Paper 6630

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Q1. a)

Random Scan Eg: pen plotter	Raster Scan Eg: Home TV sets, printers
1) Electronic beam is directed only to the parts of screen where a picture is to be drawn	Electric beam is swept from top to bottom, across the screen
2) It results in good resolution	poor resolution
3) Picture definition is stored as a set of line drawing instruction in a display file	Stored as a set ~~line~~ intensity values in a refresh buffer area
4) Refresh rate ranges from 30 to 60 frames per second	60 - 80
5) Produce smooth line drawings	Produce jagged lines that result as discrete set points

b) Resolution: No. of points per cm that can be plotted horizontally and vertically. Simply, it is the total no. of points in each direction.

Aspect Ratio: It gives the ratio of vertical to horizontal points necessary to produce equal length lines in both dir. of the screen.

GOOD WRITE

GOOD WRITE t=1 at r_4

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Q2. centre(0,0) , radius = r

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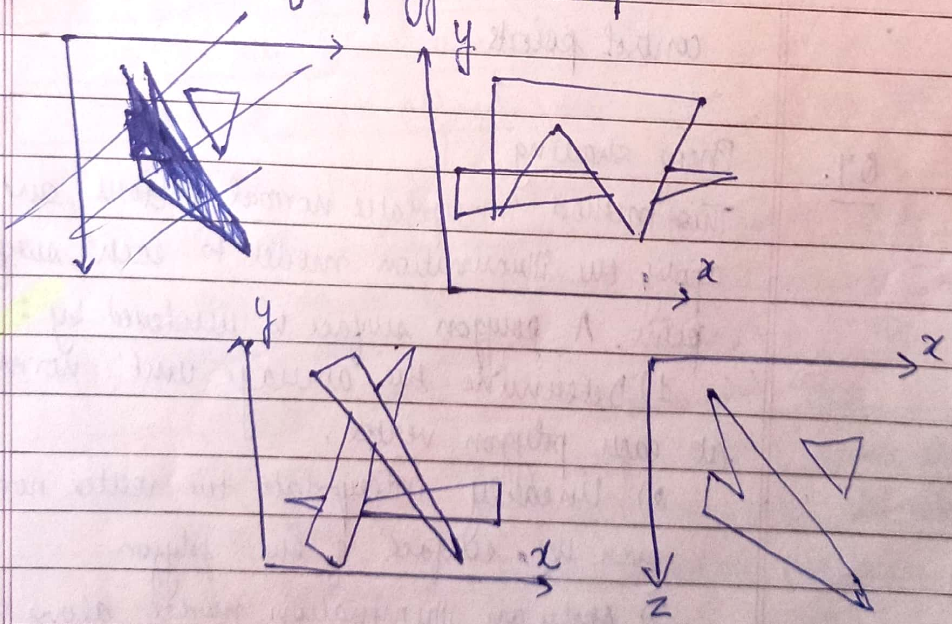
due to an all-or-nothing approach to scan conversion in which each pixel is either replaced with the primitive's color or is left unchanged.

3. (a) Aliasing : Primitives drawn have jaggies or staircasing. The phenomenon is aliasing.
Two techniques to remove this effect :
1) Increasing resolution
2) Weighted area sampling
3) Unweighted area sampling

4. (b) Two methods to specify motion between two key frames :
i) Kinematic description as a set of spline curves
2) Physically based by specifying the forces acting on the objects to be animated

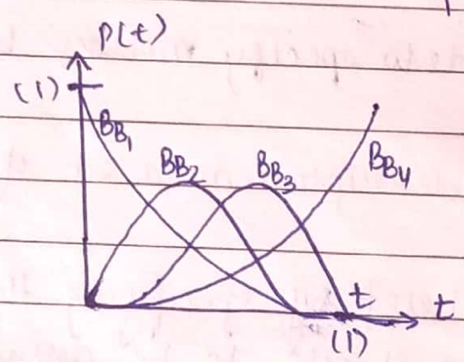
5. (b) z-extent of polygons overlap :



3) The degree of the polynomial defining the curve is one less than no. of defining polygon point i.e. for four control points, degree of polynomial is 3.

Q6. (b) Bezier curve

1. At $t=0$, only B_{B_1} is nonzero, so the curve interpolates P_1. Similarly at $t=1$, only B_{B_4} is nonzero, and the curve interpolates P_4.
where B = Bernstein polynomial



2. Each curve segment, which is sum of four control points weighted by the polynomials, is completely contained in the convex hull of the four control points.

