Measuring Planck's Constant by Observation of the Photoelectric Effect

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Abstract

In this experiment, we measured Planck's constant by observing the photoelectric effect. We determined the stopping voltage as a function of the wavelength of the incident radiation and used this to calculate a value of $h = 1.6 \times 10^{-34} \pm 0.4 \times 10^{-34}$ Js, which does not agree with the accepted value of h.

1 Introduction

1.1 Physics Motivation

Planck's constant is a fundamental constant of nature that is ubiquitous in descriptions of quantum phenomena. [1] In this experiment, we aim to determine the numerical value of Planck's constant by observing the photoelectric effect. The photoelectric effect is the emission of electrons from a metal irradiated by electromagnetic radiation. The effect is quantum mechanical in nature and the study of it helped start the quantum revolution in physics. [1]

1.2 Theoretical background

The electrons in a metal are free to move between atoms, but they are still bound to the metal by a binding energy that arises from the electromagnetic attraction of the electrons to the atomic nuclei. When a metal is irradiated

by electromagnetic radiation, the electrons in the metal absorb energy from the radiation. If an electron absorbs enough energy to escape the binding energy it can be ejected from the metal. By energy conservation, if an ejected electron absorbed an amount of energy E from the radiation it will have a kinetic energy $K = E - V_b$, where V_b is the binding energy. The binding energy varies depending on the location of the electron in the metal, so it is convenient to define the work function of the metal, Φ, to be the minimum value of the binding energy. Then the maximum kinetic energy of an electron ejected from a metal irradiated by electromagnetic radiation carrying energy E is

$$K_{\max} = E - \Phi. \tag{1}$$

This maximum kinetic energy can be measured by illuminating a metal plate (the cathode) and allowing another metal plate (the anode) to collect the ejected electrons. If we connect the anode to a wire, we will measure a current due to the ejected electrons hitting the anode. Say we apply a voltage, V, between the anode and the cathode. The ejected electrons then have a potential energy $-eV$ when they arrive at the anode, where e is the magnitude of the electron charge. By energy conservation, the highest energy electrons will arrive at the anode with kinetic energy $K_a = K_{\max} + eV$. [1] By varying the voltage between the anode and the cathode, we can find a critical value of the potential, V_s, called the stopping potential, for which none of the ejected electrons reach the anode (i.e. there is no current produced) if $V < V_s$. Determining this stopping potential allows for a calculation of $K_{\max}$ using the equation

$$K_{\max} = e|V_s| \tag{2}$$

It was experimentally discovered that increasing the intensity of the light did not increase the energy imparted to the electrons, however, increasing the frequency of the incident light did. This fact was explained by Albert Einstein in 1905. Einstein hypothesized that electromagnetic radiation does not propagate as a wave but rather as a series of discrete packets called photons, each carrying energy $E = hf$, where h is Planck's constant and f is the frequency of the radiation. [2] Under this interpretation, an increase in the intensity of the light corresponds to an increase in the number of photons, but not an increase in the energy carried by each photon. This explained why increasing the frequency of the light caused an increase in

the energy acquired by the electrons but increasing the intensity did not. [1] [1] Using Einstein's expression for the energy of electromagnetic radiation along with equations 1 and 2, we find

$$|V_s| = \frac{hc}{e}\frac{1}{\lambda} + \frac{\Phi}{e} \tag{3}$$

2 Experimental setup

2.1 Apparatus

The experimental apparatus consisted of an RCA935 phototube, which housed a $CsSb_3$ cathode and an anode to collect the ejected electrons. [4] The cathode was irradiated by light from a mercury lamp that was passed through a filter which allowed only a specific wavelength of light to pass through. The RCA935 was connected to a picoammeter in order to measure the current produced by the electrons ejected from the cathode, and a computer controlled power supply which produced a variable potential difference between the anode and cathode, allowing for the determination of the stopping potential at various wavelengths. The entire assembly was housed inside a light box in order to ensure that only light from the mercury lamp was incident on the cathode. For more detail on the experimental apparatus, see Figure 1.

