Unit 5 Exam Part 1: Integral Calculus - with calculator

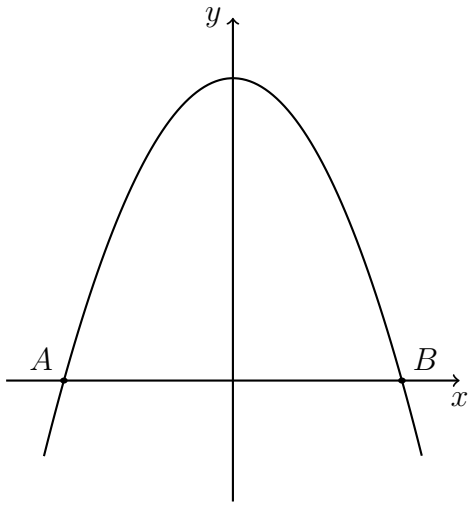
You may use a calculator on these problems **[34 marks]**

1. Let $f(x) = x^2$ and $g(x) = \sin(x + 1)$.

   (a) Solve for $f(x) = g(x)$. [3]

   (b) Find the area of the region enclosed by the graphs of f and g. [3]

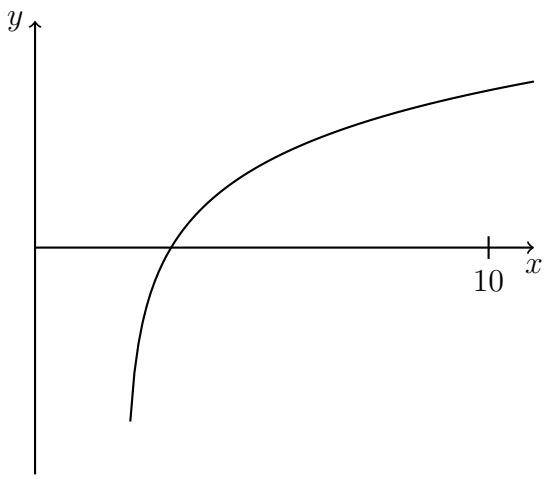
2. Let $f(x) = 6 - x^2$. Part of the graph of f is shown in the following diagram.



   (a) The graph crosses the x-axis at the points A and B.
   Find the x-coordinate of A and of B. [3]

   (b) The region enclosed by the graph of f and the x-axis is rotated 360° about the x-axis. Find the volume of the solid formed. [3]

3. Let $f(x) = 3\ln(x-2)$, for $x > 2$. The following diagram shows part of the graph of f.



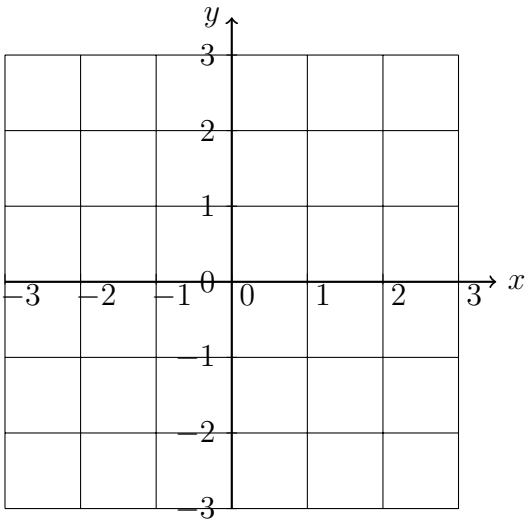
(a) Find the equation of the vertical asymptote to the graph of f. [2]

(b) Find the x-intercept of the graph of f. [2]

(c) The region enclosed by the graph of f, the x-axis, and the line $x = 8$ is rotated 360° about the x-axis. Find the volume of the solid formed. [3]

4. Let $f(x) = -x^4 + 2x^3 - \frac{1}{2}$, for $0 \leq x \leq 2$.

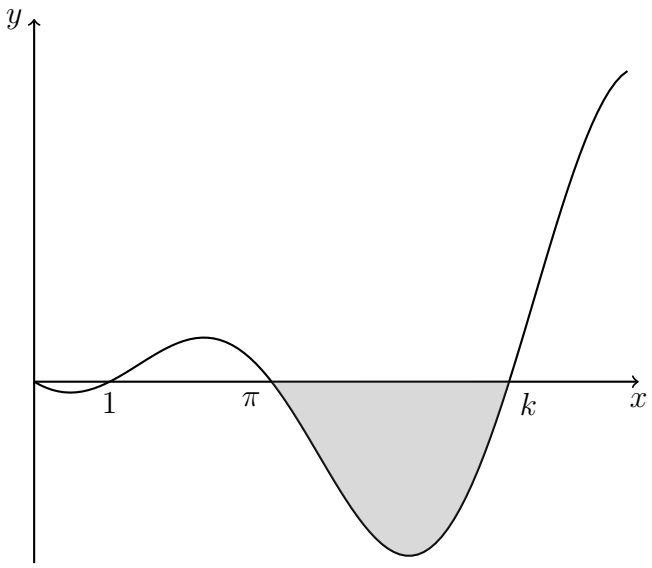
   (a) Sketch the graph of f. [3]



   (b) Solve for $f(x) = 0$. [2]

   (c) The region enclosed by the graph of f and the x-axis is rotated 360° about the x-axis. Find the volume of the solid formed. [3]

5. The graph of $y = (x-1)\sin x$, for $0 \leq x \leq \frac{5\pi}{2}$, is shown below.



(a) The graph has x-intercepts at $0, 1, \pi,$ and k. Find k. [2]

(b) The shaded region is rotated 360° about the x-axis. Let V be the volume of the solid formed.

Write down an expression for V. [3]

(c) Find V. [2]

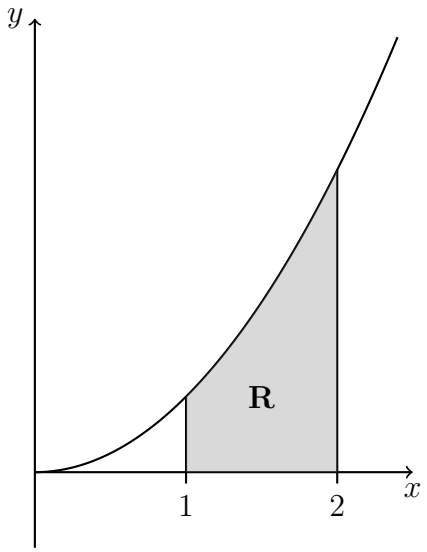
Unit 5 Exam Part 2: Integral Calculus - without calculator

No Calculator section **[38 marks]**

6. Let $f(x) = x^2$.

   (a) Find $\int_1^2 (f(x))^2 \, dx$ [4]

   (b) The following diagram shows part of the graph of f.



The shaded region R is enclosed by the graph of f, the x-axis, and the lines $x = 1$ and $x = 2$.

Find the volume of the solid formed when R is revolved 360° about the x-axis. [2]