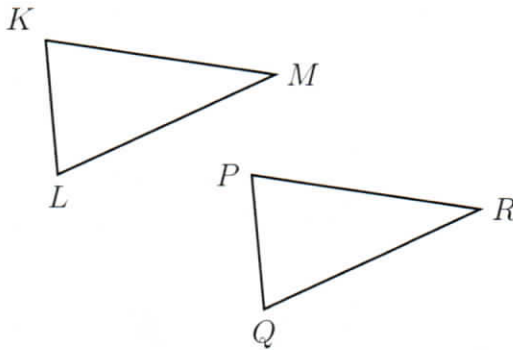**5.11 Exam: Transformational Geometry**

1. A translation maps triangle KLM onto triangle PQR.

Write each corresponding object.

(a) $L \rightarrow$ ___Q___ ✓

(b) $\angle M \cong \angle R$ ✓

(c) ___$\overline{LM}$___ ✓ $\cong \overline{QR}$

(d) Justify $\triangle KLM \cong \triangle PQR$. Use the words "rigid motion" and "translation".

because translation is a ✓
rigid motion preserving length, so $\cong$ ④
by SSS A $\cong$

2. Triangle ABC is dilated with a scale factor of k centered at A, yielding $\triangle ADE$, as shown. Given $AB = 8$, $BC = 12$, $AC = 16$, and $DE = 15$.
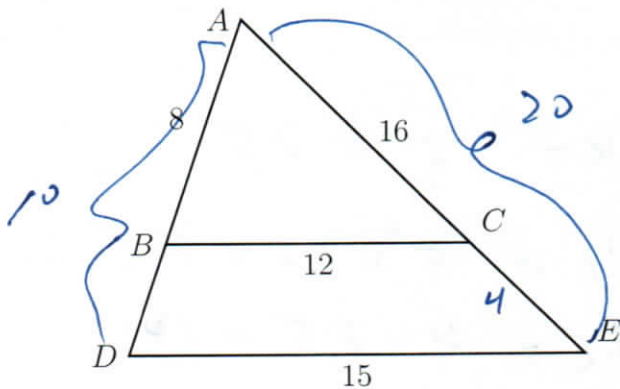
Find AD, CE, and k (the scale factor).

$\overline{BC} \rightarrow \overline{DE}$

$k = \dfrac{15}{12} = 1.25 = \dfrac{5}{4}$

$AD = 8 \times \dfrac{5}{4} = 10$

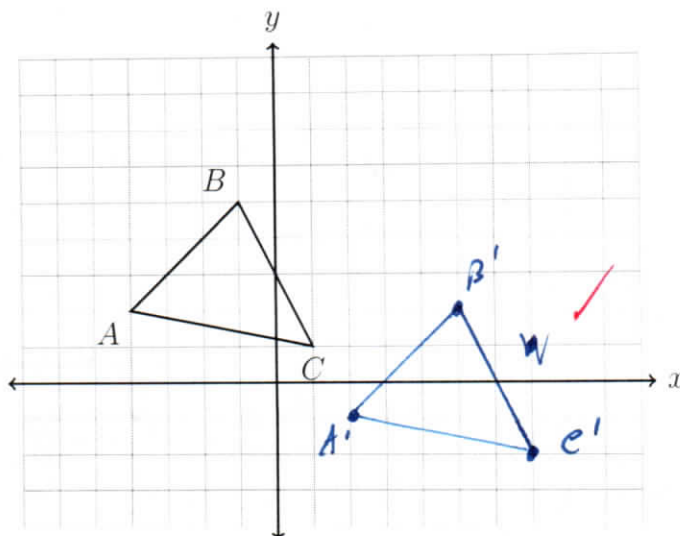$AE = 16 \times \dfrac{5}{4} = 20$

$CE = 20 - 16 = 4$

3. Find the image of $P(-4, 0)$ after the translation $(x, y) \rightarrow (x - 10, y + 2)$.
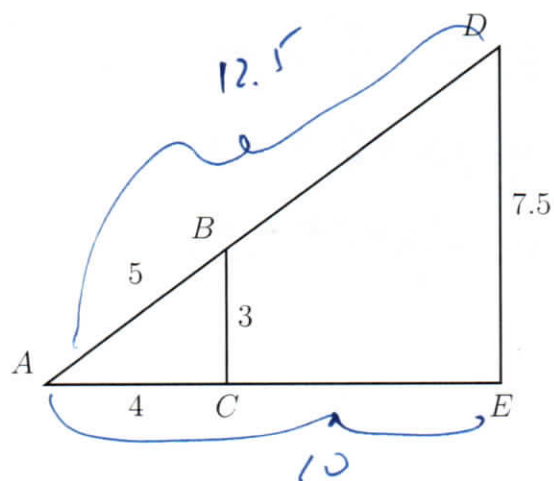
①

