BECA / Huson / 12.1 IB Math SL Name:
13 December 2017
Pretest: Vector and calculus, plus review

1a. Line L_1 passes through points $\mathrm{A}(1,\ -1,\ 4)$ and $\mathrm{B}(2,\ -2,\ 5)$.

Find $\overrightarrow{\mathrm{AB}}$. *[2 marks]*

1b. Find an equation for L_1 in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$. *[2 marks]*

1c. Line L_2 has equation $\boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} + s\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$.

Find the angle between L_1 and L_2. *[7 marks]*

1d. The lines L_1 and L_2 intersect at point C. Find the coordinates of C. *[6 marks]*

2a. The diagram shows quadrilateral ABCD with vertices A(1, 0), B(1, 5), C(5, 2) and D(4, −1).

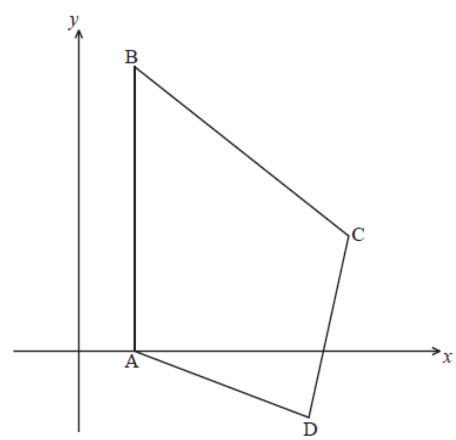


diagram not to scale

(i) Show that $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$.

(ii) Find $\overrightarrow{\mathrm{BD}}$.

(iii) Show that $\overrightarrow{\mathrm{AC}}$ is perpendicular to $\overrightarrow{\mathrm{BD}}$.

2b. The line (AC) has equation $\boldsymbol{r} = \boldsymbol{u} + s\boldsymbol{v}$.

(i) Write down vector ***u*** and vector ***v***.

(ii) Find a vector equation for the line (BD). *[4 marks]*

2c. The lines (AC) and (BD) intersect at the point $\mathrm{P}(3,\ k)$.

Show that $k = 1$. *[3 marks]*

2d. The lines (AC) and (BD) intersect at the point $\mathrm{P}(3,\ k)$.

Hence find the area of triangle ACD. *[5 marks]*

3. Let $\boldsymbol{v} = \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} k \\ -2 \\ 4 \end{pmatrix}$, for $k > 0$. The angle between ***v*** and ***w*** is $\frac{\pi}{3}$.

Find the value of k. *[7 marks]*

4a.

The line L_1 is represented by the vector equation $\boldsymbol{r} = \begin{pmatrix} -3 \\ -1 \\ -25 \end{pmatrix} + p\begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix}$.

A second line L_2 is parallel to L_1 and passes through the point B($-8, -5, 25$).

Write down a vector equation for L_2 in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$. *[2 marks]*

4b. A third line L_3 is perpendicular to L_1 and is represented by $\boldsymbol{r} = \begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix} + q\begin{pmatrix} -7 \\ -2 \\ k \end{pmatrix}$.

Show that $k = -2$. *[5 marks]*

4c. The lines L_1 and L_3 intersect at the point A.

Find the coordinates of A. *[6 marks]*

4d. The lines L_2 and L_3 intersect at point C where $\overrightarrow{\mathrm{BC}} = \begin{pmatrix} 6 \\ 3 \\ -24 \end{pmatrix}$.

(i) Find $\overrightarrow{\mathrm{AB}}$.

(ii) Hence, find $|\overrightarrow{\mathrm{AC}}|$. *[5 marks]*

5a. Let $\overrightarrow{AB} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$.

Find $\overrightarrow{BC}$. *[2 marks]*

5b. *[3 marks]*

Find a unit vector in the direction of $\overrightarrow{AB}$.

5c. *[3 marks]*

Show that $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{AC}$.

6a. *[4 marks]*

In this question, distance is in metres.