[1] It is possible for an electron to be ejected from a metal by absorbing multiple photons that wouldn't be energetic enough to eject the electron on their own. However, this is a rare occurrence because the excited electrons that remain in the metal lose energy to thermal effects very rapidly, and therefore the multiple photon collisions would have to occur almost simultaneously. This becomes more probable as the intensity of the light is increased, but in the intensities studied in this experiment, any photocurrent generated by this mechanism is negligible. [3]

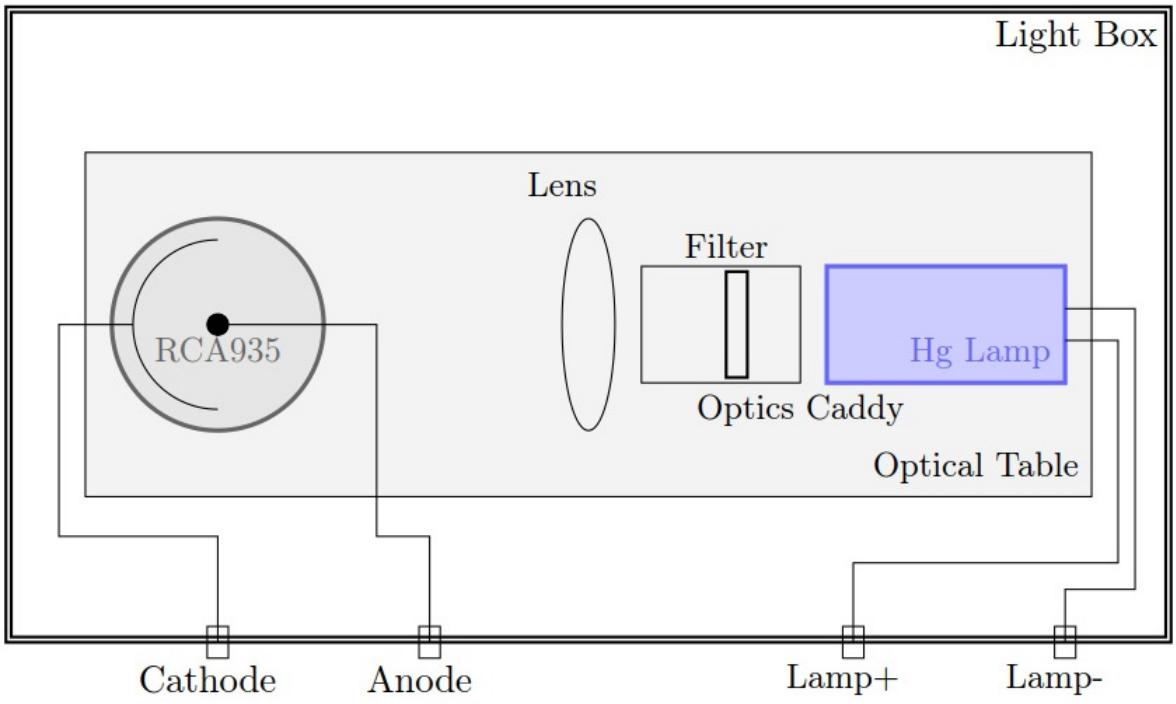

Figure 1: A schematic diagram of the experimental setup.
Light Box: A large black box that prevented external light from reaching the cathode.
Hg lamp: A mercury lamp used as the source of photons.
RCA935: An apparatus consisting of an anode and cathode that allowed for the detection of the photocurrent.
Lens: Used to focus the light from the lamp onto the cathode.
Filter: A piece of glass that only allowed a certain wavelength of light to pass through.
Adapted from *PHY 353L Course Website*

2.2 Data Collection

To collect our data, we used a LabView program to control the power supply and apply a range of different voltages to the anode-cathode assembly. This was done by specifying a starting voltage, a voltage range, and a voltage step size. For this experiment, we used a starting voltage of -2.0 V, a voltage range of 4.75 V, and a voltage step size of 0.02 V. We used the

picoammeter to measure the current at each value of the applied voltage, which allowed us to produce a set of current versus voltage data points for each wavelength of light incident on the cathode. We collected these data using six different incident wavelengths, and repeated the procedure three times for each wavelength. A typical current versus voltage graph that we obtained can be found in Figure 2.

