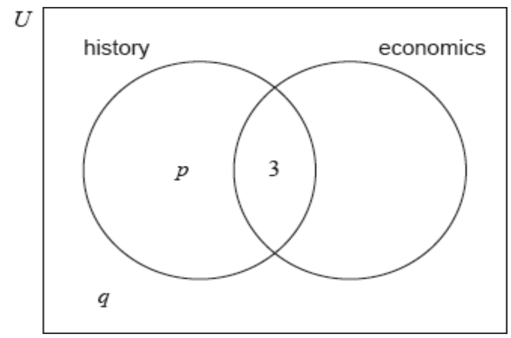
BECA / Dr. Huson / Mathematics Name:
5 February 2018

Pre-test: Probability exam problems

1a. In a group of 20 girls, 13 take history and 8 take economics. Three girls take both history and economics, as shown in the following Venn diagram. The values ***p*** and ***q*** represent numbers of girls.



Find the value of ***p***; *[2 marks]*

1b. Find the value of ***q***. *[2 marks]*

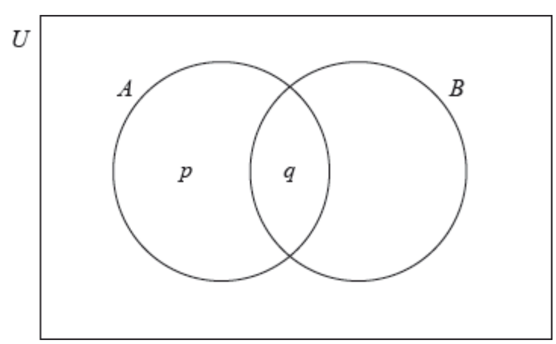
1c. A girl is selected at random. Find the probability that she takes economics but not history. *[2 marks]*

2a. Events A and B are independent with $\mathrm{P}(A \cap B) = 0.2$ and $\mathrm{P}(A' \cap B) = 0.6$.

Find $\mathrm{P}(B)$. *[2 marks]*

2b. Find $\mathrm{P}(A \cup B)$. *[4 marks]*

3a. The following Venn diagram shows the events A and B, where $\mathrm{P}(A) = 0.4,\ \mathrm{P}(A \cup B) = 0.8$ and $\mathrm{P}(A \cap B) = 0.1$. The values p and q are probabilities.



(i) Write down the value of q.

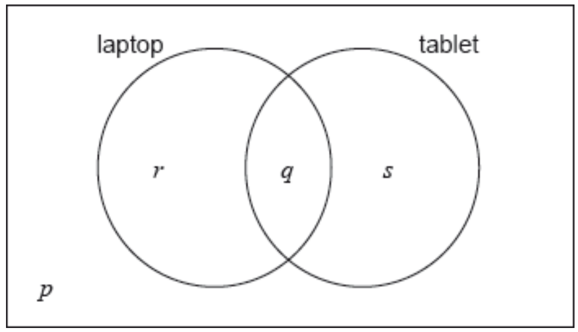
(ii) Find the value of p. *[3 marks]*

3b. Find $\mathrm{P}(B)$. *[3 marks]*

4a. In a class of 21 students, 12 own a laptop, 10 own a tablet, and 3 own neither.

The following Venn diagram shows the events "own a laptop" and "own a tablet".

The values p, q, r and s represent numbers of students.



(i) Write down the value of p.

(ii) Find the value of q.

(iii) Write down the value of r and of s. *[5 marks]*

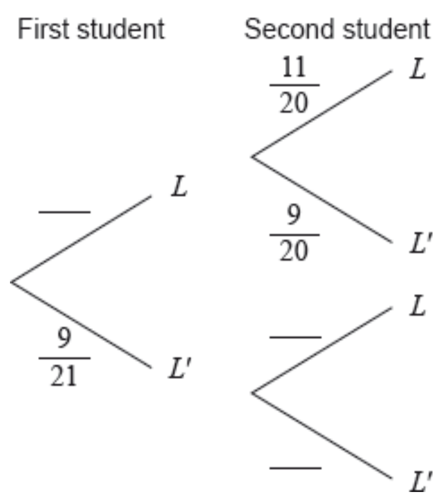
4b. A student is selected at random from the class.

