Lecture 4-2

More NumPy

Week 5 Wednesday

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Based on Python Data Science Handbook by Jake VanderPlas

In [1]: import numpy as np

```
In [2]:
    x = np.arange(4)
    y = np.arange(100, 104)
    print(x)
    print(y)

[0 1 2 3]
    [100 101 102 103]
```

```
In [2]:
         x = np.arange(4)
         y = np.arange(100, 104)
         print(x)
         print(y)
          [0 1 2 3]
          [100 101 102 103]
In [3]:
         np.concatenate([x,y])
Out[3]: array([ 0, 1, 2, 3, 100, 101, 102, 103])
        np.concatenate has an argument for axis. The axes are 0-indexed.
In [4]:
         np.concatenate([x,y], axis = 0)
Out[4]: array([ 0, 1, 2, 3, 100, 101, 102, 103])
```

Here, the array is 1 dimension so it just puts the two side by side. have to reshape to stack them or use vstack

```
In [5]:
         np.concatenate([x,y], axis = 1) # throws an error
          AxisError
                                                     Traceback (most recent call last)
          <ipython-input-5-2cd32e4cabd5> in <module>
          ----> 1 np.concatenate([x,y], axis = 1) # throws an error
          < array function internals> in concatenate(*args, **kwargs)
          AxisError: axis 1 is out of bounds for array of dimension 1
In [6]:
          x.shape # you can't use axis with index 1, because axis index 1 does not exist
Out [6]: (4,)
In [7]:
          np.vstack([x,y])
                          # vstack will vertically stack unidimensional arrays
Out[7]: array([[0, 1, 2, 3],
                 [100, 101, 102, 103]])
```

```
In [8]: x.reshape(1,4)
```

Out[8]: array([[0, 1, 2, 3]])

```
In [8]: x.reshape(1,4)
Out[8]: array([[0, 1, 2, 3]])
In [9]: y.reshape(1,4)
Out[9]: array([[100, 101, 102, 103]])
```

```
In [8]: x.reshape(1,4)
Out[8]: array([[0, 1, 2, 3]])
In [9]: y.reshape(1,4)
Out[9]: array([[100, 101, 102, 103]])
In [10]: np.concatenate([x.reshape(1,4), y.reshape(1,4)], axis = 0)
Out[10]: array([[ 0,  1,  2,  3],
```

[100, 101, 102, 103]])

note that when I concatenate along axis 0 for a 2-dimensional array, it concatenates by rows. In a 2D array, index 0 is for rows, and index 1 is for columns.

```
In [8]:
           x.reshape(1,4)
 Out[8]: array([[0, 1, 2, 3]])
 In [9]:
           y.reshape(1,4)
 Out[9]:
          array([[100, 101, 102, 103]])
In [10]:
           np.concatenate([x.reshape(1,4), y.reshape(1,4)], axis = 0)
Out[10]: array([[ 0, 1, 2, 3],
                   [100, 101, 102, 103]])
         note that when I concatenate along axis 0 for a 2-dimensional array, it concatenates by
         rows. In a 2D array, index 0 is for rows, and index 1 is for columns.
In [11]:
           np.concatenate([x.reshape(1,4), y.reshape(1,4)], axis = 1)
Out[11]: array([[ 0, 1, 2, 3, 100, 101, 102, 103]])
```

```
In [12]:
    xm = np.arange(6).reshape((2,3))
    ym = np.arange(100,106,1).reshape((2,3))
    print(xm)
    print(ym)

    [[0 1 2]
      [3 4 5]]
    [[100 101 102]
      [103 104 105]]
```

Out[13]: (2, 3)

```
In [12]:
           xm = np.arange(6).reshape((2,3))
           ym = np.arange(100,106,1).reshape((2,3))
           print(xm)
           print(ym)
            [[0 1 2]
             [3 4 5]]
            [[100 101 102]
             [103 104 105]]
In [13]:
           xm.shape
Out[13]: (2, 3)
In [14]:
           ym.shape
Out[14]: (2, 3)
```

```
In [15]: print(np.concatenate([xm,ym])) # default behavior concatenates on axis 0

[[ 0 1 2]
       [ 3 4 5]
```

[100 101 102] [103 104 105]]

```
In [15]:
            print(np.concatenate([xm,ym])) # default behavior concatenates on axis 0
             [ 3 4 5]
             [100 101 102]
             [103 104 105]]
In [16]:
            print(np.concatenate([xm,ym], axis = 0))
            # axes are reported as rows, then columns.
            # concatenating along axis 0 will concatenate along rows
             [ 3 4 5]
             [100 101 102]
             [103 104 105]]
In [17]:
            print(np.concatenate([xm,ym], axis = 1))
            # concatenating along axis 1 will concatenate along columns
                0 1 2 100 101 102]
3 4 5 103 104 105]]
```

You can always use vstack and hstack for 2D arrays.

```
In [20]:
    print(x)
    print(y)

[0 1 2 3]
    [100 101 102 103]
```

```
In [20]:
           print(x)
           print(y)
            [0 1 2 3]
            [100 101 102 103]
In [21]:
Out[21]: array([5, 6, 7, 8])
In [22]:
           x + y # elementwise addition
Out[22]: array([100, 102, 104, 106])
In [23]:
                     Element-wise multiplication
Out[23]: array([ 0, 101, 204, 309])
```

```
In [20]:
           print(x)
           print(y)
           [0 1 2 3]
           [100 101 102 103]
In [21]:
Out[21]: array([5, 6, 7, 8])
In [22]: x + y # elementwise addition
Out[22]: array([100, 102, 104, 106])
In [23]:
Out[23]: array([ 0, 101, 204, 309])
In [24]:
           np.sum(x * y) Sum all elements in the array
Out[24]: 614
```

