

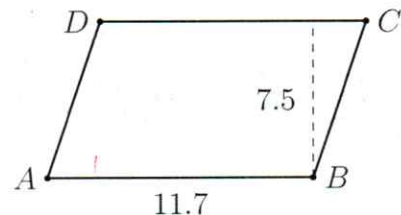
69 + 7  
73

Name: S. L. Huson

## 4.11 Exam: Transversals, volume; angle relationships

1. Find the area of the parallelogram  $ABCD$  shown below, with  $AB = 11.7$  and height  $h = 7.5$ .

$$A = 11.7 \times 7.5 \\ = 87.75 \quad (2)$$

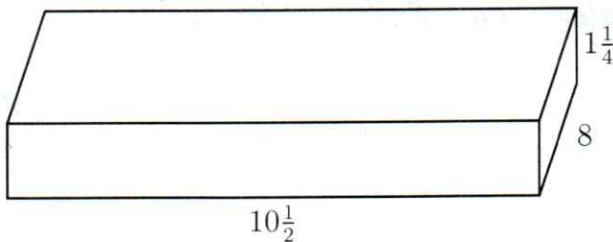


2. Find the sum of the measures of the internal angles of a hexagon. Show the formula.

$$n = 6 \\ \Sigma = (6-2) \times 180 \quad \checkmark \\ = 720 \quad \checkmark$$

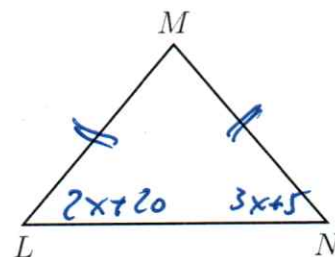
3. A wooden cutting board is  $10\frac{1}{2}$  inches long, 8 inches wide, and  $1\frac{1}{4}$  inches thick. Find the volume of the box. Show the calculation.

$$V = 10.5 \times 8 \times 1.25 \quad \checkmark \\ = 105 \quad \checkmark$$



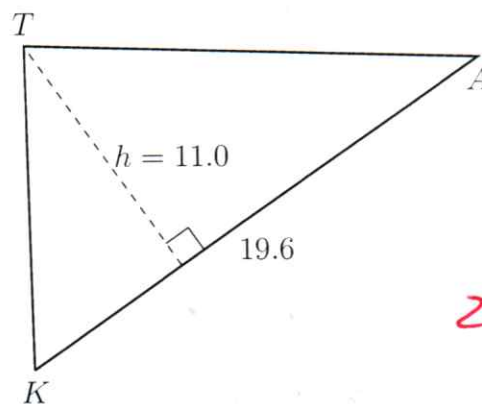
4. Given isosceles  $\triangle LMN$  with  $\overline{LM} \cong \overline{NM}$ . If  $m\angle L = 2x + 20$  and  $m\angle N = 3x + 5$ , find  $m\angle M$ .

$$2x + 20 = 3x + 5 \quad \checkmark \\ x = 15 \quad \checkmark \\ m\angle L = 2(15) + 20 \\ = 50 \quad \checkmark \\ m\angle N = 3(15) + 5 \\ = 50 \quad \checkmark \\ \left. \begin{array}{l} m\angle L = 50 \\ m\angle N = 50 \end{array} \right\} \text{check} \\ m\angle M + 50 + 50 = 180 \\ m\angle M = 80 \quad \checkmark$$



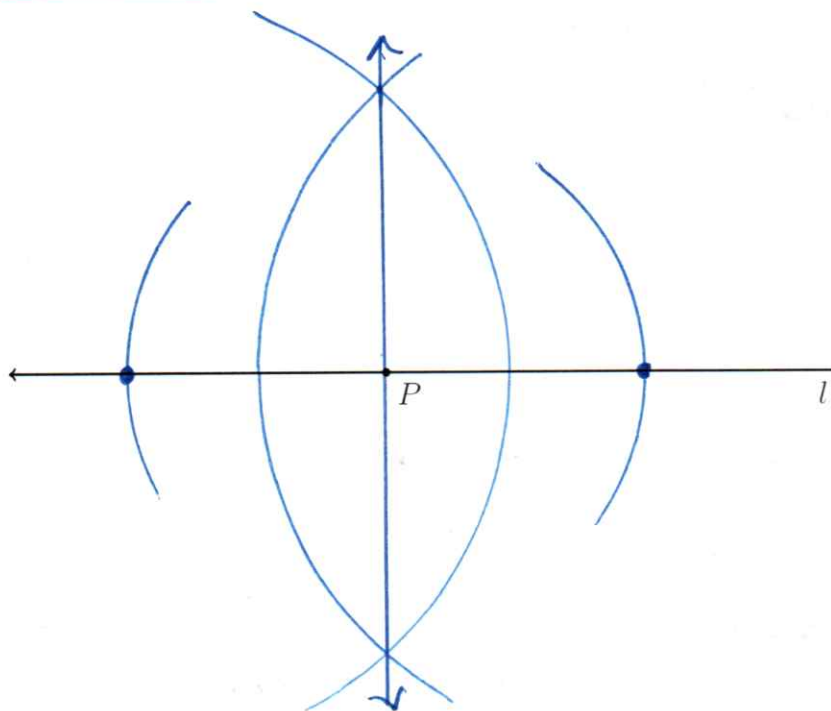
5. Find the area of  $\triangle KAT$ . The altitude  $h$  of the triangle is 11.0 centimeters and the base  $KA = 19.6$  cm. Show work by writing an equation before making the calculation.

