A theoretical model to predict the low-frequency sound absorption of a Helmholtz resonator array (L)

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A theoretical method based on mutual radiation impedance is proposed to compute the sound absorption performance of a Helmholtz resonator array in the low-frequency range. Any configuration of resonator arrangement can be allowed in the method, while all the resonators may or may not be identical. Comparisons of the theoretical predictions with those done by the past studies or experiments show that the present method can accurately predict the absorption performance in more general cases.
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I. INTRODUCTION

An array of Helmholtz resonators is often used to reduce low-frequency noise because of its resonance characteristics.[1,2] Differently tuned resonators also have been employed to decrease broadband noise.[3] One of the popular shapes of resonator array is a plane-type array illustrated in Fig. 1(a); hereafter, we call it the Helmholtz resonator array panel (for convenience, array panel). However, it has not been well understood how and how much the panel really absorbs sound energy. For instances, Zwikker *et al.*[4] regarded the top surface of the panel as a homogeneous surface and utilized Bolt's concept[5] of spatially averaged impedance to calculate normal-incidence absorption coefficient of the surface. Allard *et al.*[6,7] presented a more generalized theory considering oblique incidence for an identical-resonator array panel, and recently Kim *et al.*[8,9] applied the mutual radiation impedance between resonator orifices to evaluating absorption coefficient of a rectangular array panel with identical resonators. But the conventional methods are valid only when the cell size is very small compared to the wavelength.

In this paper, we propose a theoretical method that considers the radiation impedance of the array panel. The panel is composed of a group of resonators which may or may not be identical. Any size of the panel or cell can be allowed in the method, and there is no restriction on the resonator disposition. For numerical examples, we consider array panels with identical and differently tuned resonators and predict their absorption coefficients, which are also compared with experimental results.

II. THEORETICAL ANALYSIS

A. Description of system

As shown in Fig. 1(b), we consider an infinite array panel that is periodically arranged with a rectangular cell containing a group of resonators, which are not necessarily identical. The size of the cell is L_x by L_y, and its top surface is assumed to be acoustically rigid, except for the circular holes that correspond to the entrances of respective resonators. We can place the resonators or orifices at arbitrary positions within the cell area. Here we assume that the radii of orifices are very small compared to the wavelength of interest and that the air layer in the opening moves with uniform velocity so that it can be regarded as a plane piston.

B. Absorption coefficient of Helmholtz resonator array panel

When the top surface of the panel is positioned in the xy plane, a plane wave p_e incident on the plane [see Fig. 1(b)] can be written as

$$P_e = P_e e^{-j(k_x x + k_y y - k_z z)}, \tag{1}$$

The time factor $e^{j\omega t}$ is suppressed for simplicity. Using the Green's function (G_N) for a semi-infinite space bounded by a rigid plane, we can describe the sound pressure p on the xy plane in terms of the Kirchhoff-Helmholtz integral equation (for example, see Ref. 10). That is,

$$p = 2p_e + jk\rho c \sum_{i=-\infty}^{\infty} u_i \int_{S_i} G_N dS_i \quad \text{with } G_N = \frac{e^{-jkR}}{2\pi R}. \tag{2}$$

The subscript i in Eq. (2) represents the index of orifice, S_i is the orifice area, u_i is the orifice velocity, counted positive into the external space (z direction), ρ is the density of air, c

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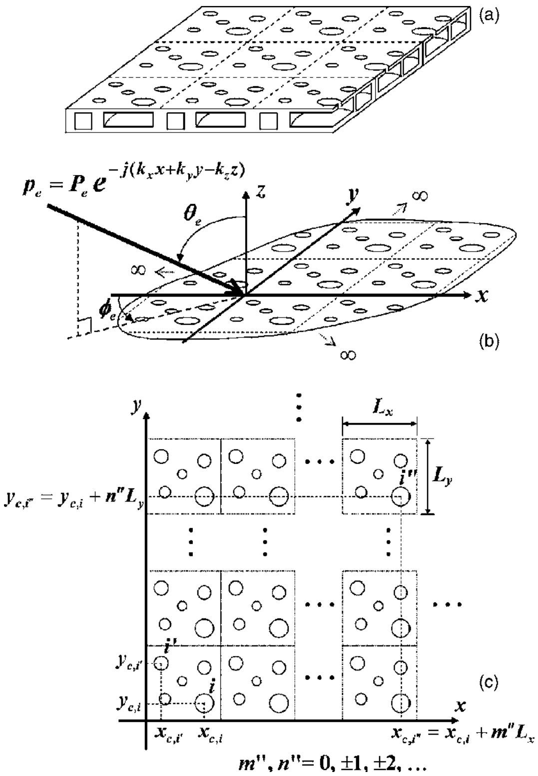