$P'(-14, 2)$

⑧

4. Translate $\triangle ABC$ by $(x, y) \to (x + 6, y - 3)$. Make a table of the coordinates and plot and label the image on the axes.

$A(-4, 2) \to A'(2, -1)$ ✓✓

$B(-1, 5) \to B'(5, 2)$

$C(1, 1) \to C'(7, -2)$

③

5. A dilation centered at A maps $\triangle ABC \to \triangle ADE$. Given the sides of the preimage, $AC = 4$, $BC = 3$, $AB = 5$, and of $DE = 7.5$ find the scale factor k and the lengths AD and AE.

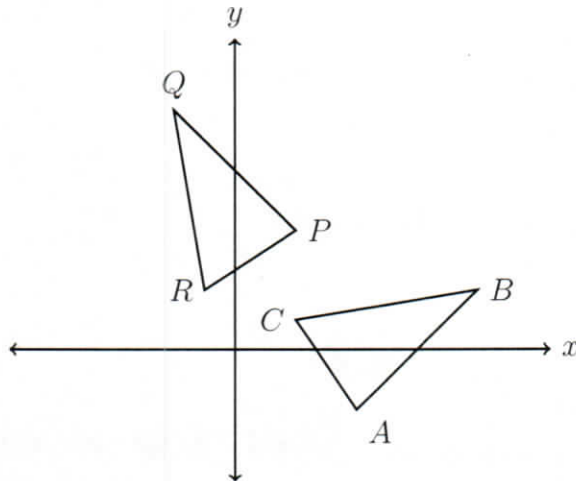$k = \dfrac{7.5}{3} = 2.5$

$AD = 5 \times 2.5 = 12.5$

$AE = 4 \times 2.5 = 10$

③

6. Given $\triangle ABC \sim \triangle DEF$. $m\angle A = 40°$ and $m\angle E = 35°$. Find the measure of $\angle C$.

$m\angle C = 105°$ ①

⑦

7. A rotation of 90° is applied to $\triangle ABC$, mapping it onto $\triangle PQR$, as shown. Which triangle has the larger area, or are they equal? Justify your answer.
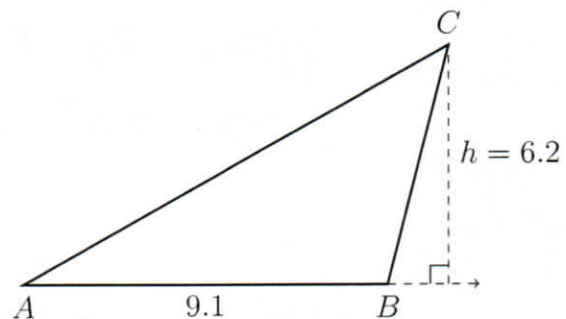
EQUAL AREA ✓
Rotation is a rigid, ✓
motion, preserving
length and area.

⌄

8. The side $\overline{AB}$ of triangle ABC is extended and an altitude to the vertex C is drawn, as shown below. The triangle's height is $h = 6.2$ and its base measures $AB = 9.1$. Find the area of the triangle.