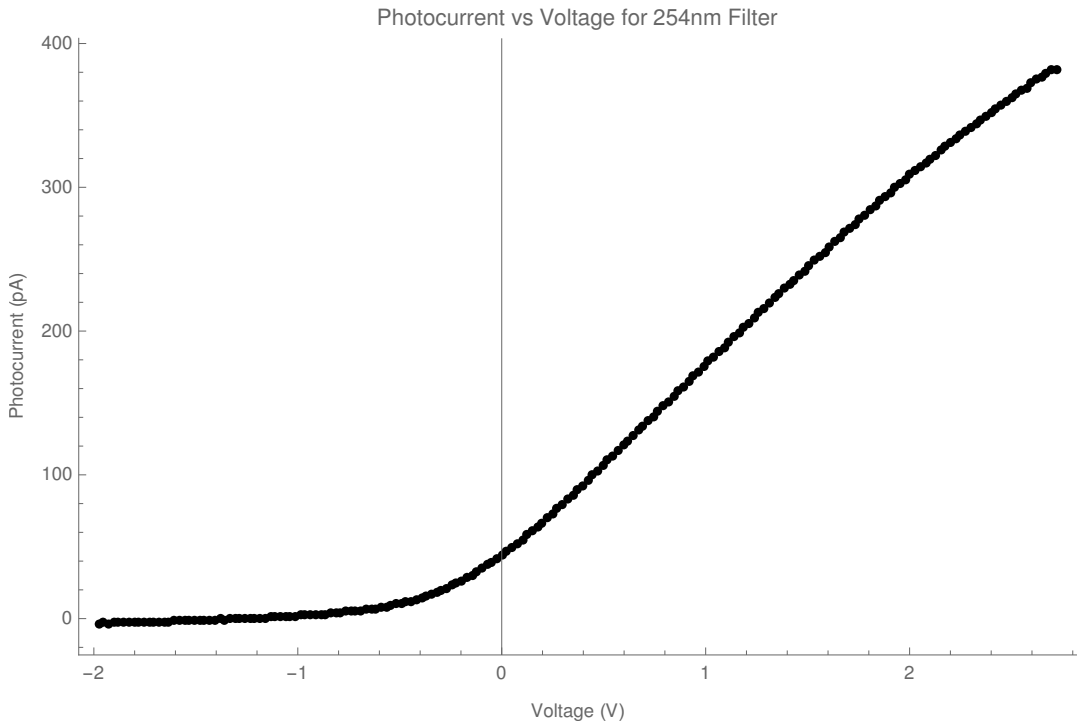

Figure 2: A graph of the measured photocurrent versus the potential difference between the anode and the cathode. Note that the error bars were suppressed to availd cluttering the plot. The uncertainty in the voltage was 0.02V and the uncertainty in the current was 0.5 pA.

3 Data Analysis and Results

In order to determine the value of h using Equation 3, we had to deduce the stopping voltage for each wavelength. Theoretically, the stopping voltage is a well defined value at which the current abruptly becomes nonzero. However, experimentally, the stopping voltage is not so easy to define due to noise in

the signal from the picoammeter and currents that come from sources other than the photoelectric effect. The noise and spurious currents cause the current to gradually rise from a noisy baseline, rather than abruptly rise from zero, as can be seen in Figure 2. To account for this difficulty, we used a statistical technique (described in Appendix A) to define the stopping voltage for each data set. This procedure resulted in three stopping voltage values for each wavelength (since three current versus voltage graphs were recorded at each wavelength), which were averaged to obtain a best estimate of V_s for each wavelength.

After obtaining these V_s values, we made a plot of the absolute value of V_s versus $1/\lambda$ and fit a line to the data according to the method described in Appendix B. We found the equation of our best fit line to be

$$|V_s| = (3.1 \times 10^{-7} \pm 0.8 \times 10^{-7} \text{ Vm})\frac{1}{\lambda} + (-0.2 \pm 0.2 \text{ V})$$

. By comparing this best fit line with Equation 3, and using the accepted value of the electron charge, $e = 1.60217662 \pm 0.00000001 \times 10^{-19}$C, we find $h = 1.6 \times 10^{-34} \pm 0.4 \times 10^{-34}$ Js, which does not agree with the accepted value of 6.626×10^{-34}Js. We also find that $\Phi = -0.2 \pm 0.2$eV. A plot of the stopping voltage versus inverse wavelength data, along with the best fit line, can be found in Figure 3. A discussion of the uncertainties in these values can be found in Appendix A.

