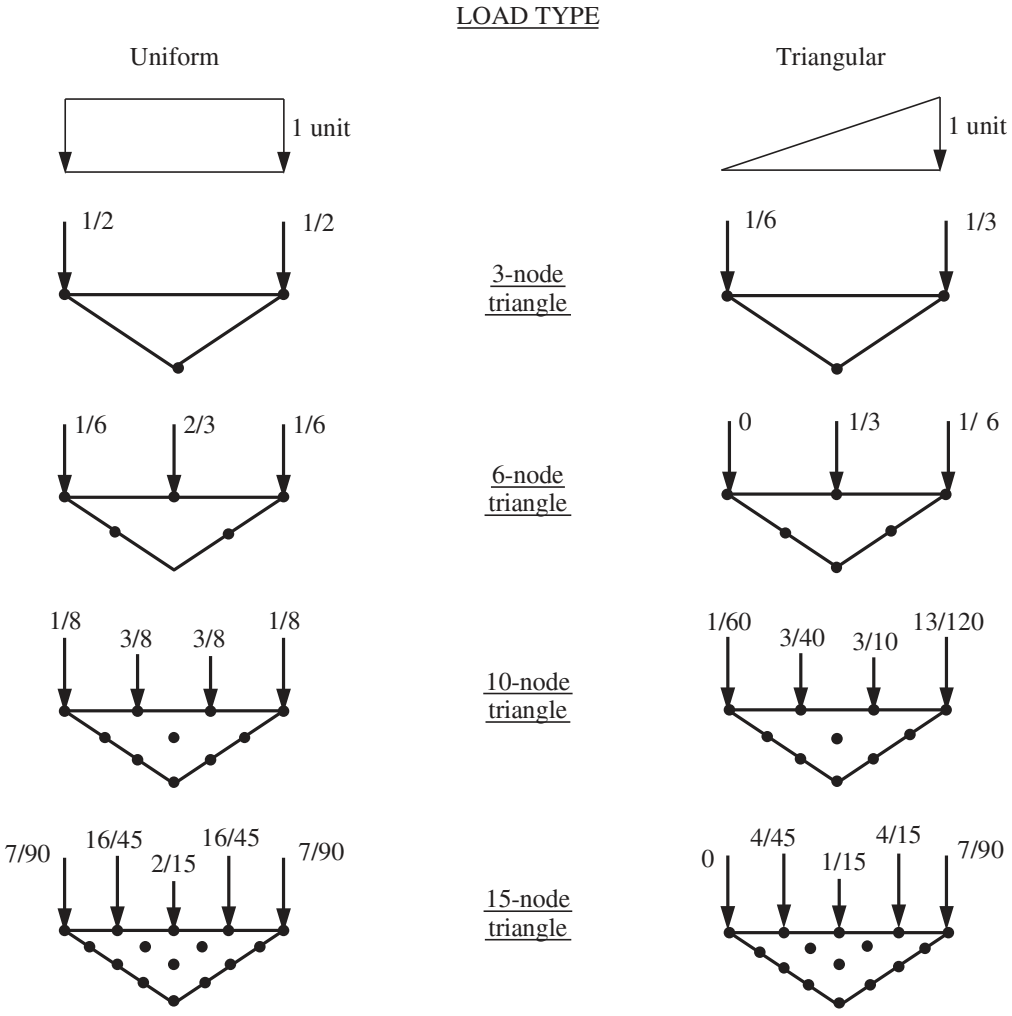
Appendix A

Equivalent Nodal Loads

Planar Elements (2D)

