Probability & Statistics 21BAI1379

DA-2 Abhishek kumar

(1) → given: $3x-2y=5$ (x on Y)

$x-4y=7$ (Y on X)

$3x-2y=5 \Rightarrow 3x = 2Y+5$

$\Rightarrow x = \frac{2}{3}Y + 5/3$

$\Rightarrow X = \frac{2}{3}Y + 5/3$ —(1)

& $X = 7+4Y \Rightarrow Y = \frac{x}{4} - \frac{7}{4}$ —(ii)

from (1) & (ii), $\frac{2}{3}Y + 5/3 = 7+4Y$

$\Rightarrow 2Y+5 = 21+12Y$

$\Rightarrow Y = -1.6$

$\therefore X = 7 + 4\times -1.6 = 0.6$

$\therefore$ mean(x) = 0.6, mean(y) = 1.6

from the eqns: $b_{xy} = \frac{2}{3} = 0.666$

$b_{yx} = \frac{1}{4} = 0.25$

$r = \sqrt{b_{xy} \cdot b_{yx}} = \sqrt{\frac{2}{3}\times\frac{1}{4}} = \sqrt{\frac{1}{6}} = 0.46$

_/_/_

(2) → Given: Accident rate = 4.5 per month

(a) P(no accident)

$$P(x=0) = \frac{e^{-\lambda} \cdot \lambda^0}{0!} = \frac{e^{-4.5} \times 4.5^0}{1} = 0.0111$$

(b) P(at least two accidents)

$$P(x \geq 2) = 1 - P(x<2)$$

$$= 1 - P(x=0) - P(x=1)$$

$$= 1 - \frac{e^{-4.5}}{0!} - \frac{e^{-4.5} \times 4.5}{1!}$$

$$= 1 - 0.0111 - 0.04990$$

$$= 0.989$$

(c) P(at most 4 accidents)

$P(x \leq 4)$

$$= P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)$$

$$= e^{-4.5}\left(1 + \frac{4.5}{1!} + \frac{(4.5)^2}{2} + \frac{(4.5)^3}{6} + \frac{(4.5)^4}{24}\right)$$

$$= 0.0111 + 0.04999 + 0.11247 + 0.1687 + 0.1898$$

$$= \boxed{0.5297}$$

(3) Given: P(wake up before alarm) = 0.4

(a) mean = $E(x) = np = 7 \times 0.4 = 2.8$

variance = $np(1-p) = 7 \times 0.4 \times 0.6$

$= 1.68$

(b) (i) $P(x \leq 1) = P(x=0) + P(x=1)$

$= {}^7C_0 (0.4)^0 (0.7)^7 + {}^7C_1 (0.4)^1 (0.6)^6$

$= 0.0279 + 0.1306$

$= 0.1585$

(ii) $P(1<x<5) = P(x=2)+P(x=3)+P(x=4)$

$= {}^7C_2(0.4)^2(0.6)^5 + {}^7C_3(0.4)^3(0.6)^4 + {}^7C_4(0.4)^4(0.6)^3$

$= 0.2612 + 0.2903 + 0.1935$

$= 0.743$

(c) $P(x>5) = P(x=6)+P(x=7)$

${}^7C_6(0.4)^6(0.6)^1 + {}^7C_7(0.4)^7(0.6)^0$

$= 0.0016 + 0.0172$

$= 0.0188$

(4) $\rightarrow P(x\geq 138) = 1-P(x\leq 138)$

$= 1-P\left(\frac{x-\mu}{\sigma} < \frac{138-100}{\sqrt{225}}\right)$

$= 1-P(Z<2.533)$

$= 1-0.9943$

$= 0.0057$

$P(x\geq 180) = 1-P(x\leq 180)$

$= 1-P\left(Z\leq \frac{180-100}{\sqrt{225}}\right)$

$= 1-0.99957$

$= 0.00043$

$0.00043\times 1800 = 0.774$

∴ At most 1 child is gifted.