$A = \frac{1}{2}(9.1)(6.2)$ ✓

$= 28.21$ ✓

$h = 6.2$ ⌄

A 9.1 B
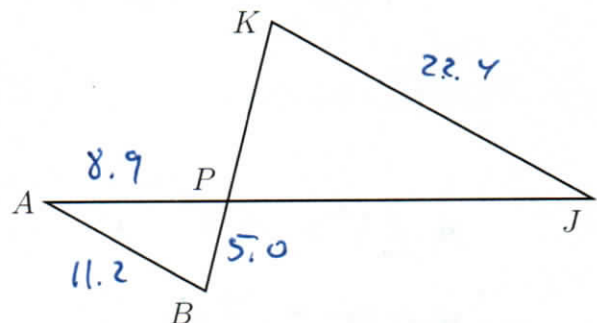
9. Given $\triangle ABP \sim \triangle JKP$ as shown below. $AB = 11.2$, $AP = 8.9$, $BP = 5.0$, and $JK = 22.4$. Find JP.
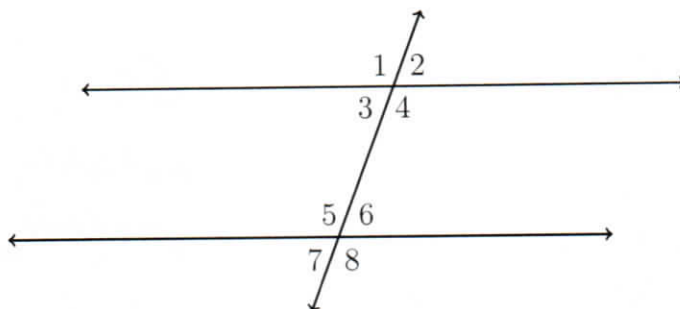
$\overline{AB} \to \overline{JK}$

$k = \frac{22.4}{11.2} = 2$ ✓

$JP = 8.9 \times 2 = 17.8$ ✓

K
22.4
8.9 P ⌄
A J
5.0
11.2
B

⑥

10. Given two parallel lines and a transversal, as shown below.

(a) State the angle corresponding with $\angle 7$. $\angle 3$ ✓

(b) What theorem would justify $m\angle 4 + m\angle 6 = 180°$? <u>SAME SIDE INTERNAL $\angle$s are</u> ✓ <u>Supplementary</u>

(c) What theorem would justify $\angle 3 \cong \angle 6$? <u>Alternate interior angles</u> ✓ are $\cong$

(d) Given $m\angle 1 = 117°$ and $m\angle 8 = (4x - 3)°$. Find x.

$$117 = 4x - 3$$
$$x = 30° ✓$$

④

11. A translation maps $X(1, 7) \rightarrow X'(-3, 9)$. What is the image of $Y(0, -3)$ under the same translation? $T_{-4, +2}$ ✓

$$Y(0, -3) \rightarrow Y'(-4, -1) ✓$$
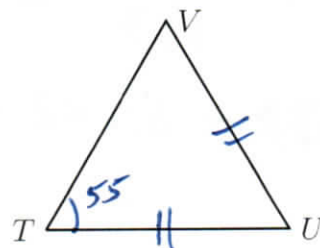
②

12. Given isosceles $\triangle TUV$ with $\overline{TU} \cong \overline{UV}$ and $m\angle T = 55$. Find $m\angle U$ and $m\angle V$.

$m\angle V = 55$ ✓
$m\angle U = 70$ ✓

(the diagram is not to scale) ②

⑧

13. The trapezoid $ABCD$, shown below, undergoes a rigid transformation carrying it onto trapezoid $WXYZ$. State the transformation. (be specific)
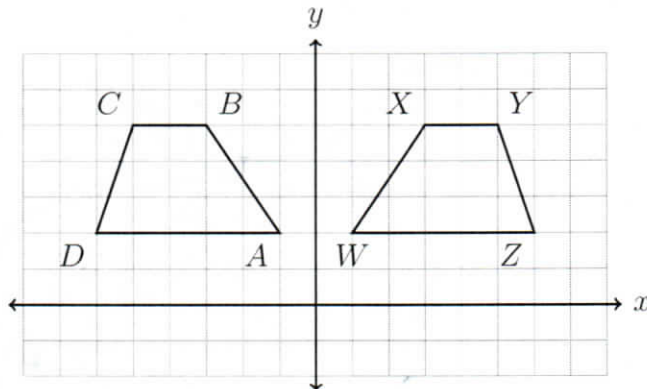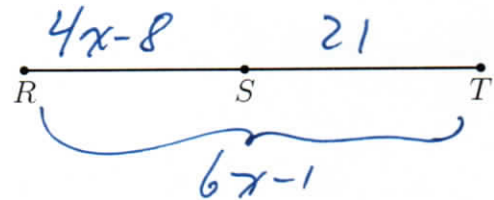
reflected across ✓ the y-axis ✓

