Project 4 Mathematics 512

Instructor: Ricardo Mancera Fall 2024

Due date: October 28th

1.

a) Use implicit finite differences method to find the price of a 5 month American put option. The stock price $S_0 = \$100$, the strike price $K = \$100$, the risk-free interest rate $r = 0.02$ and the volatility $\sigma = 0.3$. (solve the Black-Scholes PDE directly, without doing any change of variable).

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf$$

$$f(S,T) = \max(K - S, 0)$$

Hint: Suppose that S_{max} is the stock price sufficiently high that, when it is reached, the put has virtually no value ($f(S_{max}, t) = 0$) and also $f(0,t) = K$.
define $\Delta S = \frac{S_{max}}{M}$ and consider a total of $M + 1$ equally spaced stock prices: (use $M = 20$, $S_{max} = 300$)

$$0, \Delta S, 2\Delta S, \cdots, S_{max}$$

Also, divide the life of the option T into N (use N=10) equally spaced intervals of length $\Delta t = \frac{T}{N}$. A total of $N + 1$ times are considered.

$$0, \Delta t, 2\Delta t, \cdots, T$$

b) Modify your program in part a) to find the price of a European call option with the same data as above. Compare your results with the ones obtained using B-S formulas. Study the convergence of your method in part a) to the B-S results. (experiment with different values of M and N)

c) Calculate the Greek's for part b): delta, vega, rho and gamma at $S_0 = \$100$, $K = \$100$, $r = 0.02$, $\sigma = 0.3$ Compare your results with the analytical B-S formulas.