In [25]: np.dot(x,y) # 0 * 100 + 1 * 101 + 2 * 102 + 3 * 103

Out[25]: 614

```
In [25]: np.dot(x,y) # 0 * 100 + 1 * 101 + 2 * 102 + 3 * 103
Out[25]: 614
```

In [26]: x @ y # matrix multiplication

Out[26]: 614

```
In [27]:
    print(xm)
    print(ym)

[[0 1 2]
       [3 4 5]]
       [[100 101 102]
       [103 104 105]]
```

```
In [27]:
            print(xm)
            print(ym)
            [[0 1 2]
            [3 4 5]]
            [[100 101 102]
             [103 104 105]]
In [28]:
            xm + 5
Out[28]: array([[ 5, 6, 7], [ 8, 9, 10]])
In [29]:
           xm + ym # elementwise addition
Out[29]:
           array([[100, 102, 104],
```

[106, 108, 110]])

```
In [30]: print(xm)
print(ym)

[[0 1 2]
      [3 4 5]]
      [[100 101 102]
      [103 104 105]]
```

```
In [30]: print(xm)
print(ym)

[[0 1 2]
       [3 4 5]]
      [[100 101 102]
       [103 104 105]]

In [31]: xm * ym # element-wise multiplication

Out[31]: array([[ 0, 101, 204],
```

[309, 416, 525]])

```
In [30]:
           print(xm)
           print(ym)
           [[0 1 2]
            [3 4 5]]
           [[100 101 102]
            [103 104 105]]
In [31]:
           xm * ym # element-wise multiplication
Out[31]: array([[ 0, 101, 204],
                   [309, 416, 525]])
In [32]:
           np.multiply(xm, ym) # element-wise multiplication
           array([[ 0, 101, 204],
Out[32]:
```

[309, 416, 525]])

```
In [33]: print(xm) print(ym)

[[0 1 2] [3 4 5]] [[100 101 102] [103 104 105]]
```

```
In [33]: print(xm)
print(ym)

[[0 1 2]
      [3 4 5]]
      [[100 101 102]
      [103 104 105]]

In [34]: np.dot(xm, ym.T) Matrix multiplication

Out[34]: array([[ 305, 314],
```

[1214, 1250]])

```
In [33]:
            print(xm)
            print(ym)
            [[0 1 2]
            [3 4 5]]
            [[100 101 102]
             [103 104 105]]
In [34]:
           np.dot(xm, ym.T)
                                          Matrix multiplication
Out[34]: array([[ 305, 314],
                    [1214, 1250]])
In [35]:
           xm.dot(ym.T)
                                          Matrix multiplication
Out[35]: array([[ 305, 314],
```

[1214, 1250]])

```
In [33]:
            print(xm)
            print(ym)
            [[0 1 2]
             [3 4 5]]
            [[100 101 102]
             [103 104 105]]
In [34]:
           np.dot(xm, ym.T)
                                           Matrix multiplication
Out[34]: array([[ 305, 314],
                    [1214, 1250]])
In [35]:
            xm.dot(ym.T)
                                           Matrix multiplication
Out[35]: array([[ 305, 314],
                    [1214, 1250]])
In [36]:
                                           Matrix multiplication
            xm @ ym.T
Out[36]: array([[ 305, 314],
                    [1214, 1250]])
```

```
In [37]:
           x = np.arange(4)
           print(x)
           [0 1 2 3]
In [38]:
           print(x + 4)
           [4 5 6 7]
In [39]:
           print(x - 5)
           [-5 -4 -3 -2]
In [40]:
           print(x * 2)
           [0 2 4 6]
```

```
In [41]: print(x / 2)
```