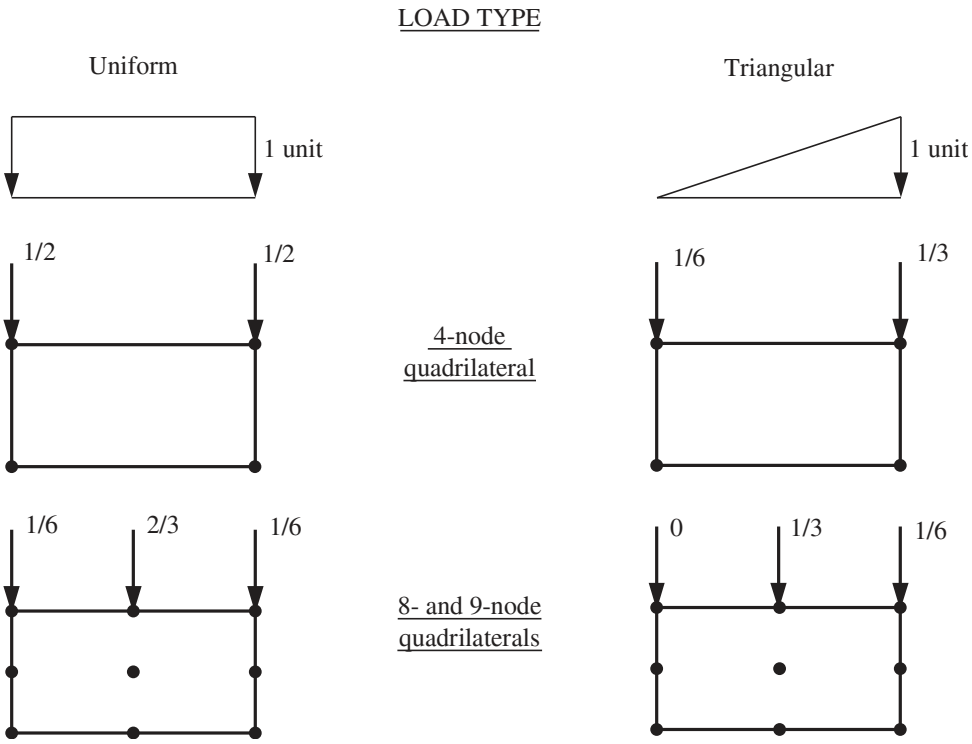
Width of loaded face = 1 unit



Programming the Finite Element Method, Fifth Edition. I. M. Smith, D. V. Griffiths and L. Margetts.

Planar Elements (2D)

Width of loaded face = 1 unit

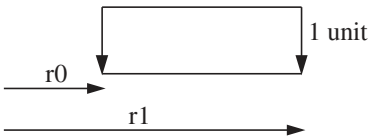


Axisymmetric Elements (2D)

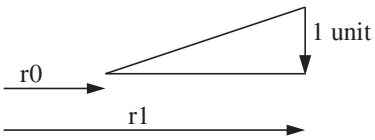
Loading over 1 radian

LOAD TYPE

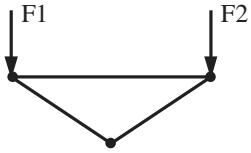
Uniform

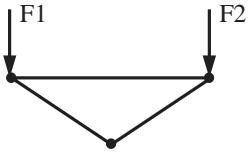


Triangular



3-node triangle





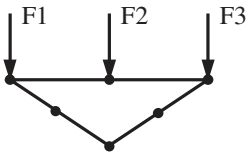
$$F_1 = \frac{1}{6}\left(r_1^2 + r_0 r_1 - 2r_0^2\right)$$

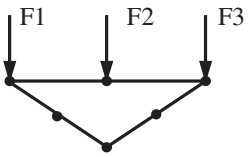
$$F_2 = \frac{1}{6}\left(2r_1^2 - r_0 r_1 - r_0^2\right)$$

$$F_1 = \frac{1}{12}\left(r_1^2 - r_0^2\right)$$

$$F_2 = \frac{1}{12}\left(3r_1^2 - 2r_0 r_1 - r_0^2\right)$$

6-node triangle





$$F_1 = \frac{1}{6}\left(r_0 r_1 - r_0^2\right)$$

$$F_2 = \frac{1}{3}\left(r_1^2 - r_0^2\right)$$

$$F_3 = \frac{1}{6}\left(r_1^2 - r_0 r_1\right)$$

$$F_1 = -\frac{1}{60}\left(r_1^2 - 2r_0 r_1 + r_0^2\right)$$

$$F_2 = \frac{1}{15}\left(3r_1^2 - r_0 r_1 - 2r_0^2\right)$$

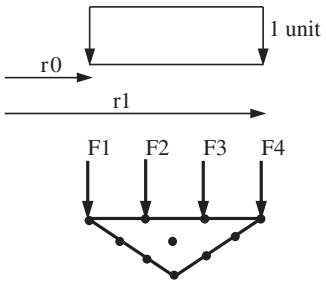
$$F_3 = \frac{1}{60}\left(9r_1^2 - 8r_0 r_1 - r_0^2\right)$$

Axisymmetric Elements (2D)

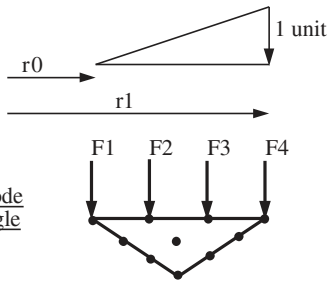
Loading over 1 radian

LOAD TYPE

Uniform



Triangular



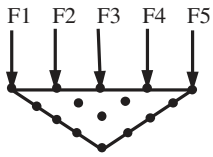
10-node triangle

$$F_1 = \frac{1}{120}\left(2r_1^2 + 11r_0r_1 - 13r_0^2\right) \qquad F_1 = \frac{1}{120}\left(r_1^2 - r_0^2\right)$$

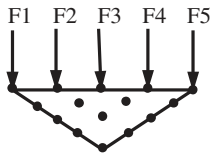
$$F_2 = \frac{1}{40}\left(3r_1^2 + 9r_0r_1 - 12r_0^2\right) \qquad F_2 = \frac{3}{40}\left(r_0r_1 - r_0^2\right)$$

$$F_3 = \frac{1}{40}\left(12r_1^2 - 9r_0r_1 - 3r_0^2\right) \qquad F_3 = \frac{3}{40}\left(3r_1^2 - 2r_0r_1 - r_0^2\right)$$

$$F_4 = \frac{1}{120}\left(13r_1^2 - 11r_0r_1 - 2r_0^2\right) \qquad F_4 = \frac{1}{120}\left(12r_1^2 - 11r_0r_1 - r_0^2\right)$$