②

14. The points R, S, and T are collinear, with $RS = 4x - 8$, $ST = 21$, and $RT = 6x - 1$.

Find RT.

show check

$4x - 8 + 21 = 6x - 1$ ✓

$x = 7$ ✓

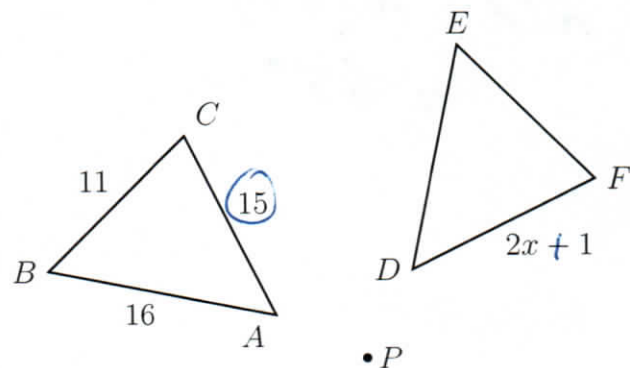$\overline{4x-8} \quad \overline{21}$
R   S   T
$6x-1$

③

$RS = 4(7) - 8 = 20$
$20 + 21 = 47$
$RT = 6(4) - 1 = 41$ ✓✓

15. In the diagram below, $\triangle ABC$ with sides of 11, 15, and 16, is mapped onto $\triangle DEF$ after a clockwise rotation of 90° about point P.

If $DF = 2x + 1$, what is the value of x?

$15 = 2x + 1$ ✓
$x = 7$ ✓

②

⑦

16. Given $\triangle ABC$ point D on $\overline{AB}$ and point E on $\overline{BC}$ such that $\triangle ABC \sim \triangle DBE$.
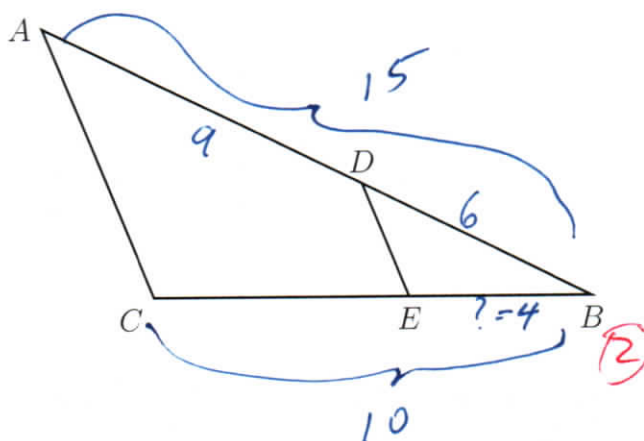
   If $AB = 15$, $BC = 10$, and $AD = 9$, what is the length of $\overline{BE}$?

$\overline{AB} \rightarrow \overline{DB}$

$15 \rightarrow 6$

$k = \frac{6}{15} = \frac{2}{5}$ ✓

$BE = 10 \times \frac{2}{5} = 4$ ✓

Diagram: Triangle with vertex A at top, C at bottom left, B at bottom right. Line from A to B passes through D. Segment DE with D on AB and E on BC. Labels: 9 (A to D), 15 (near D), 6 (D to B), C, E, ?=4, B, 10 (C to B).

(2)

17. In $\triangle ABC$ shown below, $m\angle A = (10x)°$, $m\angle B = (16x - 5)°$, and $m\angle C = (2x + 3)°$.

   Find $m\angle A$. (show the check for full credit)

$10x + 16x - 5 + 2x + 3 = 180$ ✓

$28x - 2 = 180$

$x = 6.5$ ✓

$m\angle A = 10 \cdot (6.5)$

$= 65°$ ✓

Diagram: Triangle with B at top, A at bottom left, C at bottom right. Angle at B = (16x − 5)°, angle at A = (10x)°, angle at C = (2x + 3)°.