[0. 0.5 1. 1.5]

```
In [41]: print(x / 2)
        [0. 0.5 1. 1.5]
In [42]: print(-x)
        [ 0 -1 -2 -3]
```

```
In [41]: print(x / 2)
        [0. 0.5 1. 1.5]

In [42]: print(-x)
        [ 0 -1 -2 -3]

In [43]: print(x ** 2)
        [0 1 4 9]
```

```
In [41]:
           print(x / 2)
           [0. 0.5 1. 1.5]
In [42]:
           print(-x)
           [ 0 -1 -2 -3]
In [43]:
           print(x ** 2)
           [0 1 4 9]
In [44]:
           print(x % 2) # modulo division
           [0 1 0 1]
```

```
In [41]:
           print(x / 2)
           [0. 0.5 1. 1.5]
In [42]:
           print(-x)
           [ 0 -1 -2 -3]
In [43]:
           print(x ** 2)
           [0 1 4 9]
In [44]:
           print(x % 2) # modulo division
           [0 1 0 1]
In [45]:
           print(abs(x)) # abs
           [0 1 2 3]
```

```
In [46]:
           theta = np.linspace(0, np.pi, 5)
           print(theta)
           [0.
                       0.78539816 1.57079633 2.35619449 3.14159265]
In [47]:
           print(np.sin(theta))
           [0.00000000e+00 7.07106781e-01 1.00000000e+00 7.07106781e-01
            1.22464680e-16]
In [48]:
           print(np.cos(theta))
           [ 1.00000000e+00 7.07106781e-01 6.12323400e-17 -7.07106781e-01
            -1.00000000e+001
In [49]:
           print(np.tan(theta))
           [ 0.00000000e+00 1.00000000e+00 1.63312394e+16 -1.00000000e+00
            -1.22464680e-16]
```

```
In [50]:
           x = np.array([1, 10, 100])
           print(np.log(x)) # natural Log
           print(np.log10(x)) # common Log
           [0.
                2.30258509 4.60517019]
           [0. 1. 2.]
In [51]:
           y = np.arange(3)
           print(np.exp(y)) # e^y
           [1.
                2.71828183 7.3890561 ]
In [52]:
           print(np.exp2(y)) # 2^y
           [1. 2. 4.]
In [53]:
           print(np.power(3, y)) # power ^ y
           [1 3 9]
```

```
you can use sum()
or np.sum()
np.sum() is faster than sum, but doesn't always behave the same way
```

```
you can use sum()

or np.sum()

np.sum() is faster than sum, but doesn't always behave the same way

In [54]: 

x = np.arange(100)
print(x)

[ 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47
48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71
72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95
96 97 98 99]
```

```
you can use sum()
         or np.sum()
         np.sum() is faster than sum, but doesn't always behave the same way
In [54]:
          x = np.arange(100)
          print(x)
               1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
           24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47
           48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71
           72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95
           96 97 98 99]
In [55]:
          print(sum(x))
          4950
```

```
you can use sum()
         or np.sum()
         np.sum() is faster than sum, but doesn't always behave the same way
In [54]:
           x = np.arange(100)
           print(x)
                                          9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
            24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47
            48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71
            72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95
            96 97 98 99]
In [55]:
           print(sum(x))
                             1-D structures will have same results
           4950
In [56]:
           print(np.sum(x))
           4950
```

```
In [57]:
big_array = np.random.rand(10000) Still a 1D array
%timeit
%timeit
np.sum(big_array) # the np version is much faster
```

1.58 ms \pm 15.3 μ s per loop (mean \pm std. dev. of 7 runs, 1000 loops each) 5.44 μ s \pm 88 ns per loop (mean \pm std. dev. of 7 runs, 100000 loops each)

```
In [58]:
    print(min(big_array))
    print(max(big_array))
```

6.052560173452903e-05

0.9999830003841369

0.9999830003841369

```
In [58]:
           print(min(big_array))
           print(max(big_array))
           6.052560173452903e-05
           0.9999830003841369
In [59]:
           print(np.min(big_array))
           print(np.max(big array))
           6.052560173452903e-05
           0.9999830003841369
In [60]:
           %timeit min(big array)
           %timeit np.min(big_array) # the np version is much faster
           971 \mus \pm 4.35 \mus per loop (mean \pm std. dev. of 7 runs, 1000 loops each)
           4.22 \mus \pm 24.8 ns per loop (mean \pm std. dev. of 7 runs, 100000 loops each)
```

summaries for matrices

summaries for matrices

```
In [61]:
    np.random.seed(1)
# M = np.random.random((3, 4))
M = np.arange(12)
np.random.shuffle(M)
M = np.reshape(M, (3,4))
print(M)

[[ 2  3   4  10]
        [ 1  6  0  7]
        [ 11  9  8  5]]
```

summaries for matrices

summaries for matrices

```
In [61]:
            np.random.seed(1)
            \# M = np.random.random((3, 4))
            M = np.arange(12)
            np.random.shuffle(M)
            M = np.reshape(M, (3,4))
            print(M)
            [[ 2 3 4 10]
             [1607]
              [11 9 8 5]]
In [62]:
            sum(M) # regular sum function
                                                 Base python will sum down columns
Out[62]: array([14, 18, 12, 22])
In [63]:
            np.sum(M) # np.sum function
                                              Numpy just adds literally all the numbers
Out[63]: 66
```

```
In [64]:
    print(M)

[[ 2  3  4 10]
       [ 1  6  0  7]
       [11  9  8  5]]
```

```
In [64]: print(M)

[[ 2  3  4 10]
      [ 1  6  0  7]
      [11  9  8  5]]

In [65]: np.sum(M, axis = 0) # np.sum function with axis specified
    # matrices have two dimensions
    # 0 is rows, 1 is columns
    # np.sum axis = 0, will sum over rows, so you end up getting column totals
```

Out[65]: array([14, 18, 12, 22])

axis 0 means collapse axis 0, so this is a column sum

```
In [64]:
           print(M)
           [[ 2 3 4 10]
            [ 1 6 0 7]
            [11 9 8 5]]
In [65]:
           np.sum(M, axis = 0) # np.sum function with axis specified
           # matrices have two dimensions
           # 0 is rows, 1 is columns
           # np.sum axis = 0, will sum over rows, so you end up getting column totals
Out[65]: array([14, 18, 12, 22])
In [66]:
           np.sum(M, axis = 1)
Out[66]:
           array([19, 14, 33])
```

we are collapsing axis 1, the columns, so these are row sums

```
In [64]:
           print(M)
           [[ 2 3 4 10]
            [ 1 6 0 7]
            [11 9 8 5]]
In [65]:
           np.sum(M, axis = 0) # np.sum function with axis specified
           # matrices have two dimensions
           # 0 is rows, 1 is columns
           # np.sum axis = 0, will sum over rows, so you end up getting column totals
Out[65]: array([14, 18, 12, 22])
In [66]:
           np.sum(M, axis = 1)
Out[66]: array([19, 14, 33])
In [67]:
           np.min(M, axis = 0)
Out[67]: array([1, 3, 0, 5])
```

```
In [68]: print(M)

[[ 2 3 4 10]
      [ 1 6 0 7]
      [11 9 8 5]]
```

