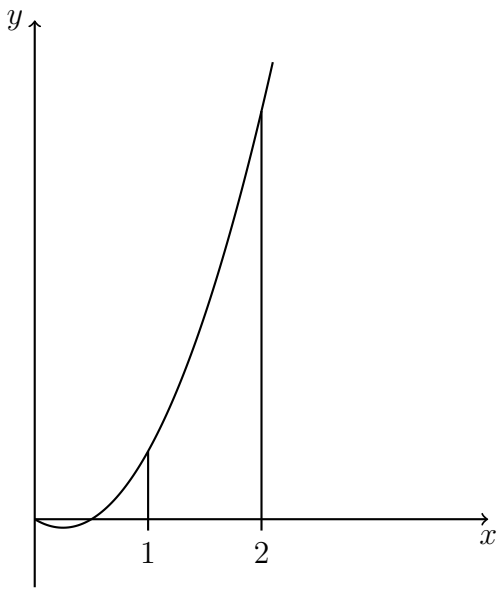
Spiral Review: 6-4+5 P1 (No Calculator) Calculus Integration

1. 16M.2.sl.TZ1.2
   Let $f(x) = x^2$ and $g(x) = 3\ln(x+1)$, for $x > -1$.

   (a) Solve $f(x) = g(x)$ [3 marks]

   (b) Find the area of the region enclosed by the graphs of f and g. [3 marks]

2. 18M.1.sl.TZ2.2
   Let $f(x) = 6x^2 - 3x$. The graph of f is shown in the following diagram.



   (a) Find $f'(x)$. [2 marks]

   (b) Find the area of the region enclosed by the graph of f, the x-axis and the lines $x = 1$ and $x = 2$. [4 marks]

3. 15N.1.sl.TZ0.3
   Let $f'(x) = 6x^2 - 5.$ and $f(2) = -3$, find $f(x)$. [6 marks]

4. 13N.1.sl.TZ0.4a
   Consider a function $f(x)$ such that $\int_1^6 f(x)\,\mathrm{d}x = 8$.

   (a) Find $\int_1^6 2f(x)\,\mathrm{d}x$. [2 marks]

   (b) Find $\int_1^6 (f(x)+2)\,\mathrm{d}x$. [4 marks]

5. 16N.2.sl.TZ0.4
   Let $f(x) = xe^{-x}$ and $g(x) = -3f(x) + 1$.
   The graphs of f and g intersect at $x = p$ and $x = q$, where $p < q$.

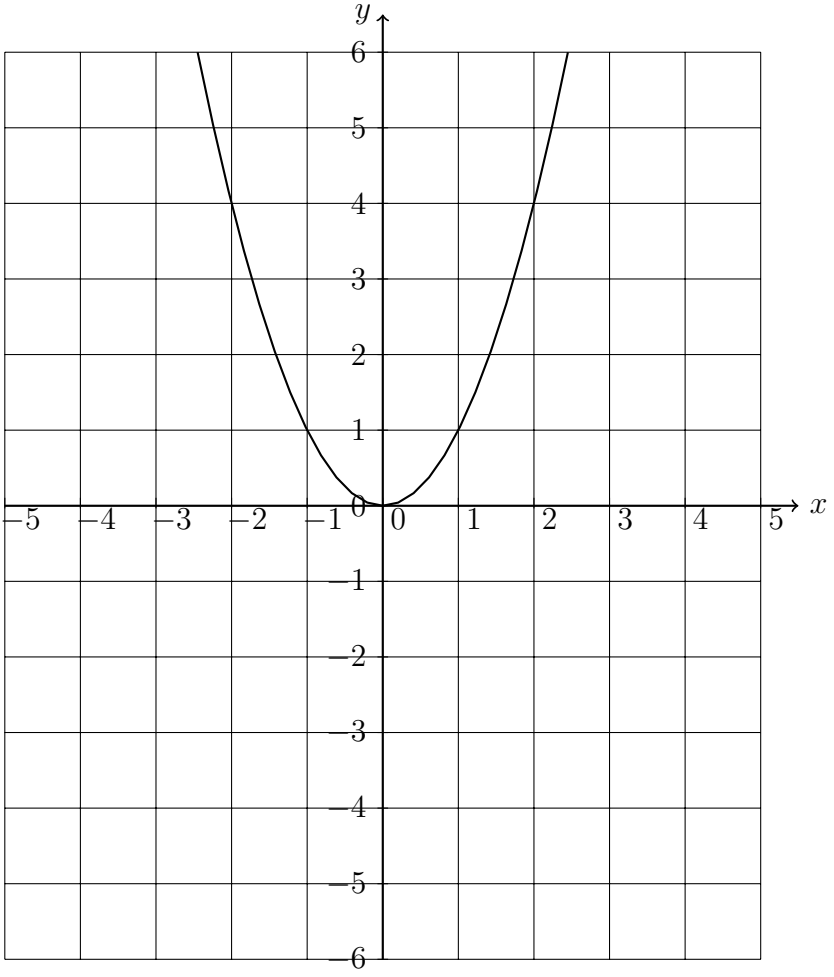
   (a) Find the values of f and q. [3 marks]

(b) Hence, find the area of the region enclosed by the graphs of f and g. [3 marks]

6. 18M.2.sl.TZ1.4
Let $g(x) = -(x-1)^2 + 5$.

(a) Write down the coordinates of the vertex of the graph of g. [1 mark]

(b) Let $f(x) = x^2$. The following diagram shows part of the graph of f.