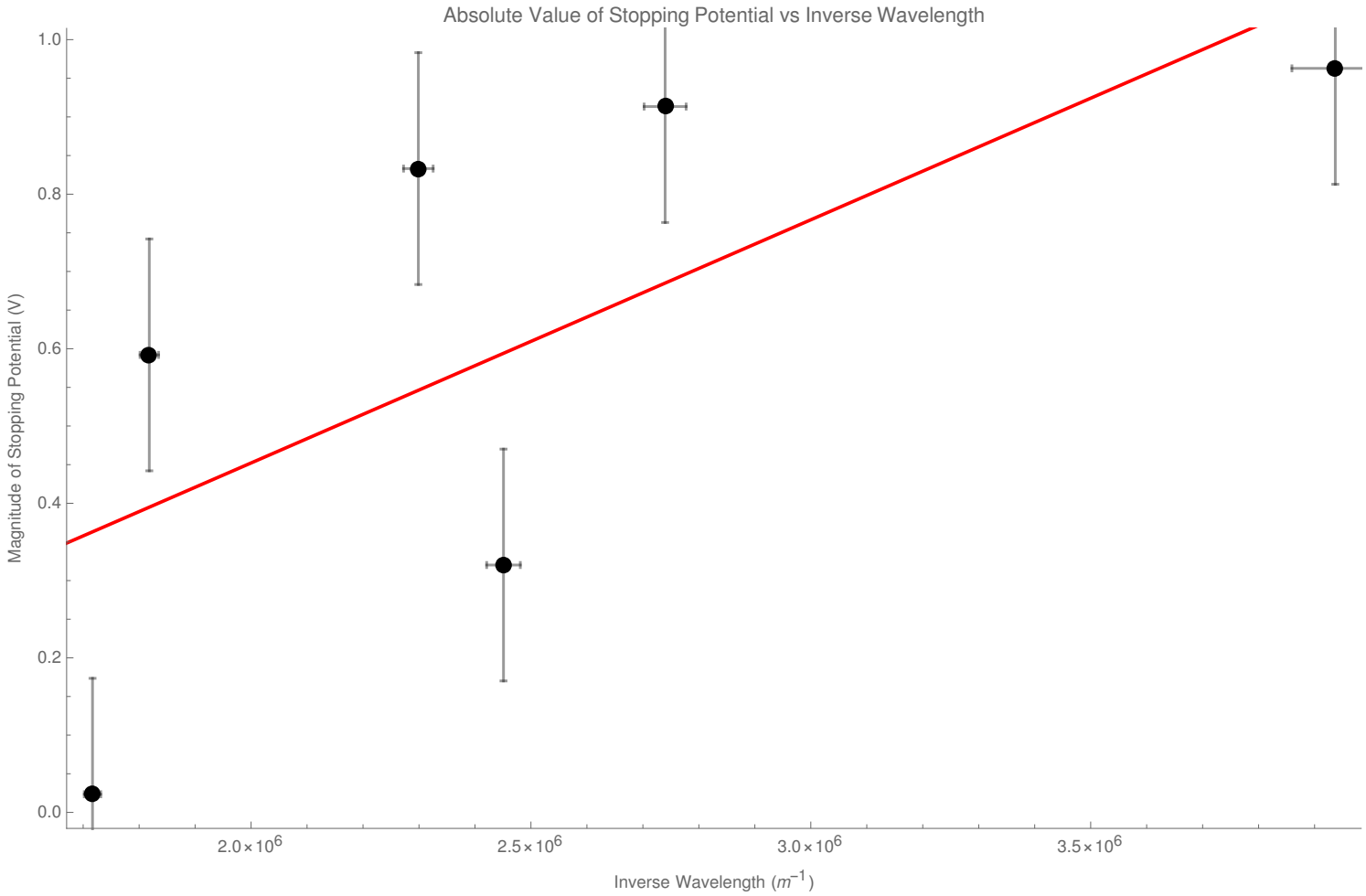

Figure 3: A graph of the absolute value of the stopping voltage versus inverse wavelength with best fit line $|V_s| = (3.1 \times 10^{-7} \pm 0.8 \times 10^{-7}\ \text{Vm})\frac{1}{\lambda} + (-0.2 \pm 0.2\ \text{V})$.

