3·1. INTRODUCTION

Index numbers are the indicators which reflect changes over a specified period of time in (*i*) prices of different commodities, (*ii*) industrial production, (*iii*) sales, (*iv*) imports and exports, (*v*) cost of living, etc. These indicators are of paramount importance to the management personnel or any government organisation or industrial concern for the purpose of reviewing position and planning action, if necessary; and in the formulation of executive decisions. They reflect the pulse of an economy and serve as indicators of inflationary or deflationary tendencies. Just as in Physics and Chemistry barometer measures atmospheric pressure or pressure of gases, so in Economics index numbers measure the pressure of economic behaviour and are rightly termed as '*economic barometers*' or '*barometers of economic activity*' since a look at some of the important indices like index numbers of wholesale prices, industrial production, agricultural production, etc., gives a fairly good idea as to what is happening to the economy of a country.

Definition. "*Index numbers are statistical devices designed to measure the relative change in the level of a phenomenon (variable or a group of variables) with respect to time, geographical location or other characteristics such as income, profession, etc.,*" In other words, these are the numbers which express the value of a variable at any given date called the '*given period*' as a percentage of the value of that variable at some standard date called the '*base period*'. The variable may be :

(*i*) the price of a particular commodity, *e.g.*, silver, iron, etc., or a group of commodities like consumer goods, foodstuffs, etc.;

(*ii*) the volume of trade, exports and imports, agricultural or industrial production, sales in a departmental store, etc.; and

(*iii*) the national income of a country or cost of living of persons belonging to particular income group/profession, etc.

For example, suppose we want to measure the general changes in the price level of consumer goods, *i.e.*, goods or commodities consumed by the people belonging to a particular section of society, say, low-income group or middle-income group or labour class, and so on. Obviously, these changes are not directly measurable as the price quotations of various commodities are available in different units, *e.g.*, wheat and sugar in rupees per quintal, milk, petrol and kerosene in rupees per litre, cloth in rupees per metre, etc. Further, the prices of some of the commodities may be increasing while those of others may be decreasing during the two periods and the rates of increase or decrease may be different for different commodities. *Index number* is a statistical device which enables us to arrive at a single representative figure which gives the general level of the price of the phenomenon (commodities) in an extensive group.

According to ***Wheldon***, "*An index number is a device which shows by its variation the changes in a magnitude which is not capable of accurate measurement in itself or of direct valuation in practice.*"

Edgeworth gave the classical definition of index numbers as follows : "*Index number shows by its variations the changes in a magnitude which is not susceptible either of accurate measurement in itself or of direct variation in practice.*"

In the words of ***Lawrence J. Kaplan***, "*An index number is a statistical measure of fluctuations in a variable arranged in the form of a series, and using a base period for making comparisons.*"

In short, "*Index numbers are specialised type of rates, ratios, percentages which give the general level of magnitude of a group of distinct but related variables in two or more situations.*"

3·2. BASIC PROBLEMS INVOLVED IN THE CONSTRUCTION OF INDEX NUMBERS

The methods of construction of index numbers warrant a cereful study of the following problems :

1. ***The Purpose of Index Number.*** An index number which is properly designed for a purpose can be most useful and powerful tool otherwise it can be equally misleading and dangerous. Thus the first and foremost problem is to determine the purpose of index number without which it is not possible to follow the steps in its construction. Moreover, precise statement of the purpose usually settles some related problems, *e.g*, if the purpose of index number is to measure the changes in the production of steel, (say), the problem of selection of items (commodities) is automatically settled.

2. ***Selection of Commodities.*** Having defined the purpose of index numbers, select only those commodities which are relevant to the index. For example, if the purpose of an index is to measure the cost of living of low income group (poor families), we should select only those commodities or items which are consumed/utilised by persons belonging to this group and due care should be taken not to include the goods/services which are ordinarily consumed by middle-income or high-income group. For such an index, selection of commodities like cosmetics and other luxury goods like scooters, cars, refrigerators, television sets, etc., will be absolutely useless.

The best solution to the problem of selection of items for any index is (*i*) to split the whole (relevant) group of commodities into various homogeneous sub-groups like cereals, milk and milk products, clothing, iron and steel, electrical appliances and fuel, etc., so that the price movement of various commodities within any sub-group follows almost the same pattern, and (*ii*) to select an adequate number of representative items from each sub-group.

Remark. It should be borne in mind that for the index number the same grade/quality of the commodities, say, wheat, rice, etc., is included at different times. In order to avoid confusion due to time-lag about qualities, it is desirable to include, as far as possible, only standardised or graded items.

3. ***Data for Index Numbers.*** The data, usually the set of prices and of quantities consumed of the selected commodities for different periods, places, etc., constitute the raw material for the construction of index numbers. The data should be collected from reliable sources such as standard trade journals, official publications, periodical special reports from the producers, exporters, etc., or through field agency. The principles of data collection, *viz.*, accuracy, comparability, sample representatives and adequacy should be borne in mind. In any case the data should strictly pertain to what is being measured. For example, for the construction of retail price index numbers, the price quotations for an adequate number of commodities (used by a particular group of people for whom the index is intended) should be obtained from superbazars, fair price shops, departmental stores, etc., and not from wholesale dealers.

4. ***Selection of Base Period.*** The period with which the comparisons of relative changes in the level of a phenomenon are made is termed as '*base period*' and the index for this period is always taken as 100. The following are basic criteria for the choice of the base period :

(i) *The 'base period' must be a 'normal period', i.e.*, a period free from all sorts of abnormalities or chance fluctuations such as economic boom or depression, labour strikes, wars, floods, earthquakes, etc. If the base period be taken as a period of economic instability or depression in which the prices of various commodities and goods, due to their scarcity, have been abnormaly high, then the comparison of price relatives in any given year will not be of much practical utility.

(ii) *The base period should not be too distant from the given period.* Since index numbers are essential tools in business planning and in formulation of executive decisions, the base period should not be too far back in the past relative to the given period because due to dynamic pace of events these days, distant base period is likely to be entirely different from the given period. Moreover, if the base year is shifted far away from the given period, it is possible that the pattern of consumption of commodities may change appreciably. For example, for deciding about grant of D.A. (dearness allowance) increment to government personnel, the prices should be compared with the period when last D.A. was announced or granted.

5. *Type of Average to be used.* Since index numbers are specialised averages, a judicious choice of average to be used in their construction is of great importance. Usually the following averages are used :

(i) Arithmetic Mean (A.M.) : simple or weighted, (ii) Geometric Mean (G.M.) : simple or weighted, (iii) Median.

Median, though easiest to calculate of all the three, completely ignores the extreme observations while arithmetic mean, though easy to calculate, is unduly affected by extreme observations. Moreover, as we shall see later, neither arithmetic mean nor median are reversible and hence do not reflect typical movements of prices or quantities. Since in the construction of index numbers we deal with ratio or relative changes and since geometric mean

(i) gives equal weights to equal ratios of change,

(ii) does not give undue weightage to extreme observations, and

(iii) G.M. based indices are reversible,

from theoretical considerations. G.M. is the most appropriate average to be used. But in spite of its theoretical claim, in practice G.M. is not used as often as A.M. because of its computational difficulties. However, in the interest of greater accuracy and precision, G.M. should be recommended.

6. *Selection of Appropriate Weights.* Generally, various items, commodities, say, wheat, rice, kerosene, clothing, etc., included in the index are not of equal importance, proper weights should be attached to them to take into account their relative importance. Thus there are two types of indices :

(i) *Unweighted Indices,* in which no specific weights are attached to various commodities, and

(ii) *Weighted Indices,* in which appropriate weights are assigned to various items.

Strictly speaking, unweighted indices can be interpreted as weighted indices, the corresponding weight for each commodity being unity. The question of allocating suitable weights is of fundamental importance but at the same time quite difficult also. The various forms of weights usually used in practice are discussed below in the various formulae for the construction of index numbers.

According to ***Bowley, A.L.***, *"The discussion of proper weight to be used has occupied a space in statistical literature out of all proportions to its significance, for it may be said at once that no great important need be attached to the special choice of weights; one of the most convenient facts of statistical theory is that, given certain condition, the same result is obtained with sufficient closeness whatever logical system of weights is applied."* By means of examples he showed that weighted and unweighted means do not materially differ from each other and also illustrated that by using different weights, the resulting averages come out to be uniform in size. He, however, is not opposed to weighting as such and advocated the arithmetic cross of Laspeyre's and Paasche's formulae discussed below.

In the words of ***Mitchell***, *"The real problem for the maker of index number is whether he shall leave weighting to chance or seek to rationalise it."*

Notations. Let

p_{ij} denote the price of jth commodity in the ith year.

q_{ij} denote the quantity of the jth commodity in the ith year,

$v_{ij} = p_{ij} \times q_{ij}$ denote the value of jth commodity in the ith year,

where $j = 1, 2, \ldots, n$ and $i = 0, 1, 2, \ldots, k$ refer to the various situations to be compared, which we have referred to as years ; '0' serving as the base year and 'i' as the given year.

In the following sequences, the summation Σ is taken over j from 1 to n, unless otherwise stated. Thus, we will write $\sum_{j=1}^{n} p_{ij} = \Sigma p_{ij}$ and $\sum_{j=1}^{n} q_{ij} = \Sigma q_{ij}$, which are the ith year price and quantity respectively. In particular, Σp_{0j} and Σq_{0j} refer to base year price and base year quantity respectively.

3·3. THE CONSTRUCTION OF INDEX NUMBERS

We describe below some methods of constructing index numbers :

3·3·1. Simple (Unweighted) Aggregate Method. This method consists of expressing aggregate of prices in any year as a percentage of their aggregate in the base year. Thus price (or quantity) index for the ith year ($i = 1, 2, \ldots, k$) as compared to the base year ($i = 0$), is given by :

$$P_{0i} = \frac{\Sigma p_{ij}}{\Sigma p_{0j}} \times 100, \quad Q_{0i} = \frac{\Sigma q_{ij}}{\Sigma q_{0j}} \times 100 \qquad \ldots (3\cdot1)$$

The *drawbacks* of this method are :

(i) The price of various commodities may be in different units, *e.g.*, per litre, per metre, per quintal, etc., and

(ii) The relative importance of various commodities is neglected.

3·3·2. Weighted Aggregate Method. This method provides for the different commodities to exert their influence in the index number by assigning appropriate weights to each. Usually the quantities consumed, sold or marketed in the base year, the given year or some typical year (which may be an average over a number of years) are used as weights. If w_j (quantity consumed) is the weight associated with the jth commodity then the *weighted aggregative price index* is given by :

$$P_{0i} = \frac{\Sigma p_{ij} w_j}{\Sigma p_{0j} w_j} \times 100 \qquad \ldots (3\cdot2)$$

By the use of different types of weights, a number of formulae have emerged for the construction of index numbers.

Laspeyre's Price Index (or Base Year) Method. If we take $w_j = q_{oj}$ in (3·2), *i.e.*, if the base year quantities are taken as weights then the *Laspeyre's aggregative price index* (after the French economist Laspeyre who formulated it in 1871) or *L-formula* is given by :

$$P_{oi}^{La} = \frac{\sum p_{ij}\, q_{oj}}{\sum p_{oj}\, q_{oj}} \times 100 \qquad \ldots(3\cdot3)$$

Paasche's Price Index (or Given year) Method. By taking given year quantities as weights, *i.e.*, $w_j = q_{ij}$ in (3·2), we get *Paasche's formula;* (after German Statistician Paasche who formulated it in 1874) or *P-formula* as :

$$P_{oi}^{Pa} = \frac{\sum p_{ij}\, q_{ij}}{\sum p_{oj}\, q_{ij}} \times 100 \qquad \ldots(3\cdot4)$$

Drobish-Bowley Price Index Number. This formula is the arithmetic mean of the Laspeyre's and Paasche's price indices and is given by

$$P_{oi}^{DB} = \frac{1}{2}\left[\frac{\sum p_{ij}\, q_{oj}}{\sum p_{oj}\, q_{oj}} + \frac{\sum p_{ij}\, q_{ij}}{\sum p_{oj}\, q_{ij}}\right] \times 100 \qquad \ldots(3\cdot5)$$

This is also sometimes known as *L-P* formula.