Out[69]: 3.452052529534663

Standard deviations of the columns

```
In [68]:
           print(M)
           [[ 2 3 4 10]
            [ 1 6 0 7]
[11 9 8 5]]
In [69]:
           np.std(M)
Out[69]:
          3.452052529534663
In [70]:
           np.std(M, axis = 0)
Out[70]: array([4.49691252, 2.44948974, 3.26598632, 2.05480467])
In [71]:
           np.mean(M, axis = 1)
Out[71]: array([4.75, 3.5, 8.25])
```

```
In [72]:
            np.random.seed(1)
            A = np.ones(24)
            np.random.shuffle(A)
            A = np.reshape(A, (2, 3, 4)) # two sheets, 3 rows, 4 columns
            print(A)
            [[1. 1. 1. 1.]
              [1. 1. 1. 1.]
              [1. 1. 1. 1.]]
                                              axis 0: sheets
                                               axis 1: rows
             [[1. 1. 1. 1.]
                                              axis 2: columns
              [1. 1. 1. 1.]
              [1. 1. 1. 1.]]
In [73]:
            np.sum(A, axis = 0) # sum across "sheets"
Out[73]: array([[2., 2., 2., 2.],
                    [2., 2., 2., 2.],
                    [2., 2., 2., 2.]
```

```
In [72]:
           np.random.seed(1)
           A = np.ones(24)
           np.random.shuffle(A)
           A = np.reshape(A, (2, 3, 4)) # two sheets, 3 rows, 4 columns
           print(A)
           [[1. 1. 1. 1.]
              [1. 1. 1. 1.]
              [1. 1. 1. 1.]]
            [[1. 1. 1. 1.]
              [1. 1. 1. 1.]
              [1. 1. 1. 1.]]
In [73]:
           np.sum(A, axis = 0) # sum across "sheets"
Out[73]: array([[2., 2., 2., 2.],
                   [2., 2., 2., 2.],
                   [2., 2., 2., 2.]
In [74]:
           np.sum(A, axis = 1) # sum across rows
Out[74]: array([[3., 3., 3., 3.],
                   [3., 3., 3., 3.]]
```

```
In [72]:
           np.random.seed(1)
           A = np.ones(24)
           np.random.shuffle(A)
           A = np.reshape(A, (2, 3, 4)) # two sheets, 3 rows, 4 columns
           print(A)
            [[1. 1. 1. 1.]
              [1. 1. 1. 1.]
              [1. 1. 1. 1.]]
             [[1. 1. 1. 1.]
              [1. 1. 1. 1.]
              [1. 1. 1. 1.]]
In [73]:
           np.sum(A, axis = 0) # sum across "sheets"
Out[73]: array([[2., 2., 2., 2.],
                   [2., 2., 2., 2.],
                   [2., 2., 2., 2.]]
In [74]:
           np.sum(A, axis = 1) # sum across rows
Out[74]: array([[3., 3., 3., 3.],
                   [3., 3., 3., 3.]]
In [75]:
           np.sum(A, axis = 2) # sum across columns
```

Out[75]: array([[4., 4., 4.], [4., 4., 4.]])

```
In [76]:
    x = float("nan") # direct creation of nan
    print(x)
    print(type(x))

nan
    <class 'float'>
```

```
In [76]:
            x = float("nan") # direct creation of nan
            print(x)
            print(type(x))
            nan
            <class 'float'>
In [77]:
            y = float("inf") # y is the float representation of infinity
            print(y / y) # these calculations will yield a nan result
            print(y - y)
            nan
            nan
In [78]:
            np.sum([x, 2])
Out[78]:
            nan
```

```
In [76]:
            x = float("nan") # direct creation of nan
            print(x)
            print(type(x))
            nan
            <class 'float'>
In [77]:
            y = float("inf") # y is the float representation of infinity
            print(y / y) # these calculations will yield a nan result
            print(y - y)
            nan
            nan
In [78]:
            np.sum([x, 2])
Out[78]:
            nan
In [79]:
            np.nansum([x, 2]) # in R you have the option na.rm = TRUE
Out[79]:
```

The following table provides a list of useful aggregation functions available in NumPy:

Function Name	NaN-safe Version	Description
np.sum	np.nansum	Compute sum of elements
np.prod	np.nanprod	Compute product of elements
np.mean	np.nanmean	Compute mean of elements
np.std	np.nanstd	Compute standard deviation
np.var	np.nanvar	Compute variance
np.min	np.nanmin	Find minimum value
np.max	np.nanmax	Find maximum value
np.argmin	np.nanargmin	Find index of minimum value
np.argmax	np.nanargmax	Find index of maximum value
np.median	np.nanmedian	Compute median of elements
np.percentile	np.nanpercentile	Compute rank-based statistics of elements
np.any	N/A	Evaluate whether any elements are true
np.all	N/A	Evaluate whether all elements are true