$$A = \frac{1}{2} (19.6) (11.0)$$
$$= 107.8$$



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6. Construct a line perpendicular to  $l$  through  $P$ .  
(make no extra marks)

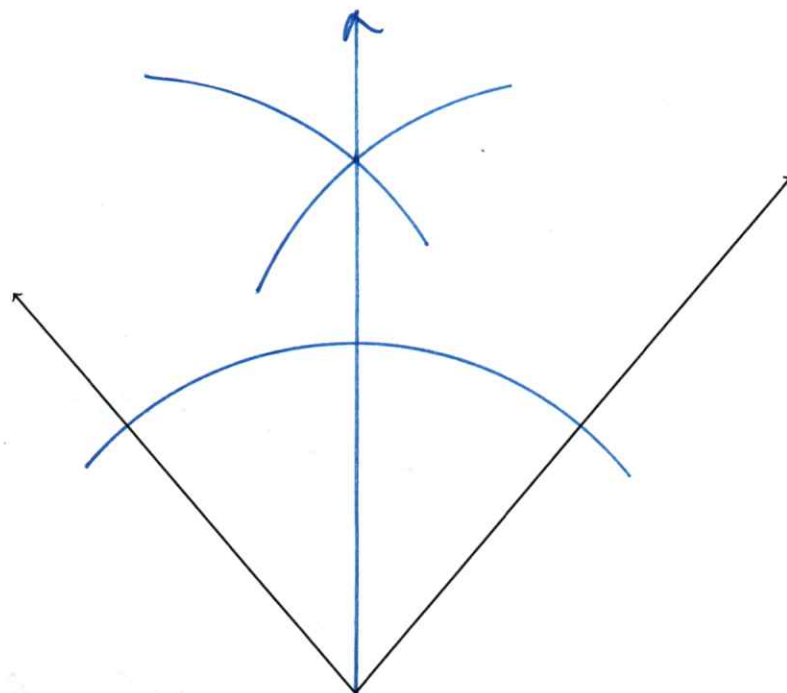


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7. Complete the construction of the bisector of the given angle.



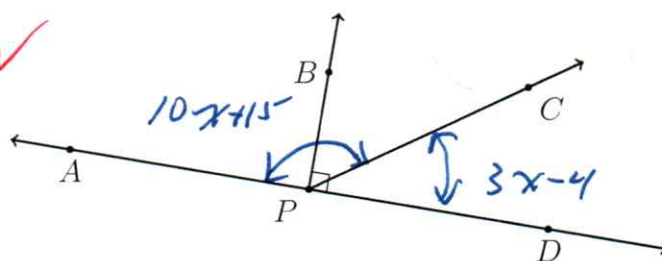
8. Angles  $APC$  and  $CPD$  form a linear pair.  $m\angle APC = 10x + 15$  and  $m\angle CPD = 3x - 4$ . Find  $m\angle CPD$ . Check your answer for full credit.

$$10x + 15 + 3x - 4 = 180 \checkmark$$

$$13x = 169$$

$$x = 13 \checkmark$$

$$\begin{aligned} m\angle CPD &= 3(13) - 4 \\ &= 35 \checkmark \end{aligned}$$



check  $\checkmark$

$$\begin{aligned} m\angle APC &= 10(13) + 15 \\ &= 145 \end{aligned}$$

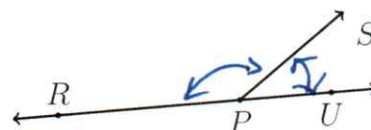
$$35 + 145 = 180 \checkmark$$

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Do Not Solve. Circle the appropriate equation, cite a justification:

- "definition of bisector"
- "linear pairs sum to  $180^\circ$ "
- "vertical  $\angle$ s are  $\cong$ "
- "alternate interior  $\angle$ s are  $\cong$ "
- "corresponding  $\angle$ s of  $\parallel$  lines are  $\cong$ "
- "same-side interior  $\angle$ s are supplementary"
- " $\perp$  rays with complementary  $\angle$ s adding to  $90^\circ$ "

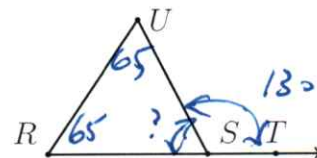
9.  $\overleftrightarrow{RP\bar{U}}$  with ray  $\overrightarrow{PS}$ .