Marshall-Edgeworth Price Index (or Base and Given Year) Method. If we take $w_j = (q_{oj} + q_{ij})/2$ in (3·2), *i.e.*, if weights are the arithmetic mean of the base year quantities and the current year quantities, Marshall-Edgeworth (M.E.) formula is obtained as :

$$P_{oi}^{ME} = \frac{\sum p_{ij}\,(q_{oj} + q_{ij})/2}{\sum p_{oj}\,(q_{oj} + q_{ij})/2} \times 100 = \frac{\sum p_{ij}\,(q_{oj} + q_{ij})}{\sum p_{oj}\,(q_{oj} + q_{ij})} \times 100 \qquad \ldots(3\cdot6)$$

Walsch Price index. Instead of using arithmetically crossed weighted aggregatives as in *M.E.* formula, Walsch used geometrically crossed weighted aggregatives [*i.e.*, $w_j = \sqrt{q_{oj} + q_{ij}}$ in (3·2)] to get

$$P_{oi}^{Wa} = \frac{\sqrt{q_{oi}\, q_{ij}}}{\sum p_{oj} \sqrt{q_{oj}\, q_{ij}}} \times 100 \qquad \ldots(3\cdot7)$$

Irving Fisher's 'Ideal' Index Number. It is given by the geometric mean of Laspeyre's and Paasche's formulae. In other words :

$$P_{oi}^{F} = \left(P_{oi}^{La} \times P_{oi}^{Pa}\right)^{1/2} = 100 \times \left(\frac{\sum p_{ij}\, q_{oj}}{\sum p_{oj}\, q_{oj}} \times \frac{\sum p_{ij}\, q_{ij}}{\sum p_{oj}\, q_{ij}}\right)^{1/2} \qquad \ldots(3\cdot8)$$

Formula (3·8) is regarded as *'ideal'* one, since it satisfies certain tests of consistency discussed in § 3·4.

Kelly's Price Index or Fixed Weight Method. If in (3·2) weights w_j are the quantities which may refer to some period (not necessarily the base year or current year) and are kept constant for all periods we get Truman L. Kelly's price index number. The average (A.M. or G.M.) of the quantities of two, three or more years can be used as weights.

An obvious and important advantage of Kelly's fixed weights index over Laspeyre's index is that in this index the change in the base period does not necessitate a corresponding change in the weights which can be kept constant until new data become available to revise the index. The fixed weight method is quite popular and is used in the construction of the Bureau of Labour wholesale price index in U.S.A.

Remarks on Various Index Number Formulae

1. If the prices of some of the items/commodities increase significantly, their consumption usually decreases and consequently the use of base year quantities as weights gives more weightage to those commodities whose prices have increased most and in this case numerator in (3·3) becomes relatively large. Applying similar argument to the use of current year quantities as weights, we infer that Laspeyre's formula is generally expected to *over estimate* or to leave an *'upward bias'* whereas Paasche's formula tends to *underestimate* and show a *'downward bias'*. However, it should not be concluded that Laspeyre's index must necessarily be larger than Paasche's index. The true value of the prices lies somewhere in between the two.

2. Although formula (3·8) is termed as *'ideal'*, it is not very popular in practice because of its computational difficulties and primarily because in its calculation new quantity weights are to be used for each period an index number is prepared. Generally it is very difficult and rather expensive to obtain correct information regarding these weights. In practice, statisticians prefer to rely on simple, though less exact, index number formulae.

3. Although many different weighting patterns or formulae for index numbers are possible and have actually been used, most published index numbers are of either Laspeyre's or Paasche's type.

4. *Quantity Index Numbers.* In the above formulae (3·3) to (3·8) we concentrated ourselves on price index numbers. By interchanging the prices (p_{ij}) and quantities (q_{ij}) in the above formula we get the corresponding formulae for the calculation of *quantity index numbers* which reflect the change in the volume of quantity or production. Thus, for example,

$$Q_{oi}^{La} = \frac{\Sigma q_{ij}\, p_{oj}}{\Sigma q_{oj}\, p_{oj}} \times 100\ ;\quad Q_{oi}^{Pa} = \frac{\Sigma q_{ij} p_{ij}}{\Sigma q_{oj}\, p_{ij}} \times 100\ ;\quad Q_{oi}^{ME} = \frac{\Sigma q_{ij}\,(p_{ij} + p_{oj})}{\Sigma q_{oj}\,(p_{ij} + p_{oj})} \times 100 \qquad \ldots(3\cdot9)$$

Quantity index numbers study the changes in the volume of goods produced (manufactured), consumed or distributed, like the indices of agricultural production, industrial production, imports and exports, etc. They are extremely helpful in studying the level of physical output in an economy.

5. *Value Index Numbers* are given by the aggregate expenditure for any given year expressed as a percentage of the same in the base year. Thus

$$V_{oi} = \frac{\Sigma p_{ij}\, q_{ij}}{\Sigma p_{oj}\, q_{oj}} \times 100 \qquad \ldots(3\cdot10)$$

However, these indices are not as common as price and quantity indices.

Example 3·1. (*a*) *From the following data calculate price index numbers for 2005 with 1995 as base by : (i) Laspeyre's, (ii) Paasche's, (iii) Marshall-Edgeworth, and (iv) Fisher's formulae :*

Commodities	1995		2005	
	Price	Quantity	Price	Quantity
A	20	8	40	6
B	50	10	60	5
C	40	15	50	15
D	20	20	20	25

(*b*) *It is stated that Marshall-Edgeworth index number is a good approximation to Fisher's ideal index number. Verify this for the data in Part (a).*

Solution. (*a*)

TABLE 3·1 : CALCUALTIONS FOR PRICE INDICES BY DIFFERENT FORMULAE

Commodities	1995		2005		$p_0 q_0$	$p_0 q_1$	$p_1 q_0$	$p_1 q_1$
	p_0	q_0	p_1	q_1				
A	20	8	40	6	160	120	320	240
B	50	10	60	5	500	250	600	300
C	40	15	50	15	600	600	750	750
D	20	20	20	25	400	500	400	500
					$\Sigma p_0 q_0 = 1{,}660$	$\Sigma p_0 q_1 = 1{,}470$	$\Sigma p_1 q_0 = 2{,}070$	$\Sigma p_1 q_1 = 1{,}790$

(*i*) *Laspeyre's Price Index* : $P_{0i}^{La} = \dfrac{\Sigma p_i\, q_0}{\Sigma p_0\, q_0} \times 100 = \dfrac{2070}{1660} \times 100 = 1{\cdot}24699 \times 100 = 124{\cdot}69$

(*ii*) *Paasche's Price Index* : $P_{0i}^{Pa} = \dfrac{\Sigma p_i\, q_i}{\Sigma p_0\, q_i} \times 100 = \dfrac{1790}{1470} \times 100 = 1{\cdot}2177 \times 100 = 121{\cdot}77$

(*iii*) *Marshall-Edgeworth Price Index* :

$$P_{0i}^{ME} = \left(\frac{\Sigma p_i\, q_0 + \Sigma p_i\, q_i}{\Sigma p_0\, q_0 + \Sigma p_0\, q_i}\right) \times 100 = \left(\frac{2070 + 1790}{1600 + 1470}\right) \times 100 = \frac{3860}{3130} \times 100 = 1{\cdot}2332 \times 100 = 123{\cdot}32$$

(*iv*) *Fisher's Price Index* : $P_{0i}^{F} = \sqrt{\dfrac{\Sigma p_i\, q_0}{\Sigma p_0\, q_0} \times \dfrac{\Sigma p_i\, q_i}{\Sigma p_0\, q_i}} \times 100 = \sqrt{\dfrac{2070}{1600} \times \dfrac{1790}{1470}} \times 100$

$$= \sqrt{1{\cdot}24699 \times 1{\cdot}2177} \times 100 = \sqrt{1{\cdot}51846} \times 100 = 1{\cdot}23226 \times 100 = 123{\cdot}23.$$

Aliter :

$$P_{0i}^{F} = \sqrt{P_{0i}^{La} \times P_{0i}^{Pa}} = \sqrt{124{\cdot}699 \times 121{\cdot}77} = \sqrt{15184{\cdot}597} = 123{\cdot}23.$$

(*b*) Since $P_{0i}^{ME} = 123{\cdot}32$ and $P_{0i}^{F} = 123{\cdot}23$ are approximately equal, Marshall-Edgeworth index number is a good approximation to Fisher's ideal index number.

Example 3·2. *Compute price index and quantity index numbers for the year 2005 with 2000 as base year, using*

(*i*) *Laspeyre's Method,*

(*ii*) *Paasche's Method, and*

(*iii*) *Fisher's method.*

Commodity	Quantity (units)		Expenditure (Rs.)	
	2000	2005	2000	2005
A	100	150	500	900
B	80	100	320	500
C	60	72	150	360
D	30	33	360	297

Solution. We are given the quantities and the values of the commodities in the base year and the current year. We know that :

$$\text{Expenditure} = \text{Price} \times \text{Quantity} \;\Rightarrow\; \text{Quantity} = \frac{\text{Expenditure}}{\text{Price}} \quad \text{or} \quad q = \frac{e}{p} \qquad \ldots (*)$$

Using (*), we shall first obtain the quantities consumed for the base year and the current year as given in the following table :

TABLE 3·2 : CALCULATIONS FOR LASPEYRE'S, PAASCHE'S AND FISHER'S INDEX NUMBERS

q_0	q_i	$e_0 = p_0 q_0$	p_0	$e_i = p_i q_i$	p_i	$p_i q_0$	$p_0 q_i$
(1)	(2)	(3)	(4) = (3) ÷ (1)	(5)	(6) = (5) ÷ (2)	(7)	(8)
100	150	500	5	900	6	600	750
80	100	320	4	500	5	400	400
60	72	150	2·5	360	5	300	180
30	33	360	12	297	9	270	396
		$\Sigma p_0 q_0 = 1{,}330$		$\Sigma p_i q_i = 2{,}057$		$\Sigma p_i q_0 = 1{,}570$	$\Sigma p_0 q_i = 1{,}726$

(*i*) *Laspeyre's Price and Quantity Indices :*

$$P_{0i}^{La} = \frac{\Sigma p_i\, q_0}{\Sigma p_0\, q_0} \times 100 = \frac{1570}{1330} \times 100 = 118{\cdot}045$$

$$Q_{0i}^{La} = \frac{\Sigma q_i\, p_0}{\Sigma q_0\, p_0} \times 100 = \frac{1726}{1330} \times 100 = 129{\cdot}744$$

(*ii*) *Paasche's Price and Quantity Indices :*

$$P_{0i}^{Pa} = \frac{\Sigma p_i\, q_i}{\Sigma p_0\, q_1} \times 100 = \frac{2057}{1726} \times 100 = 119{\cdot}177$$

$$Q_{0i}^{Pa} = \frac{\Sigma q_i\, p_i}{\Sigma q_0\, p_i} \times 100 = \frac{2057}{1570} \times 100 = 131{\cdot}019$$

(*iii*) *Fisher's Price and Quantity Indices :*

$$P_{0i}^{F} = \sqrt{P_{0i}^{La} \times P_{0i}^{Pa}} = \sqrt{118{\cdot}045 \times 119{\cdot}177} = \sqrt{14068{\cdot}248} = 118{\cdot}610$$

$$Q_{0i}^{F} = \sqrt{Q_{0i}^{La} \times Q_{0i}^{Pq}} = \sqrt{129{\cdot}774 \times 131{\cdot}019} = \sqrt{17002{\cdot}859} = 130{\cdot}395.$$

Example 3·3. *The following Table 3·3 relates to the daily pay of the wage earners on a company's pay roll :*

TABLE 3·3

	April 2000		April 2005	
	Number	Total pay	Number	Total pay
Men aged 21 and over	350	2,500	300	4,200
Women aged 18 and over	400	1,600	1,200	8,000
Youths and boys	150	450	100	560
Girls	100	250	400	1,540
Total	1,000	41,800	2,000	14,300

Construct an index of daily earnings based on 2000 as base showing the rise of earnings for all employees as one figure.

Solution. Regarding the number of wage earners as quantities and the daily wages per worker as prices, we are given the figures q_0 and $p_0 q_0$ for 2000 and the values q_i and $p_i q_i$ for 2005. The *Table 3·3A* can be easily completed.

TABLE : 3·3A : CALCULATIONS FOR INDICES OF DAILY EARNINGS

q_0	p_0	q_i	p_i	$p_0 q_0$	$p_0 q_i$	$p_i q_i$	$p_i q_0$
350	714	300	1,400	2,50,000	2,14,200	4,20,000	4,90,000
400	400	1,200	667	1,60,000	4,80,000	8,00,000	2,66,800
150	300	100	560	45,000	30,000	56,000	84,000
100	250	400	385	25,000	1,00,000	1,54,000	38,500
Total				4,80,000	8,24,200	14,30,000	8,79,300

$$P_{oi}^{La} = \frac{\Sigma p_{ij}\, q_{oj}}{\Sigma p_{oj}\, q_{oj}} \times 100 = \frac{8{,}79{,}300}{4{,}80{,}000} \times 100 = 183\,; \quad P_{oi}^{Pa} = \frac{\Sigma p_{ij}\, q_{ij}}{\Sigma p_{oj}\, q_{ij}} \times 100 = \frac{14{,}30{,}000}{8{,}24{,}200} \times 100 = 173{\cdot}5$$

$$P_{oi}^{F} = \left(P_{oi}^{La} \times P_{oi}^{Pa}\right)^{1/2} = \left(183 \times 173{\cdot}5\right)^{1/2} = 178{\cdot}3.$$

Example 3·4. *Given the data (Table 3·4) where p and q respectively stand for price and quantity and subscripts stand for time period. Find x, if the ratio between Laspeyre's (L) and Paasche's (P) index numbers is*

L : P : : 28 : 27.