FIG. 1. Helmholtz resonator array panel and its theoretical model. (a) Helmholtz resonator array panel; (b) plane wave p_e incident on the theoretical model ($k_x = k \sin\theta_e \cos\phi_e$, $k_y = k \sin\theta_e \sin\phi_e$, $k_z = k\cos\theta_e$, and k is the wave number); and (c) xy coordinates of orifice centers. For a theoretical model, the top surface of the panel is regarded as an infinite plane periodically arranged with a rectangular cell, which corresponds to a part bordered by dotted lines in this figure.

is the speed of sound, and $R = \sqrt{(x-x_0)^2 + (y-y_0)^2}$.

If we average both sides of Eq. (2) over the area of orifice i' ($S_{i'}$), the left-hand side and the first term of the right-hand side become the pressures at the center of the orifice, denoted by $p_{i'}$ and $2p_{e,i'}$, respectively, since the radius of orifice is assumed to be very small compared with the wavelength of interest. In this case, the second term can be replaced by the mutual radiation impedance[11,12] $\rho c \zeta_{i'i}$ between orifices i' and i, that is,

$$\rho c \zeta_{i'i} = \frac{jk\rho c}{S_{i'}} \int_{S_{i'}} \int_{S_i} G_N dS_i dS_{i'} . \tag{3}$$

Implying the boundary condition on the orifice i', $p_{i'} = -\rho c \zeta_{i'} u_{i'}$, where $\rho c \zeta_{i'}$ is the surface impedance on the orifice, we can obtain an infinite set of simultaneous equations

$$\sum_{i=-\infty}^{\infty} (\zeta_{i'i} + \delta_{i'i}\zeta_i)\rho c u_i = -2p_{e,i'} \quad \text{for } i' = 0, \pm 1, \ldots, \pm \infty . \tag{4}$$

where $\delta_{i'i} = 1$ for $i' = i$ and $\delta_{i'i} = 0$ for $i' \neq i$.

Consider two identical orifices i and i'' that are located at the same position in their respective cells [see Fig. 1(c)]: not only the radii of orifices but also their surface impedances are the same. Since the surface of the panel is periodic and infinite, the acoustic behaviors in front of the orifices i and i'' must be the same except for an excitation by the incident plane wave (for example, see Ref. 13). Thus the ratio between orifice velocities can be expressed as $u_i / u_{i''} = p_{e,i} / p_{e,i''}$. Substituting the ratio into Eq. (4) and rearranging the result lead to a finite set of simultaneous equations:

$$\sum_{i=1}^{N} (\bar{\zeta}_{i'i} + \delta_{i'i}\zeta_i)\rho c u_i = -2p_{e,i'} \quad \text{for } i' = 1, 2, \ldots, N, \tag{5}$$

where N is the number of orifices in a cell, and

$$\bar{\zeta}_{i'i} = \sum_{\substack{m'=-\infty \\ i''=i+m'N}}^{\infty} \zeta_{i'i''} \frac{p_{e,i''}}{p_{e,i}} . \tag{6}$$

The final expression of $\bar{\zeta}_{i'i}$ is derived in the Appendix .