Broadcasting

This is a similar concept to recyling values in R, but only works when the dimensions are compatible

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This is a similar concept to recyling values in R, but only works when the dimensions are compatible

```
In [80]:
    a = np.array([1,2,3])
    b = np.array([4,5,6])
    print(a + b)
[5 7 9]
```

Broadcasting

This is a similar concept to recyling values in R, but only works when the dimensions are compatible

```
In [80]:
           a = np.array([1,2,3])
           b = np.array([4,5,6])
           print(a + b)
           [5 7 9]
In [81]:
           c = np.array([7,8])
           print(a + c) # doesn't work
           ValueError
                                                       Traceback (most recent call last)
           <ipython-input-81-7215ac8feb02> in <module>
                 1 c = np.array([7,8])
           ----> 2 print(a + c) # doesn't work
           ValueError: operands could not be broadcast together with shapes (3,) (2,)
```

In [82]: print(a)

[1 2 3]

```
In [82]: print(a)
        [1 2 3]

In [83]: e = np.ones([3,3])
        print(e)

        [[1. 1. 1.]
        [1. 1. 1.]
        [1. 1. 1.]]
```

```
In [82]:
           print(a)
           [1 2 3]
In [83]:
           e = np.ones([3,3])
           print(e)
           [[1. 1. 1.]
            [1. 1. 1.]
             [1. 1. 1.]]
In [84]:
           print(e + a) # the array a gets 'broadcast' across all three rows
           [[2. 3. 4.]
            [2. 3. 4.]
            [2. 3. 4.]]
```

```
In [82]:
           print(a)
            [1 2 3]
In [83]:
           e = np.ones([3,3])
            print(e)
            [[1. 1. 1.]
             [1. 1. 1.]
             [1. 1. 1.]]
In [84]:
           print(e + a) # the array a gets 'broadcast' across all three rows
            [[2. 3. 4.]
             [2. 3. 4.]
             [2. 3. 4.]]
In [85]:
           print(a.reshape([3,1])) # we reshape a to be a 3x1 array
            [[1]
             [2]
             [3]]
```

```
In [82]:
            print(a)
            [1 2 3]
In [83]:
           e = np.ones([3,3])
            print(e)
            [[1. 1. 1.]
             [1. 1. 1.]
             [1. 1. 1.]]
In [84]:
            print(e + a) # the array a gets 'broadcast' across all three rows
            [[2. 3. 4.]
             [2. 3. 4.]
             [2. 3. 4.]]
In [85]:
            print(a.reshape([3,1])) # we reshape a to be a 3x1 array
            [[1]
             [2]
             [3]]
In [86]:
            print(e + a.reshape([3,1])) # the reshaped array is broadcast across columns
            [[2. 2. 2.]
             [3. 3. 3.]
             [4. 4. 4.]]
```

```
In [87]:
    d = np.vstack([a,b]) # we stack the arrays a and b vertically
    print(d)

[[1 2 3]
    [4 5 6]]
```

In [90]: print(c)

[7 8]

```
In [90]: print(c)

[7 8]

In [91]: print(d)

[[1 2 3]
  [4 5 6]]
```

```
In [90]:
           print(c)
           [7 8]
In [91]:
           print(d)
           [[1 2 3]
            [4 5 6]]
In [92]:
           print(d + c) # c does not have compatible dimensions
           ValueError
                                                      Traceback (most recent call last)
           <ipython-input-92-8c651d5d46fc> in <module>
           ----> 1 print(d + c) # c does not have compatible dimensions
           ValueError: operands could not be broadcast together with shapes (2,3) (2,)
```

```
In [90]:
           print(c)
           [7 8]
In [91]:
           print(d)
           [[1 2 3]
            [4 5 6]]
In [92]:
           print(d + c) # c does not have compatible dimensions
           ValueError
                                                        Traceback (most recent call last)
           <ipython-input-92-8c651d5d46fc> in <module>
           ----> 1 print(d + c) # c does not have compatible dimensions
           ValueError: operands could not be broadcast together with shapes (2,3) (2,)
In [93]:
           print(d + c.reshape([2,1])) # after we reshape c to be a column, we can broadcast it
           [[ 8 9 10]
            [12 13 14]]
```

[9]]

[0 1 2 3 4 5 6 7 8 9 10]

```
In [94]:
          e = np.arange(10).reshape((10, 1))
          f = np.arange(11)
          print(e)
          print(f)
           [[0]]
            [1]
            [2]
            [3]
            [4]
            [5]
            [6]
            [7]
            [8]
            [9]]
           [0 1 2 3 4 5 6 7 8 9 10]
In [95]:
          print(e + f) ## e and f are broadcast into compatible matrices and then added
                             5 6 7 8 9 10]
                   3 4 5 6 7 8 9 10 11]
                3 4 5 6 7 8 9 10 11 12]
                4 5 6 7 8 9 10 11 12 13]
                5 6 7 8 9 10 11 12 13 14]
                         9 10 11 12 13 14 15]
                      9 10 11 12 13 14 15 16]
                8 9 10 11 12 13 14 15 16 17]
                9 10 11 12 13 14 15 16 17 18]
            [ 9 10 11 12 13 14 15 16 17 18 19]]
```