Toy airplanes fly in a straight line at a constant speed. Airplane 1 passes through a point A.

Its position, *p* seconds after it has passed through A, is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix} + p\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$.

(i) Write down the coordinates of A.

(ii) Find the speed of the airplane in ms^{-1}.

6b. *[5 marks]*

After seven seconds the airplane passes through a point B.

(i) Find the coordinates of B.

(ii) Find the distance the airplane has travelled during the seven seconds.

6c. Airplane 2 passes through a point C. Its position *q* seconds after it passes through C is given by

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \\ 8 \end{pmatrix} + q\begin{pmatrix} -1 \\ 2 \\ a \end{pmatrix}, a \in \mathbb{R}.$$

The angle between the flight paths of Airplane 1 and Airplane 2 is 40°. Find the two values of *a*.

[7 marks]

7a. Let $f(x) = \frac{6x}{x+1}$, for $x > 0$.

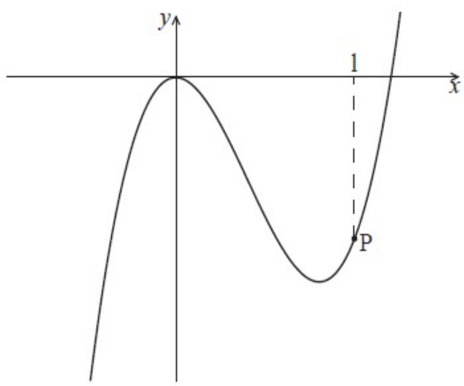
Find $f'(x)$. *[5 marks]*

7b. Let $g(x) = \ln\left(\frac{6x}{x+1}\right)$, for $x > 0$.

Show that $g'(x) = \frac{1}{x(x+1)}$. *[4 marks]*

7c. Let $h(x) = \frac{1}{x(x+1)}$. The area enclosed by the graph of *h*, the *x*-axis and the lines $x = \frac{1}{5}$ and $x = k$ is $\ln 4$. Given that $k > \frac{1}{5}$, find the value of *k*. *[7 marks]*

8a. Part of the graph of $f(x) = ax^3 - 6x^2$ is shown below.



The point P lies on the graph of f. At P, *x* = 1.

Find $f'(x)$. *[2 marks]*

8b. The graph of f has a gradient of 3 at the point P. Find the value of a. *[4 marks]*

9a. In this question, you are given that $\cos\frac{\pi}{3} = \frac{1}{2}$, and $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

The displacement of an object from a fixed point, O is given by $s(t) = t - \sin 2t$ for $0 \le t \le \pi$.

Find $s'(t)$. *[3 marks]*

9b. In this interval, there are only two values of *t* for which the object is not moving. One value is $t = \frac{\pi}{6}$.

Find the other value. *[4 marks]*

9c. Show that $s'(t) > 0$ between these two values of *t*. *[3 marks]*

9d. Find the distance travelled between these two values of *t*. *[5 marks]*

10a. Let $f(x) = \mathrm{e}^{6x}$.

Write down $f'(x)$. *[1 mark]*

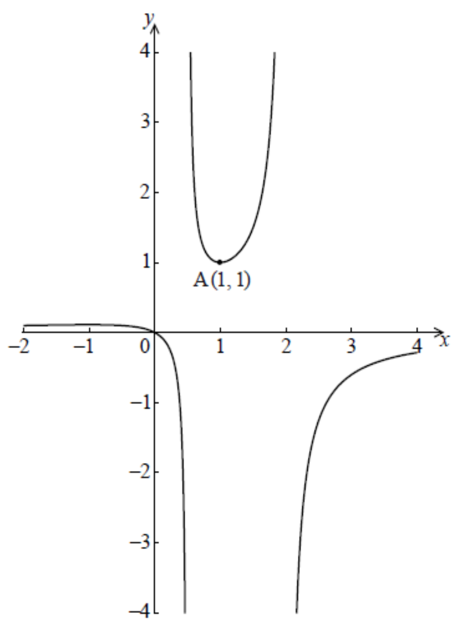
10b. The tangent to the graph of *f* at the point $\mathrm{P}(0, b)$ has gradient *m* .