$\angle RPS \cong \angle SPU$      $m\angle RPS + m\angle SPU = 180^\circ$

linear pairs sum to  $180^\circ$  2

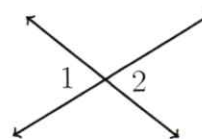
10. Given  $m\angle R = m\angle U = 65$ , and  $m\angle UST = 130$ . Find  $m\angle RSU$ .



$\angle UST \cong \angle RSU$      $m\angle UST + m\angle RSU = 180$

linear pairs sum to  $180^\circ$  2

11. Given  $m\angle 1 = 4x + 6$ ,  $m\angle 2 = 6x - 32$ . Find  $m\angle 1$ .

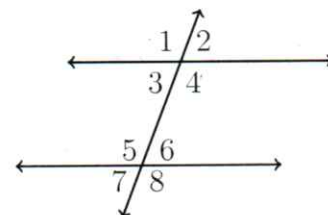


$\angle 1 \cong \angle 2$

$m\angle 1 + m\angle 2 = 180$

vertical  $\angle$ s are  $\cong$  2

12. Given two parallel lines and a transversal, as shown.

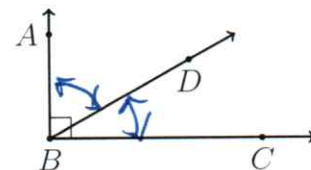


$\angle 4 \cong \angle 5$

$m\angle 3 + m\angle 6 = 180$

Alternate interior  $\angle$ s are  $\cong$  2

13. Given  $\overrightarrow{BA} \perp \overrightarrow{BC}$ ,  $m\angle ABD = 2x - 5$ , and  $m\angle DBC = x - 10$ .



$\angle ABD \cong \angle DBC$

$m\angle ABD + m\angle DBC = 90$

$\perp$  rays with complementary  $\angle$ s adding to  $90^\circ$  2

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14. The measures in degrees of the three angles of a triangle are  $3x$ ,  $\frac{1}{2}x + 7$ , and  $5x - 65$ . Find  $x$ .

$$3x + \frac{1}{2}x + 7 + 5x - 65 = 180 \quad \checkmark$$

$$8\frac{1}{2}x - 58 = 180$$

$$x = \frac{238}{8.5} = 28 \quad \checkmark$$

check

$$3(28) = 84$$

$$\frac{1}{2}(28) + 7 = 21$$

$$5(28) - 65 = 75 \quad 2$$

$$84 + 21 + 75 = 180 \quad \checkmark$$

15. Given isosceles  $\triangle RSU$  with  $\overline{UR} \cong \overline{US}$ . If  $m\angle UST = x$  and  $m\angle R = x - 80$ , find  $m\angle U$ .

$$\rightarrow m\angle RSU + x = 180$$

$$m\angle RSU = 180 - x$$

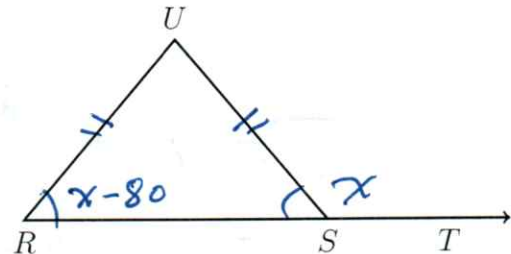
$$\text{or } \rightarrow x - 80 = 180 - x$$

$$2x = 260$$

$$x = 130 \quad \checkmark$$

$$m\angle R = 130 - 80 = 50 \quad \checkmark$$

$$\rightarrow m\angle U + 50 + 50 = 180$$



$$m\angle U = \cancel{100} \quad 80 \quad \checkmark$$

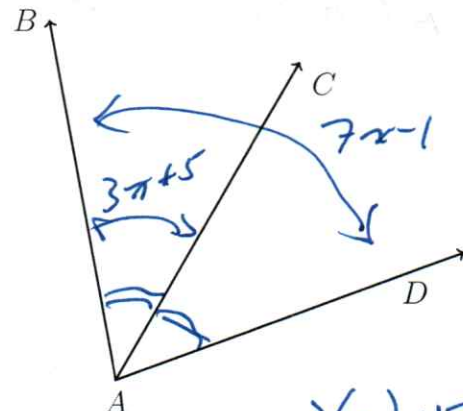
16. An angle bisector is shown below, with  $\overrightarrow{AC}$  bisecting  $\angle BAD$ . Given  $m\angle BAC = 3x + 5$  and  $m\angle BAD = 7x - 1$ , find  $m\angle BAD$ . (Show check)