The disagreement between the measured and accepted values of h, along with the negative value for the work function, are likely due to sources of error in our experiment. One such source of error could be the noise that is inherent in the signal. This noise could have affected the experimental results more than we anticipated. This could be corrected for in future experimentation by taking many current measurements at each voltage and taking an average. Another potential source of error is the scratches and

imperfections in the filters we used to isolate specific wavelengths. These imperfections could affect the wavelength of the light that reaches the cathode. This would have skewed each wavelength value and affected the relationship between the stopping voltages at different wavelengths.

4 Summary and conclusions

Our experiment measured the value of h using observations of the photoelectric effect. We determined the energy transferred to electrons in the metal in question by measuring the stopping voltage at various different wavelengths of incident light. Using this technique, we calculated the value of h to be $1.6 \times 10^{-34} \pm 0.4 \times 10^{-34}$ Js, which does not agree with the accepted value of Planck's constant.

References

[1] J. W. Rohlf, *Modern Physics from α to Z_0*, John Wiley & Sons, 1994.

[2] A. Einstein, "On a Heuristic Point of View about the Creation and Conversion of Light", *Annalen der Physik*, **17**, 132-148 (1905)

[3] M Sipila, et al., "Experimental observation of two-photon photoelectric effect from silver aerosol nanoparticles", *New Journal of Physics*, **9**, (2007)

[4] PHY 353L Course Website. The University of Texas at Austin. "Photoelectric Effect Manual." ⟨https://web2.ph.utexas.edu/ phy353l/PhotoElectricEffect/photoelectric.html⟩

[5] C. J. Sansonetti, M. L. Salit, and J. Reader, "Wavelengths of spectral lines in mercury pencil lamps", *Applied Optics*

Appendix A: Determination of Stopping Potential and Error Analysis

To determine the stopping potential for each wavelength, we found the voltage for which every subsequent data point was more than three standard deviations away from the baseline. To determine this value, we fit a line to the first thirty data points using the method described in Appendix B. We considered this line to be the best estimate of the expected value of the

current in the baseline region. Let the line be denoted by $E(V)$. We then calculated the standard deviation, σ, of these thirty baseline points with respect to the best fit line using the equation $\sigma = \sqrt{\sum_i (I_i - E(V_i))^2}$, where (V_i, I_i) is the i-th current vs voltage data point. We then calculated the line $C(V) = E(V) + 3\sigma$ and looped through the entire data set until we found the first current value I_j for which $I_k > C(x_k)$ for all $k \geq j$. We then defined the stopping voltage to be the corresponding voltage value V_j.

