Hw 6.2B

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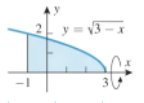
Period 7

HW 6.2B: #1, 5, 11, 12, 15, 32

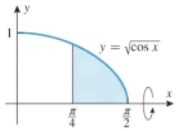
1–8 Find the volume of the solid that results when the shaded region is revolved about the indicated axis.

1.



$$V = \pi \int_{-1}^{3} (3-x)\,dx = \pi\left(3x - \frac{x^2}{2}\right)\Big|_{-1}^{3} = \pi\left(9 - \frac{9}{2} + 3 + \frac{1}{2}\right) = \boxed{8\pi}$$

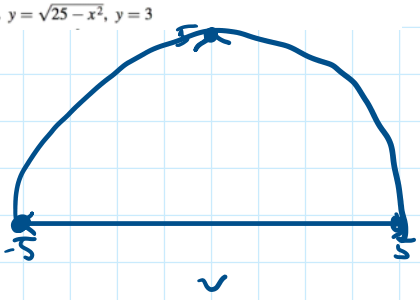
5.



$$V = \pi \int_{\pi/4}^{\pi/2} \cos x\,dx = \pi \cdot \sin x\Big|_{\pi/4}^{\pi/2} = \left(1 - \frac{\sqrt{2}}{2}\right)\pi = \boxed{\pi - \frac{\pi\sqrt{2}}{2}}$$

11–18 Find the volume of the solid that results when the region enclosed by the given curves is revolved about the x-axis.

11. $y = \sqrt{25 - x^2}$, $y = 3$



$$V = \pi \int_{-5}^{5} (25 - x^2)\,dx = \pi\left(25x - \frac{x^3}{3}\right)\Big|_{-5}^{5}$$

$$= \pi\left(125 - \frac{125}{3} + 125 - \frac{125}{3}\right)$$

$$= \pi\left(\frac{3(250)}{3} - \frac{250}{3}\right)$$

$$= \boxed{\frac{500\pi}{3}}$$

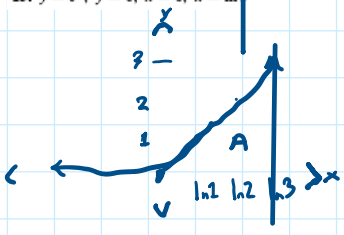
12. $y = 9 - x^2$, $y = 0$

$$V = \pi \int_{-3}^{3} (9 - x^2)^2\,dx = \pi \int_{-3}^{3} (81 - 18x^2 + x^4)\,dx = \pi\left(81x - 6x^3 + \frac{x^5}{5}\right)\Big|_{-3}^{3}$$

$$= \pi\left(486 - 324 + \frac{486}{5}\right)$$

$$= \pi\left(162 + \frac{486}{5}\right) = \boxed{\frac{1296\pi}{5}}$$

15. $y = e^x$, $y = 0$, $x = 0$, $x = \ln 3$



$$V = \pi \int_{0}^{\ln 3} (e^x)\,dx = \pi\, e^x\Big|_{0}^{\ln 3} = 3\pi - \pi = \boxed{2\pi}$$

32. Let V be the volume of the solid that results when the region enclosed by $y = 1/x$, $y = 0$, $x = 2$, and $x = b\,(0 < b < 2)$ is revolved about the x-axis. Find the value of b for which $V = 3$.

$$3 = \int_{2}^{b} \left(\tfrac{1}{x} - 0\right)dx$$

$$\ln(b) = 3 + \ln 2$$

$$b = \boxed{3e^{\ln 2}}$$

$$3 = \int_2^b \left(\frac{1}{x} - 0\right) dx$$

$$3 = (\ln x) \Big|_2^b$$

$$3 = \ln(b) - \ln 2$$

$$\ln(b) = 3 + \ln 2$$

$$\boxed{b = e^{3+\ln 2}}$$