$$2(3x + 5) = 7x - 1 \quad \checkmark$$

$$x = 11 \quad \checkmark$$

$$m\angle BAD = 7(11) - 1$$

$$= 76 \quad \checkmark$$



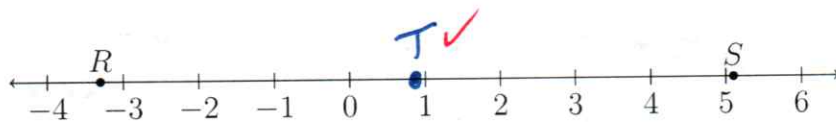
$$\text{check } m\angle BAC = 3(11) + 5$$

$$= 38 \quad \checkmark$$

$$38 \times 2 = 76 \quad \checkmark$$



17. Given  $\overleftrightarrow{RS}$  as shown on the number line, with  $R = -3.3$  and  $S = 5.1$ .



- (a) What is the exact distance on the number line between the points  $R$  and  $S$ ?

$$d = |5.1 - (-3.3)|$$

$$= 8.4 \checkmark$$

- (b) The point  $T$  bisects  $\overline{RS}$ . Find the value of  $T$ , and mark and label it on the numberline  $\overleftrightarrow{RS}$  shown above.

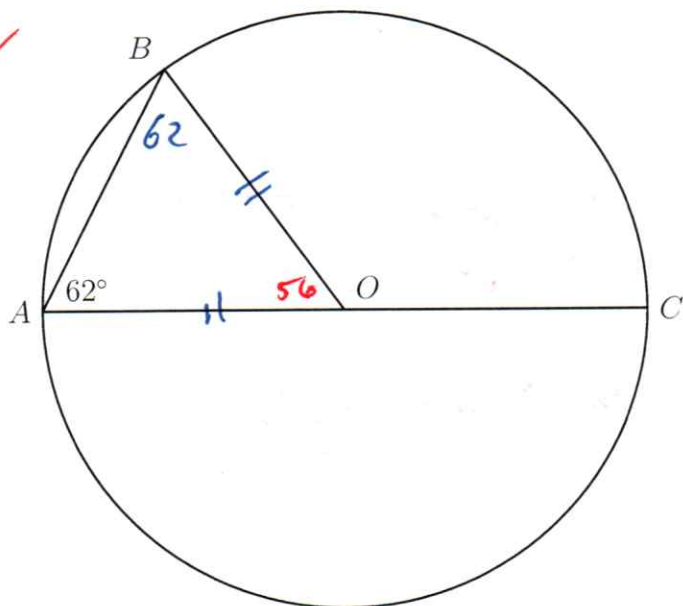
$$T = R + \frac{RS}{2}$$

$$= -3.3 + \frac{8.4}{2} = 0.9 \checkmark$$

18. The circle  $O$  is shown below with diameter  $\overline{AOC}$  and radius  $\overline{BO}$ . It is given that  $m\angle BAO = 62^\circ$ . Find the measure of the central angle  $\angle AOB$ .

$$m\angle AOB + 62 + 62 = 180 \checkmark$$

$$m\angle AOB = 56 \checkmark$$



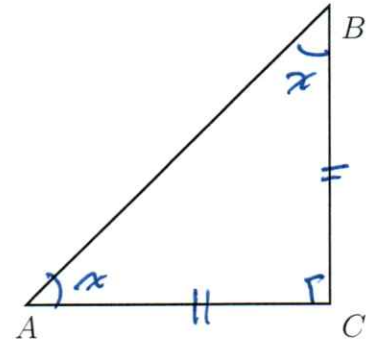
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19. Given isosceles right  $\triangle ABC$  with  $\overline{AC} \cong \overline{BC}$  and  $\overline{AC} \perp \overline{BC}$ . Find  $m\angle A$ .

$$\begin{aligned} m\angle A &= m\angle B \\ m\angle A + m\angle B + 90 &= 180 \\ 2 \times m\angle A &= 90 \\ m\angle A &= 45^\circ \end{aligned}$$