$m\angle B = 16(6.5) - 5$

$= 99$

$m\angle C = 2(6.5) + 3$

$= 16$

$65 + 99 + 16 = 180$ ✓

(4)

(6)
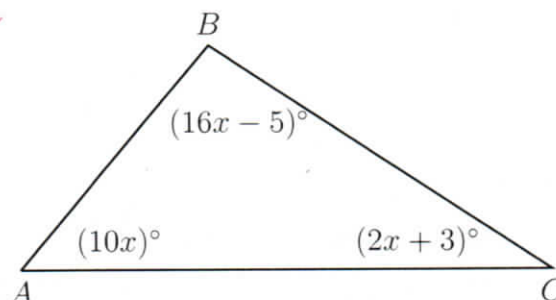
18. The shape shown below is composed of straight lines and right angles, with some lengths as marked. Find the area of the figure. (the figure is not drawn to scale)
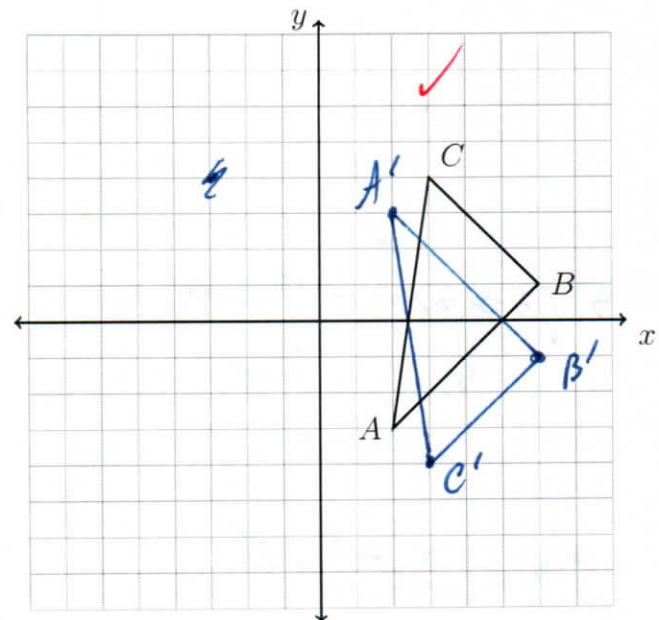
$A_{fig} = 8 + 45 + 12 + 12$

$= 77$
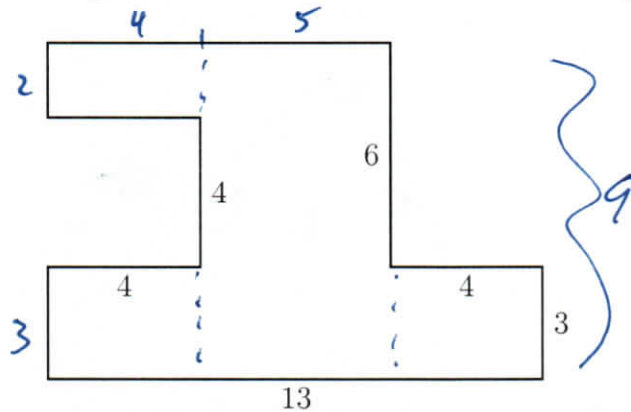
19. $\triangle ABC$ is shown with vertices $A(2, -3)$, $B(8, 1)$, and $C(-1, 8)$. Reflect the triangle across the x-axis. Write down its coordinates in a table and plot and label it on the graph.

$(6, 1)$ $(3, 4)$

$A(2, -3) \rightarrow A'(2, +3)$ ✓✓
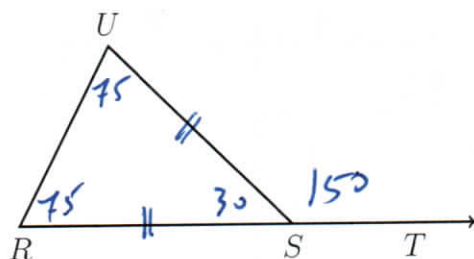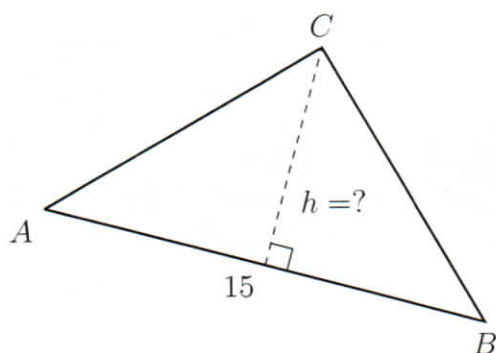
$B(6, 1) \rightarrow B'(6, -1)$

$C(3, 4) \rightarrow C'(3, -4)$

3

20. Given isosceles $\triangle RSU$ with $\overline{US} \cong \overline{RS}$. If $m\angle UST = 150$ find $m\angle U$.

$m\angle U = 75$ ✓

U
75
75   30   150
R   S   T

2

21. One side of the $\triangle ABC$ has a length $AB = 15$. The triangle's area is $71\frac{1}{4}$. Find the length of the altitude h of the triangle to vertex C and perpendicular to side $\overline{AB}$.