The absorption coefficient of the array panel can be calculated by the ratio of the absorbed effective power to the incident sound power. Because the total power absorbed on a cell area is equal to the sum of power dissipated by individual resonators and the incident power on a cell is $(L_x L_y / 2\rho c)|P_e|^2 \cos\theta_e$, the absorption coefficient α can be expressed as

$$\alpha = \sum_{i=1}^{N} \frac{\pi a_i^2}{L_x L_y} \frac{\mathrm{Re}(\zeta_i)}{\cos\theta_e} \left| \frac{\rho c u_i}{P_e} \right|^2 , \tag{7}$$

where a_i is the radius of orifice. If the surface impedances $\rho c \zeta_i$ are determined, then the absorption coefficient can be obtained by solving Eq. (5) for the unknown velocity u_i and by substituting them into Eq. (7).

III. EXPERIMENTAL VERIFICATIONS

A. Experimental setup to measure the absorption coefficient

In order to examine the accuracy of the proposed method, the normal-incidence absorption coefficients of several array panels were measured by using the transfer-function method[14] with the instrumentations that are illustrated in Fig. 2(a).

Figure 2(b) shows the dimensions of two test samples made of a PVC plate. Each specimen can be regarded as a part of a corresponding array-panel with an infinite size because the four walls of the rectangular tube are rigid and a plane wave is perpendicularly incident on the surface of the specimen.

B. Theoretical model of surface impedance

From the previous study,[15,16] the surface impedance ($\rho c \zeta$) on the opening of the resonator that has a cylindrical cavity with a concentric neck of cylindrical type can be written as

$$\zeta = k\delta_\nu(2 + l/a) + j\left[k\{l + \delta_{in} + \delta_\nu(2 + l/a)\} - \frac{\pi a^2}{\pi a_c^2} \cot k l_c \right], \tag{8}$$

where $\delta_\nu = \sqrt{2\nu/\omega}$, ν is the kinematical viscosity of air ($\nu \approx 15 \times 10^{-6}\ \mathrm{m^2/s}$), ω is the angular frequency, δ_{in} is the internal end correction of resonator, a is the inner radius

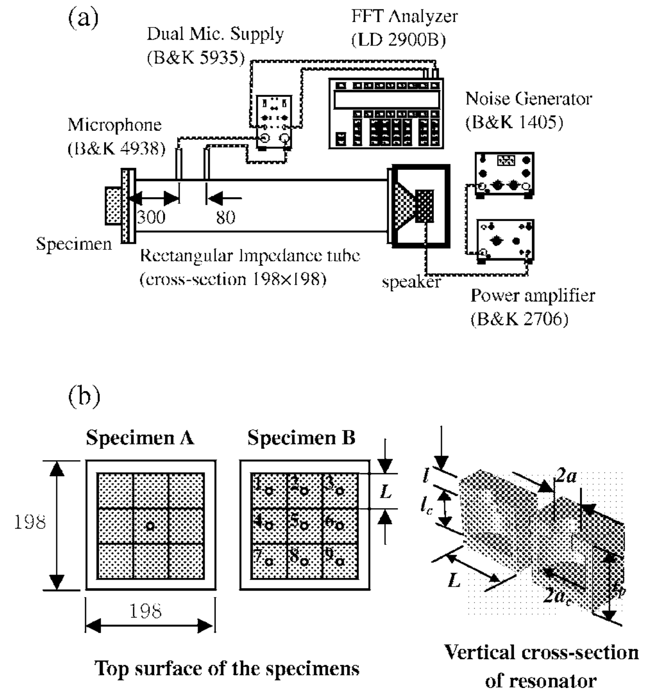

FIG. 2. (a) Experimental setup and (b) specimens to measure the normal-incidence absorption coefficient. All the dimensions of the resonators except the cavity depth l_c are the same [$a=l=5$, $a_c=25$, $t_p=110$, $L=60$ (unit: mm)]. In the case of specimen A, $l_c=64$ mm and the cavity depths of nine resonators in the specimen B are 100, 91, 82, 73, 64, 55, 46, 37, and 28 mm, respectively.