TABLE 3·4

	Commodities	
	A	B
p_0	1	1
q_0	10	5
p_i	2	x
q_i	5	2

Solution.

TABLE 3·4A : CALCULATIONS FOR LASPEYRE'S AND PAASCHE'S INDICES

Commodities	p_0	q_0	p_1	q_1	$p_1 q_0$	$p_0 q_0$	$p_1 q_1$	$p_0 q_1$
A	1	10	2	5	20	10	10	5
B	1	5	x	2	$5x$	5	$2x$	2
Total					$20 + 5x$	15	$10 + 2x$	7

$$P_{oi}^{La} = \frac{\Sigma p_{ij}\, q_{oj}}{\Sigma p_{oj}\, q_{oj}} \times 100 = \frac{20+5x}{15} \times 100 \; ; \; P_{oi}^{Pa} = \frac{\Sigma p_{ij}\, q_{ij}}{\Sigma p_{oj}\, q_{ij}} \times 100 = \frac{10+2x}{7} \times 100$$

$$\frac{P_{oi}^{La}}{P_{oi}^{Pa}} = \frac{[(10+5x)/15]}{[(10+2x)/7]} = \frac{28}{27} \text{ (given)} \;\Rightarrow\; \frac{20+5x}{15} \times \frac{7}{10+2x} = \frac{28}{27}, \text{ i.e., } x = 4 \text{ (on simplification).}$$

Example 3·5. *Calculate a suitable weighted price index from the following data :*

Commodity	Unit	Quantity Required	Price during Base year (Rs.)	Price during Current year (Rs.)
A	kg.	500	5·0	8·0
B	c.ft.	2,000	9·5	14·2
C	Dozen	50	34·0	42·0
D	Litre	20,000	12·0	24·0

Solution. Since the weights here are fixed (neither relating to current year nor to base year), we use Kelly's method for computing the index.

TABLE 3·5 : CALCULATIONS FOR KELLY'S INDEX NUMBER

Commodity	Unit	Quantity (q)	p_0	p_1	$p_0 q$	$p_1 q$
A	kg.	500	5·0	8·0	25	40
B	c.ft.	2,000	9·5	14·2	19,000	28,400
C	Dozen	50	34·0	42·0	1,700	2,100
D	Litre	20,000	12·0	24·0	240	480
Total					20,965	31,020

$$P_{oi}^{K} = \frac{\Sigma p_{ij}\, q_j}{\Sigma p_{oj}\, q_j} \times 100 = \frac{31,020}{20,965} \times 100 = 148.$$

Example 3·6. *Construct Quantity Index Numbers taking 2002 as the base :*

Commodity	Average Price	Production 2002	2003	2004	2005
A	10	62	65	66	90
B	15	138	120	110	80
C	2·5	500	540	580	800
D	22·5	10	10	10	10

Solution.

TABLE 3·6 : CALCULATIONS FOR QUANTITY INDEX NUMBERS (BASE 2002)

Commodity	Price	2002 Quantity q_0	2002 Value pq_0	2003 Quantity q_1	2003 Value pq_1	2004 Quantity q_2	2004 Value pq_2	2005 Quantity q_3	2005 Value pq_3
A	10	62	620	65	650	66	660	90	900
B	15	138	2070	120	1800	110	1650	80	1200
C	2·5	500	1250	540	1350	580	1450	800	2000
D	22·5	10	225	10	225	10	225	100	225
Total			4,165		4,025		3,985		4,325
			100		$\frac{4,025}{4,165} \times 100 = 96{\cdot}6$		$\frac{3,985}{4,165} \times 100 = 95{\cdot}7$		$\frac{4,325}{4,165} \times 100 = 103{\cdot}8$

Example 3·7. *With regard to Laspeyre's and Paasche's price index numbers it is maintained that "if the prices of all the goods change in the same ratio, the two indices will be equal, for then the weighting system is irrelevant ; or, if the quantities of all the goods change in the same ratio, they will be equal, for then the two weighting systems are the same relatively." (Karmal) Prove the given statement.*

Solution. In the usual notations, we have to prove that if,

$$\frac{p_{ij}}{p_{oj}} = \text{constant}, k, \text{(say)} \;\forall j = 1, 2, ..., n \quad ...(*) \quad \text{or} \quad \frac{q_{ij}}{q_{oj}} = \text{constant}, m, \text{(say)} \;\forall j = 1, 2, ..., n \quad ...(**)$$

If then (*) holds, then

$$P_{oi}^{La} = \frac{\Sigma p_{ij}\, q_{oj}}{\Sigma p_{oj}\, q_{oj}} \times 100 = \frac{k\, \Sigma p_{oj}\, q_{oj}}{\Sigma p_{oj}\, q_{oj}} \times 100 = 100\, k \quad [\text{From } (*)]$$

$$P_{oi}^{Pa} = \frac{\Sigma p_{ij}\, q_{ij}}{\Sigma p_{oj}\, q_{ij}} \times 100 = \frac{k\, \Sigma p_{oj}\, q_{ij}}{\Sigma p_{oj}\, q_{ij}} \times 100 = 100\, k \quad [\text{From } (*)] \quad \therefore \quad P_{oi}^{La} = P_{oi}^{Pa}$$

Similarly, if (**) holds, then

$$P_{oi}^{La} = \frac{\Sigma p_{ij}\, q_{oj}}{\Sigma p_{oj}\, q_{oj}} \times 100 = \frac{\Sigma p_{ij}\, (q_{ij}/m)}{\Sigma p_{oj}\, (q_{ij}/m)} \times 100 \quad [\text{From } (**)] = \frac{\Sigma p_{ij}\, q_{ij}}{\Sigma p_{oj}\, q_{ij}} \times 100 = P_{oi}^{Pa}$$

Example 3·8. *If L (p) and P(q) represent respectively Laspeyre's index number for prices and Paasche's index number for quantities, show that*

$$L(p)/L(q) = P(p)/P(q).$$

Solution. By *def.*, we have in the usual notations :

$$L(p) = \frac{\Sigma p_{ij}\, q_{oj}}{\Sigma p_{oj}\, q_{oj}} \times 100 \; ; \quad P(p) = \frac{\Sigma p_{ij}\, q_{ij}}{\Sigma p_{oj}\, q_{ij}} \times 100$$

$$L(q) = \frac{\Sigma p_{oj}\, q_{ij}}{\Sigma p_{oj}\, q_{oj}} \times 100 \; ; \quad P(q) = \frac{\Sigma p_{ij}\, q_{ij}}{\Sigma p_{ij}\, q_{oj}} \times 100$$

$$\therefore \quad L(p) \times P(q) = \left[\frac{\Sigma p_{ij}\, q_{oj}}{\Sigma p_{oj}\, q_{oj}} \times 100\right] \times \left[\frac{\Sigma p_{ij}\, q_{ij}}{\Sigma p_{ij}\, q_{oj}} \times 100\right] = \frac{\Sigma p_{ij}\, q_{ij}}{\Sigma p_{oj}\, q_{oj}} \times 100^2$$

Similarly, we obtain $\quad L(q) \times P(p) = \dfrac{\Sigma p_{ij}\, q_{ij}}{\Sigma p_{oj}\, q_{oj}} \times 100^2$

From (*) and (**), we get $\quad L(p) \times P(q) = L(q) \times P(p) \Rightarrow \dfrac{L(p)}{L(q)} = \dfrac{P(p)}{P(q)}$.

***Example* 3·9.** *Let*

X_j = *Price relative* $= \dfrac{p_{ij}}{p_{oj}}$; Y_j = *Quantity relative* $= \dfrac{q_{ij}}{q_{oj}}$; V_{oi} = *Value index number* $= \dfrac{\sum\limits_i p_{ij}\, q_{ij}}{\sum\limits_i p_{oj}\, q_{oj}}$

and $\quad w_j = p_{oj}\, q_{oj}$ *be the weights of* X_j *and* Y_j $(j = 1, 2, ..., n)$. *Show that*

$$\frac{P^{La}_{oi}}{P^{Pa}_{oi}} = 1 - \frac{r_{XY}\,\sigma_X\sigma_Y}{V_{oi}},$$

where r_{XY} *is the weighted correlation coefficient between X and Y,* σ_X *and* σ_Y *being the weighted standard deviations of X and Y respectively. Deduce the conditions under which Laspeyre's price index* P^{La}_{oi} *is greater than, equal to or less than Paasche's price index,* P^{Pa}_{oi}.

Solution. By *def.*, we have

$$r_{XY}\,\sigma_X\,\sigma_Y = \text{Cov}\,(X, Y) = \frac{\Sigma w_j X_j Y_j}{\Sigma w_j} - \left(\frac{\Sigma w_j X_j}{\Sigma w_j}\right)\left(\frac{\Sigma w_j Y_j}{\Sigma w_j}\right)$$

summation extending over j from 1 to n.

$$r_{XY}\,\sigma_X\,\sigma_Y = \frac{\Sigma p_{ij}\, q_{ij}}{\Sigma p_{oj}\, q_{oj}} - \frac{\Sigma p_{ij}\, q_{oj}}{\Sigma p_{oj}\, q_{oj}} \times \frac{\Sigma p_{oj}\, q_{ij}}{\Sigma p_{oj}\, q_{oj}} = \frac{\Sigma p_{ij}\, q_{ij}}{\Sigma p_{oj} q_{oj}}\left[1 - \frac{\Sigma p_{ij}\, q_{oj}}{\Sigma p_{oj}\, q_{oj}} \cdot \frac{\Sigma p_{oj}\, q_{ij}}{\Sigma p_{ij}\, q_{ij}}\right]$$

$$\therefore \quad r_{XY}\,\sigma_X\,\sigma_Y = V_{oi}\left(1 - \frac{P^{La}_{oi}}{P^{Pa}_{oi}}\right) \quad \Rightarrow \quad \frac{P^{La}_{oi}}{P^{Pa}_{oi}} = 1 - \frac{r_{XY}\,\sigma_X\,\sigma_Y}{V_{oi}}$$

***Remarks* 1.** If $r_{XY} > 0$, then from (*), we get : $\left(P^{La}_{oi} / P^{Pa}_{oi}\right) < 1 \quad \Rightarrow \quad P^{La}_{oi} < P^{Pa}_{oi}$

and if $r_{XY} < 0$, then we have : $\quad \left(P^{La}_{oi} / P^{Pa}_{oi}\right) > 1 \quad \Rightarrow \quad P^{La}_{oi} > P^{Pa}_{oi}$

Hence, if the correlation between the price relatives X and the quantity relatives Y is positive (negative), then Laspeyre's index is less (greater) than Paasche's index.

2. If either $r_{XY} = 0$, *or* if $\sigma_X = 0$, (*i.e.*, all price movements are same for all commodities *or* if $\sigma_Y = 0$, (*i.e.*, all quantity movements are same for all commodities), then we get $\quad P^{La}_{oi} = P^{Pa}_{oi}$.

3. In practice, under normal economic conditions, we have

$$-1 \le r_{XY} < 0 \text{ and consequently } \quad P^{La}_{oi} > P^{Pa}_{oi}.$$

***Example* 3·10.** *Prove that Fisher's ideal index number lies between Laspeyre's and Paasche's index numbers.*

Solution. Let us consider two real numbers $a > 0, b > 0$.

Let	$a < b$		Also	$a < b$	
$\Rightarrow$	$a^2 < ab$	$(\because a > 0)$	$\Rightarrow$	$ab < b^2$,	$(\because b > 0)$
$\Rightarrow$	$a < \sqrt{ab}$	$(\because a > b)$	$\Rightarrow$	$\sqrt{ab} < b$,	$(\because b > 0)$
$\therefore$		$a < b$	$\Rightarrow$	$a < \sqrt{ab} < b$	

Thus, the geometric mean of two real positive numbers lies between them. The desired **result now immediately follows, since Fisher's ideal index number is the geometric mean between Laspeyre's and Paasche's index numbers.**

$$\therefore \quad P^{La}_{oi} < P^{Pa}_{oi} \quad \Rightarrow \quad P^{La}_{oi} < P^{F}_{oi} < P^{Pa}_{oi}$$

$$\text{and} \quad P^{Pa}_{oi} < P^{La}_{oi} \quad \Rightarrow \quad P^{Pa}_{oi} < P^{F}_{oi} < P^{La}_{oi}$$

In particular, if $P^{La}_{oi} = P^{Pa}_{oi}$, then Laspeyre's, Paasche's and Fisher's indices are all equal.

Remark. Since Fisher's ideal index lies between Laspeyre's index (which has an upward bias) and Paasche's index (which has a downward bias), it provides a better estimate of price changes than these two indices.

***Example* 3·11.** *Show that Marshall Edgeworth index number lies between Laspeyre's and Paasche's index numbers. More specifically,*

(*a*) *if* $\quad P^{La}_{oi} < P^{Pa}_{oi}$ *then* $\quad P^{La}_{oi} < P^{ME}_{oi} < P^{Pa}_{oi}$, *and*

(*b*) *if* $\quad P^{Pa}_{oi} < P^{La}_{oi}$ *then* $\quad P^{Pa}_{oi} < P^{ME}_{oi} < P^{La}_{oi}$

Solution. To establish this result we shall first prove the following lemma.