C
$h = ?$
A
15
B

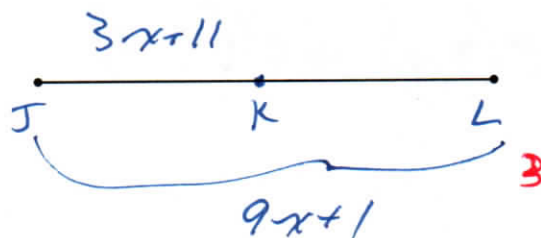$A = \frac{1}{2}(15)h = 71.25$ ✓

$h = \frac{71.25 \times 2}{15}$ *

2

$= 9.5$ ✓

22. The point K is the midpoint of $\overline{JL}$, $JK = 3x + 11$, and $JL = 9x + 1$. Find JK.

$2(3x+11) = 9x+1$ ✓

$21 = 3x$

$x = 7$ ✓

$JK = 3(7)+11$

$= 32$ ✓

$3x+11$

J   K   L

3

$9x+1$

check
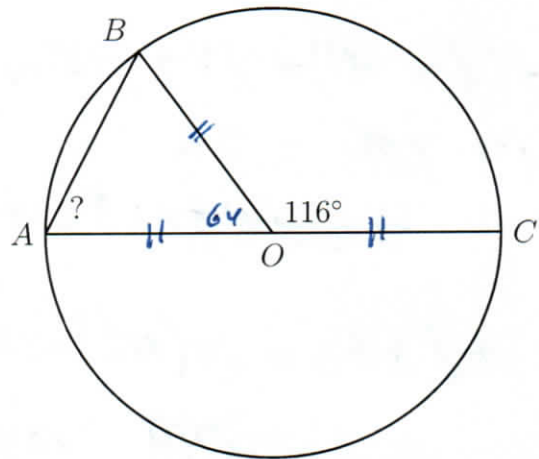$JL = 9(7)+1 = 64$
$32 \times 2 = 64$ ✓

7

Early finishers

23. The circle O is shown below with diameter $\overline{AOC}$ and radius $\overline{BO}$. Given that the central angle $m\angle COB = 116°$. Find the measure of angle A, that is, $m\angle BAO$.

$m\angle AOB = 64$

$m\angle A = 58$ ✓

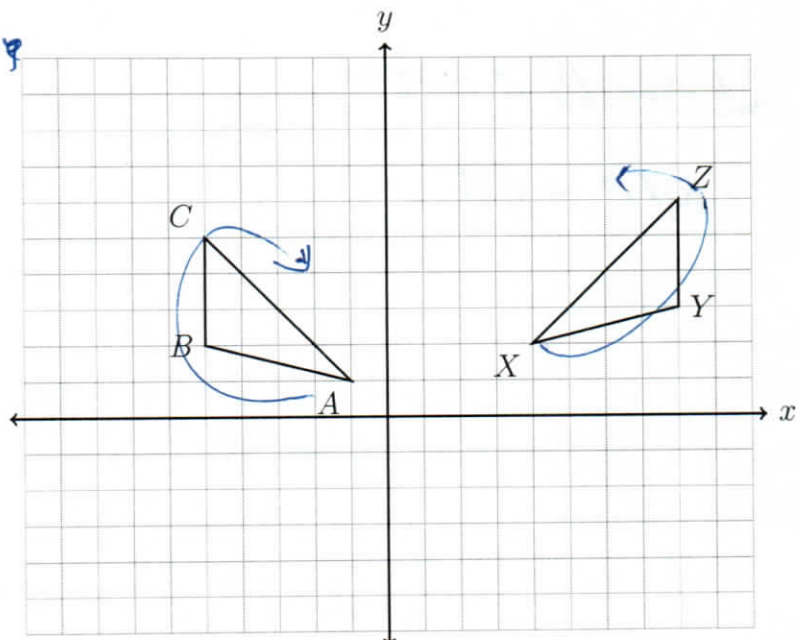B

A ? H 64 116° H C
O

2

24. The triangle ABC, shown below, undergoes two rigid motions carrying it onto triangle XYZ. State the two isometric transformations. (be specific)