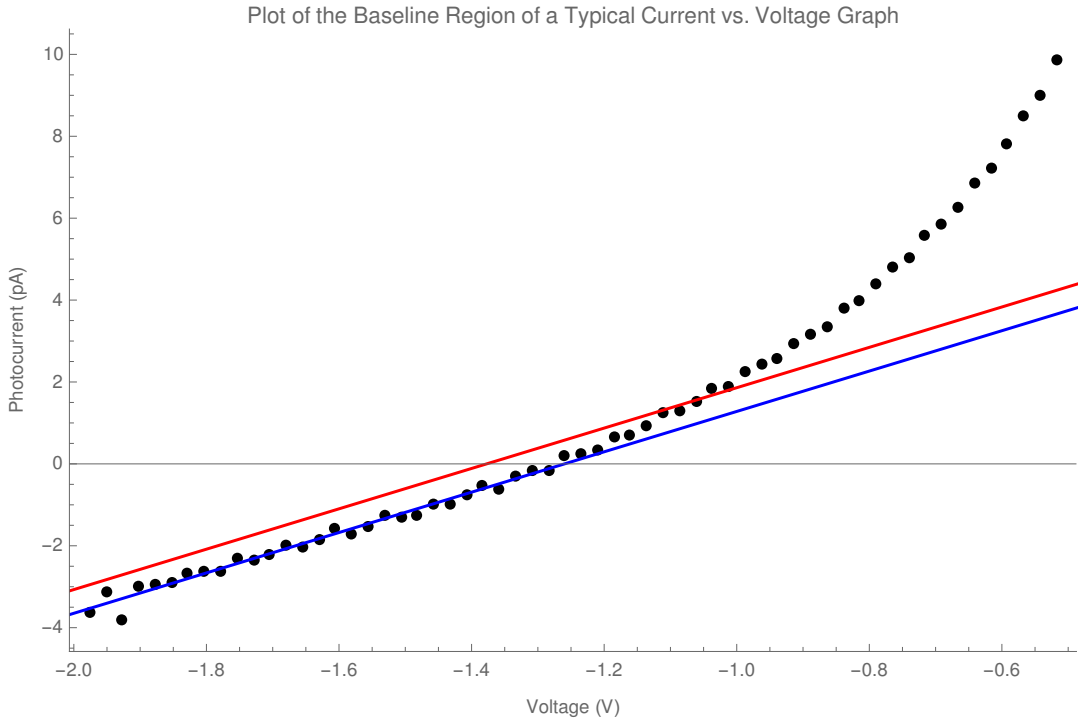

Figure 4: A graph of the first sixty points of the current vs voltage graph for the 254nm filter. The blue line is $E(V)$, the best fit line to the first thirty points (which we took to be a best estimate of the baseline), and the red line is $C(V)$, which lies three standard deviations above this best fit line. We defined the stopping voltage to be the abscissa of the data point for which all subsequent points lie above the red line.

To determine the uncertainty in these stopping voltage measurements, we found the range of voltage values for which the corresponding currents were within one standard deviation of the line $C(V)$. We then took half

of this range to be the uncertainty in the stopping voltage. We also estimated the uncertainty in the individual current measurements to be σ, the standard deviation of the current baseline with respect to the best fit line $E(V)$, which we found to be 0.5 pA. We estimated the uncertainty of the individual voltage measurements to be 0.02 V, which was the size of our voltage increments. We estimated the uncertainty in the wavelengths to be 5 nm. Although the emission wavelengths of mercury are known to an uncertainty of 0.0001 nm for some types of lamps, we cannot ensure that the mercury lamp we used had the exact same emission wavelengths (e.g. due to impurities in the mercury used in the lamp). [5] In addition, we do not know how the filters we used affected the wavelength of the light. For this reason, we chose a conservative error estimate of 5nm. The error in $1/\lambda$ was then calculated using standard error propagation techniques.

Appendix B: Linear Least Squares Regression

To find the best fit parameters for the linear function $y = ax+b$, when fitting to data points (x_i, y_i), we used the equations $a = \frac{\left(\sum \frac{x_i}{\sigma_i^2}\right)\left(\sum \frac{y_i}{\sigma_i^2}\right) - \left(\sum \frac{x_i y_i}{\sigma_i^2}\right)\left(\sum \frac{1}{\sigma_i^2}\right)}{\left(\sum \frac{x_i}{\sigma_i^2}\right)^2 - \left(\sum \frac{x_i^2}{\sigma_i^2}\right)\left(\sum \frac{1}{\sigma_i^2}\right)}$ and $b = \frac{\left(\sum \frac{x_i y_i}{\sigma_i^2}\right) - a\left(\sum \frac{x_i^2}{\sigma_i^2}\right)}{\sum \frac{x_i}{\sigma_i^2}}$, where σ_i is the uncertainty in y_i. The uncertainties in these parameters are given by $\delta a = \sqrt{\frac{\sum \frac{1}{\sigma_i^2}}{\left(\sum \frac{x_i^2}{\sigma_i^2}\right)\left(\sum \frac{1}{\sigma_i^2}\right) - \left(\sum \frac{x_i}{\sigma_i^2}\right)^2}}$ and $\delta b = \sqrt{\frac{\sum \frac{x_i^2}{\sigma_i^2}}{\left(\sum \frac{x_i^2}{\sigma_i^2}\right)\left(\sum \frac{1}{\sigma_i^2}\right) - \left(\sum \frac{x_i}{\sigma_i^2}\right)^2}}$.