Lemma. If a, b, c and d are positive numbers, then $\dfrac{a}{b} < \dfrac{c}{d} \Rightarrow \dfrac{a}{b} < \dfrac{a+c}{b+d} < \dfrac{c}{d}$ ... (*)

Proof. If $\dfrac{a}{b} < \dfrac{c}{d}$, then $ad < bc$. ... (1)

Adding ab to both sides, we get

$$a(b + d) < b(a + c) \quad \Rightarrow \quad \frac{a}{b} < \frac{a+c}{b+d}, \qquad ...(2)$$

since all the quantities are positive.

Adding cd to both sides of (1), we get

$$(a + c)d < c(b + d) \quad \Rightarrow \quad \frac{a+c}{b+d} < \frac{c}{d} \qquad ... (3)$$

Equations (2) and (3) establish (*).

(*a*) If $P^{La}_{oi} < P^{Pa}_{oi}$, then $\quad \dfrac{\Sigma p_{ij} q_{oj}}{\Sigma p_{oj} q_{oj}} < \dfrac{\Sigma\, p_{ij} q_{ij}}{\Sigma\, p_{oj} q_{ij}}$, ... (**)

summation extending over j from 1 to n.

Hence on using (*), we get from (**)

$$\frac{\Sigma p_{ij} q_{oj}}{\Sigma p_{oj} q_{oj}} < \frac{\Sigma p_{ij} q_{oj} + \Sigma p_{ij} q_{ij}}{\Sigma p_{oj} q_{oj} + \Sigma p_{oj} q_{ij}} < \frac{\Sigma p_{ij} q_{ij}}{\Sigma p_{oj} q_{ij}} \quad \Rightarrow \quad P^{Pa}_{oi} < P^{ME}_{oi} < P^{La}_{oi}$$

(*b*) If, $P^{Pa}_{oi} < P^{La}_{oi}$, then $\quad \dfrac{\Sigma\, p_{ij} q_{ij}}{\Sigma\, p_{oj} q_{ij}} < \dfrac{\Sigma\, p_{ij} q_{oj}}{\Sigma\, p_{oj} q_{oj}} \quad \Rightarrow \quad P^{Pa}_{oi} < P^{ME}_{oi} < P^{La}_{oi}$

From (*a*) and (*b*), we conclude that Marshall-Edgeworth index lies between Laspeyre's and Paasche's indices.

3·3·3. Average of Price Relatives. The indices by averaging the price relatives are obtained as follows :

(*i*) Calculate the price relatives (p_{ij} / p_{oj}) ; $(j = 1, 2, ..., n)$ for each commodity. Then, $100(p_{ij} / p_{oj})$ which are the price relatives in percentages, give the simple price index number of the jth commodity in the year 'i' relative to '0' as base year. Price relatives are pure numbers independent of units of measurement.

3·4. THE CRITERIA OF A GOOD INDEX NUMBER

Mathematical Tests. The components of errors in the construction of index numbers can be broadly classified as :

(*a*) Formula Error, (*b*) Sampling Error, and (*c*) Homogeneity Error.

Formula error arises due to the usage of different formulae, none of which measures the price changes or quantity changes with perfection or exactitude. Sampling error results from the sampling of the commodities to be included in the index for measuring the 'price changes' or 'quantity changes'. Change in the composition of commodities in the two periods of comparison gives rise to homogeneity error. [For details see § 3·4·1.]

As a measure for the formula error, a number of mathematical tests, discussed below, have been suggested.

3·4·1. Unit Test. (Freund and Williams : *Modern Business Statistics*) This requires the index numbers to be independent of the units in which the prices and quantities of various commodities are quoted. This test is satisfied by all the formulae (discussed in § 3·3) except the formula (3·1).

3·4·2. Time Reversal Test. This is one of the two very important tests proposed by Irving Fisher as tests of consistency for a good index number. He argued that any formula to be accurate must maintain time consistency by working both forward and backward *w.r.t.* time. According to ***Fisher***. *"The test is that the formula for calculating an index number should be such that it will give the same ratio between one point of comparison and the other, no matter which of the two is taken as base. Or, putting it another way, the index number reckoned forward should be the reciprocal of that reckoned backward, except for a constant of proportionality."*

Thus, if the time script (say, Price) of any index formula be interchanged then the resulting index should be the reciprocal of the original index. Symbolically,

$$P_{ii'} = (1/P_{i'i}),\ (i \neq i' = 0, 1, 2, \ldots, k) \qquad \ldots(3\cdot17)$$

For example, for the Laspeyre's formula,

$$P_{oi}^{La} = \frac{\Sigma p_{ij}\, q_{oj}}{\Sigma p_{oj}\, q_{oj}} \quad \text{and} \quad P_{io}^{La} = \frac{\Sigma p_{oj}\, q_{ij}}{\Sigma p_{ij}\, q_{ij}}$$

$$\therefore \quad P_{oi}^{La} \times P_{io}^{La} = \frac{\Sigma p_{ij}\, q_{oj}}{\Sigma p_{oj}\, q_{oj}} \times \frac{\Sigma p_{oj}\, q_{ij}}{\Sigma p_{ij}\, q_{ij}} \neq 1$$

Hence, *Laspeyre's formula does not satisfy Time Reversal Test.* Similarly it can be seen that *Paasche's formula also does not satisfy this test.*

For the Fisher's index formula,

$$P_{oi}^{F} = \left[\frac{\Sigma p_{ij}\, q_{oj}}{\Sigma p_{oj}\, q_{oj}} \times \frac{\Sigma p_{ij}\, q_{ij}}{\Sigma p_{oj}\, q_{ij}}\right]^{1/2} \quad \text{and} \quad P_{oi}^{F} = \left[\frac{\Sigma p_{oj}\, q_{ij}}{\Sigma p_{ij}\, q_{ij}} \times \frac{\Sigma p_{oj}\, q_{oj}}{\Sigma p_{ij}\, q_{oj}}\right]^{1/2}$$

$$\therefore \quad P_{oi}^{F} \times P_{io}^{F} = 1$$

Hence, *Fisher's index formula satisfies Time Reversal Test.*

It can be easily verified that the index numbers based on :

(*i*) The simple geometric mean of price relatives, *i.e.*, formula (3·12), and

(*ii*) Marshall-Edgeworth formula (3·6) (without the factor 100), also satisfy Time Reversal Test.

3·4·3. Factor Reversal Test. This is the second test of consistency suggested by ***I. Fisher***. In his words : *"Just as our formula should permit the interchange of two items without giving inconsistent results, so it ought to permit interchanging the prices and quantities without giving inconsistent results, i.e., the two results multiplied together should give the true value ratio, except for a constant of proportionality."*

Symbolically, we should have

$$P_{oi} \times Q_{oi} = \frac{\Sigma V_{ij}}{\Sigma V_{oj}} = \frac{\Sigma p_{ij}\, q_{ij}}{\Sigma p_{oj}\, q_{oj}} \qquad \ldots(3\cdot18)$$

For example, for Fisher's index,

$$P_{oi}^{F} = \left[\frac{\Sigma p_{ij}\, q_{oj}}{\Sigma p_{oj}\, q_{oj}} \times \frac{\Sigma p_{ij}\, q_{ij}}{\Sigma p_{oj}\, q_{ij}}\right]^{1/2} \quad \text{and} \quad Q_{oi}^{F} = \left[\frac{\Sigma q_{ij}\, p_{oj}}{\Sigma q_{oj}\, p_{oj}} \times \frac{\Sigma q_{ij}\, p_{ij}}{\Sigma q_{oj}\, p_{ij}}\right]^{1/2}$$

$$\Rightarrow \quad P_{oi}^{F} \times Q_{oi}^{F} = \frac{\Sigma p_{ij}\, q_{ij}}{\Sigma p_{oj}\, q_{oj}}, \text{ (on simplification)}$$

Hence, Fisher's index satisfies Factor Reversal Test. It may be pointed out that none of the other formulae satisfies the factor reversal test.

***Remarks* 1.** In verification of these tests, various formulae of § 3·2 are taken without the factor 100.

2. Since *Fisher's index satisfies both Time Reversal and Factor Reversal Test,* it is termed as *'ideal'* index number.

3. If Laspeyre's price index is equal to Paasche's price index, then in the usual notations, we have

$$P_{0i}^{La} = P_{0i}^{Pa} \Rightarrow \frac{\Sigma p_i\, q_0}{\Sigma p_0 q_0} \times 100 = \frac{\Sigma p_i\, q_i}{\Sigma p_0 q_i} \times 100 \quad i.e., \quad (\Sigma p_1 q_0)(\Sigma p_0 q_i) = (\Sigma p_0 q_0)(\Sigma p_i q_i), \quad \ldots(*)$$

the summation being taken over different commodities.

If (*) holds, then it can be easily verified that (without factor 100) :

$$(i) \quad P_{0i}^{La} \times P_{i0}^{La} = 1 \quad \text{and} \quad P_{0i}^{Pa} \cdot P_{0i}^{Pa} = 1 \qquad \ldots(**)$$

$$(ii) \quad P_{0i}^{La} \times Q_{0i}^{La} = V_{0i} \text{ and } \quad P_{0i}^{Pa} \cdot P_{i0}^{Pa} = V_{0i} \qquad \ldots(***)$$

From (*), (**) and (***), we conclude that :

(*a*) If Laspeyre's price index is equal to Paasche's price index, then both of these index numbers satisfy the time reversal test.

(*b*) If Laspeyre's price index is equal to Paasche's price index, then both of these index numbers satisfy the factor reversal test.

3·4·4. Circular Test. This is another test for the adequacy of an index number. This test is based on the shiftability of the base and is an extension of the time reversal test. The test is that

$$P_{oi} \times P_{ii'} \times P_{i'0} = 1, \quad i \neq i' \neq 0 \quad \text{or} \quad P_{ab} \times P_{bc} \times P_{ca} = 1, \quad a \neq b \neq c \qquad \ldots(3\cdot19)$$

This test is satisfied *only* by the indices based on :

(*i*) Simple geometric mean of the price relatives, *viz.*, the formula (3·12) and

(*ii*) Kelly's fixed weight method.

For example, in case of simple G.M. of price relatives, price indices (without factor 100) are given by :

$$P_{ab} = \left[\prod_{j=1}^{n}\left(\frac{p_{bj}}{p_{aj}}\right)\right]^{1/n}, \quad P_{bc} = \left[\prod_{j=1}^{n}\left(\frac{p_{cj}}{p_{bj}}\right)\right]^{1/n}, \quad P_{ca} = \left[\prod_{j=1}^{n}\left(\frac{p_{aj}}{p_{cj}}\right)\right]^{1/n}$$

$$P_{ab} \times P_{bc} \times P_{ca} = \left[\prod_{j=1}^{n}\left(\frac{p_{bj}}{p_{aj}}\right) \times \prod_{j=1}^{n}\left(\frac{p_{cj}}{p_{bj}}\right) \times \prod_{j=1}^{n}\left(\frac{p_{aj}}{p_{cj}}\right)\right]^{1/n} = 1$$

Hence, circular test holds in this case.

Kelly's fixed base index (without factor 100) also gives :

$$P_{ab} \times P_{bc} \times P_{ca} = \frac{\Sigma w_j p_{bj}}{\Sigma w_j p_{oj}} \times \frac{\Sigma w_j p_{oj}}{\Sigma w_j p_{bj}} \times \frac{\Sigma w_j p_{aj}}{\Sigma w_j p_{cj}} = 1$$

Since the two formulae satisfying this test are not frequently used, the test need not be considered seriously.

Remark. It should be borne in mind that Fisher's index number formula, though known as ideal index number, does not satisfy the circular test.

Example 3·19. *Prepare price and quantity index numbers for 2005 with 2002 as base year form the following data [Table 3·14] by using :*

(i) Laspeyre's, (ii) Paasche's, (iii) Marshall-Edgeworth, and (iv) Fisher's Method.

Year	*Article I*		*Article II*		*Article III*		*Article IV*	
	Price	*Qty.*	*Price*	*Qty.*	*Price*	*Qty*	*Price*	*Qty*
2002	5·00	5	7·75	6	9·63	4	12·50	9
2005	6·50	7	8·80	10	7·75	6	12·75	9

With reference to the above, verify that the Factor Reversal Test and Time Reversal Test are satisfied by Fisher's formula.

