ASTR 1040 Jupyter Notebook Exam Template

This is a template file you can use to complete your exam. The first cell (below after FAQ section) contains lots of helpful constants you may need. Make sure to run it before you start working!

Below I've also included examples of how to do common `astropy` / math things:

Astropy FAQ

This is a template file you can use to do your homeworks in. I recommend copying this each time you start a homework. The first cell (below) contains lots of helpful constants you may need to use throughout the semester. Make sure to run it each time before you start working!

For reference, all of the `astropy` constants as well as examples can be found here. Likewise all of the units and their names / how to access them can be found here.

To create a variable with units you need to multiply by the corresponding unit class:

```
d = 1 * units.au
```

You can then convert unitful quantities to other units by calling the `to` method and passing the desired end unit class:

```
d_m = d.to(units.m)
```

If you have a ratio of quanities where all the units should cancel out, you can obtain the dimensionles number using the `dimesionless_unscaled` method. For example writing:

```
d_m/d
```

yields $1.4959787 \times 10^{11} \frac{\mathrm{m}}{\mathrm{AU}}$, but doing:

```
(d_m/d).to(units.dimensionless_unscaled)
```

returns `1.0` as expected.

It may also be useful to convert quantities to SI units, and you can do this on any quantity that has units by adding `.si` to it. For example:

```
d.si
```

returns $1.4959787 \times 10^{11}\ \mathrm{m}$ as this is an AU in SI units (SI unit of distance is the m).

If you ever need to obtain just the *number* of a quantity (without any units) you can do:

```
d.value
```

which returns 1 (no units, just the number 1 since we said `d = 1 * units.au`).

Math FAQ

Let's say we have two numbers assigned to variables `x` and `y`, i.e.:

```
x = 1.
y = 2.
```

```
Addition/subtraction/multiplication/division work largely how you
would expect:
```

To add:

```
x_plus_y = x+y
print(x_plus_y)
>>> 3.
```

To subtract:

```
x_minus_y = x-y
print(x_minus_y)
>>> -1.
```

To multiply:

```
x_times_y = x*y
print(x_times_y)
>>> 2.
```

To divide:

```
x_over_y = x/y
print(x_over_y)
>>> 0.5
```

Unfortunately exponents are probably *not* what you would expect, but to raise to a power you do:

```
y_tothe_x = y**x #note the ** for exponents
```

Oftentimes in astrophysics we have very large numbers, i.e. the mass of the supermassive black hole at the center of our galaxy (Sag A*) is $\sim 4 \times 10^6 \mathrm{M}_\odot$. To write this number in python we can use the convenient "e" syntax:

```
from astropy.constants import M_sun
SagAMass = 4e6*M_sun
print(SagAMass)
>>> 7.953639482792203e+36 kg
```

To take the nth root of something raise it to the power of 1/n, i.e. to take the cubic root of `x`:

```
cubeRoot = x**(1/3)
```

numpy

Oftentimes we will also use `numpy` to do math, as it provides convenient functionality to interface with all of the trig things we will need. This is also where you will get π from! A few examples:

```
import numpy as np
pi = np.pi
sinx = np.sin(x)
sinx_plus_2pi = np.sin(x+2*pi) #should be the same as sinx
angle = np.atan2(y/x) #arctangent, use atan2 if you care about
which quadrant!
h = np.sqrt(x**2+y**2) #let x and y define sides of triangle
y_trig = h*np.cos(angle) #should be the same as y
```

In [ ]:

```
#SETUP CELL (modify at your own peril)
from astropy import units #access units by doing units.<unit> (i.e. units.au
from astropy import constants
import numpy as np #common math functions (i.e. np.sin(x)) and better arrays
import matplotlib.pyplot as plt #plotting functions (i.e. plt.plot(x,y))
G = constants.G # gravitational constant
M_sun = constants.M_sun # mass of the sun
R_sun = constants.R_sun # radius of the sun
L_sun = constants.L_sun # luminosity of the sun
M_earth = constants.M_earth # mass of the earth
R_earth = constants.R_earth # radius of the earth
M_jup = constants.M_jup # mass of jupiter
R_jup = constants.R_jup # radius of jupiter
sigma_sb = constants.sigma_sb # Stefan–Boltzmann constant
c = constants.c # speed of light
h = constants.h # Planck constant
k_B = constants.k_B # Boltzmann constant
m_e = constants.m_e # mass of electron
m_p = constants.m_p # mass of proton
m_n = constants.m_n # mass of neutron (basically just the mass of a proton b
g0 = constants.g0 # standard gravity, 9.8 m/s^2
e = constants.e # absolute value of electron/proton charge
```