(i) Show that $m = 6$.

(ii) Find *b* . *[4 marks]*

10c. Hence, write down the equation of this tangent. *[1 mark]*

11a. Let $f(x) = \frac{x}{-2x^2+5x-2}$ for $-2 \le x \le 4$, $x \ne \frac{1}{2}$, $x \ne 2$. The graph of f is given below.



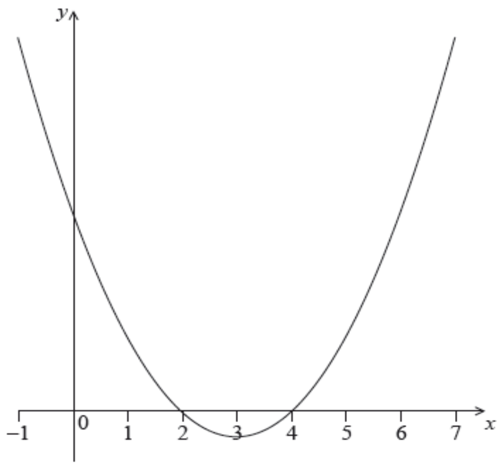
The graph of f has a local minimum at A(1, 1) and a local maximum at B.

Use the quotient rule to show that $f'(x) = \frac{2x^2-2}{(-2x^2+5x-2)^2}$. *[6 marks]*

11b. Hence find the coordinates of B. *[7 marks]*

11c. Given that the line $y = k$ does not meet the graph of *f* , find the possible values of *k* . *[3 marks]*

12a. The following diagram shows part of the graph of a quadratic function f.



The vertex is at $(3, \ -1)$ and the x-intercepts at 2 and 4.

The function f can be written in the form $f(x) = (x - h)^2 + k$.

Write down the value of h and of k. *[2 marks]*

12b. The function can also be written in the form $f(x) = (x - a)(x - b)$.

Write down the value of a and of b. *[2 marks]*

12c. Find the y-intercept. *[2 marks]*

13. Three consecutive terms of a geometric sequence are $x - 3$, 6 and $x + 2$.

Find the possible values of x. *[6 marks]*

14a. Let $f(x) = x^2$ and $g(x) = 3\ln(x + 1)$, for $x > -1$.

Solve $f(x) = g(x)$. *[3 marks]*

14b. Find the area of the region enclosed by the graphs of f and g. *[3 marks]*

15a. A population of rare birds, P_t, can be modelled by the equation $P_t = P_0 e^{kt}$, where P_0 is the initial population, and t is measured in decades. After one decade, it is estimated that $\frac{P_1}{P_0} = 0.9$.

(i) Find the value of k.

(ii) Interpret the meaning of the value of k. *[3 marks]*

15b. Find the least number of **whole** years for which $\frac{P_t}{P_0} < 0.75$. *[5 marks]*

16a. The price of a used car depends partly on the distance it has travelled. The following table shows the distance and the price for seven cars on 1 January 2010.

Distance, x **km**	11 500	7500	13 600	10 800	9500	12 200	10 400
Price, y **dollars**	15 000	21 500	12 000	16 000	19 000	14 500	17 000

The relationship between x and y can be modelled by the regression equation $y = ax + b$.

(i) Find the correlation coefficient.

(ii) Write down the value of a and of b. *[4 marks]*

16b. On 1 January 2010, Lina buys a car which has travelled $11\,000$ km.

Use the regression equation to estimate the price of Lina's car, giving your answer to the nearest 100 dollars. *[3 marks]*

16c. The price of a car decreases by 5% each year.

Calculate the price of Lina's car after 6 years. *[4 marks]*

16d. Lina will sell her car when its price reaches $10\,000$dollars.

Find the year when Lina sells her car. *[4 marks]*

17a. Let $f(x) = \frac{1}{x-1} + 2$, for $x > 1$.