reflect ✓ across the
y-axis ✓

Translate ✓
3 right ✓
1 up

$T_{+3,+1} \circ r_{y-axis}$

y

C

B

A

Z

Y

X

x

4

⑥

25. An angle bisector is shown below, with $\overrightarrow{AC}$ bisecting $\angle BAD$. Given $m\angle BAC = 6x+1$ and $m\angle BAD = 14x - 15$, find $m\angle BAD$. (Show check)
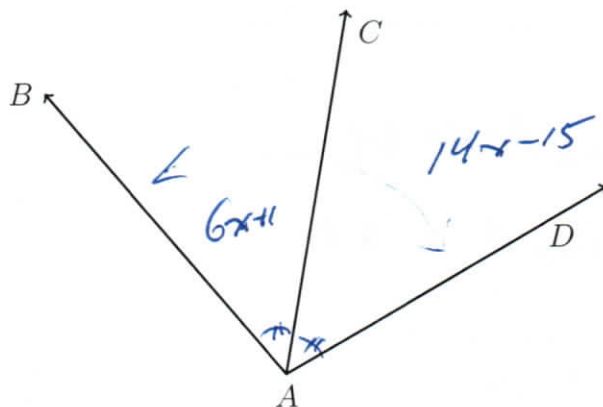
$$2(6x+1) = 14x - 15 \checkmark$$

$$17 \; \cancel{37} = 2x$$

$$\cancel{x = 17} \; x = 8\frac{1}{2} \checkmark$$

$$m\angle BAD = 14\left(8\frac{1}{2}\right) - 15$$

$$= \cancel{234} \quad 104 \checkmark$$

$$m\angle BAC = 6\left(8\frac{1}{2}\right) + 1$$

$$= 52$$

B C

$6x+1$ $14x-15$ D

A

④
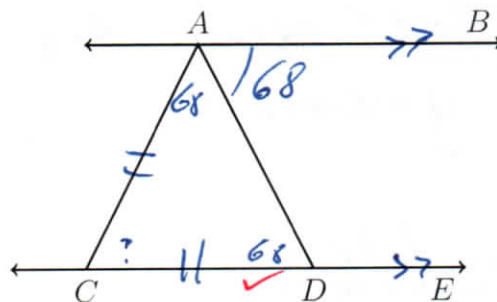
$52 \times 2 = 104 \checkmark$

26. Given parallel lines $\overleftrightarrow{AB} \parallel \overleftrightarrow{CDE}$ with $\overline{AC} \cong \overline{CD}$. If $m\angle BAD = 68$ find $m\angle ACD$.