Solution. TABLE 3·14A : CALCULATIONS FOR PRICE AND QUANTITY INDICES

Article	2002		2005		p_1q_0	p_0q_0	p_1q_1	p_0q_1
	p_0	q_0	p_i	q_i				
I	5·00	5	6·50	7	32·50	25·00	45·50	35·00
II	7·75	6	8·80	10	52·80	46·50	88·00	77·50
III	9·63	4	7·75	6	31·00	38·52	46·50	57·78
IV	12·50	9	12·75	9	114·75	112·50	114·75	112·50
Total					231·05	222·52	294·75	282·78

$$P_{oi}^{La} = \frac{\Sigma p_{ij} q_{oj}}{\Sigma p_{oj} q_{oj}} \times 100 = \frac{231·05}{222·52} \times 100 = 103·8 \;;\; Q_{oi}^{La} = \frac{\Sigma q_{ij} p_{oj}}{\Sigma q_{oj} p_{oj}} \times 100 = \frac{282·78}{222·52} \times 100 = 127·8$$

$$P_{oi}^{Pa} = \frac{\Sigma p_{ij} q_{ij}}{\Sigma p_{oj} q_{ij}} \times 100 = \frac{294·75}{282·78} \times 100 = 104·23 \;;\; Q_{oi}^{Pa} = \frac{\Sigma q_{ij} p_{ij}}{\Sigma p_{oj} p_{ij}} \times 100 = \frac{294·75}{231·05} \times 100 = 127·57$$

$$P_{oi}^{ME} = \frac{1}{2}\left(P_{0i}^{La} + P_{0i}^{Pa}\right) = 104·15 \;;\; Q_{oi}^{ME} = \frac{1}{2}\left(Q_{0i}^{La} + Q_{0i}^{Pa}\right) = 127·325$$

$$P_{oi}^{F} = \left(P_{oi}^{La} \times P_{oi}^{Pa}\right)^{1/2} = \left(\frac{231·05}{222·52} \times \frac{294·75}{282·78}\right)^{1/2} \times 100 = 104·1 \text{ (on simplification)}$$

$$Q_{oi}^{F} = \left(Q_{oi}^{La} \times Q_{oi}^{Pa}\right)^{1/2} = \left(\frac{282·78}{222·52} \times \frac{294·75}{231·05}\right)^{1/2} \times 100 = 1·2724$$

$$\therefore \quad P_{oi}^{F} \times P_{io}^{F} = \left(\frac{\Sigma p_{ij} q_{oj}}{\Sigma p_{oj} q_{oj}} \times \frac{\Sigma p_{ij} q_{ij}}{\Sigma p_{oj} q_{ij}}\right)^{1/2} \times \left(\frac{\Sigma p_{oj} q_{ij}}{\Sigma p_{ij} q_{ij}} \times \frac{\Sigma p_{oj} q_{oj}}{\Sigma p_{ij} q_{oj}}\right)^{1/2}$$

$$= \left(\frac{231·05}{222·52} \times \frac{294·75}{282·78} \times \frac{282·78}{294·75} \times \frac{222·52}{231·05}\right)^{1/2} = 1$$

$\Rightarrow$ Fisher's index satisfies Time Reversal Test

$$\text{and } P_{oi}^{F} \times Q_{oi}^{F} = \left(\frac{\Sigma p_{ij} q_{oj}}{\Sigma p_{oj} q_{oj}} \times \frac{\Sigma p_{ij} q_{ij}}{\Sigma p_{oj} q_{ij}}\right)^{1/2} \times \left(\frac{\Sigma q_{ij} p_{oj}}{\Sigma q_{oj} p_{oj}} \times \frac{\Sigma q_{ij} p_{ij}}{\Sigma q_{oj} p_{ij}}\right)^{1/2}$$

$$= \left(\frac{231·05}{222·52} \times \frac{294·75}{282·78} \times \frac{282·78}{222·52} \times \frac{294·75}{231·05}\right)^{1/2} = \frac{294·75}{222·52} = \frac{\Sigma p_{ij} q_{ij}}{\Sigma p_{oj} q_{oj}} = V_{oi}$$

$\Rightarrow$ Fisher's index satisfies Factor Reversal Test.

Hence Fisher's Ideal Index number satisfies both time reversal and factor reversal tests.

3·4·5. Errors in the Measurement of Price and Quantity Index Numbers and Their Control. Index numbers P_{oi} for prices are constructed from m commodities of a group under consideration. The m commodities are common to both base year (0) and given year (i). In practical calculations, we might not be in a position to consider all possible common commodities, say, M in number, to both years. Normally m commodities form a *part or sample* of M commodities. When we talk of a value index V_{oi}, it can be either in terms of M commodities $V_{oi}(M)$, or in terms of its sample with certain m commodities, $V_{oi}(m)$.

But in many economic analysis, when we talk of a value index (or ratio), the values for the group under consideration for two years may be calculated on the basis of total coverages of commodities in base and given years including certain unique commodities that are not common to both the years. Thus we may have a non-homogeneous group of commodities some of which are not present in both the years. The value index computed by individual year's total coverage of commodities will be denoted by $V_{oi}(\mu)$. If we are now given the problem of measuring the price and quantity factors $P_{oi}(\mu)$ and $Q_{oi}(\mu)$ of this value index $V_{oi}(\mu)$, they are subject to an error of measurement, and this error has the following three components :

(*i*) A formula error, (*ii*) Sampling error, and (*iii*) A homogeneity error.

The *formula error* arises from the fact that there is no universally accepted formula that will measure the price change or quantity change of a given body of data with exactitude. The stated mathematical tests are towards finding a solution to this type of error. If we denote by D, the difference between P_{oi}^{La} and P_{oi}^{Pa} or between Q_{oi}^{La} and Q_{oi}^{Pa}, D provides a measure for the formula consistency. Note that when D is negligible, the formula error tends to be absent.

The ***sampling error*** **of any price or quantity index based upon any given formula arises from the fact that the formula operates upon the commodity list of $V_{oi}(m)$** rather than upon that of $V_{oi}(M)$. The differences $P_{oi}(M) - P_{oi}(m)$ and $Q_{oi}(M) - Q_{oi}(m)$ are, therefore, the sampling errors. There are well-established methods of estimating and minimising this error. Mostly the index numbers in practical use can be expressed as averages of *relatives* and hence we are concerned with the sampling distribution of an average. Following features require explanation in a full statement of sampling error of an index number :

(*i*) Error of an average of a random sample. (*ii*) Effect of weighting.

(*iii*) Effect of sampling from a finite population. (*iv*) Effect of sample stratification.

The sampling error is generally controlled through increase in size of the sample. The formal rule is that error varies inversely with square root of sample size. In the present case as it is a finite sample, error decreases with the percentage the sample size bears with the total population (*i.e.*, $100m/M$). Most index number samples are stratified, hence it is assumed that the strata are known and that the sample is actually drawn proportionally from each stratum.

The third component is a *homogeneity error*. If it were possible to obtain an exact measurement of it, this magnitude would represent the difference between $P_{oi}(M)$ and $P_{oi}(\mu)$ and also between $Q_{oi}(M)$ and $Q_{oi}(\mu)$. No direct measure of the homogeneity error is available since there is no direct measure yet on $P_{oi}(\mu)$ and $Q_{oi}(\mu)$. Let us denote this error by E.

$$E = [P_{oi}(M) - P_{oi}(\mu)] \quad \text{or} \quad [Q_{oi}(M) - Q_{oi}(\mu)]$$

As there is no direct method of calculating E, we shall be content with indirect measurement of E by a measure R in actual situations.

The R-test for homogeneity. If R is defined for any pair of comparison periods, 0 and i as :

$$R = \frac{(\mu_i - M) + (\mu_o - M)}{\mu_i + \mu_o} = \frac{\mu_i + \mu_o - 2M}{\mu_i + \mu_o}$$

where μ_k = number of all commodities for the group under consideration in kth period, $k = 0, i$;

and M = number of commodities of the group that are common to both periods (*i.e.*, binary commodities).

Here $(\mu_i - M)$ and $(\mu_0 - M)$ are the numbers of unique commodities in periods i and 0. Obviously, we have $0 \le R \le 1$, where $R = 0$ represent complete homogeneity. When R tends to zero, the homogeneity error tends to be negligible and in such a case one can substitute $P_{oi}(M)$ for $P_{oi}(\mu)$ and $Q_{oi}(M)$ for $Q_{oi}(\mu)$. Note that when the base and the given periods are close, the value of R tends to zero and as such the homogeneity error is controlled through short period comparisons. For a long period comparison, in that case, one has to split the long period into several suitable short periods and their cumulative effect would measure the change over long period with a reduction in homogeneity error. It should be noted that the short period comparisons also reduce the formula error, as the value of D gets reduced there by in most cases.

The *fourth kind of error is the errors in the basic data* of index numbers which is common to any empirical analysis.

3·5. CLASSIFICATION OF INDEX NUMBERS

Broadly speaking there are three types of index numbers :

(*i*) ***Price index numbers*** **which measure the general changes in the retail or wholesale price level of a particular commodity or group of commodities.**

(*ii*) ***Cost of living index numbers*** **are intended to study the effect of change in the price level on the cost of living of different classes of people.**

(*iii*) *Quantity index numbers* which are indices to measure the changes in the quantity of goods manufactured in a factory, *e.g.*, the indices of *industrial production or agricultural production*.

In the following sequences, we shall discuss these index numbers briefly.

3·5·1. Economic Adviser's Wholesale Price Index Number. The official, general purpose index number of wholesale prices in India was issued by the Economic Adviser, Ministry of Commerce and Industry, in two series, one started in January 1947 with the year ending August 1939 as base year and the other (revised series) started in April 1956 with 1952-53 as base year. We shall discuss below, very briefly, the latter index number with 1952-53 as base.

Revised Index Number of Wholesale Prices (1952-53 = 100). This index number compiled by office of the Economic Adviser is based on 112 commodities classified into 6 groups according to the Standard International Trade classification and comprising 555 individual quotations as in *Table 3·15*.

TABLE 3·15 : COMMODITIES, MARKETS AND QUOTATIONS IN THE REVISED SERIES OF INDEX NUMBER OF WHOLESALE PRICES

Group	*No. of Commodities*	*No. of Markets*	*No. of Quotations*
1. Food	31	105	216
2. Liquor and Tobacco	3	5	10
3. Fuel, Power, Light and Lubricants	8	7	24
4. Industrial Raw Materials	23	37	84
5. Manufactured Articles :			
(*a*) Intermediate Products	14	7	44
(*b*) Finished Products	33	22	177
Total	112	183	555

For this revised index number, the prices data was collected for 295 quotations through *official sources* (like State Governments, State Bank of India, etc.) and for the remaining 260 quotations from *unofficial sources*, (like chamber of commerce, trade associations and leading business houses).

The weights were assigned to various commodities according to their production in the post-partition period 1948-49. Thus *while the comparison base for the revised index is 1952-53, the weight base is 1948-49.*

The weekly price quotations for various commodities, prevailing on or about Friday, were collected by the Office of the Economic Adviser. The *Commodity Indices* were obtained as the simple arithmetic mean of the price relatives of varieties; *sub-group* or *group indices* were obtained as the weighted arithmetic mean of commodity indices. Finally, the *general index* or *all commodities index* was obtained as a weighted *A.M.* of the group indices. These index numbers were issued through "Index Numbers of Wholesale Prices in India, revised series", published weekly by the Economic Adviser. Monthly indices were obtained as the arithmetic mean of weekly indices and annual indices were obtained as the arithmetic mean of the monthly indices.

New Series of Index Numbers of Whole-sale Prices (Base : 1961-62 = 100). The revised series of index numbers discussed above was replaced, on the recommendations of 'Whole-sale Price Index Revision Committee', by a new series on index numbers of wholesale prices in India with base 1961-62. It covers 139 commodities comprising 774 quotations. The *Table 3·16* gives group-wise distribution of commodities, markets, and quotations.

TABLE 13·16 : COMMODITIES, MARKETS AND QUOTATIONS AND WEIGHTS IN THE NEW REVISED SERIES OF WPI NUMBERS (1961 – 62 = 100)

Group	*No. of Commodities*	*No. of Markets*	*No. of Quotations*	*Weights* New Series 1961-62 = 100	*Weights* Revised Series 1952-53 = 100
1. Food Articles	38	128	275	413	504
2. Liquor and Tobacco	3	6	12	25	21
3. Fuel, Power, Light and Lubricants	10	7	28	61	30
4. Industrial Raw Materials	25	47	106	121	155
5. *Chemicals	11	6	16	7	—
6. *Machinery and Transport Equipment	7	18	83	79	—
7. Manufactured Articles :	45	43	254	294	290
(a) Intermediate Products	(13)	(7)	(43)	(57)	(41)
(b) Finished Products	(32)	(36)	(211)	(237)	(249)
Total	139	(225)	774	1,000	

* New Group as compared with Revised Index Number.

The new series of index numbers is also computed as the weighted arithmetic mean of the price relatives. The weekly price quotations for various varieties are collected on or about each Friday and the commodity indices, group and sub-group indices, and general (all commodity) indices are obtained exactly in the similar way as in the series of revised index numbers.