Q7. Phong shading

This method interpolates normal vectors, and then apply the illumination model to each surface point. A polygon surface is rendered by:

1) Determine the average unit normal vector at each polygon vertex.

2) Linearly interpolate the vertex normals over the surface of the polygon

3) Apply an illumination model along each scan line to calculate projected pixel intensities for the surface points.

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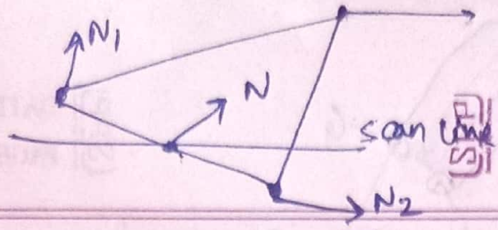
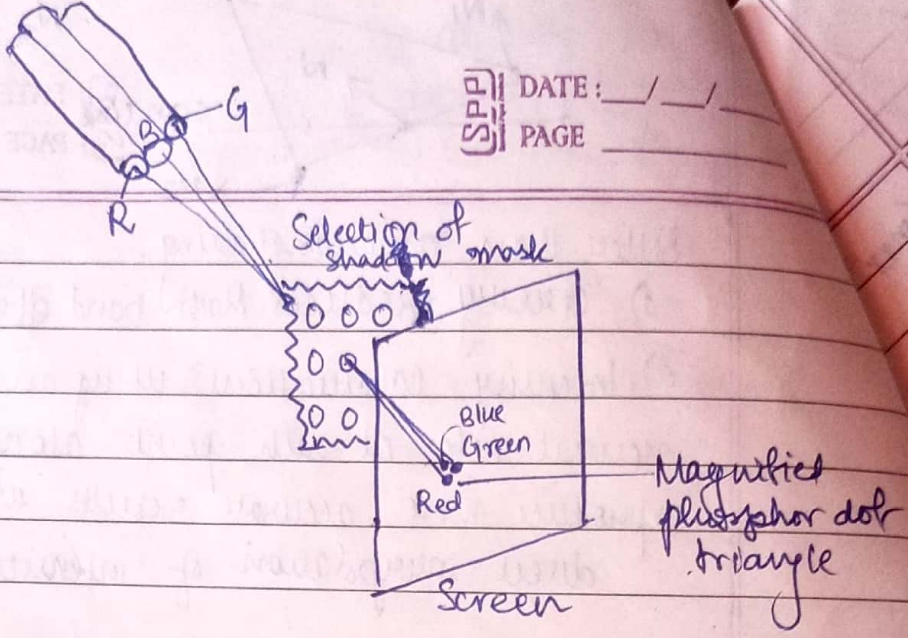


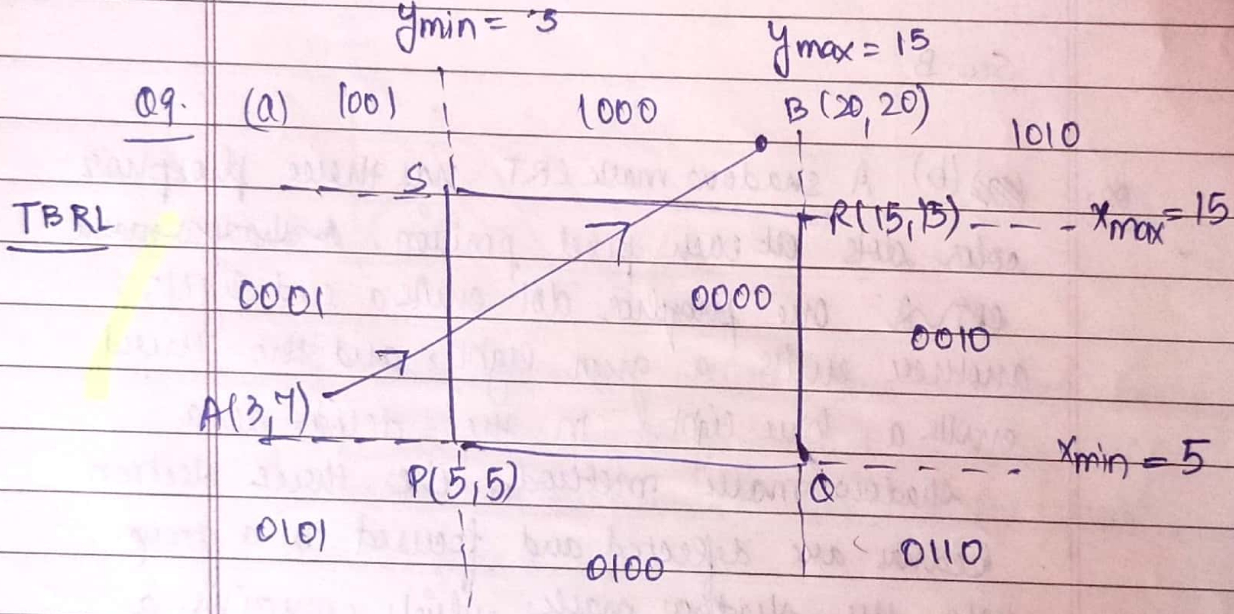
$$N = \frac{y - y_2}{y_1 - y_2} N_1 + \frac{y_1 - y}{y_1 - y_2} N_2$$