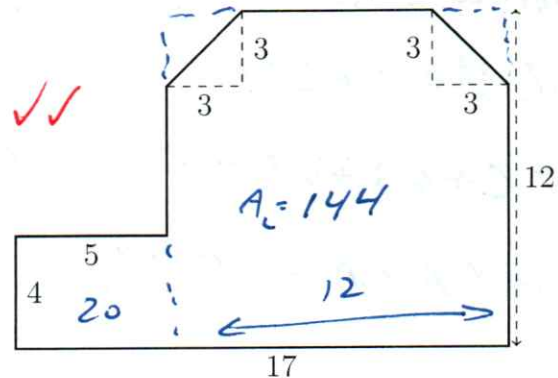
20. A sheet metal part is cut with square corners and two  $45^\circ$  cutouts as shown with lengths marked in centimeters.

- (a) Find the area of the figure. (the drawing is not to scale)

$$A_{\text{small}} = 4 \times 5 = 20$$

$$A_{\text{large}} = 12 \times 12 = 144$$

$$A_{\text{cut}} = \frac{1}{2} (3 \times 3) = 4\frac{1}{2}$$



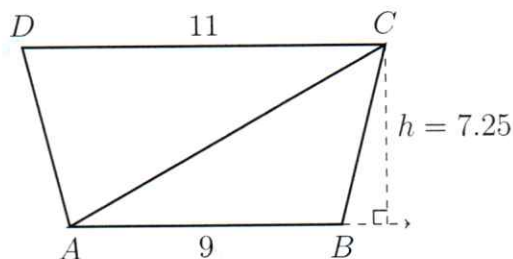
$$A_{\text{Fig}} = 20 + 144 - 2(4\frac{1}{2}) = 155$$

- (b) Spicy: The weight of the sheet metal is 2.25 grams per square centimeter. Find the weight of the part.

$$W = 155 \text{ cm}^2 \times \frac{2.25 \text{ g}}{1 \text{ cm}^2} = 348.75 \text{ g}$$

⑥

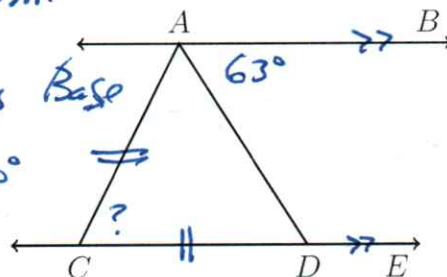
21. The trapezoid  $ABCD$  has two parallel sides,  $\overline{AB} \parallel \overline{CD}$  with lengths  $AB = 9$  and  $CD = 11$ . The trapezoid's height is  $h = 7.25$ . Find the area of the trapezoid. ✓



$$A = \frac{1}{2}(9)(7.25) + \frac{1}{2}(11)(7.25) \\ = 72.5 \quad \checkmark$$

22. Given parallel lines  $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$  with  $\overline{AC} \cong \overline{CD}$ . If  $m\angle BAD = 63$  find  $m\angle ACD$ .

$m\angle ADC = m\angle PAD = 63^\circ$  Alt. Interior  
 $m\angle CAD = m\angle ADC = 63^\circ$  Isosceles Base  
 $m\angle C + 63 + 63 = 180$  ✓  $\triangle$  Sum 180  
 $m\angle ACD = 54^\circ$  ✓

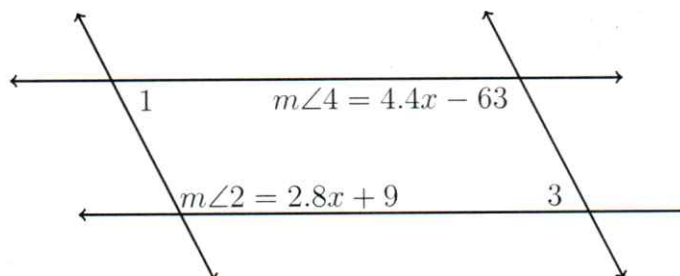


23. Two parallel lines intersect a second set of parallel lines. Given  $m\angle 2 = 2.8x + 9$  and  $m\angle 4 = 4.4x - 63$ , find the measure of  $\angle 1$ .