The graph of g intersects the graph of f at $x = -1$ and $x = 2$.
On the grid above, sketch the graph of g for $-2 \leq x \leq 4$. [3 marks]

(c) Find the area of the region enclosed by the graphs of f and g. [3 marks]

7. 14M.1.sl.TZ2.5
The graph of a function h passes through the point $(\frac{\pi}{12}, 5)$.
Given that $h'(x) = 4\cos 2x$, find $h(x)$. [6 marks]

8. (#19) 17N.1.sl.TZ0.8
Let $f(x) = x^2 - x$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f.

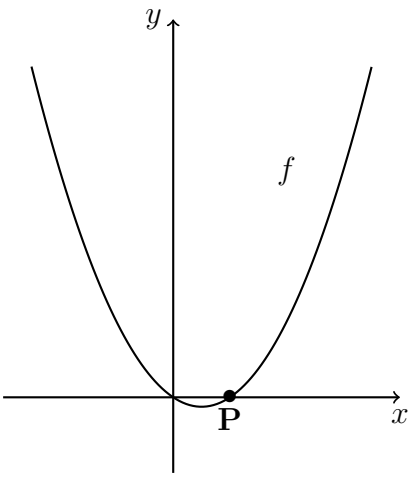


diagram not to scale

The graph of f crosses the x-axis at the origin and at the point $P(1, 0)$.

(a) Show that $f'(1) = 1$. [3 marks]

(b) The line L is the normal to the graph of f at P.
Find the equation of L in the form $y = ax + b$. [3 marks]

(c) The line L intersects the graph of f at another point Q, as shown in the following diagram.

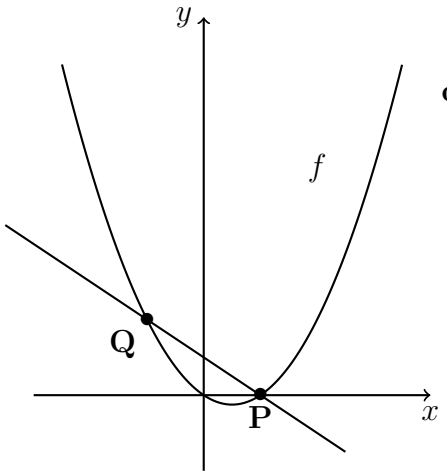


diagram not to scale

Find the x-coordinate of Q.[4 marks]

(d) Find the area of the region enclosed by the graph of f and the line L. [6 marks]

9. (#23) 17M.1.sl.TZ2.10
Let $f(x) = x^2$. The following diagram shows part of the graph of f.

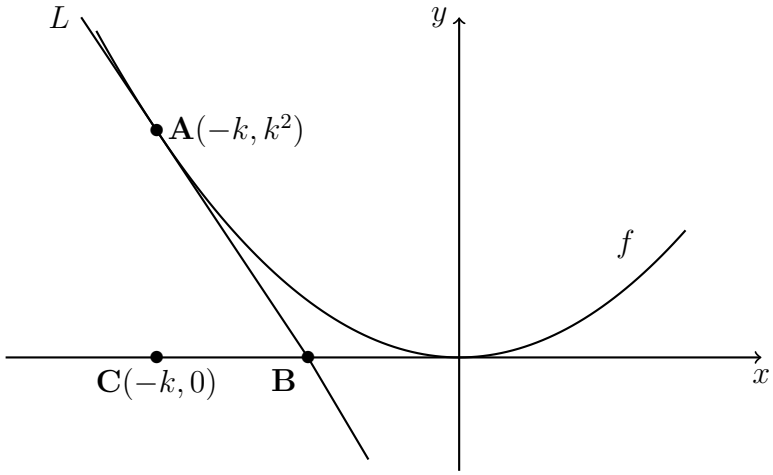


diagram not to scale

The line L is the tangent to the graph of f at the point $A(-k, k^2)$, and intersects the x-axis at point B. The point C is $(-k, 0)$.

(a) Write down $f'(x)$. [1 mark]

(b) Find the gradient of L. [2 marks]

(c) Show that the x-coordinate of B is $-\frac{k}{2}$. [5 marks]

(d) Find the area of triangle ABC, giving your answer in terms of k. [2 marks]

(e) The region R is enclosed by L, the graph of f, and the x-axis. This is shown in the following diagram.

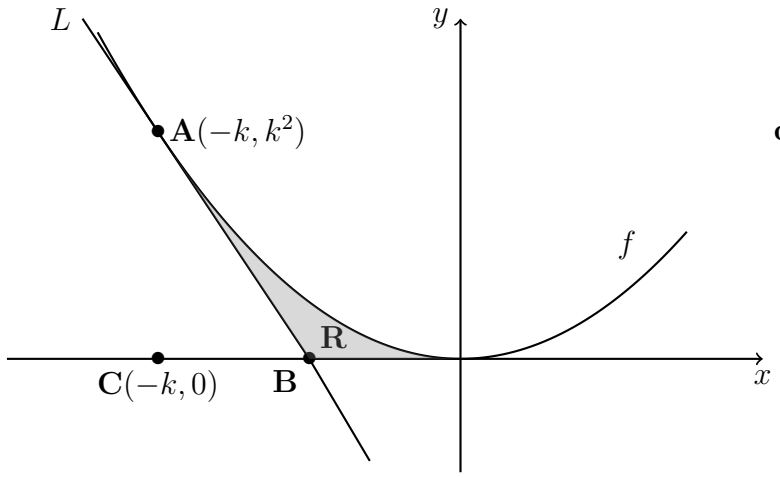


diagram not to scale

Given that the area of triangle ABC is p times the area of R, find the value of p. [7 marks]