Project 2 Mathematics 512

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Presentation and report due date: Monday, Sep. 23rd

1.

a) Use the random number generator $\boldsymbol{x_n \equiv (ax_{n-1} + c)\, mod(m)}$ with $\boldsymbol{a = 7^5,\ c = 0\ \ and\ m = 2^{31} - 1}$ to generate 10000 uniformly distributed random numbers on $[\mathbf{0}, \mathbf{1}]$ and plot the histogram**.** Do the numbers generated seem to be random? If so, how do you explain this? Perform 1 or 2 simple statistical tests to check if they are U[0,1].

b) Do the following by hand: Take $\boldsymbol{a = 6}$ and $\boldsymbol{m = 11}$ and $\boldsymbol{x_0 = 3}$ and generate $\boldsymbol{x_n \equiv (ax_{n-1} + c)\, mod(m)}$ what sequence do you get? What is the period? Try now $\boldsymbol{a = 6}$ and $\boldsymbol{m = 10}$ and $\boldsymbol{x_0 = 3}$ what sequence do you get? What is the period?

c) Generate 10000 uniformly distributed random numbers on $[\mathbf{0}, \mathbf{1}]$ using the built-in function of MATLAB or another built in function in the computer language you are working.

d) Compare the histograms obtained in parts a) and c) and comment on the results.

e) Call the random numbers generated in part a) $u_1, u_2, \cdots$. Plot the pairs (u_1, u_2), (u_2, u_3),(u_3, u_4), etc. Do you see any particular pattern? You might need to zoom on a much smaller subinterval of [0,1] to see a pattern.

f) As a conclusion of this problem, what advantages and disadvantages of this random numbers generator did you observe?

2.

a) Use the numbers generated in problem 1, to generate 10000 discrete random numbers with the following probability distribution:

$$P(X = k) = \begin{cases} 0.3 \text{ for } k = 1 \\ 0.2 \text{ for } k = 2 \\ 0.35 \text{ for } k = 3 \\ 0.15 \text{ for } k = 4 \end{cases}$$

b) Plot distribution obtained, and comment on the results.

3.

Generate 5000 Binomial distributed $(n = 100, p = 0.8)$ random numbers by doing:

a) Generate Bernoulli random variables and add the results. Plot the histogram and use your data to calculate the probability that the Binomial random variable is less than or equal to 50. Compare with the theoretical answer.

b) Use the inverse transformation method mentioned in class.

c) Compare the histograms obtained in parts a) and b) and the computing times required in each method.

4.

Generate 5000 Poisson random variables with mean $\lambda = 2$. Plot the histogram of your results. Compare with the theoretical Poisson density.

5.

Generate 5000 Exponentially distributed with mean $\lambda = 5$ random numbers using the inverse transformation method mentioned in class. Plot the histogram of your results. Compare with the theoretical exponential density.

6.

Generate 6000 Cauchy distributed random numbers using the inverse transformation method mentioned in class. Plot the histogram of your results. Compare with the theoretical Cauchy density.

7.

A deck of 100 cards (numbered 1, 2, . . . ,100) is shuffled and then turned over one card at a time. Say that a "hit" occurs whenever card *i* is the *i* th card to be turned over, *i* = 1, . . . ,100. Write a simulation program to estimate the expectation and variance of the total number of hits. Run the program. Find the exact answers and compare them with your estimates.

8.

A pair of fair dice are to be continually rolled until all the possible outcomes 2, 3, . . . , 12 have occurred at least once. Develop a simulation study to estimate the expected number of dice rolls that are needed.

9.

A random selection of *10* balls is to be made from an urn that contains *100* balls, 20 of which have color type *1*, 30 of color type *2*, 40 of color type *3* and 10 of type *4*. Simulate $X_1, \cdots, X_4$, where X_i denotes the number of withdrawn balls that have color type *i*.

10.

Generate 10000 independent samples from a standard normal distribution, by using:

a) The Box-Muller method.
b) The Marsaglia-Bray method.
c) The acceptance-rejection method.
d) An available built in function in your computer.
e) Compare histograms and times required for the methods in a), b), c) and d) and the theoretical Gaussian density.