(i) Write down the probability that this student owns a laptop.

(ii) Find the probability that this student owns a laptop or a tablet but not both. *[4 marks]*

4c. Two students are randomly selected from the class. Let L be the event a "student owns a laptop".

(i) **Copy** and complete the following tree diagram. (Do **not** write on this page.)



(ii) Write down the probability that the second student owns a laptop given that the first owns a laptop.

[4 marks]

5a. A factory has two machines, A and B. The number of breakdowns of each machine is independent from day to day.

Let A be the number of breakdowns of Machine A on any given day. The probability distribution for A can be modelled by the following table.

a	0	1	2	3
$\mathrm{P}(A = a)$	0.55	0.3	0.1	k

Find k. *[2 marks]*

5b. (i) A day is chosen at random. Write down the probability that Machine A has no breakdowns.

(ii) Five days are chosen at random. Find the probability that Machine A has no breakdowns on exactly four of these days. *[3 marks]*

5c. Let B be the number of breakdowns of Machine B on any given day. The probability distribution for B can be modelled by the following table.

b	0	1	2	3
$\mathrm{P}(B = b)$	0.7	0.2	0.08	0.02

Find $\mathrm{E}(B)$. *[2 marks]*

5d. On Tuesday, the factory uses both Machine A and Machine B. The variables A and B are independent.

(i) Find the probability that there are exactly two breakdowns on Tuesday.

(ii) Given that there are exactly two breakdowns on Tuesday, find the probability that both breakdowns are of Machine A. *[8 marks]*

6a. Let C and D be independent events, with $\mathrm{P}(C) = 2k$ and $\mathrm{P}(D) = 3k^2$, where $0 < k < 0.5$.

Write down an expression for $\mathrm{P}(C \cap D)$ in terms of k. *[2 marks]*

6b. Find $\mathrm{P}(C'|D)$. *[3 marks]*

Topic 4—Vectors

4.1	Magnitude of a vector	$\lvert \boldsymbol{v} \rvert = \sqrt{v_1^{\,2} + v_2^{\,2} + v_3^{\,2}}$
4.2	Scalar product Angle between two vectors	$\boldsymbol{v} \cdot \boldsymbol{w} = \lvert\boldsymbol{v}\rvert \lvert\boldsymbol{w}\rvert \cos\theta$ $\boldsymbol{v} \cdot \boldsymbol{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$ $\cos\theta = \dfrac{\boldsymbol{v} \cdot \boldsymbol{w}}{\lvert\boldsymbol{v}\rvert \lvert\boldsymbol{w}\rvert}$
4.3	Vector equation of a line	$\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$

Topic 5—Statistics and probability

5.2	Mean of a set of data	$\bar{x} = \dfrac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$
5.5	Probability of an event A Complementary events	$\mathrm{P}(A) = \dfrac{n(A)}{n(U)}$ $\mathrm{P}(A) + \mathrm{P}(A') = 1$
5.6	Combined events Mutually exclusive events Conditional probability Independent events	$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$ $\mathrm{P}(A \cap B) = \mathrm{P}(A)\,\mathrm{P}(B \mid A)$ $\mathrm{P}(A \cap B) = \mathrm{P}(A)\,\mathrm{P}(B)$
5.7	Expected value of a discrete random variable X	$\mathrm{E}(X) = \mu = \sum_{x} x\,\mathrm{P}(X = x)$
5.8	Binomial distribution Mean Variance	$X \sim \mathrm{B}(n, p) \quad \Rightarrow \quad \mathrm{P}(X = r) = \binom{n}{r} p^r (1-p)^{n-r},\ r = 0, 1, \ldots, n$ $\mathrm{E}(X) = np$ $\mathrm{Var}(X) = np(1-p)$
5.9	Standardized normal variable	$z = \dfrac{x - \mu}{\sigma}$