Cheat sheet area

If you have opted to make a cheat sheet as a Jupyter notebook cell, copy and paste your markdown content into the empty markdown cell below and any code content into the empty code cell below:

Star's thermostat	Mass Range of Stars	Uncertainty Principle	Electron Degeneracy Pressure	Exclusion Principle

|- Ability to fuse H into He in core via P-P chain
- Hydrostatic Equilibrium & Energy Balance $\implies$ self-regulating | [0.08$M_{\odot}$, 100$M_{\odot}$] |- uncertainty in *location* $\times$ uncertainty in *momentum* ≈ planck's constant (*h*)
- uncertainty in *energy* $\times$ uncertainty in *time* ≈ planck's constant (*h*)|- There is lots of

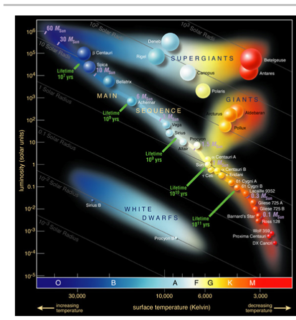
space between nuclei in normal matter
- Under enormous pressure, matter compresses
- This compresses the *electron cloud* that surrounds the nucleus, but not the nucleus itself
- Compression ceases when electron's quantum states are filled (*Pauli Exclusion Principle*)
  - **Big Picture**: Squeezing matter restricts ***locations*** of its particles, increasing their uncertainty in ***momentum***.
  - **Applications**:
    - In Low-M stars EDP can prevent a collapsing gas cloud from becoming a star $\rightarrow$ brown dwarfs
    - Determines how Solar Mass stars begin He Flash
    - White Dwarfs are supported by EDP | - *Two fermions cannot occupy the same quantum state at the same time*

Deriving Values

> Only measured values we have are *Intensity / Flux*, *Parallax Angle:* α, & *Peak Wavelength:* λ_{max}.

Value	What to Use		Form	Notes
d [parsec or lyr]	α	$d = \frac{1[as]}{\alpha[as]}$	parsec	- Can only find d this way in *our* galaxy. - Use the small angle theorem!
L [W]	d and I	$L = (4\pi d^2) * I$ $I = \frac{L}{4\pi d^2}$ $d = \sqrt{\frac{L}{4\pi I}}$	[W]	$L \propto M^{3.5}$
T [K]	λ_{max} and Wien's Law	$T = \frac{b}{\lambda_{max}}$ $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$		

Value	What to Use	Form	Notes
R [m] & A [m^2]	σ (Stefan-Boltzmann) and T	$I = \frac{P}{A} = \sigma T^4$ $A = \frac{L}{P/A}$ $R = \sqrt{\frac{A}{4\pi}}$	1. Find L from I and use **inverse square law for light**. 2. Find I_s at surface from S-B law
M [kg]	- Find P and *seperation*, a, for **Total Mass** - Measure ratio of d to *center of mass* - OR ratio of *velocities*	$(M_1 + M_2)P^2 = a^3$ $\frac{M_1}{M_2} = \frac{d_{CM_2}}{d_{CM_1}}$ $\frac{M_1}{M_2} = \frac{v_2}{v_1}$ OR	- Needs to be a *Binary System*! - [UNITS *MUST* BE in $M_\odot$, yrs, and AU] - Need 2/3 observable quantities (orbital period (P), seperation (a or R), or orbital velocity (v)): $\frac{2\pi R}{P} = v$, for circular orbits

HR Diagram	High vs Low M Stars	Notes
	**High-M** - High L & T - Shorter lives (10^6 yrs) - Bluer **Low-M** - Low L & T - Longer lives (10^{11} yrs) - Redder	**Lifetime** = Fuel available / Rate of burning $\rightarrow$ ex: Lifetime(O) / Sun = (Fuel(O) / Fuel(S)) / (Rate(O) / Rate(S)) **How does M impact Lifetime?** $\rightarrow\uparrow M \rightarrow\uparrow T_{core} \rightarrow\uparrow$ amt of fusion - 2 types of clusters: Globular (old) and Open (young) - Determine cluster age from "main seq. turn off": when a star runs out of fuel $\rightarrow$ older means bluer, low-M stars have died and thus the cluster is "redder" **Star Birth** - Stars form from molecular clouds which