Better than gouraud shading:
1) Greatly reduces Mach band effect
2) Intensity calculation using an approximated normal vector at each point along the scan line produce more accurate results than the direct interpolation of intensities.

Sec. B

Q8. (b) A shadow mask CRT has three phosphor color dots at each pixel position. ~~A shadow mask CRT~~ One phosphor dot emits a red light, another emits a green light, and the third emits a blue light. In the delta-delta shadow mask method, the three electron beams are deflected and focused as a group onto the shadow mask, which contains a series of holes aligned with the phosphor-dot patterns. When the three beams pass through a hole in the shadow mask, they activate a dot triangle, which appears a small color spot on the screen. The phosphor dots in the triangles are arranged so that each electron beam can activate only its corresponding color dot when it passes through the shadow mask.





Given :

$^{w}x_{min} = 5$ $\quad$ $^{w}y_{min} = 5$

$^{w}x_{max} = 15$ $\quad$ $^{w}y_{max} = 15$

~~1) Clipping point A against boundary line $^{w}y_{min} = 5$.~~

There are four possible cases when processing vertices in sequence around the polygon perimeter :

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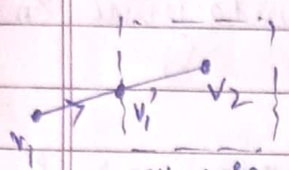
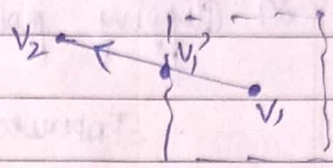
Test 1



out → in

o/p → intersect pt + destination pt.
$= V_1' V_2$

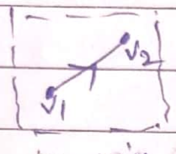
Test 2



in → out

o/p → intersect point
$= V_1'$

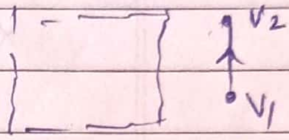
Test 3



in → in

o/p → dest. pt
→ V_2

Test 4



out → out

o/p → NULL

1) clipping against boundary line $y_{min} = 5$
(to push) to 0)

$(x_1, y_1) = (3, 7)$ $\quad (x_2, y_2) = (20, 20)$

$$\therefore m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{20 - 7}{20 - 3} = \frac{13}{17}$$

(x_{min}, y')

Intersection point $I_1(x', y')$:

$$y' = y_1 + m(x_{min} - x_1)$$
$$= 7 + \frac{13}{17}(5 - 3)$$

$$\boxed{y' = 8.43}$$

$\therefore I_1(x', y') = (5, 8.43)$

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2) Clipping point B against boundary line $x_{max} = 15$

Intersection
New point $I_1(x', y')$ and $B(x_2, y_2)$ form a line with point B lying outside the window

$\therefore$ Intersection point $I_2(x'', y'')$: (x'', y_{max})

$$x'' = x_2 + \left(\frac{y_{max} - y_2}{m}\right)$$

$$= 20 + \left(\frac{15 - 20}{17}\right) 13$$

$$= 20 + \left(\frac{(-5)(13)}{17}\right)$$

$$= 20 - 3.82$$

$$= 16.18$$

$\therefore I_2(x'', y'') = (16.18, 15)$

3) Finally, clipped line $I_1(5, 8.43)$ to $I_2(16.18, 15)$ has region code 0000 and lies within the window.

