Homework 1

1. Let $A, B \subset X$. Prove the following two versions of the De-Morgan law:

$$\begin{aligned}(A \cap B)^c &= A^c \cup B^c, \\ (A \cup B)^c &= A^c \cap B^c.\end{aligned}$$

2. Define the following operation: $A - B = \{x : x \in A \text{ and } x \notin B\}$. Prove the following:

   (a) $(A - B) \cap C = (A \cap C) - B$.

   (b) $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$.

   (c) $A - (B - C) = (A - B) \cup (A \cap B \cap C)$.

   (d) $(A - B) \times C = (A \times C) - (B \times C)$.

3. Let $f : X \to Y$ be a function. Let $A, B \subset X$ and $C, D \subset Y$. Prove the following:

   (a) $f(A \cup B) = f(A) \cup f(B)$.

   (b) $f(A \cap B) \subset f(A) \cap f(B)$, is the opposite inclusion true as well. If yes prove it, if no give a counter example.

   (c) $f^{-1}(C \cup D) = f^{-1}(C) \cup f^{-1}(D)$.

   (d) $f^{-1}(C \cap D) = f^{-1}(C) \cap f^{-1}(D)$.

4. Prove that composition of injective functions is injective. Prove that composition of surjective functions is surjective.

5. Let X be a set with n elements. How many functions from X to itself? How many injective functions? How many surjective funcitons?

6. Let $f : \mathbb{R} - \{1\} \to \mathbb{R}$, given by $f(x) = \frac{x+1}{x-1}$. Prove that f is injective. Compute its image. Consider the bijective function $f : \mathbb{R} - \{1\} \to \operatorname{Im} f$ and compute the inverse function $f^{-1} : \operatorname{Im} f \to \mathbb{R} - \{1\}$.