



# Julia and optimization: Part 1

15.S60: Computing in Optimization and Statistics

Session #6: Tuesday, January 22



## Goals of this session

- Share and take away some tips and tricks for coding in Julia
- Build familiarity with Julia's capabilities in optimization and methods for addressing scalability issues
- Learn how to manage data and structure optimization code bases in Julia



# Agenda for today

## Part 1: Julia intro + tips/tricks

- Why do we use Julia?
- Issues with Julia
- Tips and tricks

## Part 2: Intro to JuMP

- Coding LPs in Julia
- Data management
- Coding best practices

## Part 3: Integer optimization

- Branch-and-bound
- Interpreting Gurobi
- Speeding up the solver

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# Part 1: Introduction to Julia + tips and tricks

# History

- Julia was founded at MIT by 4 co-founders:



Stefan Karpinski



Alan Edelman



Jeff Bezanson



Viral Shah

- First developed in 2009, with the first stable release in 2012
- Goals were to combine the benefits and iron out the disadvantages of each programming language into one open- sourced, liberally-licensed language



## Why Julia?

High-level and  
easy to use

Julia is fast!

Good for  
optimization



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## High-level and easy to use

- High-level, with readable, math-like syntax similar to Python/Matlab
- Possible to do complex computations quickly, e.g., matrix multiplication:

```
A = [1 2 3  
     2 1 2  
     3 2 1]  
  
b = [1,1,1]  
A \ b
```

```
3-element Vector{Float64}:  
 0.24999999999999997  
 8.326672684688673e-17  
 0.25
```

- Julia is also interactive, which makes it possible to run Julia in Jupyter notebooks or in the REPL





## Why Julia?

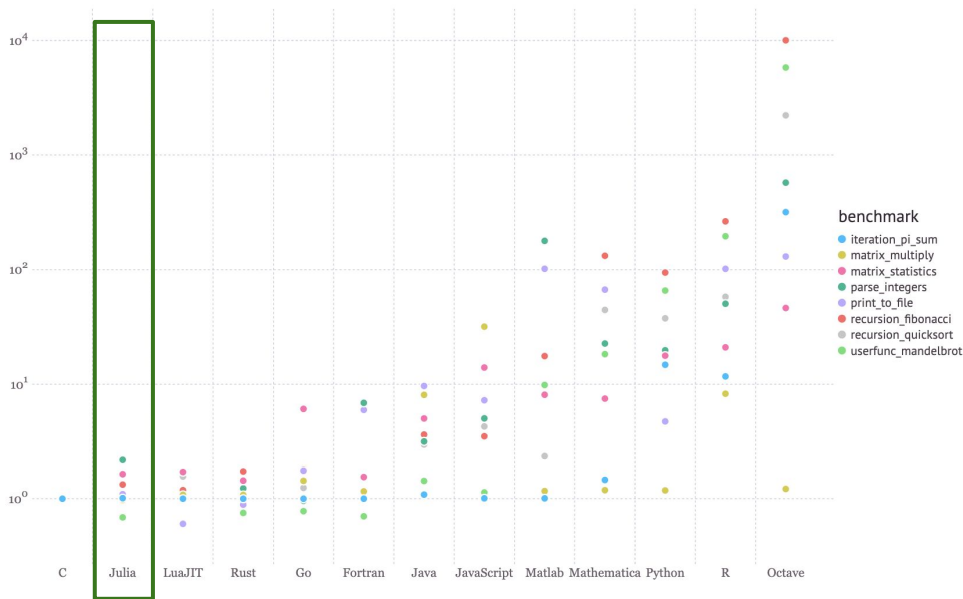
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# Julia is fast!

