Project 6 Mathematics 512

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Due date: Dec. 6

1. Lookback option price

Consider an American lookback put option on a non-dividend-paying stock. If exercised at time τ, this pays off the amount by which the maximum stock price between time 0 and time τ exceeds the current stock price. Suppose that the initial stock price is \$50, the stock price volatility is 40% per annum, the risk-free interest rate is 10% per annum, the total life of the option is six months. Price this option.

2. Beyond Geometric Brownian Motion Models

Consider closing daily prices for crude oil for at least the latest six months (you can get this data from yahoo finance, or other finance websites). Is this price sequence consistent with the assumption that prices follow a geometric Brownian motion (gBm)? Hint: check for normality and independence of returns. In case you conclude that gBm is not a good model, propose an alternative model.

3. Dynamic Programming

Inventory control problem: Let us denote

$x_k =$ stock available at the beginning of the kth period. (nonnegative integer)

$u_k =$ stock ordered and delivered at the beginning of the kth period. (nonnegative integer)

$w_k =$ demand during the kth period $with$ given probability distribution. (nonnegative integer)

$h =$ holding cost per unit item remaining unsold at the end of the kth period.

$c =$ cost per unit stock ordered.

$p =$ shortage cost per unit demand unfilled.

Assume that $w_0, \cdots w_{N-1}$ are independent random variables, and that excess demand is backlogged and filled as soon as additional inventory becomes available.

The cost functional (representing expected loss) to be minimized is given by

$$E\left\{\sum_{k=0}^{N-1}\left[cu_k + p\max(0, w_k - x_k - u_k) + h\max(0, x_k + u_k - w_k)\right]\right\}$$

And the stock dynamics is represented by

$$x_{k+1} = x_k + u_k - w_k$$

Assume also that there is an upper bound on the stock $x_k + u_k$ that can be stored and is equal to 3 units, the planning horizon is $N = 4$ periods. Take $c = 2,\ p = 3\ and\ h = 4$. Furthermore, the demand w_k has the same probability distribution for all periods, given by

$$P(w_k = 0) = 0.1\ ;\ P(w_k = 1) = 0.6\ ;\ P(w_k = 2) = 0.3$$

Also, the initial stock x_0 is zero and the cost at the final period N is zero. Find the optimal controls that minimize the cost functional.

4. Value at risk

Suppose that an investor in the United States owns on April 24, 2020 a portfolio worth $5 million consisting of investments consisting of 3 indices: the S&P 500 (3 million), the FTSE 100 in the UK (1 million) and the Nikkei 225 in Japan (1 million). Calculate the 1-day 99% VaR. (use the most recent 501 days of data and the historical approach).