$m\angle 2 = m\angle 4$   
 $2.8x + 9 = 4.4x - 63$  ✓  
 $1.6x = 72$

$x = 45$  ✓  
 $m\angle 2 = 2.8(45) + 9$   
 $= 135$  ✓

$m\angle 1 + 135 = 180$   
 $m\angle 1 = 45$  ✓



check  
 $m\angle 4 = 4.4(45) - 63$   
 $= 135$  ✓

(9)



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**Do Not Solve!**

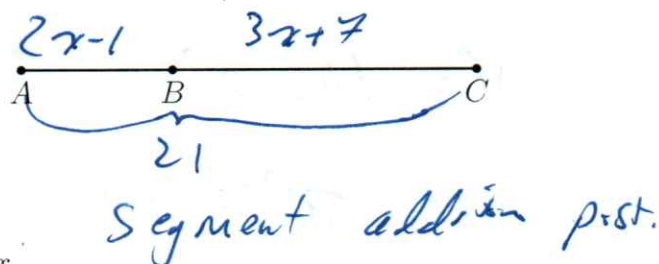
**Model the situation with an equation in terms of  $x$ .**

24. Given  $\overline{ABC}$ , with  $AB = 2x - 1$ ,  $BC = 3x + 7$ , and  $AC = 21$ . Find  $x$ .

$$2x - 1 + 3x + 7 = 21$$

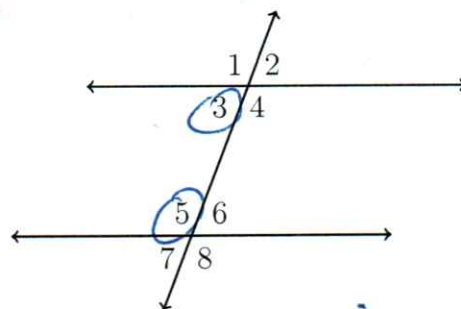
$$5x = 15$$

$$x = 3$$



25. Given  $m\angle 3 = x + 35$  and  $m\angle 5 = 4x - 25$ . Find  $x$ .

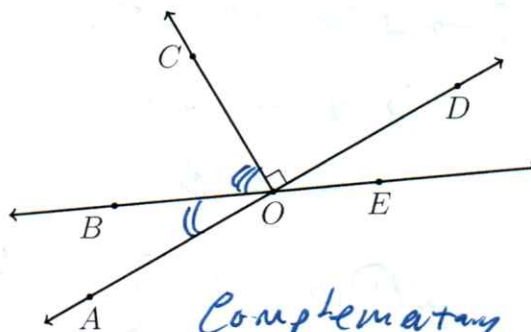
$$x + 35 + 4x - 25 = 180$$



Same-side interior angles

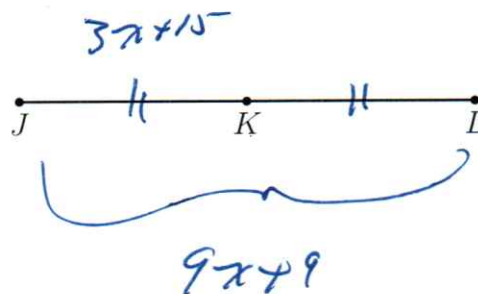
26. In the diagram below  $m\angle AOB = 6x + 5$  and  $m\angle COB = 8x + 15$ . Find  $x$ .

$$6x + 5 + 8x + 15 = 90$$



27. The point  $K$  is the midpoint of  $\overline{JL}$ ,  $JK = 3x + 15$ , and  $JL = 9x + 9$ . Find  $x$ .

$$2(3x + 15) = 9x + 9$$



4

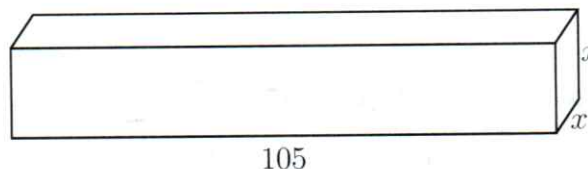
28. A feeding trough in the shape of a rectangular prism is 105 inches long. The trough's cross section is square. If its volume is 15,120 cubic inches, what is the dimension of each side of its square end,  $x$ ? (drawing not to scale)

$$V = 105x^2 = 15,120 \checkmark$$

$$x^2 = 144$$

$$x = 12 \checkmark$$

(N2)



29. Given  $\overrightarrow{BA} \perp \overrightarrow{BC}$ ,  $m\angle ABD = 2x$ , and  $m\angle DBC = x - 15$ . Find  $m\angle DBC$ .

