

### Quiz: Functions and Quadratics

*Take home, open notes, open book (including Wikipedia and other online materials). No online calculators or human help. Due Tuesday.*

Answer this section's problems on lined paper using IB standards. (and no ragged edges)

## Solve for the roots or zeros of a quadratic function, $f(x) = 0$

### Factoring

Factor each function then state the function's zeros.

1.  $f(x) = x^2 - 5x$
2.  $f(x) = x^2 + 5x + 6$
3.  $f(x) = 2x^2 - 15x + 7$
4.  $f(x) = \frac{1}{2}x^2 + 4x - 10$

### Using the quadratic formula

Find an exact solution by using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

5.  $x^2 + 3x - 5 = 0$
6.  $3x^2 + 7x = 2$

Use the discriminant in the following two problems.  $D = b^2 - 4ac$

7. Show that the function  $f(x) = -2x^2 - 6x + 5$  has two distinct zeros.
8. Solve for  $k$  such that the function  $g(x) = x^2 - kx + 25$  has a single (double) root.

### Completing the square

Rewrite the function in vertex form:  $f(x) = a(x - h)^2 + k$ . Include the step showing the  $(-\frac{b}{2a})^2$  term. State the vertex as an ordered pair and the equation for the axis of symmetry.

9.  $f(x) = x^2 + 6x + 4$
10.  $f(x) = x^2 - 12x + 20$

### The inverse of a function

Derive the inverse of each function. Simplify the expression.

11.  $f(x) = 2x + 1$
12.  $f(x) = \sqrt{x + 2}$

### Function substitution

13. Given  $f(x) = 3x^2 - x + 17$ . Simplify  $f(-3x)$ .
14. Given  $f(x) = 5 - (x^2 + 4x)$ . Simplify  $f(\frac{1}{3}x + 1)$ .

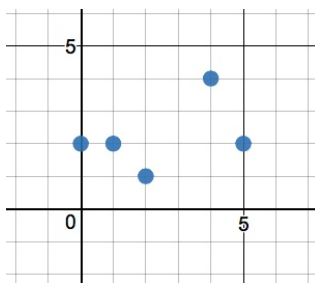
### Function composition

In each exercise, perform the composition  $f \circ g$  and simplify.

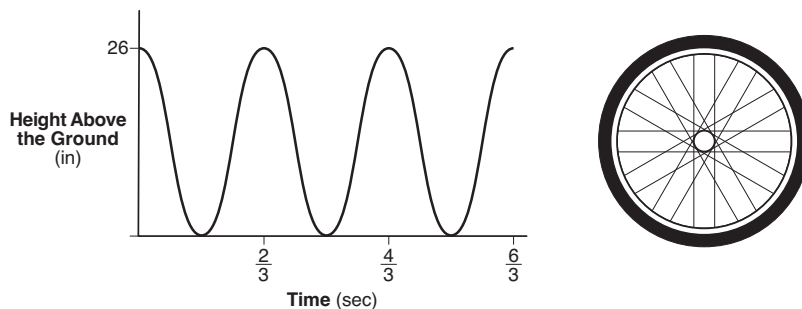
15. Given  $f(x) = x^2 - x$  and  $g(x) = 3x - 1$
16. Given  $f(x) = 1 - \frac{2x}{x^2 - x}$  and  $g(x) = 2x$

### Domain and range of a function

17. Write down the domain and range of the function graphed below.



18. What is the range of the given function modeling a bicycle wheel?



### Function transformations

Describe how the functions below have transformed from the parent function  $f(x) = |x|$ .