of neck, l is the neck length, and a_c and l_c represent the radius of air cavity and its depth, respectively. For the internal end correction, there have been numerous studies, but, for simplicity, we employ the approximated expression suggested by Selamet *et al.*,[17]

$$\delta_{in} = 0.82a(1-1.33a/a_c) \quad \text{for } a/a_c < 0.4. \tag{9}$$

C. Theoretical and experimental results

First of all, we consider an array panel that has one resonator in a cell: we will call such a panel an '$N=1$' array panel. In this case, the absorption coefficient can be obtained from Eqs. (5) and (7) as

$$\alpha = \frac{\pi a_1^2}{L_x L_y}\frac{1}{\cos\theta_e}\frac{4\,\mathrm{Re}(\zeta_1)}{|\bar{\zeta}_{11}+\zeta_1|^2}, \tag{10}$$

and $\bar{\zeta}_{11}$ represents the net radiation impedance[11,12] $\zeta_{\mathrm{rad},1}$ on the orifice:

$$\zeta_{\mathrm{rad},1} = \bar{\zeta}_{11} = \frac{\pi a_1^2}{L_x L_y}\cdot\frac{1}{\cos\theta_e} + \frac{\pi a_1^2}{L_x L_y}\sum_{\substack{m=-\infty\\ m,n\neq 0,0}}^{\infty}\sum_{n=-\infty}^{\infty}\frac{jk}{\sqrt{\beta_{mn}^2-k^2}}\left(\frac{2J_1(a_1\beta_{mn})}{a_1\beta_{mn}}\right)^2, \tag{11}$$

where $\beta_{mn}=\sqrt{(2\pi m/L_x-k_x)^2+(2\pi n/L_y-k_y)^2}$. If all β_{mn} except β_{00} are larger than k, i.e.,

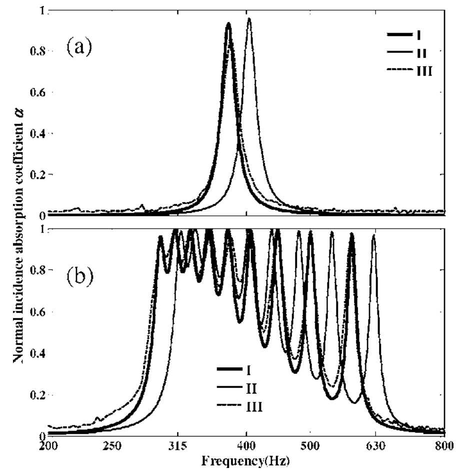

FIG. 3. Comparison of predicted and measured absorption coefficients for the specimens (a) A and (b) B shown in Fig. 2(b). Theoretical predictions are obtained by using the proposed method (graph I) and the conventional one (graph II); graph III represents the experimental result.

$$(m\lambda/L_x - \sin\theta_e\cos\phi_e)^2 + (n\lambda/L_y - \sin\theta_e\sin\phi_e)^2 > 1 \quad \text{for } (m,n)\neq(0,0), \tag{12}$$

where $\lambda=2\pi/k$ is the wavelength, the double summation term of Eq. (11) becomes purely imaginary, and so the use of external end correction $\delta_{ex,1}$, which equals the imaginary part of net radiation impedance divided by k, gives

$$\alpha = \frac{4\mathrm{Re}(\zeta_1)\cos\theta_e/\epsilon_1}{|1+(jk\delta_{ex,1}+\zeta_1)\cos\theta_e/\epsilon_1|^2} \quad \text{with } \epsilon_1=\frac{\pi a_1^2}{L_x L_y}. \tag{13}$$

The above equation is identical to those used in the past studies[4,7,8] except that in the present method, $\delta_{ex,1}$ can be calculated for all angles of incidence. It is also found that in the case of $L_x=L_y$, the proposed formula of $\delta_{ex,1}$ becomes equal to that done by Allard[7] at the special incidence angle that he considered. It is noteworthy that the expressions such as Eq. (13) can be adopted for all angles only if $\max(L_x,L_y)/\lambda<0.5$ because the allowed range of L_x and L_y in Eq. (12) varies with the angle of incidence.