HR Diagram	High vs Low M Stars	Notes
		collapse in on own gravity; very dense **Jean's Instability**: When a cloud/nebula becomes unstable (Large + cold => become unstable), then small lumps in cloud start to gain mass. **Jean's Mass**: the minimum mass a cloud must have to begin star formation collapse. $M_J = 18 M_\odot \sqrt{\frac{T^3}{n}} \rightarrow$ at T = 30K and n = 300 cm^{-3}, $M_J \approx 171 M_\odot$

Gravitational Contraction			
1. *Power* is radiated from cloud	$P_{rad} = \sigma T^4$	2. E decreases $\rightarrow$ Pressure P decreases	$P_{pres} = n k_B T$
3. As P decreases, cloud contracts $\rightarrow R$ decreases	3.5. region collapses $\rightarrow$ gains $E_k \rightarrow$ randomizes to become $E_{Thermal}$	$E_g = \frac{-GMm}{r}$, $E_k = \frac{1}{2} m v^2$, $E_k + E_g = C$	4. after contraction, T increases
5. Same # of particles, but now in *smaller volume* $\rightarrow$ density increases	6. T and n increase $\rightarrow P$ increases:	$P_{pres} = n k_B T$	7. [?] Why does cloud keep contracting if P is increasing? $\rightarrow R$ is smaller, so F_g increases!
[?] Why does contraction eventually stop? $\rightarrow$ cloud gets dense enough that radiation can't escape $\rightarrow T$ rises, P rises, contraction slows. $\rightarrow$ This produces a *protostar*!	* The primary energy source for heating a protostar comes from the gravitational energy released during collapse	Protostars are *Lower Right* of HR diagram.	A Protostar becomes a real star *faster* if it has *more mass*, because it can get hotter faster and start fusion * The more massive a WD is $\rightarrow$ smaller R

Low-M Life After Main Sequence			
1. Stops fusing H in core $\rightarrow$ becomes He core	2. He core contracts $\rightarrow$ heats up	3. H in shell ignites, layer outside of He core.	4. Shell burning is furious $\rightarrow$ star expands and photosphere cools due to increased pressure from shell burning *core contracts while photosphere expands*
5. Star enters *Red Giant Phase*	6. Core temps reach $100 m \circ K \rightarrow$ He fusion ignites	* Since degeneracy pressure supports He core, He ignition	7. After He flash, H shell fusion lessens $\rightarrow$ star contracts, lives

Low-M Life After Main Sequence			
		is sudden, known as the Helium Flash	short 2nd life on Horizontal Branch
8. Core is depleted of He $\rightarrow$ expands again as a *double shell burning star* (He & H) with a core primarily made of C	9. The C core of a Low-M star *Will NOT* reach 600 mil $\circ K$ to ignite carbon.	* instead, outer layers of star are ejected into a planetary nebula	10. Exposed C is called a *White Dwarf*, supported by EDP and no longer undergoing fusion.

In [ ]:
```
# CODE EXAMPLES / Clicker Questions:
# 1.3. You are running to catch a squirrel at a speed of $20 \pm 1$ $\rm{\fr
# a. What is the corresponding quantum limit on the uncertainty in your posi
m = 50*units.kg
Δv = 1*units.km/units.hr
Δp = m*Δv
Δx = h/(4*np.pi*Δp) # uncertainty in position

# Q6. How long will the sun last?
# a. find # of protons in sun -> M_sun / m_p
```

Review Questions/Topics (NOT comprehensive, but meant to give an overview to get you started):

1. What are the key properties of the sun? Put them in a context that makes them meaningful.

> Mass, size, surface temperature, and composition are all important properties of stars. Our Sun is pretty 'average' in all of these.

2. How does $E = mc^2$ power stars?

> $E = mc^2$ is the equation for Energy that we use with regard to *nuclear fusion*. The tiny difference in mass in the reaction ($4^1H + 2e$ and 4^4He) results in energy being created. Stars then use fusion reactions to power their cores and continue the process of nuclear burning inside.

3. Why does nuclear fusion require high temperatures and pressures?

> Nuclear fusion requires very high temperatures and pressures because **higher temps = higher speeds**. Higher speeds overcome the electromagnetic repulsion experienced by nuclei when they come close to each other, and they thus are able to bind together.

> In addition, nuclear fusion relies on **quantum tunneling**. Ignoring quantum tunneling, if we try to calculate the required thermal energy to begin fusion, we will get a number significantly higher than that of our own Sun.