[0 9 18 27 36 45 54 63 72 81 90]]

```
In [96]:
           print(e * f) ## e and f are broadcast into compatible matrices and then multiplied element-wise
                0 0 0 0 0 0 0 0 0 0 0]
1 2 3 4 5 6 7 8 9 10]
              0 2 4 6 8 10 12 14 16 18 20]
                3 6 9 12 15 18 21 24 27 30]
                4 8 12 16 20 24 28 32 36 40]
                5 10 15 20 25 30 35 40 45 50]
            [ 0 6 12 18 24 30 36 42 48 54 60]
            [ 0 7 14 21 28 35 42 49 56 63 70]
            [ 0 8 16 24 32 40 48 56 64 72 80]
            [ 0 9 18 27 36 45 54 63 72 81 90]]
In [97]:
           print(d)
           [[1 2 3]
            [4 5 6]]
```

```
In [96]:
          print(e * f) ## e and f are broadcast into compatible matrices and then multiplied element-wise
                0 0 0 0 0 0 0 0 0 0 0]
1 2 3 4 5 6 7 8 9 10]
               2 4 6 8 10 12 14 16 18 20]
               3 6 9 12 15 18 21 24 27 30]
             0 4 8 12 16 20 24 28 32 36 40]
               5 10 15 20 25 30 35 40 45 50]
            [ 0 6 12 18 24 30 36 42 48 54 60]
            [ 0 7 14 21 28 35 42 49 56 63 70]
            [ 0 8 16 24 32 40 48 56 64 72 80]
            [ 0 9 18 27 36 45 54 63 72 81 90]]
In [97]:
          print(d)
          [[1 2 3]
           [4 5 6]]
In [98]:
          d.reshape((1,6)) + d.reshape((6,1))
          array([[ 2, 3, 4, 5, 6, 7],
Out[98]:
                  [3, 4, 5, 6, 7, 8],
                  [4, 5, 6, 7, 8, 9],
                  [5, 6, 7, 8, 9, 10],
                  [6, 7, 8, 9, 10, 11],
                  [7, 8, 9, 10, 11, 12]])
```

```
In [99]:
           x = np.arange(6)
           print(x)
           [0 1 2 3 4 5]
In [100]:
           print(x < 3)</pre>
           [ True True False False False]
In [101]:
           print(x >= 3)
           [False False True True]
In [102]:
           print(x == 3)
           [False False True False False]
```

```
In [103]:
# the results can then be used to subset
print(x[x >= 3])
```

[3 4 5]

```
In [103]: # the results can then be used to subset
print(x[x >= 3])

[3 4 5]

In [104]: np.sum(x >= 3) # True = 1, False = 0, so sum counts how many are true
```

Out[104]: 3

```
In [103]: # the results can then be used to subset
print(x[x >= 3])

[3 4 5]

In [104]: np.sum(x >= 3) # True = 1, False = 0, so sum counts how many are true

Out[104]: 3

In [105]: np.mean(x >= 3) # finds the proportion that is True
```

Out[105]: 0.5

```
In [103]:
             # the results can then be used to subset
             print(x[x >= 3])
             [3 4 5]
In [104]:
             np.sum(x \ge 3) # True = 1, False = 0, so sum counts how many are true
Out[104]: 3
In [105]:
             np.mean(x \ge 3) # finds the proportion that is True
Out[105]:
             0.5
In [106]:
             print(\sim(x == 3)) # use the tilde for negation of boolean values
             [ True True True False True True]
```

```
In [107]: print(~x == 3) # be careful if you leave off parenthesis
```

[False False False False False]

```
In [109]:
          y = np.arange(12).reshape([3,4])
           print(y)
           [[ 0 1 2 3]
           [ 4 5 6 7]
            [ 8 9 10 11]]
In [110]:
           print(y >= 6)
           [[False False False]
            [False False True True]
            [ True True True]]
In [111]:
           np.sum(y >= 6)
Out[111]: 6
```

```
In [109]:
           y = np.arange(12).reshape([3,4])
           print(y)
            [[0 1 2 3]
            [ 4 5 6 7]
             [8 9 10 11]]
In [110]:
           print(y >= 6)
            [[False False False]
             [False False True True]
             [ True True True ]
In [111]:
           np.sum(y >= 6)
Out[111]: 6
In [112]:
           np.sum(y >= 6, axis = 0) # you can perform sums and other aggregate functions axis-wise on the bool
Out[112]: array([1, 1, 2, 2])
```

```
In [109]:
           y = np.arange(12).reshape([3,4])
            print(y)
            [[0 1 2 3]
             [ 4 5 6 7]
             [ 8 9 10 11]]
In [110]:
           print(y >= 6)
            [[False False False]
             [False False True True]
             [ True True True ]]
In [111]:
           np.sum(y >= 6)
Out[111]: 6
In [112]:
           np.sum(y >= 6, axis = 0) # you can perform sums and other aggregate functions axis-wise on the bool
Out[112]: array([1, 1, 2, 2])
In [113]:
           np.sum(y >= 6, axis = 1)
Out[113]: array([0, 2, 4])
```

```
In [114]:
    a = np.array([True, True, False, False])
    b = np.array([True, False, True, False])
    print(a)
    print(b)

[ True True False False]
    [ True False True False]
```

```
In [114]:    a = np.array([True, True, False, False])
    b = np.array([True, False, True, False])
    print(a)
    print(b)

    [ True True False False]
    [ True False True False]

In [115]:    print(a & b) # bitwise and
    [ True False False False]

In [116]:    print(a | b) # bitwise or
    [ True True True False]
```

```
In [114]:
            a = np.array([True, True, False, False])
            b = np.array([True, False, True, False])
            print(a)
            print(b)
             [ True True False False]
             [ True False True False]
In [115]:
            print(a & b) # bitwise and
            [ True False False False]
In [116]:
            print(a | b) # bitwise or
            [ True True False]
In [117]:
            print(a ^ b) # bitwise xor (exclusive or)
            [False True True False]
```

```
In [118]: print(~a) # bitwise not
```