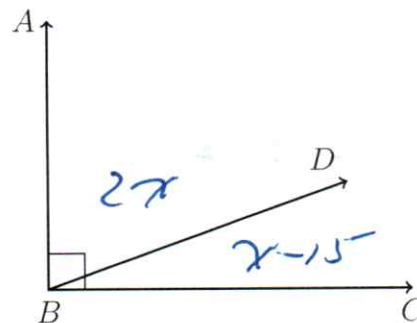
For full credit, show the check using both angle measures.

$$2x + x - 15 = 90 \checkmark$$

$$3x = 105$$

$$x = 35 \checkmark$$

$$m\angle DBC = 35 - 15 = 20 \checkmark$$



check

$$m\angle ABD = 2(35) = 70 \checkmark$$

$$20 + 70 = 90 \checkmark$$

⑥

## Early finishers

30. In the diagram below  $m\angle AOB = 3x + 11$  and  $m\angle DOE = 5x - 3$ . Find  $m\angle DOE$ .

(Calculate  $m\angle AOB$  as a check)

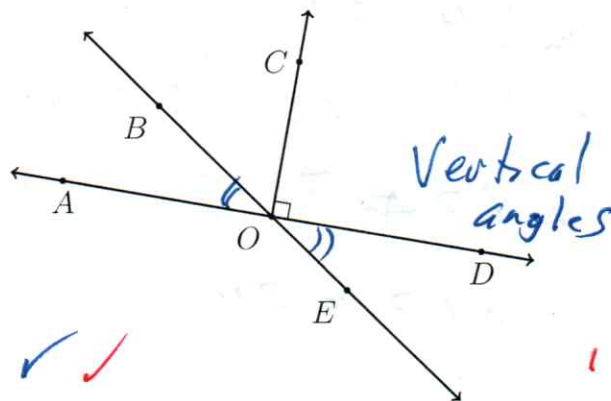
$$3x + 11 = 5x - 3$$

$$x = 7$$

$$m\angle DOE = 5(7) - 3 = 32$$

check

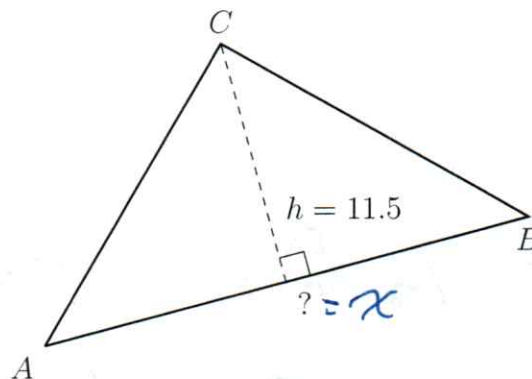
$$m\angle AOB = 3(7) + 11 = 32 \checkmark \checkmark$$



31. One side of the  $\triangle ABC$  has a height  $h = 11.5$ . The triangle's area is 103.5. Find the length of the side  $\overline{AB}$ .

$$A = \frac{1}{2}(11.5)x = 103.5$$

$$x = 18 \checkmark$$



32. Of two complementary angles, the measure of  $\angle A$  is five times that of  $\angle B$ . Find  $m\angle A$ .

$$m\angle B = x$$

$$m\angle A = 5x$$

$$5x + x = 90 \checkmark$$

$$x = 15$$

$$m\angle A = 5(15)$$

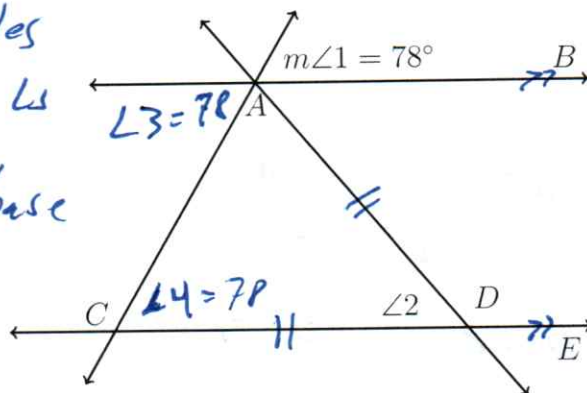
$$= 75 \checkmark$$

check  $15 + 75 = 90 \checkmark$

33. Given parallel lines  $\overleftrightarrow{AB} \parallel \overleftrightarrow{CDE}$  with  $\overline{AD} \cong \overline{CD}$ . If  $m\angle 1 = 78$  find  $m\angle 2$ .

$m\angle 3 = 78$   
 $m\angle 4 = 78$   
 $m\angle A = 78$   
 $m\angle 2 \neq 78 + 78 = 180$   
 $m\angle 2 = 24$  ✓