15-node triangle



$$F_1 = \frac{7}{90}\left(r_0r_1 - r_0^2\right) \qquad F_1 = -\frac{1}{252}\left(r_1^2 - 2r_0r_1 + r_0^2\right)$$

$$F_2 = \frac{4}{45}\left(r_1^2 + 2r_0r_1 - 3r_0^2\right) \qquad F_2 = \frac{4}{315}\left(3r_1^2 + r_0r_1 - 4r_0^2\right)$$

$$F_3 = \frac{1}{15}\left(r_1^2 - r_0^2\right) \qquad F_3 = \frac{1}{105}\left(r_1^2 + 5r_0r_1 - 6r_0^2\right)$$

$$F_4 = \frac{4}{45}\left(3r_1^2 - 2r_0r_1 - r_0^2\right) \qquad F_4 = \frac{1}{315}\left(17r_1^2 - 13r_0r_1 - 4r_0^2\right)$$

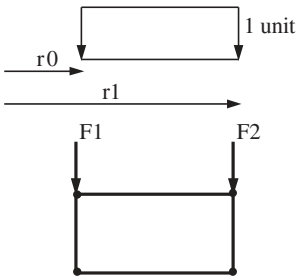
$$F_5 = \frac{7}{90}\left(r_1^2 - r_0r_1\right) \qquad F_5 = \frac{1}{1260}\left(93r_1^2 - 88r_0r_1 - 5r_0^2\right)$$

Axisymmetric Elements (2D)

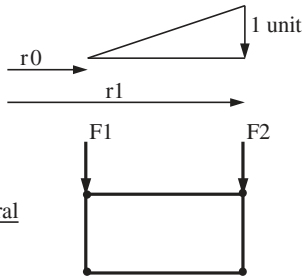
Loading over 1 radian

LOAD TYPE

Uniform



Triangular



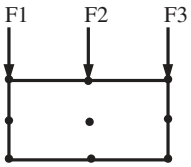
4-node quadrilateral

$$F_1 = \frac{1}{6}\left(r_1^2 + r_0 r_1 - 2r_0^2\right)$$

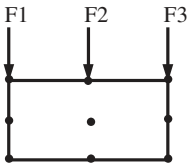
$$F_2 = \frac{1}{6}\left(2r_1^2 - r_0 r_1 - r_0^2\right)$$

$$F_1 = \frac{1}{12}\left(r_1^2 - r_0^2\right)$$

$$F_2 = \frac{1}{12}\left(3r_1^2 - 2r_0 r_1 - r_0^2\right)$$



8 and 9-node quadrilaterals



$$F_1 = \frac{1}{6}\left(r_0 r_1 - r_0^2\right)$$

$$F_2 = \frac{1}{3}\left(r_1^2 - r_0^2\right)$$

$$F_3 = \frac{1}{6}\left(r_1^2 - r_0 r_1\right)$$

$$F_1 = -\frac{1}{60}\left(r_1^2 - 2r_0 r_1 + r_0^2\right)$$

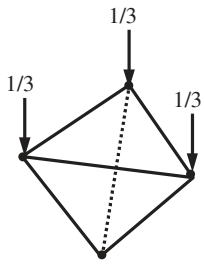
$$F_2 = \frac{1}{15}\left(3r_1^2 - r_0 r_1 - 2r_0^2\right)$$

$$F_3 = \frac{1}{60}\left(9r_1^2 - 8r_0 r_1 - r_0^2\right)$$

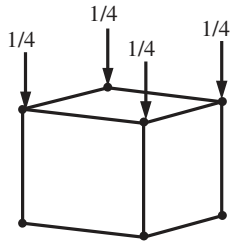
Three Dimensional Elements (3D)

Area of loaded face = 1 unit
Unit stress applied

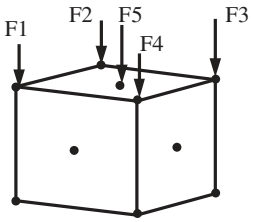
4-node
tetrahedron



8-node
hexahedron



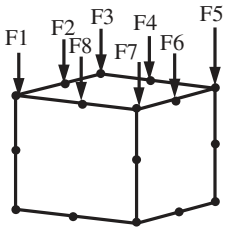
14-node
hexahedron (type 5)



$F_1 = F_2 = F_3 = F_4 = \frac{1}{12}$

$F_5 = \frac{2}{3}$

20-node
hexahedron



$F_1 = F_3 = F_5 = F_7 = -\frac{1}{12}$

$F_2 = F_4 = F_6 = F_8 = \frac{1}{3}$