- Julia is fast thanks to:
  - Multiple dispatch
  - Just-in-time compilation
  - No two-language problem



Julia micro-benchmarks test performance



# Multiple dispatch

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- **Single dispatch:** for other high-level languages (e.g. Python), method execution is based on the type of one argument
- **Multiple dispatch:** for Julia, method execution is based on the types of all inputs
  - Julia creates methods that are specialized to the exact argument types
  - Leads to extensible code, where you can add new types to a function without modifying existing code



# Multiple dispatch

2 methods for a generic function `my_function`:

```
function my_function(x)
  println("Default output")
end
```

```
function my_function(x::Int) # only called when x is an integer
  println("You gave me an integer!")
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What will: `my_function(1.0)` return?



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An error?

“Default output”

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```

What will: `my_function(1)` return?

What about: `my_function("ORC")`?



# Just-in-time compilation

- Multiple dispatch: when functions are called with specific argument types, Julia creates specialized methods for those types
- **Just-in-time compilation** is when code is compiled into machine code at runtime, rather than ahead of time
- These methods are cached, so future calls with the same argument types reuse the compiled code and will run fast



# No two-language problem

- **Two language problem** is the workflow where two different programming languages are used for the same task:
  - A high-level language (e.g. Python, Matlab, R) for prototyping and quick development
  - A low-level language (e.g., C, C++) for performance-sensitive code



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- This requires you to transfer / re-write code in a faster language if performance is an issue
  - Leads to more implementation, more complex codebases, etc.
- In Julia, high-level code compiles to efficient machine code
  - You can use Julia for prototyping, and *stay in Julia* while optimizing it for fast performance
  - More readable code, faster iterations, and fewer bugs



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  - JuMP is an API that allows you to build and solve optimization problems with many different solvers, such as Gurobi, CPLEX, Mosek, and GLPK
  - Define all your constraints, variables, and objectives through JuMP syntax, which enables flexibility

```
@variable(model, x >= 0)
@constraint(model, A * x == b)
@objective(model, Min, c' * x)
```





# Performance in optimization

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```
@variable(model, x >= 0)
@constraint(model, A * x == b)
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```

- Because of the no two-language problem, we don't need to re-write our code in low-level languages for performance improvements



## Issues with Julia

Debugging can  
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Slow startup  
time

Immature  
package system



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# Debugging

- Improvements in syntax error messages since Julia 1.10:

Unclear where the error is :/

```
julia> [[], [], [,] [], []]  
ERROR: syntax: unexpected ","
```

Pinpoints where the error occurred :)

```
julia> [[], [], [,] [], []]  
ERROR: ParseError:  
# Error @ REPL[1]:1:11  
[[], [], [,] [], []  
#      └─┬─ unexpected ``,`
```



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```

- Improvements in the stack trace since Julia 1.10 to make it more concise

Very verbose and hard to read :/

```
Stacktrace:  
[1] error()  
  @ Base ./error.jl:44  
[2] f(g::Function, a::Int64; kw::Base.Pairs{Symbol, Union{}, Tuple{}, NamedTuple{(), Tuple{}}})  
  @ Main ./REPL[1]:1  
[3] f(g::Function, a::Int64)  
  @ Main ./REPL[1]:1  
[4] #f#16  
  @ ./REPL[2]:1 [inlined]  
[5] f(a::Int64)  
  @ Main ./REPL[2]:1  
[6] top-level scope  
  @ REPL[6]:1
```

More concise :)

```
Stacktrace:  
[1] error()  
  @ Base ./error.jl:44  
[2] f(g::Function, a::Int64; kw::@Kwargs{})  
  @ Main ./REPL[1]:1  
[3] f(a::Int64)  
  @ Main ./REPL[2]:1  
[4] top-level scope  
  @ REPL[3]:1
```



# Debugging

- Unfortunately, even in more recent versions of Julia, error messages can still be ambiguous:

```
f(x::Int, y::Int) = x + y
f(x::Int, y::String) = parse(Int, y) + x

f(3, missing)
```

```
MethodError: no method matching f(::Int64, ::Missing)
The function `f` exists, but no method is defined for this combination of argument types.
```

```
Closest candidates are:
```

```
f(::Int64, ::String)
  @ Main In[2]:2
f(::Int64, ::Int64)
  @ Main In[2]:1
```