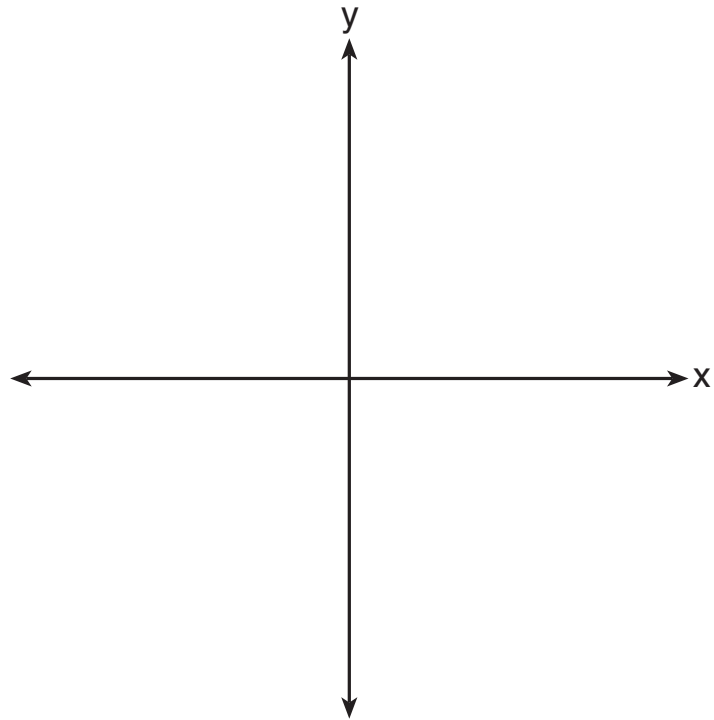
19.  $g(x) = |x + 2|$
20.  $h(x) = -|x| + 2$

### Sketching a quadratic function

Answer in the space provided. (you may also use additional lined paper)

21. Given  $f(x) = -(x - 3)^2 + 16$

- (a) Write down the vertex of the function as an ordered pair.
- (b) Write down the equation of the axis of symmetry.
- (c) Expand the function from vertex form to standard form,  $ax^2 + bx + c$  where  $a, b, c \in \mathbb{R}$ .
- (d) Write down the value of  $f(0)$ . Explain what this represents on the graph.
- (e) Hence factor the function. Write down the roots.
- (f) Sketch the function, labeling the intercepts with values and the vertex as an ordered pair. Show the axis of symmetry as a dotted line and label it with its equation.



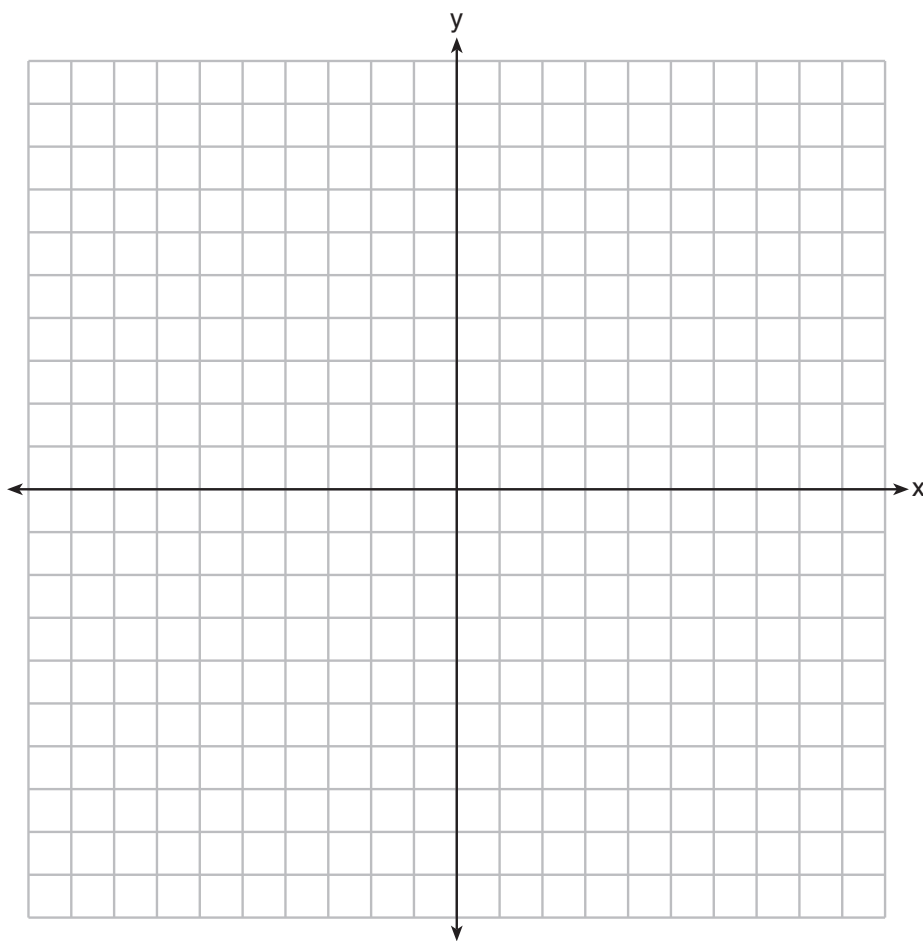
- (g) Write down the domain and range of the function.