[False False True True]

Out[119]: True

Out[120]: False

fancy indexing

Regular lists in python do not support fancy indexing, but NumPy does!

fancy indexing

Regular lists in python do not support fancy indexing, but NumPy does!

```
In [121]:
    np.random.seed(1)
    x = np.random.randint(100, size = 10)
    print(x)

[37 12 72 9 75 5 79 64 16 1]
```

fancy indexing

Regular lists in python do not support fancy indexing, but NumPy does!

```
In [123]:
    a = [1, 4, 7]
    b = [2, 3, 8]
    ind = np.vstack([a,b])
    print(ind)

[[1 4 7]
    [2 3 8]]
```

```
In [123]:
            a = [1, 4, 7]
            b = [2, 3, 8]
            ind = np.vstack([a,b])
            print(ind)
            [[1 4 7]
             [2 3 8]]
In [124]:
            print(x[ind])
            [[12 75 64]
             [72 9 16]]
In [125]:
            X = np.arange(12).reshape((3, 4))
            print(X)
            [[ 0 1 2 3]
             [ 4 5 6 7]
             [ 8 9 10 11]]
```

```
In [123]:
            a = [1, 4, 7]
            b = [2, 3, 8]
            ind = np.vstack([a,b])
            print(ind)
            [[1 4 7]
             [2 3 8]]
In [124]:
            print(x[ind])
            [[12 75 64]
             [72 9 16]]
In [125]:
            X = np.arange(12).reshape((3, 4))
            print(X)
            [[ 0 1 2 3]
             [ 4 5 6 7]
             [ 8 9 10 11]]
In [126]:
            row = np.array([0, 1, 2])
            col = np.array([2, 1, 3])
            X[row, col]
Out[126]: array([ 2, 5, 11])
```

- np.sort()
- np.argsort() gives the indexes of the values to have the proper sorting

- np.sort()
- np.argsort() gives the indexes of the values to have the proper sorting

```
In [127]:
    np.random.seed(2)
    x = np.arange(5)
    np.random.shuffle(x)
    print(x)
```

[2 4 1 3 0]

- np.sort()
- np.argsort() gives the indexes of the values to have the proper sorting

- np.sort()
- np.argsort() gives the indexes of the values to have the proper sorting

```
In [127]:
             np.random.seed(2)
             x = np.arange(5)
             np.random.shuffle(x)
             print(x)
             [2 4 1 3 0]
In [128]:
             x.sort() # sorts x in place
             print(x)
             [0 1 2 3 4]
In [129]:
             y = np.array([5, 2, 1, 4])
             print(y)
             print(y.argsort())
             [5 2 1 4]
             [2 1 3 0]
```

- np.sort()
- np.argsort() gives the indexes of the values to have the proper sorting

```
In [127]:
             np.random.seed(2)
             x = np.arange(5)
             np.random.shuffle(x)
             print(x)
             [2 4 1 3 0]
In [128]:
             x.sort() # sorts x in place
             print(x)
             [0 1 2 3 4]
In [129]:
             y = np.array([5, 2, 1, 4])
             print(y)
             print(y.argsort())
             [5 2 1 4]
             [2 1 3 0]
In [130]:
             d = y.argsort()
             y[d]
Out[130]: array([1, 2, 4, 5])
```

```
In [131]:
            np.random.seed(1)
            X = np.random.randint(0, 10, (4, 6))
            print(X)
             [[5 8 9 5 0 0]
              [1 7 6 9 2 4]
              [5 2 4 2 4 7]
              [7 9 1 7 0 6]]
In [132]:
            # sort each column of X
            # np.sort returns a copy of X after sorted. It does not modify X
            np.sort(X, axis=0)
Out[132]: array([[1, 2, 1, 2, 0, 0],
                    [5, 7, 4, 5, 0, 4],
                    [5, 8, 6, 7, 2, 6],
                    [7, 9, 9, 9, 4, 7]])
```

[0, 1, 6, 7, 7, 9]])

```
In [131]:
            np.random.seed(1)
            X = np.random.randint(0, 10, (4, 6))
            print(X)
            [[5 8 9 5 0 0]
             [1 7 6 9 2 4]
              [5 2 4 2 4 7]
              [7 9 1 7 0 6]]
In [132]:
            # sort each column of X
            # np.sort returns a copy of X after sorted. It does not modify X
            np.sort(X, axis=0)
Out[132]: array([[1, 2, 1, 2, 0, 0],
                    [5, 7, 4, 5, 0, 4],
                    [5, 8, 6, 7, 2, 6],
                    [7, 9, 9, 9, 4, 7]])
In [133]:
            # sort each row of X
            np.sort(X, axis=1)
Out[133]: array([[0, 0, 5, 5, 8, 9],
                    [1, 2, 4, 6, 7, 9],
                    [2, 2, 4, 4, 5, 7],
```