Figure 3(a) illustrates the comparison between the theoretical and measured results for an $N=1$ array panel [specimen A in Fig. 2(b)], whose cell area is equal to the cross-section area of the impedance tube ($L_x=L_y=198$ mm). The theoretical results in the figure were obtained by using respectively the proposed method and the conventional one[4] based on the spatially averaged impedance and $\delta_{ex,1}=8a/3\pi$: the value $8a/3\pi$, which is widely used in many applications, is an approximate external-end-correction of a single resonator with infinite baffle. As mentioned above, Allard's formula is equal to the proposed one in this case. Figure 3(b) shows the results for the specimen B in Fig. 2(b). The corresponding array panel has 36 resonators (nine differently tuned resonators) within a cell, whose size is $L_x=L_y=396$ mm because the period of the panel become doubled to take into account the mirror imaging effect of the side wall; Allard *et al.*[6,7] did not present any way to deal with such a panel. The comparisons shown in Figs. 3(a) and 3(b) represent that the theoretical prediction agrees better with the measurement in the case of the proposed method than the other theory,[4] particularly in respect to the resonance frequency at which the maximum absorption occurs. These results certainly confirm that the model, which we have proposed, is capable of predicting the absorption coefficients of array panels with more various resonator arrangment.

IV. CONCLUSIONS

We have presented a method that can compute absorption coefficients of Helmholtz resonator array panels. Any size cell can be allowed in the method, and the number of resonators and their disposition in the cell can be also selected without restriction. Comparisons of the theoretical predictions with the measurements have confirmed that the present method can more accurately predict the absorption coefficient in general cases than the conventional method does.

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APPENDIX: DERIVATION OF RADIATION IMPEDANCE ON ORIFICE

In Eq. (3), the use of spatial Fourier transform of the Green's function G_N (for example, see Ref. 18), and Bessel's integral[19] leads to

$$\zeta_{i'i}=\frac{a_i}{a_{i'}}\int_0^\infty \frac{jk}{\sqrt{\beta^2-k^2}}\cdot\frac{2J_1(a_{i'}\beta)J_1(a_i\beta)J_0(R_{i'i}\beta)}{\beta}d\beta, \qquad \text{(A1)}$$

where $R_{i'i}=\sqrt{(x_{c,i'}-x_{c,i})^2+(y_{c,i'}-y_{c,i})^2}$, J_n represents the Bessel function of the first kind of order n, and x_c and y_c are respectively the x and y coordinates at the center of orifice area. Substituting Eqs. (1) and (A1) and the identities of $a_{i''}=a_i$, $x_{c,i''}-x_{c,i'}=m''L_x$, and $y_{c,i''}-y_{c,i'}=n''L_y$ [see Fig. 1(c)] into Eq. (6) gives

$$\bar{\zeta}_{i'i}=\frac{a_i}{a_{i'}}\int_0^\infty d\beta\cdot\frac{jk}{\sqrt{\beta^2-k^2}}\cdot\frac{2J_1(a_{i'}\beta)J_1(a_i\beta)}{\beta} \cdot\sum_{m''=-\infty}^{\infty}\sum_{n''=-\infty}^{\infty}J_0(R_{i'i}\beta)\cdot e^{-j(m''k_xL_x+n''k_yL_y)}. \qquad \text{(A2)}$$

Here, the double summation can be rewritten from Fourier series expansion as

$$\sum_{m''=-\infty}^{\infty}\sum_{n''=-\infty}^{\infty}J_0(R_{i'i''}\beta)e^{-j(m''k_xL_x+n''k_yL_y)} =\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\frac{2\pi}{L_xL_y}e^{j\{(2\pi m/L_x-k_x)(x_{c,i'}-x_{c,i})+(2\pi n/L_y-k_y)(y_{c,i'}-y_{c,i})\}} \times\int_0^\infty J_0(R\beta)J_0(R\beta_{mn})RdR \qquad \text{(A3)}$$

where $\beta_{mn}=\sqrt{(2\pi m/L_x-k_x)^2+(2\pi n/L_y-k_y)^2}$, and so the utilization of Fourier-Bessel integral[20] produces the final expression of $\bar{\zeta}_{i'i}$

$$\bar{\zeta}_{i'i}=\frac{\pi a_i^2}{L_xL_y}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\frac{jk}{\sqrt{\beta_{mn}^2-k^2}}\cdot\frac{2J_1(a_{i'}\beta_{mn})}{a_{i'}\beta_{mn}}\cdot\frac{2J_1(a_i\beta_{mn})}{a_i\beta_{mn}} \cdot e^{j\{(2\pi m/L_x-k_x)(x_{c,i'}-x_{c,i})+(2\pi n/L_y-k_y)(y_{c,i'}-y_{c,i})\}}. \qquad \text{(A4)}$$

[1] J. M. Garrelick, "The transmission loss of a wall incorporating an array of resonators," J. Acoust. Soc. Am. **65**, S52 (1979).