Vertical angles  
 alt. interior ls  
 Isosceles base



34. The volume of the rectangular prism shown is 120 cubic feet. Its length is length is ten feet longer than its height  $x$ . Its depth is 5 feet. Find the length of the prism.  
 (not drawn to scale)

$$V = (x+10)5x = 120$$

$$x^2 + 50x = 24$$

$$x^2 + 50x - 24 = 0$$

$$(x+12)(x-2) = 0$$

$$x = -12, +2$$

$$x = 2$$

$$l = x + 10 = 12$$
 ✓

disregard negative length

check

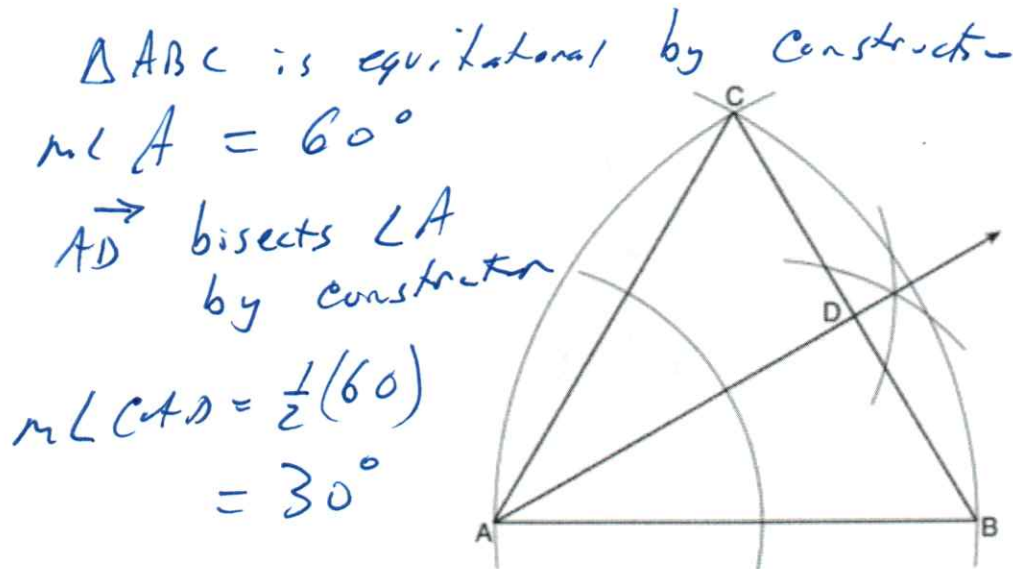
$$12 \times 5 \times 2 = 120$$
 ✓

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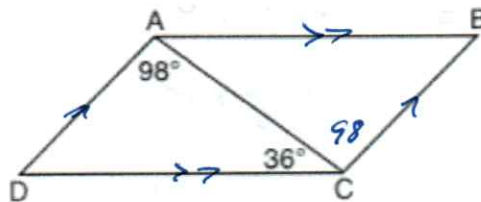
35. Regents problem 1

Using the construction below, state the degree measure of  $\angle CAD$ . Explain why.



36. Regents problem 2

In parallelogram ABCD shown below  $m\angle DAC = 98^\circ$  and  $m\angle ACD = 36^\circ$ .



What is the measure of angle B? Explain why.

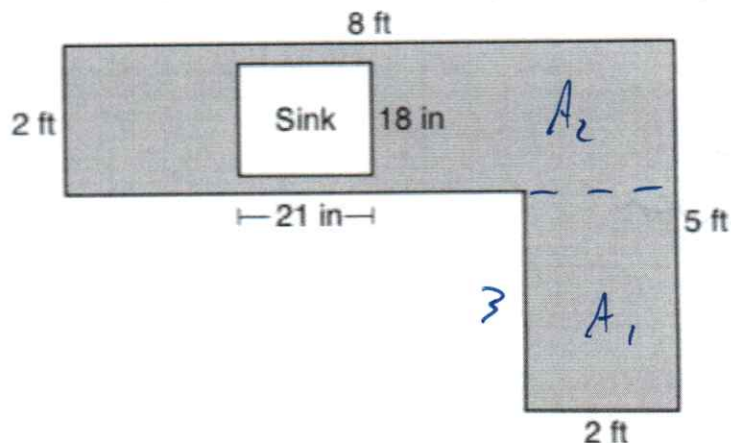
$m\angle ACB = 98^\circ$  Alt. interior angles  
 $m\angle DCB = 36 + 98$   
 $= 134$   
 $m\angle B + 134 = 180$   
 $m\angle B = 46$

angle sum  
 consecutive parallelogram  
 interior angles



37. Regents problem 3

A countertop for a kitchen is modeled with the dimensions shown below. An 18-inch by 21-inch rectangle will be removed for the installation of the sink.



What is the area of the top of the installed countertop, to the nearest square foot?

$$A_1 = 2 \times 3 = 6$$

$$A_2 = 2 \times 8 = 16$$

$$A_s = 1\frac{1}{2} \times 1\frac{3}{4}$$
$$= 2.625$$

$$A_c = 6 + 16 - 2.625$$
$$= 19.375$$
$$\approx 19 \text{ sq. ft.}$$