Write down the equation of the horizontal asymptote of the graph of f. *[2 marks]*

17b. Find $f'(x)$. *[2 marks]*

17c. Let $g(x) = ae^{-x} + b$, for $x \geqslant 1$. The graphs of f and g have the same horizontal asymptote.

Write down the value of b. *[2 marks]*

17d. Given that $g'(1) = -e$, find the value of a. *[4 marks]*

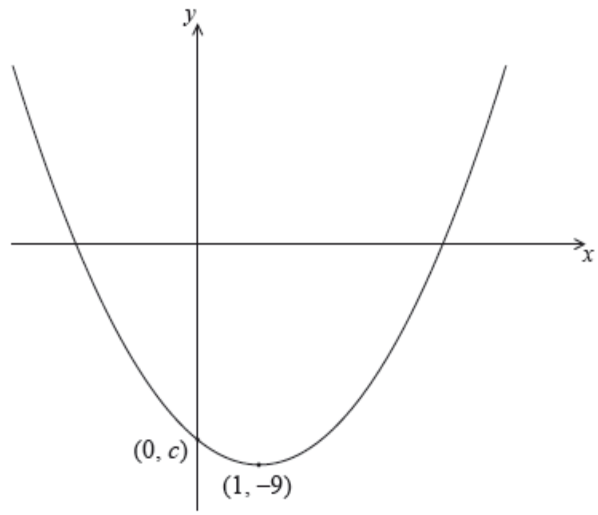
17e. There is a value of x, for $1 < x < 4$, for which the graphs of f and g have the same gradient. Find this gradient. *[4 marks]*

18a. Let $f(x) = (x-5)^3$, for $x \in \mathbb{R}$.

Find $f^{-1}(x)$. *[3 marks]*

18b. Let g be a function so that $(f \circ g)(x) = 8x^6$. Find $g(x)$. *[3 marks]*

19a. The following diagram shows part of the graph of a quadratic function f.



The vertex is at $(1, \ -9)$, and the graph crosses the *y*-axis at the point $(0, \ c)$.

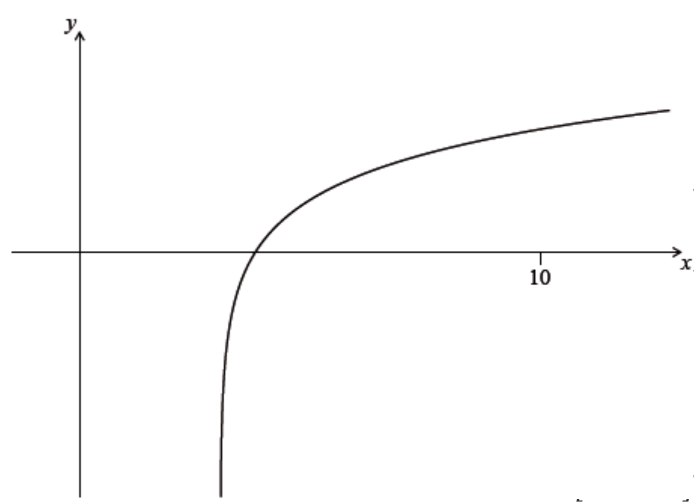
The function can be written in the form $f(x) = (x-h)^2 + k$.

Write down the value of h and of k.

19b. Let $g(x) = -(x-3)^2 + 1$. The graph of g is obtained by a reflection of the graph of f in the x-axis, followed by a translation of $\begin{pmatrix} p \\ q \end{pmatrix}$.

Find the value of p and of q. *[5 marks]*

20a. Let $f(x) = 2\ln(x-3)$, for $x > 3$. The diagram shows part of the graph of f. Find the equation of the vertical asymptote to the graph of f.



20b. Find the x-intercept of the graph of f.

21a. The first three terms of a geometric sequence are $u_1 = 0.64, \ u_2 = 1.6$, and $u_3 = 4$.

Find the value of r.

21b. Find the value of S_6. *[2 marks]*

21c. Find the least value of n such that $S_n > 75\,000$. *[3 marks]*