(5) mean $= \lambda = 2$

$K = 4$

$$f(x) = \frac{\lambda^K x^{K-1} e^{-\lambda x}}{\Gamma(K)}$$

$$= \frac{(2)^4 x^3 e^{-2x}}{3!} = \frac{8 e^{-2K} x^3}{3}$$

$$\text{mean} = \frac{K}{\lambda} = \frac{4}{2} = 2$$

$$\text{variance} = \frac{K}{\lambda^2} = \frac{4}{2^2} = 1$$

—/—/—

(6) Given: $\alpha = 2,\ \beta = 3$

$$\text{Mean} = \left(\frac{1}{\alpha}\right)^{1/\beta} \Gamma\left(1 + \frac{1}{\beta}\right)$$

$$= \left(\frac{1}{2}\right)^{1/3} \left(\Gamma\left(1 + 1/3\right)\right)$$

$$= \left(\frac{1}{2}\right)^{1/3} \left(\Gamma\, 4/3\right) = (0.795)(1.354) = 1.077$$

$$\underline{\text{Variance}} = \left(\frac{1}{\alpha}\right)^{2/\beta} \Gamma\left(1 + 2/\beta\right) - \left(\left(\frac{1}{\alpha}\right)^{1/\beta} \Gamma\left(1 + \frac{1}{\beta}\right)\right)^2$$

$$\left(\frac{1}{2}\right)^{2/3} \Gamma\left(5/3\right) - \left(1.077\right)^2$$

$$= 0.629 \times 0.897 - 1.159$$

$$= \left(\frac{1}{2}\right)^{2/3} \Gamma\left(5/3\right) - \left[\left(\frac{1}{2}\right)^{1/3} \Gamma\left(\frac{4}{3}\right)\right]^2$$

(b) $$P(x \le 5) = \int_5^\infty (2)(3)\, x^{3-1} e^{-2x^3}\, dx$$

$$2x^3 = t \Rightarrow 6x^2\, dx = dt$$

$$\Rightarrow x = 5 \rightarrow t = 150$$

$$x = \infty \rightarrow t = \infty$$

$$\int_{150}^\infty e^{-t}\, dt = \left[-e^{-t}\right]_{150}^\infty$$

$$= \left[-e^{-150} + e^{-\infty}\right] = e^{-150}$$

(c) $P(1.8 < x < 5)$

$$= \int_{1.8}^{5} (2)(3) x^{3-1} e^{-2x^3} \, dx$$

$$= \int_{1.8}^{5} 6x^2 e^{-2x^3} \, dx = \left[-e^{-t} \right]_{11.664}^{180}$$

$$= -e^{-180} + e^{-11.664}$$

---/---/---

(7) Given: $n = 49$

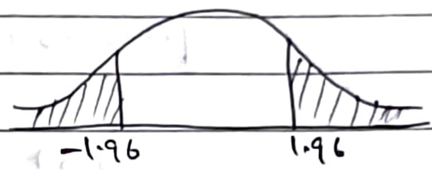
Mean $= 15200 = \bar{x}$

Population mean $= 15150 = \mu_0$

SD $= 1200$

$\alpha = 0.05$

$Z_{\alpha/2} = \pm 1.96$



$H_0: \mu = 15150$

$H_1: \mu \neq 15180$

$$Z = \frac{\bar{x} - \mu_0}{\frac{S}{\sqrt{n}}} = \frac{15200 - 15180}{\frac{1200}{\sqrt{49}}}$$

$$= \frac{50}{\frac{1200}{7}} = \frac{50}{171.428} = 0.2916$$

$Z < Z_{\alpha/2}$

$\therefore$ H_0 is accepted.

(8) Given: $P_1 = \frac{20}{200} = 0.1$ $\quad P_2 = \frac{12}{200} = 0.06$

$\alpha = 1\% \Rightarrow \alpha = 0.01$

$$P = \frac{200 \times \frac{20}{200} + 200 \times \frac{12}{200}}{400} = \frac{32}{400} = 0.08$$

$$Q = 1 - 0.08 = 0.92$$

$H_0: P_1 = P_2$

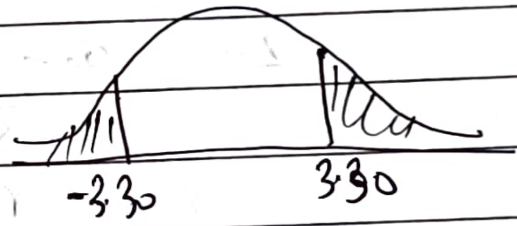
$H_1: P_1 \neq P_2$

$$Z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{0.1 - 0.06}{\sqrt{0.08 \times 0.92 (0.01)}} = \frac{0.04}{\sqrt{0.000736}}$$

$$= \frac{0.04}{0.02713} = 1.4743$$

$Z_{\alpha/2} = \pm 3.30$



$\therefore Z < Z_{\alpha/2}$

So, H_0 is accepted.