> But our Sun is fusing, so what else is happening in there? Quantum tunneling builds on the uncertainty principle.

4. What is quantum tunneling and why is it important in the sun?

> Quantum tunneling is important for fusion because it is how nuclei overcome the electromagnetic repulsive barrier between each other. They instead just 'sneak' through the barrier via tunneling.

5. What kind of equilibrium governs the Sun? Why is this important?

> Hydrostatic equilibrium: core is at high temperature & high density preventing gravitational collapse. Energy balance: between fusion and surface radiation. Solar thermostat: tight negative feedback loop for fusion which prevents a runaway reaction.

6. Where in the Sun is the energy generated, and by what process? How does energy get transported out of the Sun?

> In the core by fusion. Energy is transported out of the Sun by radiative diffusion. amount of energy produced by fusion = surface radiation

7. What is the photosphere of the Sun? What is its temperature? How is the temperature measured?

> The photosphere of the Sun is the surface of the Sun and thus its T is the Sun's surface temperature. Temperature is measured by using the peak wavelength measured and Wien's law.

8. What is helioseismology, and what can be learned from it?

> Helioseismology is the study that measures Doppler maps of the Sun and Stars to determine its direction of rotation. > Blue shift = coming towards us in rotation, Red = away Used to probe the structure of the Sun We can compare these measurements with neutrino flux against our mathematical models to check how accurate our predictions of the Solar structure are.

9. What causes solar activity/space weather?

10. How does the “solar thermostat” keep energy generation in the Sun relatively constant?

11. Why is the sun getting brighter?

> Each nuclear reaction releases 1He -> core shrinks -> core temperature increases -> rate of fusion increases

12. Why does the Sun emit neutrinos? What was the solar neutrino problem, and what is our current state of understanding of its solution?

    > Emitted through fusion. Neutrino problem = onyl detect about 1/3 of expected neutrinos here on Earth from the Sun. Neutrinos change "flavors"

13. Explain the basis of the stellar classification scheme that involves both “spectral type” (O, B, A, F, G, K or M) and “luminosity class” (I through V). How do the colors of these stars vary? What is being measured by the spectral type? What is indicated by the luminosity class? How are stellar mass and the order of spectral types related?

    > The stellar classification scheme is visually represented on the HR diagram. The main sequence follows a slight S curve, and is the sequence stars are within if they are *actively fusing hydrogen*.

    > Spectral types are in ascending order of Surface Temperature: [Only Boys Accepting Feminism Get Kissed Meaningfully].

    > Luminosity classes are as follows: - I: Supergiant - II: Bright giant - III: Giant - IV: Subgiant - V: Main Sequence

    > High mass stars: High luminosity, hotter temps, shorter lives, Bluer color

    > Low mass stars: Lower L, lower T, longer lives, Redder color

14. What is the difference between “apparent” and “absolute” magnitude when describing the brightness of a star? How is the magnitude system defined? What is the rule of thumb relationship between magnitude and brightness?

    > Absolute Magnitude: how bright a star would be if seen from a standard distance Apparent Magnitude: how bright a star is from Earth Lower magnitude = brighter

15. What is the relationship between flux and luminosity? Between flux and temperature? Between flux and distance?

16. What is the H-R diagram and why is it important? What is the range of surface temperatures and of luminosities of “main sequence” stars as plotted on H–R diagrams?

17. What is stellar parallax? How does it help us determine distances to stars?

18. How do we measure the masses of stars using binaries?

19. How do we determine the radius of stars?

20. How does star birth begin in gas clouds?

21. How do stars form?
22. Are there more low mass or high mass stars in general? Why?

> More Low-Mass stars in general $\rightarrow$ NEW (open) clusters have more Low-M stars. Low-M stars *LIVE LONGER*! Low-M are also LESS BRIGHT.

