The addition rule



Distributions

Tables and charts used to summarize a problem situation

A frequency distribution displays the number of times each event in the sample space occurs, either in tabular or graphical form.

A probability distribution shows the same data, normalizing the totals to one.

Combinatorics formulas

Combinations, when order doesn't matter

$$_{n}C_{r} = \frac{n!}{(n-r)!r!} \qquad \text{"n pick r"}$$

Permutations, when order does matter

$$_{n}P_{r} = \frac{n!}{(n-r)!}$$

Definition of theoretical probability

The theoretical probability of an event A is $\mathrm{P}(A) = \dfrac{n(A)}{n(U)}$

where $n(A)$ is the number of ways an event can occur

and $n(U)$ is the total number of possible outcomes (p. 65)

Theoretically, in n trials, one would expect the event to occur $n \times \mathrm{P}(A)$ times

Probabilities are between 0 and 1, inclusive. $0 \leq \mathrm{P}(X) \leq 1$

Empirical (experimental) probability

The relative frequency of an event can be used as an estimate of its probability.

$$\mathrm{P}(A) = \frac{\text{number of occurrences of event } A}{\text{total number of trials}}$$

The larger the number of trials the more reliable the estimate of probability.

Independence and mutual exclusivity

Two events are independent if the occurrence of one does not affect the probability of the other.

$$\mathrm{P}(\text{both } A \text{ and } B \text{ occur}) = \mathrm{P}(A) \times \mathrm{P}(B)$$

Two events are mutually exclusive if they never occur together.

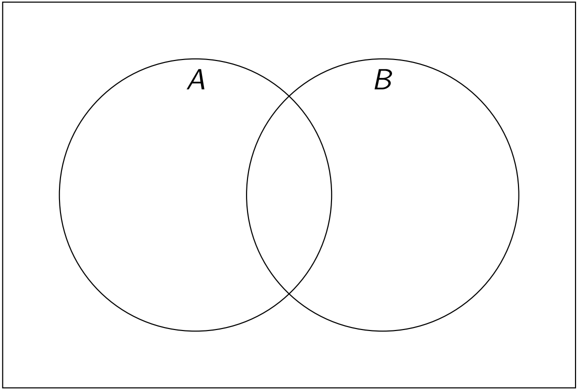
$$\mathrm{P}(\text{both } A \text{ and } B \text{ occur}) = 0 \qquad \text{and}$$

$$\mathrm{P}(\text{either } A \text{ or } B \text{ occur}) = \mathrm{P}(A) + \mathrm{P}(B)$$

Venn diagrams

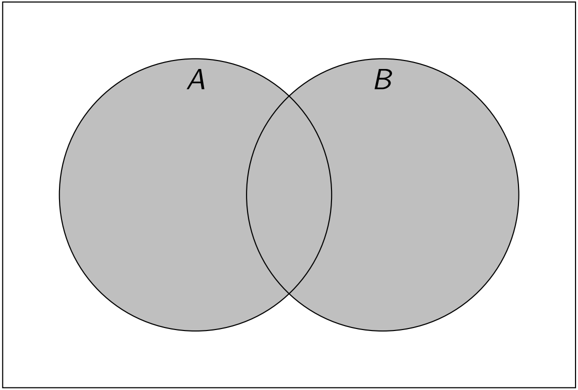
For organizing compound events

When two events can occur, and perhaps both, or neither.



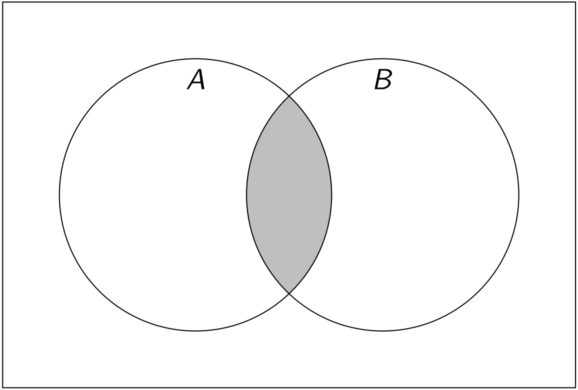
The union of sets: $A \cup B$

That A happens, or B happens, or both



The intersection of sets: $A \cap B$

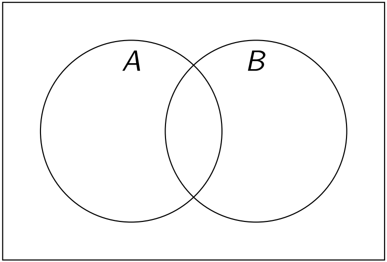
That both A and B happen



The addition rule

That A or B or both occur

When two events can occur, and perhaps both



$$P(\text{either } A \text{ or } B \text{ occur}) = P(A) + P(B) - P(\text{both } A \text{ and } B \text{ occur})$$

Vocabulary for probability & statistics

event, experiment, random

probability, P(A), values [0,1]

theoretical, empirical, subjective

sample space, U; frequency, trials

n(U) = number of possibilities

P(A) = n(A)/n(U); expected = n * P