## Graphing quadratics

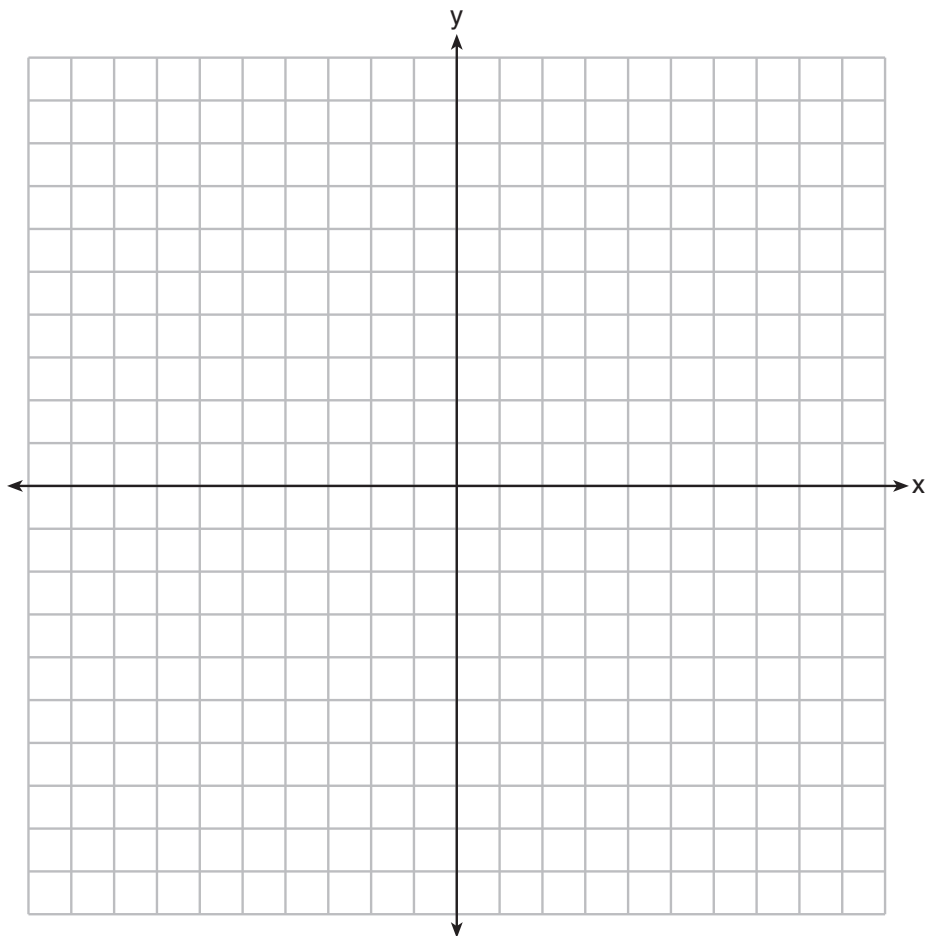
Answer on lined paper. Graph the function on the grid shown below.