Series of Index Numbers of Wholesale Prices. (Base : (970-71 = 100). This series of index numbers was introduced since the first week of January 1977 and has an enlarged coverage and improved weighting system. In the new classification, the commodities are grouped under three major groups *viz.,*

1. Primary Articles, 2. Fuel, Power, Light and Lubricants, and 3. Manufactured products.

The following *Table 3·17* gives a comparative study of the number and distribution of commodities and quotations, along with their weights for the two series 1961-62 = 100 and 1970-71 = 100.

TABLE 3·17 : COMPARATIVE STUDY OF COMMODITIES, QUOTATIONS AND WEIGHTS FOR WPI SERIES 1961-62 = 100 AND 1970-71 = 100

Group	*No. of Commodities* 1961-62 Series	*No. of Commodities* 1970-71 Series	*No. of Quotations* 1961-62 Series	*No. of Quotations* 1970-71 Series	*Weights* 1961-62 Series	*Weights* 1970-71 Series
I. Primary Articles	61	80	340	405	416·67	425·4
(a) Food Articles	31	39	234	260	297·99	292·4
(b) Non-food Articles	22	26	92	115	106·21	129·6
(c) Minerals	8	15	14	30	12·47	3·4
II. Fuel, Power, Light and Lubricants	8	10	23	28	84·59	60·1
III. Manufactured Products	149	270	404	842	498·75	514·5
All Commodities	218	360	767*	1275	1000	1000

* Seven quotations have been dropped in the revised series (1970-71 = 100).

The WPI (1970-71 = 100) is calculated on the principle of weighted arithmetic mean according to the Laspeyre's formula in its modified form :

$$I = \frac{\sum_i I_i w_i}{\sum_i w_i},$$

where I_i is the index of the ith group and w_i is the weight allotted to the ith group.

Price relatives are calculated as the percentage ratios which current price quotations bear to those prevailing in the base year, *i.e.,* dividing the current price quotations by the base price (average of 1970-71) and multiplying it by 100. The commodity index is arrived at as the simple arithmetic average of price relatives of varieties or quotations in different markets. The index for sub-group and major groups is, in turn, derived from the weighted arithmetic mean of commodity indices. Similarly, the all commodities index is computed as the weighted arithmetic mean of major group indices.

The WPI numbers (Base : 1970-71 = 100) of the Economic Adviser to the Government of India (now in the Ministry of Industries), since the first week of January 1977 and were published in the monthly bulletin.

Wholesale Price Index Number Series (Base : 1981-82 = 100)

The WPI series with base 1981-82 = 100 was introduced in July 1989. In this series the commodities are divided into 3 groups and 16 sub-groups.

This series covers a total of 447 commodities as against 360 commodities in the earlier WPI series with base 1970-71. The break-up of the commodities in 3 major groups is given in the *Table 3·18.*

TABLE 3·18 : BREAK-UP OF COMMODITIES (1981-82 = 100)

Group	*No. of Commodities*
I. Primary articles	93
(a) Food articles	44
(b) Non-food articles	28
(c) Minerals	21
II. Fuel, Power, Light and Lubricants	20
III. Manufactured Products	334
Total	447

Weekly price quotations for the selected items/commodities for compilation of WPI are collected each week. Price-data are collected through official as well as non-official sources. The official sources include : Directorate of Economics and Statistics, Ministry of Agriculture, Agricultural Marketing Departments of Central and State Governments, State Directorate of Economics and Statistics, and other agencies belonging to State Governments. The non-official sources include the Chambers of Commerce, Trade Associations, leading manufacturers and business houses.

Some figures for WPI numbers (1981-82 = 100) are given in the *Table 3·19.*

TABLE 3·19 : INDEX NUMBERS OF WHOLESALE PRICES IN INDIA—BY GROUPS AND SUB-GROUPS (MONTH-END/YEAR-END DATA) (BASE : 1981-82 = 100)

Last week of month/ year ended Saturday	Weight	1990-91	1996-97	1997-98	1997	1998							
		(April-March)			Oct.	May	June.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
1	2	3	4	5	6	7	8	9	10	11	12	13	14
ALL COMMODITIES	100.000	191.8	320.1	337.1	331.6	344.6	350.0	353.2	354.3	357.1	359.8	358.5	354.8
I. Primary Articles	32.295	195.5	329.6	37.8	337.6	363.8	375.9	382.2	381.8	388.8	397.1	395.4	385.1
(A) Food Articles	17.386	210.8	377.9	393.2	382.6	422.9	438.1	445.9	443.3	460.2	475.2	464.8	446.7
(B) Non-Food Articles	10.081	210.5	328.5	356.4	342.1	355.9	368.5	379.0	382.0	375.4	376.1	388.3	386.1
(C) Minerals	4.828	109.0	158.1	166.5	166.4	167.5	167.6	159.8	159.8	159.8	159.8	160.0	160.0
II. Fuel, Power, Light & Lubricants	10.663	188.6	344.9	384.1	377.4	380.4	380.2	379.8	379.9	381.7	382.1	381.7	382.1
III. Manufactured Products	27.042	190.3	310.0	322.3	310.7	327.0	329.7	331.8	334.0	334.5	334.6	333.3	332.5

Current Series of Wholesale Price Indices (Base : 1993-94 = 100). As the base period of WPI series 1981-82 = 100 was getting quite distant, it was again revised with a view to reflect adequately the economic growth in various sectors of the economy. The current WPI series uses 1993-94 (=100) as base year. In this series also, the various commodities are divided into 3 major groups and 16 sub-groups as in 1981-82 WPI series.

Tables 3·20 and 3·21 give the figures for WPI numbers (1993-94 = 100) for some years.

TABLE 3·20 : INDEX NUMBER OF WHOLESALE PRICES
(By Major Groups and Sub-groups)
(Base : 1993-94 = 100)

Major Group/Sub-groups	*1994-95*	*1999-00*	*2000-01*	*2001-02*
I. Primary articles	115.8	158.0	162.5	168.4
1. Food articles	112.8	165.5	170.5	176.1
2. Non-food articles	124.2	143.0	146.5	152.9
3. Minerals	104.9	110.4	113.5	119.3
II. Fuel, power, light and lubricants	108.9	162.0	208.1	226.7
III. Manufactured products	112.3	137.2	141.7	144.3
1. Food products	114.1	151.3	145.7	145.4
2. Beverages, tobacco and tobacco products	118.3	174.1	179.8	193.8
3. Textiles	118.2	115.0	119.9	119.3
4. Wood and wood products	110.9	193.9	180.0	174.4
5. Paper and paper products	106.1	149.6	165.4	172.8
6. Leather and leather products	109.7	154.6	149.6	141.0
7. Rubber and plastic products	106.4	123.6	125.5	126.0
8. Chemicals and Chemical products	116.6	155.2	164.4	169.0
9. Non-metallic mineral products	110.9	127.4	133.9	144.0
10. Basic metals, alloys and metal products	108.4	135.0	140.3	140.7
11. Machinery and machine tools including electrical machinery	106.0	116.1	123.0	129.1
12. Transport equipments and parts	107.4	135.4	143.4	146.8
All Commodities	112.6	145.3	155.7	161.3

Source : Office of the Economic Adviser, Ministry of Industry (*Statistical Pocket Book India 2002*, Page 207)

TABLE 3·21 : INDEX NUMBERS OF WHOLESALE PRICES (Base : 1993-94 = 100)

	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.
All Commodities (100.00)												
2001-02	159.9	160.3	160.8	161.1	161.7	161.7	162.5	162.3	161.8	161.0	160.8	161.9
2002-03	162.3	162.8	164.7	165.6	167.1	167.4	167.5	167.8	167.2	167.8	169.4	171.6
2003-04	173.1	173.4	173.5	173.4	173.7							
Primary Articles (22·03)												
2001-02	165.1	167.1	169.7	168.6	169.4	170.2	170.8	170.0	169.3	166.4	166.7	167.3
2002-03	169.0	168.9	171.9	172.3	175.4	176.1	175.2	176.5	174.7	173.8	176.5	177.9
2003-04	179.9	180.8	183.8	180.5	178.6							
Fuel, Power, Light & Lubricants (14.23)												
2001-02	222.7	222.6	222.5	222.3	226.1	226.3	230.4	230.6	229.0	228.1	227.3	233.0
2002-03	230.4	230.4	233.8	238.4	237.8	238.8	240.9	240.8	238.8	241.8	244.3	254.1
2003-04	254.2	247.6	246.1	249.3	249.5							
Manufactured Products (63.75)												
2001-02	114.2	144.1	144.0	144.9	144.5	144.3	144.4	144.4	144.2	144.2	143.9	144.1
2002-03	144.9	145.5	146.8	147.1	148.5	148.6	148.4	148.5	148.6	149.2	150.3	151.0
2003-04	152.6	154.3	~~153.8~~	~~154.1~~	~~155.1~~							

Source : *CSO, Monthly Abstract of Statistics,* October 2003 :

3·5·2. Cost of Living (Consumer Price) Index Number. Cost of living index numbers are constructed to study the effect of changes in the prices of a basket of goods and services on the purchasing power of a particular class of people during current period as compared with some base period. Change in the cost of living of an individual between two periods means the change in his money income which will be necessary for him to maintain the same standard of living in both periods. Thus the cost of living index numbers are intended to measure the average increase in the cost of maintaining the *same* standard in a given year *as* in the base year. Since the consumption habits of people differ widely from class to class (such as poor, low income, middle income, rich, etc.) and even within the same class from region to region, the changes in the level of prices affect different classes differently and consequently the general price index numbers usually fail to reflect the effects of changes in the general price level on the cost of living of different classes of people. Cost of living index numbers are, therefore compiled, to get a measure of the general price-movement of the commodities consumed by different classes of people.

Remark. *'Cost of Living'* index numbers should not be interpreted as a measure of *'Standard of Living'*. There is no doubt that the prices, on which cost of living index number is based affect the purchasing habits of the people but 'price' is only one of the factors on which the standard of living of people depends. The other important factors which account for the standard of living, *viz.*, the size of the family, its age composition, its income or occupation, the place or the region, etc., are not taken into account while computing cost of living index. Accordingly the Sixth International Conference of Labour Statisticians, held under the auspices of I.L.O. in 1949, recommended that the term *'Cost of Living* 'index' should more appropriately be replaced by the term *'Consumer Price Index'* or *'Retail Price Index'*.

Main Steps on the Construction of Cost of Living Index Number :

• (a) *Scope and Coverage.* Fist step is to specify the particular population group or the class for which the index number is intended such as industrial workers, government employees, low income or middle income class people, etc. together with a well-defined geographical region of their stay such as a city or an industrial area or a particular locality in a city. As far as possible, the class should form a homogeneous group of people with respect to income.

(b) *Family Budget Enquiry.* Having decided about the class of people explained in (a), we conduct a sample family budget enquiry, *i.e.*, we select a sample of families from the class of people for whom the index is intended and scrutinise their budgets in detail. The enquiry should cover a reasonably adequate number of families and should be conducted during a normal period, *i.e.*, a period free from economic boom or depression. The purpose of the enquiry is to determine the amount an average family spends on different items of consumption. The family budget enquiry yields the following information :

(1) *The nature, quality and quantity of the commodities consumed by the people, the commodities being classified under the following heads :*

(*i*) Food, (*ii*) Clothing, (*iii*) Fuel and Lighting, (*iv*) House rent, and (*v*) Miscellaneous.

Each of these groups may further be divided into sub-groups, *e.g.*, 'Food' may be sub-divided into wheat, rice, pulses, ghee, sugar, etc. The head 'Miscellaneous' includes items such as education, recreation, medicine and medical charges, gifts, charities, dhobi, etc. The list however does not include non-consumption monetary transactions such as payment towards insurance premiums, provident fund, purchase of savings certificates and bonds, etc.

It should be borne in mind that for the computation of living index number, only those commodities should be included which are primarily used by the class of people for which it is meant, *i.e.*, the commodities should represent the tastes, habits and customs of the selected class of people.

(2) *The proportion which the expenditure on each item (in a group) bears to the expenditure on the whole group.*

(3) *The proportion, which expenditure on each group bears to the total expenditure on all the groups.*

(e) *The retail price of different commodities.* The question of obtaining retail price quotations, though very important, is quite difficult and tedious since the retail prices vary form place to place, shop to shop and person to person. The price quotations should be obtained from the 'local markets' where the class of people reside or from super bazars, fair price shops or departmental stores from which they usually make their purchases.

Remark. In order to draw a sample of the families which would be representative of the class for which index number is intended, the following procedure may be adopted :

(*i*) Stratify the families into groups *w.r.t.* income, *e.g.*, income below Rs. 100 P.M.; between Rs. 100 and 150, etc., if such information can be easily obtained.

(*ii*) Depending on families to be covered, *i.e.*, the sample size n being fixed in advance (due to certain considerations), a suitable sampling fraction may be decided upon ~~for~~ each stratum and the families may be sampled by the method of systematic sampling with a random start (*c.f.* Chapter 7).