[2] R. A. Prydz, L. S. Wirt, and H. L. Kuntz, "Transmission loss of a multilayer panel with internal tuned Helmholtz resonators," J. Acoust. Soc. Am. **87**, 1597–1602 (1990).

[3] S.-H. Seo and Y.-H. Kim, "Silencer design by using array resonators for low-frequency band noise reduction," J. Acoust. Soc. Am. **118**, 2332–2338 (2005).

[4] C. Zwikker and C. W. Kosten, *Sound Absorbing Materials* (Elsevier, New York, 1949), Chap. 7, 157–163.

[5] R. H. Bolt, "On the design of perforated facings for acoustic materials," J. Acoust. Soc. Am. **19**, 917–921 (1947).

[6] P. Guignouard, M. Meisser, J. F. Allard, P. Rebillard, and C. Depollier, "Prediction and measurement of the acoustic impedance and absorption coefficient at oblique incidence of porous layers with perforated facings," Noise Control Eng. J. **36**, 129–135 (1991).

[7] J. F. Allard, *Propagation of Sound in Porous Media* (Elsevier, London, 1993), Chap. 10.

[8] S. R. Kim, M.-S. Kim, Y.-H. Kim, and Y.-W. Kim, "*Absorptive characteristics of resonator panel for low frequency noise control, and its application*," on the CD-ROM Seogwipo, Korea, August 25–28, 32nd International Congress and Exposition on Noise Control Engineering (ISBN 89-952189-1-6 98060, Proceedings of Inter-Noise 2003), paper N932.

[9] M.-S. Kim, "Absorption characteristics of resonator array panel or low frequency applications," M.S. dissertation, Mechanical Engineering Department, Korea Advanced Institute of Science and Technology, 2003.

[10] P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill, New York, 1953), Chap. 7, pp. 804–806.

[11] R. L. Pritchard, "Mutual Acoustic impedance between radiators in an infinite rigid plane," J. Acoust. Soc. Am. **32**, 730–737 (1960).

[12] E. Skudrzyk, *The Foundations of Acoustics: Basic Mathematics and Basic Acoustics* (Springer-Verlag, Wien, 1971), Chap. XXVIII, pp. 663–676.

[13] Y. W. Lam, "A boundary integral formulation for the prediction of acoustic scattering from periodic structures," J. Acoust. Soc. Am. **105**, 762–769 (1999).

[14] ISO 10534-2: 1998 Acoustics - Determination of Sound Absorption Coefficient and Impedance in Impedance Tubes - Part 2: Transfer-Function Method First Edition.

[15] U. Ingard, "On the theory and design of acoustic resonators," J. Acoust. Soc. Am. **25**, 1037–1061 (1953).

[16] L. L. Beranek and I. L. Ver, *Noise and Vibration Control Engineering, Principle and Application* (Wiley, New York, 1992), Chap. 8, pp. 232–234.

[17] A. Selamet and Z. L. Ji, "Circular asymmetric Helmholtz resonators," J. Acoust. Soc. Am. **107**, 2360–2369 (2000).

[18] E. Skudrzyk, *The Foundations of Acoustics: Basic Mathematics and Basic Acoustics* (Springer-Verlag, Wien, 1971), Chap. XXVII, pp. 659–660.

[19] G. N. Watson, *A Treatise on the Theory of Bessel Functions*, 2nd ed. (Cambridge U.P., London, 1966), Chap. II, pp. 20–22.

[20] G. N. Watson, *A Treatise on the Theory of Bessel Functions*, 2nd ed. (Cambridge U.P., London, 1966), Chap. XIV, pp. 453–453.