[0, 1, 6, 7, 7, 9]])

```
In [131]:
            np.random.seed(1)
            X = np.random.randint(0, 10, (4, 6))
            print(X)
            [[5 8 9 5 0 0]
             [1 7 6 9 2 4]
              [5 2 4 2 4 7]
              [7 9 1 7 0 6]]
In [132]:
            # sort each column of X
            # np.sort returns a copy of X after sorted. It does not modify X
            np.sort(X, axis=0)
Out[132]: array([[1, 2, 1, 2, 0, 0],
                    [5, 7, 4, 5, 0, 4],
                    [5, 8, 6, 7, 2, 6],
                    [7, 9, 9, 9, 4, 7]])
In [133]:
            # sort each row of X
            np.sort(X, axis=1)
Out[133]: array([[0, 0, 5, 5, 8, 9],
                    [1, 2, 4, 6, 7, 9],
                    [2, 2, 4, 4, 5, 7],
```

In [134]: x[0,:] # selecting a row

Out[134]: array([5, 8, 9, 5, 0, 0])

[0, 1, 6, 7, 7, 9]])

```
In [131]:
            np.random.seed(1)
            X = np.random.randint(0, 10, (4, 6))
            print(X)
            [[5 8 9 5 0 0]
             [1 7 6 9 2 4]
              [5 2 4 2 4 7]
              [7 9 1 7 0 6]]
In [132]:
            # sort each column of X
            # np.sort returns a copy of X after sorted. It does not modify X
            np.sort(X, axis=0)
Out[132]: array([[1, 2, 1, 2, 0, 0],
                    [5, 7, 4, 5, 0, 4],
                    [5, 8, 6, 7, 2, 6],
                    [7, 9, 9, 9, 4, 7]])
In [133]:
            # sort each row of X
            np.sort(X, axis=1)
Out[133]: array([[0, 0, 5, 5, 8, 9],
                    [1, 2, 4, 6, 7, 9],
                    [2, 2, 4, 4, 5, 7],
```

[0, 1, 6, 7, 7, 9]])

```
In [131]:
            np.random.seed(1)
            X = np.random.randint(0, 10, (4, 6))
            print(X)
            [[5 8 9 5 0 0]
             [1 7 6 9 2 4]
              [5 2 4 2 4 7]
              [7 9 1 7 0 6]]
In [132]:
            # sort each column of X
            # np.sort returns a copy of X after sorted. It does not modify X
            np.sort(X, axis=0)
Out[132]: array([[1, 2, 1, 2, 0, 0],
                    [5, 7, 4, 5, 0, 4],
                    [5, 8, 6, 7, 2, 6],
                    [7, 9, 9, 9, 4, 7]])
In [133]:
            # sort each row of X
            np.sort(X, axis=1)
Out[133]: array([[0, 0, 5, 5, 8, 9],
                    [1, 2, 4, 6, 7, 9],
                    [2, 2, 4, 4, 5, 7],
```

```
In [134]: X[0,:] # selecting a row

Out[134]: array([5, 8, 9, 5, 0, 0])
In [135]: print(X)

        [[5 8 9 5 0 0]
        [1 7 6 9 2 4]
        [5 2 4 2 4 7]
        [7 9 1 7 0 6]]

In [136]: X[:,1].argsort() # the argsort for the column index 1

Out[136]: array([2, 1, 0, 3], dtype=int64)
```

[0, 1, 6, 7, 7, 9]])

```
In [131]:
            np.random.seed(1)
            X = np.random.randint(0, 10, (4, 6))
            print(X)
            [[5 8 9 5 0 0]
             [1 7 6 9 2 4]
              [5 2 4 2 4 7]
              [7 9 1 7 0 6]]
In [132]:
            # sort each column of X
            # np.sort returns a copy of X after sorted. It does not modify X
            np.sort(X, axis=0)
Out[132]: array([[1, 2, 1, 2, 0, 0],
                    [5, 7, 4, 5, 0, 4],
                    [5, 8, 6, 7, 2, 6],
                    [7, 9, 9, 9, 4, 7]])
In [133]:
            # sort each row of X
            np.sort(X, axis=1)
Out[133]: array([[0, 0, 5, 5, 8, 9],
                    [1, 2, 4, 6, 7, 9],
                    [2, 2, 4, 4, 5, 7],
```

```
In [134]:
           X[0,:] # selecting a row
Out[134]: array([5, 8, 9, 5, 0, 0])
In [135]:
            print(X)
            [[5 8 9 5 0 0]
             [1 7 6 9 2 4]
             [5 2 4 2 4 7]
             [7 9 1 7 0 6]]
In [136]:
            X[:,1].argsort() # the argsort for the column index 1
Out[136]: array([2, 1, 0, 3], dtype=int64)
In [137]:
            print(X[ X[:,1].argsort() , : ]) # 'subset' X by the argsort to arrange X by the column
            [[5 2 4 2 4 7]
             [1 7 6 9 2 4]
             [5 8 9 5 0 0]
             [7 9 1 7 0 6]]
```