Construction of Cost of Living Index. Cost of living index number is constructed by the following formulae :

(*i*) *Aggregate Expenditure (or Weighted Aggregates) Method.* In this method weights to the assigned to various commodities are provided by the quantities consumed in the base year. Thus, in the usual notations :

$$\text{Cost of Living Index} = \frac{\sum_j p_{ij}\, q_{oj}}{\sum_j p_{oj}\, q_{oj}} \times 100 \qquad \ldots(3{\cdot}20)$$

$$= \frac{\text{Total expenditure in current year with base year quantities as weights}}{\text{Total expenditure in base year}} \times 100$$

Remark. This is nothing but Laspeyres' index and is the most popular method of constructing cost of living Index numbers.

(*ii*) *Family Budget Method or the Method of Weighted Relatives* : In this method, Cost of Living Index is obtained on taking the weighted average of price relatives, the weights being the values of quantities consumed in the base year. Thus in the usual notations, if we write

$$\text{Price Relative} = P_j = \frac{p_{ij}}{p_{oj}} \times 100 \quad \text{and} \quad w_j = p_{oj}\, q_{oj} \;;\; j = 1, 2, \ldots, n,$$

then

$$\text{Cost of Living Index} = \frac{\sum_j w_j P_j}{\sum_j w_j} \qquad \ldots(3{\cdot}21)$$

***Remarks* 1.** It may be noted that the cost of living index numbers by both the methods agree, since

$$\frac{\sum w_j P_j}{\sum w_j} = \frac{\sum p_{oj}\, q_{oj}\, (p_{ij}/p_{oj}) \times 100}{\sum p_{oj}\, q_{oj}} = \frac{\sum p_{ij}\, q_{oj}}{\sum p_{oj}\, q_{oj}} \times 100,$$

which is same as (3·20).

2. The Sixth International Conference of Labour Statisticians held in Geneva in 1946 suggested that the period of enquiry of family budgets and the base periods should be identical as far as possible.

Uses of Cost of Living Index Number. **1.** Cost of living index numbers indicate whether the real wages are rising or falling, money wages remaining unchanged. In other words they are used for the calculation of real wages and for determining the change in the purchasing power of the money. We have :

$$\text{Purchasing Power of Money} = \frac{1}{\text{Cost of Living Index Number}} \qquad \ldots(3{\cdot}21a)$$

$$\text{Real Wages} = \frac{\text{Money Wages}}{\text{Cost of Living Index}} \times 100 \qquad \ldots(3{\cdot}21b)$$

2. Cost of living indices are used by the administrators for :

(*a*) Regulation of dearness allowance or the grant of bonus to the workers so as to enable them to meet the increased cost of living.

(*b*) Settling disputes relating to wages and salary.

3. These indices are also used for deflation of income and value series in national accounts.

4. A contracts manager in a construction company uses them in price adjustment clauses of building contracts.

5. Cost of living index numbers are used widely in wage negotiations and wage contracts. For example, they are used for automatic adjustment of wages under '*Escalator clauses*' in collective bargaining agreements. Escalator clause provides for certain point automatic increase in the wages corresponding to a unit increase in the consumer price index.

6. By itself, cost of living index number does not throw much light on the inflationary or deflationary trend on the soundness of an economy but in conjunction with other tools such

as the indices of wholesale prices, wages, profits, production, employment, etc., it serves as an economic indicator for the analysis of price situation.

7. Tax authorities need them to compute cost inflation indices to determine capital gains, and thereby capital gains tax to be imposed on the tax-payers.

3·5·3. Consumer Price Indices (CPI)—Indian Scenario. In India, a number of consumer price indices are compiled at national and regional levels, and disseminated for general use. The important consumer price indices, compiled at national level in India are :

(*i*) Consumer Price Index for Urban Non-Manual Employees [CPI (UNME)]

(*ii*) CPI for Industrial Workers [CPI (IW)]

(*iii*) CPI for Agricultural Labours [CPI (AL)]

(*iv*) CPI for Rural Labourers [CPI (RL)].

Among these indices, the first index [CPI (UNME)] is compiled by CSO and the last three are compiled by the Labour Bureau.

A number of States are also regularly compiling and publishing consumer price indices for specified target populations. For example, West Bengal Government is compiling CPI for selected urban centres in the State. A 'Rural Consumer Price Index' (RCPI) is compiled by the Government of Uttar Pradesh. The Governments of Maharashtra and Karnataka bring out Retail Price Index (RPI) for urban and rural populations separately. Haryana compiles CPI (R) and CPI (IW) for the State. Andhra Pradesh and Tamil Nadu bring out CPI (IW) for the respective States.

1. Consumer Price Index for Urban Non-Manual Employees. [CPI (UNME)] The reference population of CPI (UNME) is restricted to households obtaining predominantly their income from gainful employment in non-manual work in non-agriculture sector in urban areas.

The CSO has been compiling CPI (UNME) since 1960. The base year of the earlier series was 1960. The CPI (UNME) series, with 1960 as the base year, was based on a Middle Class Family Living Survey (MCFLS) conducted by the CSO in 1958-59. The survey covered about 36,000 families of non-manual employees, residing in 45 cities and towns spread throughout the country. This survey provided the base-data for deriving the weights required for construction of the index. The centres were selected by taking into account their administrative importance, middle class concentration, and regional representation. The number of centres allotted to various states were broadly in proportion to the urban population of the respective State. The index for each centre involved approximately 180 items of goods and services.

The current CPI(UNME) series is having the base year 1984-85. These indices are separately compiled on monthly basis for 59 urban centres, together with the corresponding All-India general index. The weights for the current CPI(UNME) series are based on a Family Living Survey (FLS), conducted during 1982-83 in 59 selected urban centres spread all over the country. The survey covered 45,000 families of urban non-manual employees, who, by definition, derived 50 per cent or more of their income from gainful employment on occupations of one or more of its members doing non-manual work in the non-agricultural sector. **Prior to regular price-data collection, market surveys were carried out during the year 1983 in each of the centres. Using the amended weights, and results of the market survey, the revised CPI(UNME) series, with 1984-85 base, is being compiled and released every month for use by general public since November 1987.**

CPI(UNME) series are regularly compiled and published in the Monthly Abstract of Statistics, brought out by CSO, Department of Statistics, Ministry of Planning and Programme Implementation, Government of India. Summary-values of the CPI(UNME) series are also published in some other publications such as monthly RBI Bulletin and the Indian Labour Journal.

The *Tables 3·22 and 3·23* give CPI(UNME) figures for some years in India.

TABLE 3·22 : CONSUMER PRICE INDEX FOR URBAN NON-MANUAL EMPLOYEES

(Base : 1984-85 = 100)

Centre	*Weight*	*1990-91*	*1999-00*	*2000-01*	*2001-02*
Ahmedabad	1.68	153	316	337	350
Bangalore	2.75	161	365	389	412
Bhopal	1.64	166	343	361	375
Kolkata	6.92	164	328	344	355
Chandigarh	0.78	176	429	445	464
Chennai	3.86	168	386	420	456
Cuttack (1)	0.92	154	357	365	379
Delhi (2)	6.56	156	359	381	398
Hyderabad (3)	1.99	164	357	383	410
Jaipur	1.39	165	357	371	388
Lucknow	1.53	158	326	342	364
Mumbai	8.40	154	353	375	395
Patna	1 28	167	340	344	359
Shillong	0.22	179	359	382	406
Shimla	0.40	163	356	377	394
Srinagar	0.32	150	364	393	403
Triruvananthapuram	1.36	152	338	362	384
All India (4)	100.00	161	352	371	390

Source : Central Statistical Organisation

(1) Includes Bhubaneswar.

(2) Includes New Delhi.

(3) Includes Secunderabad.

(4) All India Index covers 59 centres including the selected centres listed above.

Secondary Source : STATISTICAL POCKET BOOK INDIA-2002.

TABLE 3·23 : PRICE INDEX NUMBER OF CONSUMER PRICES (URBAN NON-MANUAL EMPLOYEES (*BASE : 1984-85 = 100*)

	Apr.	*May*	*June*	*Jul*	*Aug.*	*Sep.*	*Oct.*	*Nov.*	*Dec.*	*Jan.*	*Feb.*	*Mar.*
All-India (100.00)												
2001-02	379	382	386	391	393	392	393	395	394	393	392	395
2002-03	396	398	402	406	407	408	408	408	405	406	408	410
2003-04	413	415	417	421	420	420	423					

Source : *CSO, Monthly Abstract of Statistics, October 2003.*

II. (CPI For Industrial Workers. [CPI (IW)]. The consumer price indices for industrial workers [CPI (IW)] with the original base of 1960 was having as its 'source of weights' a Family Living Survey (FLS) that was conducted during 1958-59 in 50 selected centres. A second FLS was conducted in 1981-82 in 70 centres; which enabled the revision of the services to 1982 base. The current series of CPI (IW) is prepared with Base : 1982 = 100. The following *Table 3·24.* gives the CPI (IW) for selected centres in India for the years 1999 to 2001.

TABLE 3·24 : INDEX NUMBER OF CONSUMER PRICES—INDUSTRIAL WORKERS

(Selected Centres)

(Base : 1982 = 100)

Centre	*1991*	*1999*	*2000*	*2001*
Ahmedabad	218	422	441	460
Ajmer	217	411	433	452
Alwaye	195	423	442	458
Amritsar	204	379	388	403
Balaghat	213	382	385	409
Bangalore	204	405	425	438
Barbil	207	390	411	420
Bhilai	196	373	390	407
Kolkata	218	437	451	492
Chennai	208	446	475	487
Coimbatore	197	402	432	441
Delhi	218	480	514	529
Guwahati	208	436	460	471
Hyderabad	203	395	419	438
Indore	222	425	445	470
Jamshedpur	204	397	405	419
Jharia	198	363	363	365
Kanpur	222	428	428	447
Mumbai	226	468	505	528
Nagpur	223	438	461	483
Nasik	228	432	465	498
Pune	217	466	493	516
Raniganj	201	373	380	399
Rourkela	194	396	406	407
Srinagar	202	471	480	520
Yamunanagar	201	392	412	428
All India (1)	212	424	441	458

Source : ***Labour Bureau.***

(1) All India Index covers 70 centres including the above.

Secondary Source : ***Statistical Pocket Book India, 2002.***

The following ***Table 3·25*** **gives CPI (IW) on All India Basis for the years 2001-02, 2002-03 and 2003-04**

TABLE 3·25 : INDEX NUMBERS OF CONSUMER PRICES (INDUSTRIAL WORKERS)

(Base : 1982 = 100)

	Apr.	*May*	*June*	*July*	*Aug.*	*Sep.*	*Oct.*	*Nov.*	*Dec.*	*Jan.*	*Feb.*	*March*
All-India (100.00)												
2001-02	448	451	457	463	466	465	468	472	469	467	466	468
2002-03	469	472	476	481	484	485	487	489	484	483	484	487
2003-04	493	494	497	501	499	499	503					
All-India : Food (57.00)												
2001-02	450	455	464	468	473	469	474	479	473	464	460	462
2002-03	463	463	473	479	482	484	486	487	479	474	475	479
2003-04	488	490	494	499	494	493	500					
All-India : Clothing (8.54)												
2001-02	322	322	323	323	323	325	325	326	327	328	328	330
2002-03	330	363	332	332	332	333	331	331	331	333	335	335
2003-04	337	337	337	339	339	338	335					

Source : *CSO : Monthly Abstract of Statistics*, October 2003.

Consumer Price Index for Agricultural and Rural Labours. The Labour Bureau has been compiling and publishing Consumer Price Index Numbers for Agricultural Labourers [CPI (AL)] since September, 1964 on base 1960-61 = 100. This old series of index members was updated to new base 1986-87 = 100 with enlarged coverage and enhanced sample size w.e.f. November, 1995. While the old series was being compiled for Agriculturer Laboures alone, in the new Series (Base 1986-87 = 100) two separate indices are compiled for Rural Labourers [CPI (RL)] and Agricultural Labourers [CPI (AL)]. The rural retail prices for these two index numbers are the same, but the weighting diagrams for each of the two series are different.

For the construction of [CPI (RL)] and its subset [CPI (AL)] (Base 1986-87 = 100), rural retail price data from a fixed set of 600 representative sample villages spread over 66 NSS regions in 20 States are collected every month by NSSO since July 1986.

The weights for all the 20 States and all-India have been derived from the results of the consumer expenditure survey conducted by National Sample Survey Organisation (NSSO) during its Thirty Eight (38th) Round held in 1983. The Survey recorded the consumer expenditure on a wide range of goods and services, which have been categorised into the following five groups :

(*i*) Food,

(*ii*) Pan, Supari, Tobacco and Intoxicants,

(*iii*) Fuel and Light,

(*iv*) Clothing, bedding and footwear, and

(*v*) Miscellaneous.