```
Stacktrace:
```

```
[1] top-level scope
     @ In[2]:4
```

- Important to test code frequently to catch bugs!



## Issues with Julia

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Immature  
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# Slow startup time

- **Packages:** compiled when using them, so it might take a really long time to set up your working environment
  - Known as the “Time to first \_\_\_\_” problem (TTFX)
  - Julia versions 1.10 and onwards have sped up the performance of loading packages





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  - Known as the “Time to first \_\_\_\_” problem (TTFX)
  - Julia versions 1.10 and onwards have sped up the performance of loading packages
- **Functions:** because compilation happens at runtime (just-in-time compilation), Julia can be slower on the first call to the function
- When benchmarking code, it’s important to ignore compilation time for packages and functions



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## Immature package system

- “Mega packages” in Python like `numpy` and `pandas` don’t exist to the same scale in Julia
- However, Julia is a new and fast-growing community, and packages are being created at a fast pace



# Immature package system

- “Mega packages” in Python like `numpy` and `pandas` don’t exist to the same scale in Julia
- However, Julia is a new and fast-growing community, and packages are being created at a fast pace
- Some comprehensive packages for scientific computing:
  - Statistics: `StatsBase.jl` and `Statistics.jl`
  - Machine learning: `MLJ.jl`, `Flux.jl`, and `Knet.jl` for deep learning
  - Data tools: `DataFrames.jl`, `CSV.jl`, `Arrow.jl`, and `Spark.jl` for big data
  - Data visualization: `Plots.jl`, `Makie.jl`
  - Optimization: `JuMP.jl` and `Optim.jl`
  - Differential equations: `DifferentialEquations.jl`

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# Julia tips and tricks



# Type stability

- A function is **type stable** if the compiler can infer the type of its return value from the types of its input arguments



# Type stability

- A function is **type stable** if the compiler can infer the type of its return value from the types of its input arguments
- If a function is type stable...
  - Julia can create efficient machine code
- If a function is not type stable...
  - Compiler will produce machine code full of “ifs,” covering all options of variable types
  - This slows down machine code :/





# Type stability

- `Number` is an abstract type which contains many concrete types, such as `Int8` and `Float64`
- Is this function type stable?

```
function add(x::Number, y::Number)
    z = x + y
    return z
end
```



# Type stability

- `Number` is an abstract type which contains many concrete types, such as `Int8` and `Float64`
- Is this function type stable?

```
function add(x::Number, y::Number)
    z = x + y
    return z
end
```

- What about this function?

```
function largest(a::Float64, b::Int64)
    if a > b
        c = a
    else
        c = b
    end
    return c
end
```



# Type stability

- Make functions type stable:

```
function largest_stable(a::Float64, b::Int64)
    b = b*1.0
    if a > b
        c = a
    else
        c = b
    end
    return c
end
```

Cast `b` to a float

```
function largest_stable_new(a::Number, b::Number)
    a, b = promote(a, b)
    if a > b
        return a
    else
        return b
    end
end
```

Use `promote`

- Useful to check the type stability of functions (especially for more complex ones) using `@code_warntype <your_function>`



# Broadcasting

- **Broadcasting** is a way to apply operations element-wise over arrays or other objects using the `.` (dot) syntax

```
x = [1, 2, 3]
y = [4, 5, 6]
x .+ y          # elementwise addition
x .* y          # elementwise multiplication
```

- Can also be applied to functions, avoiding the need for loops

```
y = [1, 2, 3]
g(x) = x^2 + 1
g.(y)
```

- Benefits are in readability, as performance is the same as a `for` loop



# Dictionaries

- Dictionaries are a type of data structure for when you need fast lookups of *values* using *keys*
  - Order is not preserved, unless using an ordered dictionary explicitly
- When type-specified, dictionaries are fast, for example:

```
typedict = Dict{String, Int}()
```

- Iteration is also fast, and can be done by:

```
for (k, v) in typedict
    ...
end
```

- Useful operations: `haskey` (to check existence of keys), `get` (to safely access values), `delete!` (to delete entries)



# Data structures

- A `struct` is similar to a Python class, which is a user-defined data structure to store related data with concrete types
  - Immutable structs: cannot change the values of fields after creating an instance of the struct
  - Mutable structs: field can change after creation of instance, but this is less memory efficient



# Data structures

- A `struct` is similar to a Python class, which is a user-defined data structure to store related data with concrete types
  - Immutable structs: cannot change the values of fields after creating an instance of the struct
  - Mutable structs: field can change after creation of instance, but this is less memory efficient
- Naturally results in type stable code, which leads to efficient machine code and optimized loops
- Useful in large-scale data-oriented projects, where you need to manage different kinds of data
- We will see this in action in Part 2!

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**Any other tips and tricks from you  
all?**