22. Given the function  $f(x) = -x^2 - x + 6$ .

- (a) Write down the  $y$ -intercept.
- (b) State whether the parabola opens upward or downward. Explain how you know this from the function expressed in standard form.
- (c) Express the function in factored form. Hence state the solutions to  $f(x) = 0$ .
- (d) Show that the axis of symmetry of the parabola is  $x = -\frac{1}{2}$ .
- (e) Hence state the vertex as an ordered pair.
- (f) Graph the function. Mark the vertex as an ordered pair and label each intercept with its value. Plot the axis of symmetry as a dotted line and label it with its equation.
- (g) Write down the domain and range of the function.



23. (a) Graph the parent function  $f(x) = x^2$ . Mark the point  $P(3, f(3))$  on the graph  
(b) The function  $g(x)$  is the function  $f$  after being translated to the right 5 and down 4. Graph  $g$ .  
(c) Mark the point on the function  $g$ ,  $Q$ , that represents the point  $P$  after the translation.



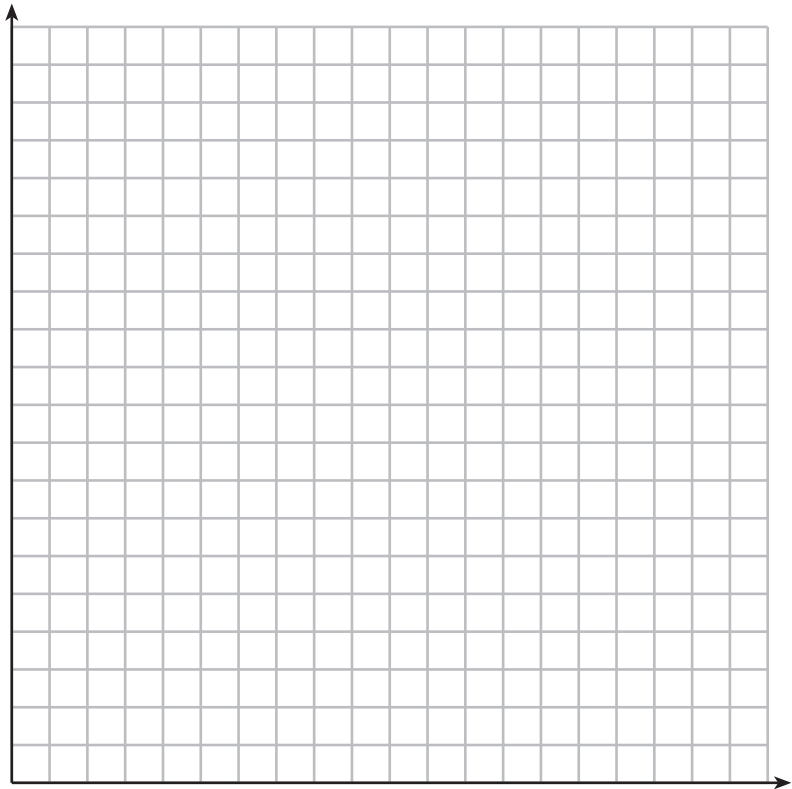
## Model situations with quadratic functions

24. The path of a diver is given by

$$f(x) = -5x^2 + 12x + 9$$

where  $y$  is the height (in meters) and  $x$  is time in seconds.

- (a) On the grid below, graph the function over the domain where  $x \geq 0$  and the range where  $f(x) \geq 0$ . Use a horizontal scale of 5 squares equals one second and vertical scale of 1 square equals one meter. Label the intercepts and vertex.
- (b) What is the maximum height of the diver? Label the point on the graph with the word “max.”
- (c) What is the time when the diver enters the water? Label the point on the graph representing this with the word “splash.”



## Honor pledge

I have not received human help with this test, nor have I used calculators (including Desmos) except for an approved graphing calculator. Signed: