Estimation of curve similarity using turning functions

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ESTIMATION OF CURVE SIMILARITY USING TURNING FUNCTIONS

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Abstract

The process of classifying objects is a fundamental feature of most human pursuits, and the idea that people classify together those things that people find similar is both intuitive and popular across a wide range of disciplines. Estimation of difference between curves (curve matching) is an useful and often necessary technique in many applications, including: pattern recognition, image object recognition, robotic applications, computational geometry, etc.

In this paper, three methods for curve matching using turning functions are presented. While the first two, called plain and polygonal method, are based on a simple adaptation of the existing approaches, the third one, called penalty method, is a new one and tries to overcome some important problems from the first two. The advantages and essential problems of the proposed methods are also discussed. A number of examples are presented to show major differences among the methods and their potential usefulness.

AMS Subject Classification: 68T45, 68T10

Key Words: shape, difference, matching, turns

1 Introduction

Similarity is fundamental for learning, knowledge and thought, for only our sense of similarity allows us to order things into kinds so that these can function as stimulus meanings. Reasonable expectation depends on the similarity of circumstances and on our tendency to expect that similar causes will have similar effects (Quine [22]). Tversky [26] describes the similarity concept as an organizing principle by which individuals classify objects, form concepts, and make generalization.

Many authors agree that special sciences develop their own highly refined notions of similarity. Goodman [9] claims that there is nothing like overall similarity that can be universally measured, but we always have to say in what respects two things are similar. Similarity judgments will thus become crucially dependent on the context in which they occur.

According to Popper [21] (see Fig. 1), two things which are similar are always similar in certain respect and generally, similarity with it repetition, always presupposes the adoption of a point of view.

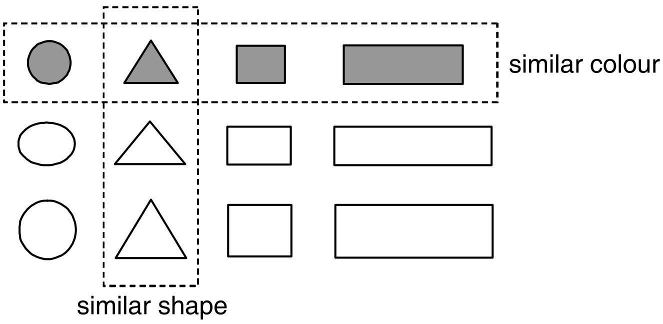

Figure 1: Illustration of Popper's different respects

Fig. 1 illustrates how certain respects can effect similarity. Some of the objects are similar with respect to colour, while others are similar with respect to either shape or area. So, the context of the view on the considered objects has a major influence on the similarity concept.

Estimation of similarity or difference between curves (curve matching) is an active research field in applied mathematics, computational geometry and computer science. Many efforts have been spent to design a good technique for curve matching. Most of the attempts have been limited to find reliable method for matching of closed curves because they represent object boundaries produced after image pre-processing (segmentation). In such cases, matching according to the curve shapes corresponds to matching according to the curve areas. Thus, the problem is focussed on matching techniques that covers just one context, curve shape. However, most of these methods are not suitable for matching of "free" curves (non-closed and complex curves with intersections and more endpoints), although some of them could be extended in that direction. In case of "free" curves, to be intuitively acceptable, matching techniques have to consider not only shape, but sometimes imaginable area of the curve.

The general purpose shape matching, i.e. the estimation of the difference *Diff(A,B)* between the curves A and B has been shown to be a difficult problem. Various matching approaches designed for specific application and kind of curves have been proposed. Generally, the approaches can be classified as: transformational, geometrical, structural and quantitative.

Transformational approaches are based on the comparison of various curve representations, including: fourier descriptors (Gorman et al [10]), turning functions (Arkin et al [1]), autoregressive coefficient (Dubois et al [8]), stochastic labelling (Bhanu et al [6]), convolution (Schwartz et al [23]), Hough transformation (Turney et al [25]), curve bending function and variations (Huang et al [11]), etc. Methods that use dynamic curve transformation (e.g. Azencott et al [3], Singh et al [24]) are also in this group.

Geometrical approaches try to find optimal position and scaling for the match (usually for polygons), and then calculate the difference in the areas, boundaries or contour line segments using an optimal correspondance (see Kashyap et al [14], Atallah et al [4], Ventura et al [27], Adamek et al [2], Petrakis et al [20]).

Structural approaches are based on string or graph matching (Wolfson [28], Maes [17], Avis et al [5]) and use corresponding curve representation. Some methods decompose the curve and then use transformational approach to match curve parts (Latecki et al [16]). The syntactic approaches (see Pavlidis [19], Nishida [18], Kupeev et al [13]) also belong in this group.

Quantitative approaches (e.g. Ireton et al [12]) are mainly adopted for closed curves and use various shape descriptors like: ratio perimeter/area^2, average angle changes, ratio of perpendicular chords, etc.

A good overview of possible shape descriptors can be found in the book by Costa and Cesar [7].

All general purpose matching methods should satisfy some criteria to be reliable and robust:

- As first, the most important of all, the matching result should correspond to the human intuition.
- Each matching method should be invariant under the elementary transformations (translation, rotation and uniform scaling).
- It should satisfy the basic distance criteria and the triangle inequality (*Diff(A,A)* $= 0$, *Diff(A,B)* > 0 for $A \neq B$, *Diff(A,B)* = *Diff(B,A)* and *Diff(A,B)* + *Diff(B,C)* $\geq$ *Diff(A,C)*).
- It should be continuous function (small changes in the shape should produce small changes in the matching result).
- It should be universal in the sense that there is no restrictions on curves are assumed (e.g. to be able to deal with closed as well as open curves).
- It should abstract from distortions (e.g. digitalized noise or segmentation errors) although this is in essence a preprocessing step.
- The matching method should have reasonable performance.

Unfortunately, there is no reliable matching method that satisfy all of these criteria, particularly for "free" curves.

In this paper, three methods for "free" curve matching using turning functions for curve representations will be presented. While the first two are based on an adaptation of the existing approaches, the third one is a new and tries to overcome some important problems from the first two. A number of examples will be presented to show the major advantages and weakness of the proposed methods and their potential usefulness.

2 Basic Method

We will consider polygonal curves assuming that they are obtained by choosing some special points (polygonal transformation) or by choosing some discrete number of points along the curve. This is motivated by the fact that any digital curve can be regarded as a polygonal without loss of information (with possibly a large number of vertices). The possible curve distortions can be neglected by approximating the original curve with one that has a similar perceptual appearance. For that purpose, one of the known techniques could be used (e.g. Latecki et al [15]).

The curves are re-scaled so that the total length is 1. The proposed matching techniques use turning functions $\phi(s)$ for curve representations. $\phi(s)$ measures the change of the angle ϕ along the curve as a function of the curve length s. It increases with counter-clockwise turn and decreases with clockwise turn, accumulating the turns along the curve. The function $\phi(s)$ is: translation independent, scaling independent, rotation corresponds to a vertical shift and the choice of starting point (for closed curves) corresponds to a horizontal shift of the function (see Fig. 2).

The curve matching is based on the distance between turning functions. It is most reasonable to use *L1* or *L2* metrics to calculate these distances. *L2* has several advantages over *L1* as it was pointed out in Arkin et al [1].

$$L1 = \int_0^1 |\phi_1(s) - \phi_2(s)| ds \qquad L2 = [\int_0^1 (\phi_1(s) - \phi_2(s))^2 ds]^{\frac{1}{2}}$$

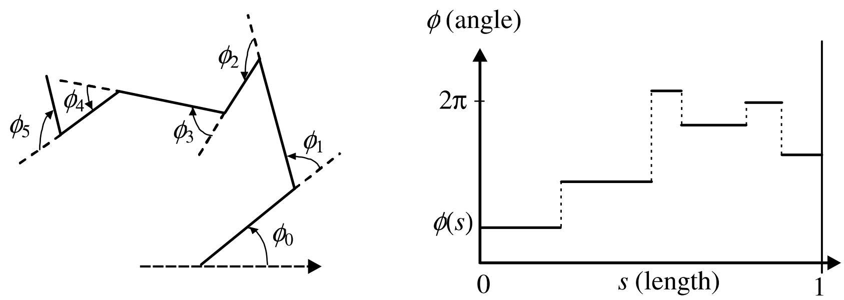

Figure 2: A turning function and calculation of the distance

To define a reliable metric for curve matching the *L2* metric should be optimized by the curve orientations and choice of the starting point in the curve representations.

3 Plain Method

The first and simplest matching method is based on direct comparison of the turning functions. We called this method, plain method. The difference between two curves A and B is defined by

$$Diff(A,B) = \min_{\alpha \in \Re}[\int_0^1 (\phi_1(s) - \phi_2(\pm s) + \alpha)^2 ds]^{\frac{1}{2}}$$

where the optimal α is $\alpha = \int_0^1 (\phi_1(s)ds - \int_0^1 (\phi_2(\pm s)ds$

and $\pm s$ denotes that the representation of the second curve is taken from both sides. It means that we are looking for the best orientation α between the curves to achieve the minimal distance between their turning functions in a least squares sense (Fig. 3).

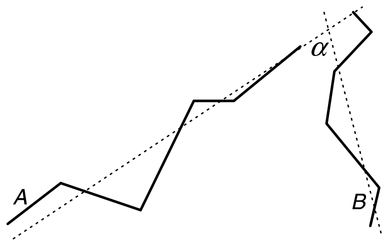

Figure 3: The optimal α in the plain method

This method is the fastest possible. If the curves have m and n line segments respectively, the optimal α can be calculated in O(m+n) operations. Also, this method is metric. Unfortunately, it does not work well for some kind of curves. The typical examples are: closed curves, near-to-closed curves and curves with strong turns in different directions (see Fig. 4).

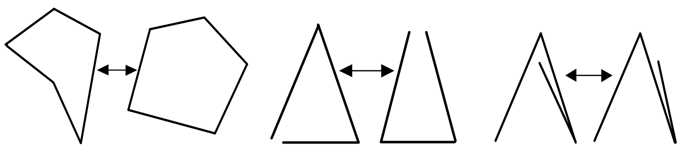

Figure 4: Examples where the plain method fails

It is clear that the plain method does not work well for closed curves, because it does not optimize the distance between turning functions by the choice of starting point in the curve representations. The problem with near-to-closed curves is that two context (shape and area) should be considered if we want to provide matching that corresponds to human intuition. It is not clear when the human view on a curve will be mainly based on the curve shape or on the curve area. In the second case, near-to-triangle curves from Fig. 4 will be very similar (viewed as triangles) although their shapes examined from one of the two sides are significantly different. The problem with the last two curves from Fig. 4 is the turn in different direction that increases their difference too much (see examples in Fig. 8).

4 Polygonal Method

The main idea behind the polygonal method is to consider the curves as polygons by "travelling" along them forward and backward (see Fig. 5). In this way, the similarity measure is deduced to polygon matching.

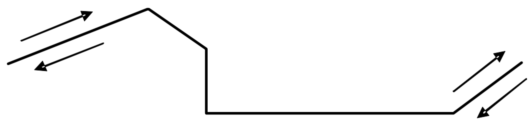

Figure 5: Curve representation as a polygon

We used a well-known method for polygon matching described in Arkin et al [1] where the difference between two poligons A and B is defined by

$$\mathit{Diff}(A,B) = \min_{\substack{\alpha\in\Re \\ u\in[0,1]}} [\int_0^1 (\phi_1(s) - \phi_2(s+u) + \alpha)^2 ds]^{\frac{1}{2}}$$

where the optimal α is $\alpha = \int_0^1 [\phi_1(s) - \phi_2(s)]ds - 2\pi u$

Here, u corresponds to choice of the starting point in the curve representation (the curve is considered as closed) and α is the angle for optimal match.

In this approach, we are looking for the best angle (orientation) between the curves and the best choice of the starting points to achieve the minimal distance between their turning functions in a least squares sense. The polygonal method is metric. It is relatively fast and has good theoretical foundation. A simple implementation takes $O(m \cdot n \cdot (m+n))$, while the best implementation takes $O(m \cdot n \cdot \log(m \cdot n))$ operations (see Arkin et al [1]). Unfortunately, this approach also does not work well for some kind of curves. Again, typical examples are near-to-closed curves and curves with strong turns in different directions (see Fig. 6).

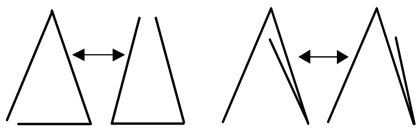

Figure 6: Examples where the polygonal method fails

The experience with the plain and the polygonal methods shows that it is difficult to design a reliable matching method using a "static" curve representation. In attempt to overcome these problems we will use a "dynamic" curve representation based on a snake-like curve moving.

5 Penalty Method

The third proposed method, designed to overcome the weakness of the first two we called the penalty method. It is based on snake-like moving of the original curves, finding optimal position for matching and penalizing the result by the distance between original curve and its snake-like variation. In the snake-like moving of the curve the first line segment is translated at the end of the curve, then the second one is translated at the end, and so on, until the original curve is reached (see Fig 7.).

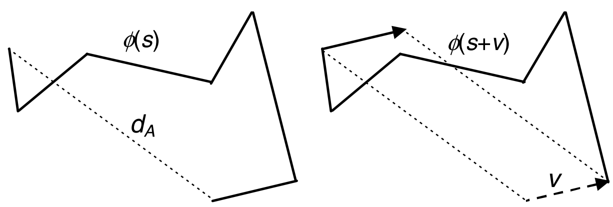

Figure 7: A snake-like moving of a curve

The difference in the penalty method is defined by

$$Diff(A,B) = \min_{\substack{\alpha\in\Re\\ u\in A\\ v\in B}}[(\int_0^1 (\phi_1(s+u) - \phi_2(s\pm v) + \alpha)^2 ds)^{\frac{1}{2}} +$$

$$+\, d_A \cdot \min_{\beta\in\Re}(\int_0^1 (\phi_1(s+u) - \phi_1(s) + \beta)^2 ds)^{\frac{1}{2}} +$$

$$+\, d_B \cdot \min_{\gamma\in\Re}(\int_0^1 (\phi_2(s+v) - \phi_2(s) + \gamma)^2 ds)^{\frac{1}{2}}]$$

where α, β and γ are the angles for optimal match, u and v denote the line segments in snake-like moving of the curves A and B respectively, d_A and d_B are penalty factors measuring the degree of closeness of the curves and typically depend on the distance between the first and the last point ($d_A = d_B = 0$ for closed curves). The expression $\phi(s+u)$ denotes the turning function for the curve obtained by the translation of the next segment u from the beginning at the end of the curve. The optimal α, β and γ are determined in same way as in the polygonal method.

The penalty method works well for curves where the plain and the polygonal methods fail. The main problem is triangle inequality, which can not be proved. Although slower than the polygonal method, the complexity remains to be O($m{\cdot}n{\cdot}\log(m{\cdot}n)$).

6 Examples

It seems more appropriate to present examples of the working of the proposed methods in terms of similarity, instead in terms of difference between curves. To transform the difference into similarity we can use a suitable decreasing function. In the same time, we will make an adjustment of the similarity measure based on the proposition that by increasing of the complexity of the curves the difference becomes more relative, and consequently, the similarity should be in corresponding way increased. The following formula is used to transform the difference into similarity

$$Com = \sum_{i=0}^{n-1} [\phi(s_{i+1}) - \phi(s_i)]$$

$$Sim(A,B) = 2^{\frac{-Diff(A,B)}{(k+Com_A+Com_B)}}$$

where *Com* measures the complexity of the curves, summarizing the changes of the angles along the curve. Parameter k is used to translate the similarity results in the desired interval. In our examples, the parameter $k = 500$, providing more uniform distribution of the similarity results in (0,1] interval.

As the polygonal curves are composed of limited number of line segments, the implementation of optimization steps in all methods is deduced to simple examination of all possible "situations".

The following examples (see Fig. 8) show the potential usefulness of the methods. The start of the curves is marked with '+' and the end with '*'. The similarity results obtained by the plain, polygonal and penalty method are presented on the right side of each pair of curves.