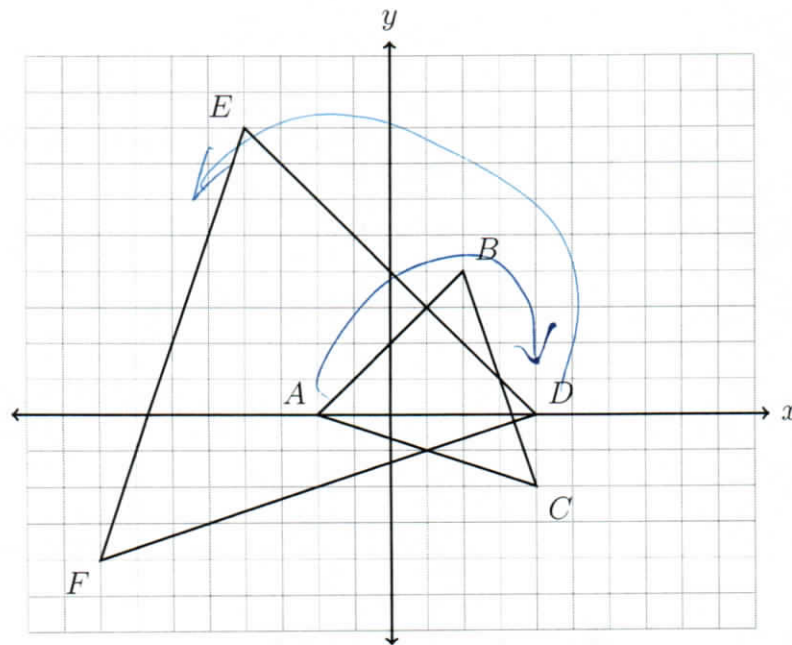$$m\angle C + 68 \times 2 = 180°$$

$$m\angle C = 44 \checkmark$$

A B

68 68

C 68 D E 2

⑥

27. Of two supplementary angles, the measure of $\angle A$ is five times that of $\angle B$. Find $m\angle A$.

$$m\angle B = x$$
$$m\angle A = 5x$$
$$x + 5x = 180$$
$$x = 30$$
$$m\angle A = 5(30) = 150 \checkmark$$

2

28. On the set of axes below, $\triangle ABC$ has vertices at $A(-2,0)$, $B(2,4)$, $C(4,-2)$, and $\triangle DEF$ has vertices at $D(4,0)$, $E(-4,8)$, $F(-8,-4)$.

Which transformations map $\triangle ABC \rightarrow \triangle DEF$? Mark each statement True or False

correct

$4 \rightarrow 2$
$3 \rightarrow 1$

(a) A dilation with a scale factor of -2 centered at the origin   True   False

(b) A dilation with a scale factor of $\frac{1}{2}$ centered at point A True   False

(c) A dilation with a scale factor of 2 centered at the origin, followed by a rotation of 180° about the origin   True   False

(d) A dilation with a scale factor of 2 centered at the origin, followed by a reflection across the y-axis   True   False
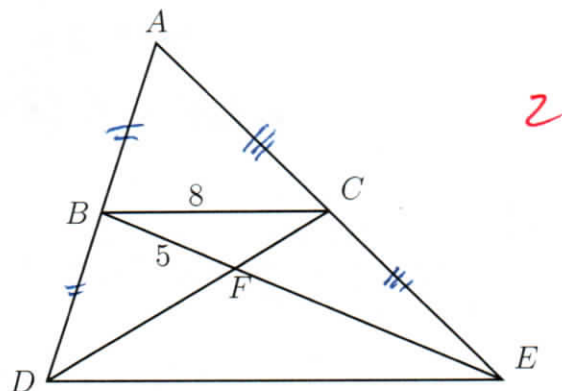
4

29. Triangle ADE and its midline $\overline{BC}$ are drawn, with B the midpoint of $\overline{AD}$ and C the midpoint of $\overline{AE}$. The two medians $\overline{AE}$ and $\overline{AE}$ are drawn, as shown, intersecting in point F, the centroid.

$\triangle FCB \sim \triangle FDE$ with scale factor $k = 2$. Given $BC = 8$ and $BF = 5$.
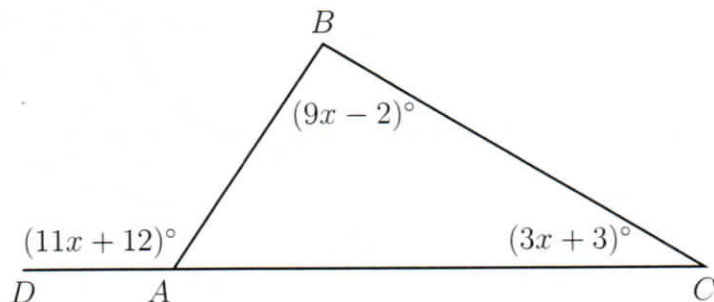
Find DE and FE.

$DE = 8 \times 2 = 16$ ✓

$FE = 5 \times 2 = 10$ ✓

2

30. In $\triangle ABC$ shown below, side $\overline{AC}$ is extended to point D with $m\angle DAB = (11x + 12)°$, $m\angle C = (3x + 3)°$, and $m\angle B = (9x + 2)°$.

What is $m\angle BAC$?

$11x + 12 = 9x - 2 + 3x + 3$

$11 = x$ ✓

$m\angle BAC = 180 - 133$

$= 47$ ✓

$m\angle DAB = 11(11) + 12$

$= 133$

$m\angle B = 9(11) - 2)$

$= 97$

$m\angle C = 3(11) + 3$

$= 36$

$36 + 97 = 133$ ✓

2

④