---

23. What wavelength ranges of radiation are best for observing the process of star formation, and why?

---

24. What is the "mass-luminosity relation" for main sequence stars? How much more luminous than the Sun are the massive O-type stars?
25. Despite their greater fuel supply, massive stars have shorter lifetimes than low mass stars. Explain why with some detail provided (like using the mass-luminosity relation to estimate the rate of 'fuel usage').
26. Sketch an "H-R diagram" for stars. Label the x-axis with temperature and spectral type, and the y-axis with luminosity and absolute magnitude. Roughly indicate the locations on the diagram of: a) a main sequence star of one solar mass; b) a main sequence star of 0.1 solar mass; c) a main sequence star of 25 solar masses; d) a red giant star; e) a white dwarf star; f) a red supergiant star; g) the approximate shape of the curve corresponding to the main-sequence.
27. What defines the main sequence? How are the lives of different main sequence stars similar and different? Compare the timescales involved. What is the key intrinsic quantity that drives their differences?
28. How are H-R diagrams constructed from observables? I.e. what do we really observe as a proxy for temperature, luminosity, etc.
29. How are the ages of star clusters determined? Compare the H–R diagram for a globular cluster with that for an open cluster like the Pleiades, and explain how this tells us about their relative ages.
30. What is "degeneracy pressure", how does it differ from ordinary pressure, and what is its relevance to the structure and evolution of stars? What kinds of stars are supported by degeneracy pressure?
31. What is the uncertainty principle? Explain how it works in something like quantum tunneling. Why don't you notice its effects in your daily life? How does it relate to degeneracy pressure?
32. What is the exclusion principle? How does it relate to degeneracy pressure?

In [ ]:

```

```

Exam instructions:

For the multiple choice and short answer questions you have two options:

1. Answer these on the actual test (i.e. circle the multiple choice answer you select and write out by hand the answers to short answer questions)
2. Clearly label and type them into markdown cells (i.e. for MC type 1: a if you picked a for q1)

For the quantitative questions, if you opt to use the notebook you have two options:

1. Just use the notebook to do calculations, writing your final answers down by hand on the physical test. If you do this make sure you note this on the test so I know to check your work here!
2. Clearly label and fully answer the questions here, with a combination of markdown and code cells as you see fit.

After you have finished the test, create a PDF from your work (just like you have done with your homework) and upload it to the corresponding Midterm 1 Canvas assignment.

2. Multiple Choice

   A. d
   B. c
   C. *d*
   D. c
   E. c
   F. b
   G. a
   H. d
   I. *d*
   J. d
   K. b
   L. c
   M. b
   N. e
   O. d
   P. a
   Q. c
   R. a
   S. d
   T. c
3. Short Answer Questions

   A. The "solar thermostat" is a concept meant to describe how the Sun is able to sustain self-regulation through a tight, negative feedback loop for fusion that prevents a runaway reaction from occuring. Additionally, the Sun maintains Hydrostatic Equilibrium, which prevents its gravitational collapse, as well as an

Energy Balance between fusion and surface radiation. The Sun is getting gradually brighter because within its core, each nuclear fusion reaction releases Energy, causing the core to shrink and its temperature to tise, which increases the rate of fusion and thus causes an increase in brightness over time.

B. *There are more stars with masses less than / close to the mass of our Sun because there fundamentally must be more low-mass stars in the Universe than high-mass stars. This is because it takes less energy to form more massive stars, and because the cloud from which a star forms must have more mass.*

C. *The lifespan of the first generation of stars was* ...

D. a. The axes of the color-magnitude are transferrable because the x-axis for Color measures blue-red magnitudes, which tell us information about the temperature of a star. In general, blue stars are hotter than red stars, so we can use this data to infer about a star's temperature, which is plotted on the HR diagram. The y-axis looks at absolute magnitude, which is essentially the brightness of a star as seen from some standard point in the sky. Thus, we can use this data to estimate a star's luminosity, as seen on the HR diagram.

   b. One difference between the diagrams is that the Gaia diagram does not display white dwarfs well. This may be because they are simply harder to observe as they are dimmer. Another difference is that there is a slight difference in the actual shape of the data, which may be from the fact that the Gaia diagram is only a sample of data, which may cause the skewing from the HR model.

In [ ]:
```
# 4. Quantitative

# 1.a
λ_max = 250 * units.nm
T = (constants.b_wien / λ_max).si

alpha = (0.130 * units.arcsec).to(units.rad)
d = 1*units.rad / alpha

I = 2e-8 * units.W / units.m**2
L = (4*np.pi*(d**2) * I).si
L = L / L_sun

A = L / I

R = (np.sqrt(A / (4*np.pi))).si
R = R / R_sun

print(T, L, R)

# b. My answers imply that the spectral designation of this star is around a
```

```
# 2. a.
```

```
11591.087820740688 K 1.6528425794132107e-21 kg / (W s3) 1.1656651511743358e-
16 s(3/2) / (kg(1/2) m2)
```