Note. The base period for the new series (1986-87 = 100) has been taken as agricultural year (July, 1986 to June, 1987). Though there is a gap between the weight base (1983) and the comparison base **(1986-87), no adjustment in weights due to difference in price level between two periods has been made.**

Compilation of Indices

The indices for both the series [CPI (AL) and CPI (RL)] are compiled separately for each of the 20 States at general as well as at disaggregated (*i.e.*, group/sub-group) level. In the compilation of the State level index, the price relative of each item in respect of each village is worked out. A simple average of village-wise price relatives is worked out to arrive at index number is constructed with triennium ending 1981-82 as base. The proportion of total value of production of each crop during the base year is the weight assigned to each crop.

TABLE 3·54 : INDEX NUMBERS OF AGRICULTURAL PRODUCTION
(Base : Triennium Ending 1981 – 82 = 100)

	Weight	1970–71	1980–81	1990–91	1991–92	1992–93	1993–94	1994–95	1999–00	2000–01	2001–02
1	2	3	4	5	6	7	8	9	10	11	12
A. Food grains	62.92	87.9	104.9	143.7	137.6	144.3	150.2	155.9	169.7	157.5	171.0
(*a*) Cereals	54.98	84.1	105.0	144.2	140.4	146.9	153.0	158.3	175.1	165.1	176.5
Rice	29.74	84.4	107.8	149.4	150.2	146.5	161.5	164.5	180.3	170.07	184.2
Wheat	14.45	67.7	103.2	156.6	158.2	162.5	170.0	186.8	217.0	195.4	203.0
Coarse cereals	10.79	105.4	99.8	113.1	89.4	127.0	106.7	103.2			
(*b*) Pulses	7.94	113.6	104.1	140.5	118.6	126.5	131.0	139.0	132.1	105.4	133.0
Gram	3.07	126.3	105.4	130.2	100.2	107.4	121.1	156.5	124.4	85.6	129.3
B. Non-food grains	37.08	82.6	97.4	156.3	158.8	163.7	169.4	180.9	187.7	176.8	185.2
(*a*) Oilseeds	12.64	97.1	95.1	179.5	181.9	193.6	203.4	208.4	193.3	176.4	194.1
Groundnut	5.60	101.8	83.4	125.3	118.3	142.8	130.5	134.4	87.7	103.7	122.7
Rapeseed and mustard	2.41	97.2	113.0	256.3	287.3	235.4	261.1	282.2	283.7	206.2	227.9
(*b*) Fibers	5.09	65.6	94.2	128.2	128.5	144.8	137.0	151.2	149.5	127.6	151.7
Cotton	4.37	63.4	93.2	130.9	129.2	151.6	142.8	158.1	153.3	128.3	155.4
Jute	0.55	76.5	100.8	122.6	138.3	116.0	113.9	123.8	145.9	141.9	149.2
Mesta	0.14	77.3	96.7	76.7	79.0	64.0	62.5	63.2	65.1	65.7	67.3
(*c*) Plantation crops	2.29	73.2	76.0	144.9	146.1	162.2	163.5	174.5	188.1	185.1	188.1
Tea	1.46	74.7	101.6	132.3	125.5	125.5	132.6	134.4	143.6	143.6	143.6
Coffee	0.44	79.1	85.1	122.3	129.5	116.2	149.7	129.6	148.2	148.2	148.2
Rubber	0.39	60.8	101.1	217.6	242.0	259.7	287.3	311.4	399.3	399.3	399.3
(*d*) Others											
Sugarcane	8.11	81.2	98.8	154.3	162.6	145.9	147.0	176.3	191.6	191.5	187.0
Tobacco	1.12	75.5	100.2	115.8	121.7	124.2	117.2	118.0	109.1	101.7	101.7
Potatoes	2.09	50.2	103.9	163.3	176.0	163.6	186.8	186.9	265.4	237.8	237.8
C. All Commodities	100.00	85.9	102.1	148.4	145.5	151.5	157.3	165.2	176.4	164.7	176.3

Source : *Directorate of Economics & Statistics, Department of Agriculture & Co-operation Ministry of Agriculture* and Statistical Pocket Book 2002

Example 3.34. *Calculate Index Numbers of foodgrains production for the years 1980-81 and 1981-82 with base 1979-80 = 100 from the following data:*

Foodgrains	*Weight*	*Production (million tonnes)*		
		1979 – 80	*1980 – 81*	*1981 – 82*
Rice	*34*	*42*	*53*	*54*
Wheat	*12*	*29*	*32*	*35*
Jowar	*5*	*11*	*12*	*12*
Bajra	*3*	*6*	*5*	*6*
Other cereals	*6*	*11*	*12*	*13*
Pulses	*10*	*10*	*11*	*12*

Solution. The indices of food grains production for the years 1980-81 and 1981-82 with base year 1979-80, using fixed weights are obtained by Kelly's fixed weight formula.

TABLE 3·55 : CALCULATIONS FOR FOODGRAINS PRODUCTION INDICES

Foodgrains	*Weight* (w)	*Production (million tonnes)* 1979-80	1980-81	1981-82	*Weighted Production*		
		q_0	q_1	q_2	q_0w	q_1w	q_2w
1. Rice	34	42	53	54	1,428	1,802	1,836
2. Wheat	12	29	32	35	348	384	420
3. Jowar	5	11	12	12	55	60	60
4. Bajra	3	6	5	6	18	15	18
5. Other cereals	6	1	12	13	66	72	78
6. Pulses	10	10	11	12	100	110	120
					2015	2443	2532

$$Q_{01}{}^K = \frac{\Sigma q_1 w}{\Sigma q_0 w} \times 100 = \frac{2,443}{2,015} \times 100 = 121{\cdot}2\ ;\ Q_{02}{}^K = \frac{\Sigma q_2 w}{\Sigma q_0 w} \times 100 = \frac{2,532}{2,015} \times 100 = 125{\cdot}6$$

Hence, indices of foodgrains production for the years 1980-81 and 1981-82 *w.r.t.* base 1979-80 are 121·2 and 125·6 respectively.

3.9. USES OF INDEX NUMBERS

Though originally designed to study the general level of prices or accordingly purchasing power of money, today index numbers are extensively used for a variety of purposes in economics, business management, etc. and for quantitative data relating to production, consumption, profits, personnel phenomenon for two periods, places, etc. The main uses of index numbers can be summarised as follows :

1. *Index Numbers as Economic Barometers.* In the words of **G. Simpson and Kafka**, *"Index numbers are today one of the most widely used statistical devices... They are used to take the pulse of the economy and they have come to be used as indicators of inflationary or deflationary tendencies."* They are indispensable tools for the management personnel and in business planning and formulation of executive decisions. The indices of prices (wholesale and retail), output (volume of trade, import and export, industrial, and agricultural production) and bank deposits, foreign exchange and reserves etc., throw light on the nature of variation in the general economic and business activity of the country and gives us a fairly good appraisal of the general trade, economic development and business activity of the country.

2. *Index Numbers help in Studying Trends and Tendencies.* Since the index numbers study the relative changes in the level of phenomenon at different periods of time, they are specially useful for the study of the general trend for a group phenomenon in a time series data. The indices of output (industrial and agricultural production), volume of trade, import and export, etc., are extremely useful for studying the changes in the level of phenomenon due to the various components of a time series *viz.*, secular trend, seasonal and cyclical variations and irregular components and reflect upon the general trend of production and business activities. As a measure of average change in extensive group, the index numbers can be used to forecast future events.

3. *Index Numbers help in Formulating Decisions and Policies.* Index numbers of the data relating to prices, production, profits, imports and exports, personnel and financial matters are indispensable for any organisation in efficient planning and formulation of executive decisions. For example, the cost of living index numbers are used by the government and the industrial and business concerns for the regulation of dearness allowance (D.A.) or grant of bonus to the workers so as to enable them to meet the increased cost of living from time to time. Although index numbers are now widely used to study the general economic and business numbers are now widely used to study the general economic and business conditions of the society, they are also applied with advantage by sociologists (population indices), psychologists (I Q's), health and educational authorities, etc., for formulating and revising their policies from time to time.

4. Consumer price indices are useful for : (*i*) Measuring the purchasing power of money, (*ii*) deflation of income and (*iii*) wage negotiations and wage contracts.

For details see § 3·5·2.

3.10. LIMITATIONS OF INDEX NUMBERS

Although index numbers are indispensable tools in economics, business management, etc., they have their limitations and proper care should be taken in using and interpreting them. Some of their limitations are enumerated below:

1. Since index numbers are computed from sample data, all the errors inherent in any sampling procedure creap in its construction. Hence the index numbers reflect only approximately changes in the relative level ~~of a phenomenon.~~

2. At each stage of the construction of the index numbers, starting from selection If commodities to the choice of formula there is likelihood of the error being introduced. An attempt should be made to minimise these errors, as far as possible. [For details see terms 2 to 7, § 3.2].

3. Due to rapid advancements in science and technology these days, there is a rapid change in the tastes, customs and fashions and consequently in the pattern of consumption of various commodities among the people in a society. Hence index numbers, may not be able to keep pace with the changes in the nature and quality of the commodities consumed at the two periods being considered and hence may not be truly representative.

4. None of the formula for the construction of index numbers is exact and contains the so-called '*Formula error*'. For example, Laspeyer's index has an upward bias while Paasche's index has a downward bias.

5. Index numbers are special type of averages. Since the various averages (mean, median, geometric mean) have their relative limitations, their indiscriminate use may also introduce some error. [For details, see item 5, § 3·2].

6. By subjective selection of base year, commodities, price and quantity quotations, index numbers are liable to be manipulated by selfish persons to obtain the desired results.

7. Also see § 3·4·5. Errors in the measurement of Price and Quantity index numbers and their control.

In spite of the above limitations, the index numbers if properly constructed with caution are extremely useful devices.

DISCUSSION AND REVIEW QUESTIONS

1. What is an index number ? Describe briefly the problems that are involved in the construction of an index number of prices.

2. (*a*) "Index numbers are economic barometers." Elucidate

(*b*) "The index numbers are responsible to indicate an upward or downward trend in the value of the phenomenon (or variate) under study which help in the study of economic prosperity of the people of a certain area and so they are sometimes known as 'Economic Indicators'. Discuss.

3. (*a*) Discuss the importance and limitations of index numbers Explain the various stages in the construction of the wholesale price index numbers of a set of a group of 'related variables'.

(*b*) "Index numbers are devices for measuring differences in the magnitude of a group of related variables." Discuss the statement and point out the important uses of Index Numbers.

(*c*) Discuss the problem of the construction of Index Number of wholesale price with special reference to (*i*) selection of base period, (*ii*) selection of commodities and markets, (*iii*) selection of the type of average, and (*iv*) weighting.

4. (*a*) "Weighting is only one of the many problems in the construction of an index number." Discuss. What are the other major problems ?

(*b*) Explain the importance of using appropriate weights in framing index numbers and point out some important systems of weighting.

(*c*) Describe Laspeyre's and Paasche's methods of weighting index relatives. Which one would you prefer and why ?

5. (*a*) Define the following Index Numbers and discuss their merits and demerits :

(*i*) Laspeyre's Index number, (*ii*) Paasche's Index Number, and (*iii*) Fisher's Ideal Index Numbers.

(*b*) "It is sometimes stated that Laspeyer's price index tends to overestimate price changes while Paasche's price index tends to underestimate price changes." Substantiate this statement.

6. (*a*) (*i*) Explain the concept of an index number.

(*ii*) Distinguish between aggregrative type and average type index number formulae.

Develop Fisher's Ideal Index and examine its merits and demerits.

(*b*) What are Marshall-Edgeworth, Laspeyre's and Paasche's index numbers? Prove that the Marshall-Edgeworth index number lies between Laspeyre's and Paasche's index numbers.

7. (*a*) Let $x = (p_1/p_0)$, $y = (q_1/q_0)$, $f_{x,y} = p_0\, q_0$, $V_{01} = (\Sigma p_1\, q_1 / \Sigma\, p_0 q_0)$. Show that

$$\frac{([\Sigma\, p_1 q_0) / (\Sigma\, p_0 q_0)]}{[(\Sigma\, p_1 q_0) / (\Sigma\, p_0 q_0)]} = 1 - \frac{r_{xy}\, S_x\, S_y}{V_{01}}$$

Interpret this result in regard to the Laspeyre's and Paasche's Index Numbers.

(*b*) If $L(p)$ and $P(q)$ represent Laspeyre's index number for prices and Paasche's index number for quantities, show that $\quad L(p) \times P(q) = V_{01},$

where V_{01} is the value index number. Hence or otherwise show that $\quad \dfrac{L(p)}{L(q)} = \dfrac{P(p)}{P(q)}$.

Show also that if $A(p) = \frac{1}{2}[L(p) + P(q)]$ then $A(p)$ is greater than the Fisher's Ideal Index Number.

(*c*) Explain what is meant by 'bias' in an index number. Give an example of an index having an 'upward bias'.

What is meant by reversibility of an index number ? What index numbers are reversible ?

8. (*a*) Define the terms: price relative, quantity relative and value relative and show how they are related to one another.

(*b*) Explain briefly the methods of computing index numbers :

(*i*) by the simple average of relatives method, (*ii*) by weighted aggregrative method, and

(*iii*) by simple aggregrative method.

Give an example for each method.