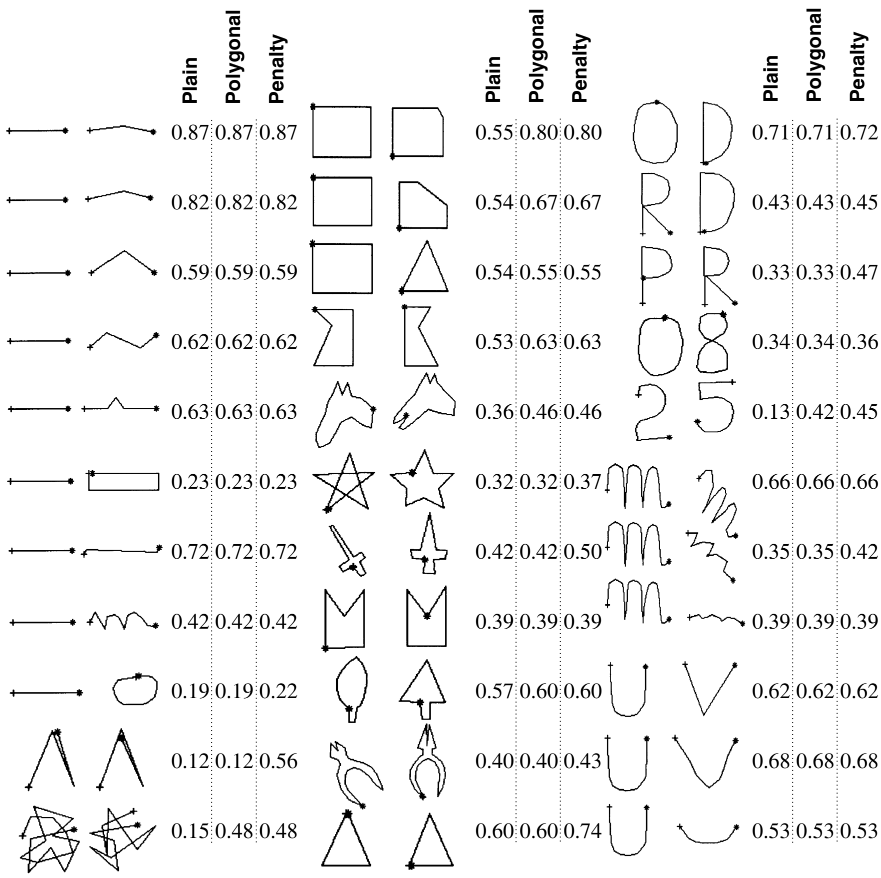

Figure 8: The similarity results for some curves

Generally speaking, the similarity results show that in most cases the penalty method is most reliable. The plain method is useful for simpler kind of curves. The results obtained by the polygonal method are closed to those obtained by the penalty method. However, the penalty method makes slight correction of the similarity results in positive direction giving more reliable results in the cases of "problematic" curves.

7 Conclusion

In this paper, three methods for curve matching based on turning functions are presented.

The following table summarize the principal features of the discussed methods.

METHOD **FEATURE**	**Plain**	**Polygonal**	**Penalty**
Invariance under elementary transformations	Yes	Yes	Yes
Basic distance criteria	Yes	Yes	Yes
Metric	Yes	Yes	Yes?
Continuity	Yes	No (the problem is in transition from open to close)	No (the problem is the penalty factor)
Generality	Low	High	High
Complexity	O(m+n)	O($m\cdot n\cdot\log(m\cdot n)$)	O($m\cdot n\cdot\log(m\cdot n)$)
Reliability	Lowest	Medium	Highest

Like the other methods for curve matching, the proposed methods could have serious problems with change-of-scale of the curves. This problem was avoided by the curve rescaling to the same total length, assuming that the scale of the curve is known in advance. If portions and scale of the curve are unknown, than it is unclear how this rescaling should be done. However, it is difficult to design a reliable matching method, in cases of partially occluded curves or unknown scale.

It is possible to use the proposed methods for matching of complex curves (with intersections) and curves with more endpoints. In the second case, the *Diff(A,B)* should be optimized by the second curve representations starting from all endpoints. In addition, it is necessary to design an assumption for the way of travelling along the curves, allowing some of the line segments to be passed more times.

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