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(b) Data structures used :

1) ET : Edge table contains the list of all edges by storing their endpoint coordinates. Within each bucket, edges are kept in order of inc. x coord. of lower endpoint.

2) AET : Active edge list contains all those edges of the polygon that are intersected by ~~the~~ current scan line. The edges are dropped into the table in a sorted manner i.e. ~~increasing~~ intersection value of x.

Q10. (a) Step 1: Rotate an object (anticlockwise) about y axis by an angle $90°$

Step 2: Reflect through yz plane

Step 3: Uniformly double the size.

Matrix 1: $\theta = 90°$

$$[R_y] = \begin{bmatrix} \cos 90 & 0 & -\sin 90 & 0 \\ 0 & 1 & 0 & 0 \\ +\sin 90 & 0 & \cos 90 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Matrix 2 : yz plane

$$[Ref_{yz}] = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

GOOD WRITE $t=1$ at r_4

Matrix 3 : $s = 2$

$$[S] = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ Final transformation matrix obtained :

$$[T] = [R_y][Ref_{yz}][S]$$

$$= \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -2 & 0 \\ 0 & 2 & 0 & 0 \\ -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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(b) $[X^*] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

Step 1: Rotate by $\phi = 30^\circ$ about y axis
Step 2: Translate by -2 units in y dir.
Step 3: Single point pers. proj on $z=0$ plane from centre of proj. at $z = z_1 = 3$ units
$r = -1/z = -1/3 = -0.33$

$[T] = [R_y][Tr_y][Pr_z]$

$= \begin{bmatrix} \cos 30 & 0 & -\sin 30 & 0 \\ 0 & 1 & 0 & 0 \\ \sin 30 & 0 & \cos 30 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -0.33 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} \sqrt{3}/2 & 0 & -1/2 & 0 \\ 0 & 1 & 0 & 0 \\ 1/2 & 0 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -0.33 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

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$$= \begin{bmatrix} \sqrt{3}/2 & 0 & -1/2 & 0 \\ 0 & 1 & 0 & 0 \\ 1/2 & 0 & \sqrt{3}/2 & 0 \\ 0 & -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -0.33 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[T] = \begin{bmatrix} \sqrt{3}/2 & 0 & -1/2 & 1/6 \\ 0 & 1 & 0 & 0 \\ 1/2 & 0 & \sqrt{3}/2 & -1/2\sqrt{3} \\ 0 & -2 & 0 & 1 \end{bmatrix}$$

$\frac{\sqrt{3}}{2} \times \frac{1}{2\sqrt{3}}$

New triangle :

$$[X^*] = [X][T]$$

$$= \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & 0 & -1/2 & 1/6 \\ 0 & 1 & 0 & 0 \\ 1/2 & 0 & \sqrt{3}/2 & -1/2\sqrt{3} \\ 0 & -2 & 0 & 1 \end{bmatrix}$$

$\frac{1}{6} - \frac{1}{2\sqrt{3}} + 1$

0.288

$\frac{-1}{2\sqrt{3}} + 1$

$$= \begin{bmatrix} 0.5 & -2 & 0.866 & 0.712 \\ 1.366 & -2 & 0.366 & 0.878 \\ 1.366 & -1 & 0.366 & 0.878 \\ 0.5 & -1 & 0.866 & 0.712 \\ 0 & -2 & 0 & 1 \\ 0.866 & -2 & -0.5 & 1.166 \\ 0.866 & -1 & -0.5 & 1.166 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

$0.28 + 1$

$\frac{\sqrt{3}}{2} + \frac{1}{2}$

2.73

$-\frac{1}{2} + \frac{\sqrt{3}}{2}$

As unity $h \neq 1$ here, we convert the transformed triangle:

$R_1 \rightarrow R_1/0.712$ $\quad$ $R_2 \rightarrow R_2/0.878$

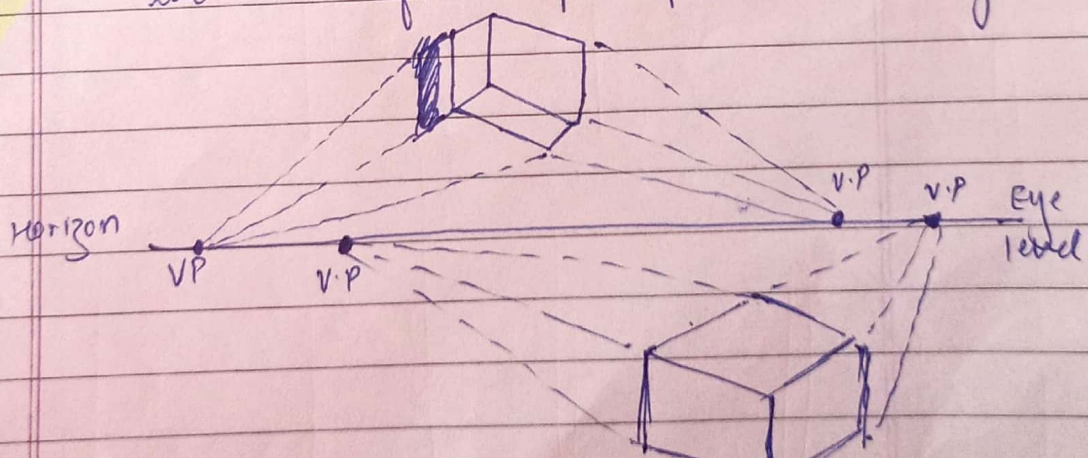
$R_4 \rightarrow R_4/0.712$ $\quad$ $R_3 \rightarrow R_3/0.878$

$R_6 \rightarrow R_1/1.166$

$R_7 \rightarrow R_7/1.166$

$$[X^*] = \begin{bmatrix} 0.702 & -2.80 & 1.216 & 1 \\ 1.55 & -2.27 & 0.416 & 1 \\ 1.55 & -1.13 & 0.416 & 1 \\ 0.702 & -1.40 & 1.216 & 1 \\ 0 & -2 & 0 & 1 \\ 0.742 & -1.71 & -0.428 & 1 \\ 0.742 & -0.85 & -0.428 & 1 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

Q11. (a) Vanishing points: Points on the horizontal reference line at which lines originally $\parallel$ to the untransformed principal axes converge.



 t=1 at r4

(c)

Step 1 : Defining G_{H_x}, the x component of the Hermite Geometry matrix :

$$G_{H_x} = \begin{bmatrix} P_1 \\ P_4 \\ R_1 \\ R_4 \end{bmatrix}_x$$

where P_1 P_4 are end points of curve and R_1, R_4 are tangent vectors at P_1 and P_2 respectively

Step 2 : Rewriting $x(t)$:

$$x(t) = a_x t^3 + b_x t^2 + c_x t + d_x$$
$$= T \cdot C_x$$
$$= T \cdot M_H \cdot G_{H_x}$$

where $x(t)$ is one of the three cubic polynomials that define a curve segment $Q(t)$.
$0 \leq t \leq 1$ with $T = [t^3\ t^2\ t\ 1]$
and matrix of coefficients :

$$C_x = \begin{bmatrix} a_x \\ b_x \\ c_x \\ d_x \end{bmatrix}$$

Now, $x(t) = [t^3\ t^2\ t\ 1] \cdot M_H \cdot G_{H_x}$ — ①

For the curve end points P_1 and P_4, values of t are :

$t = 0$ at P_1

$t = 1$ at P_4

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Step 3: The constraints $x(0)$ and $x(1)$ are found by direct substitution:

$$x(0) = P_{1x} = [0 \ 0 \ 0 \ 1] \cdot M_H \cdot G_{Hx} \quad —② $$

$$x(1) = P_{4x} = [1 \ 1 \ 1 \ 1] \cdot M_H \cdot G_{Hx} \quad —③ $$

Step 4: Differentiating eq ①:

$$x'(t) = [3t^2 \ 2t \ 1 \ 0] \cdot M_H \cdot G_{Hx} \quad —④ $$

Step 5: Putting $t = 0 (R_1)$ and $t = 1 (R_4)$

∴ The tangent vector constraint eq can be written as:

$$x'(0) = R_{1x} = [0 \ 0 \ 1 \ 0] \cdot M_H \cdot G_{Hx} \quad —⑤ $$
$$x'(1) = R_{4x} = [3 \ 2 \ 1 \ 0] \cdot M_H \cdot G_{Hx} \quad —⑥ $$

~~Step 6~~ Now the constraints ②, ③, ⑤ and ⑥ can be written in matrix form as:

$$\begin{bmatrix} P_1 \\ P_4 \\ R_1 \\ R_4 \end{bmatrix}_x = G_{Hx} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 3 & 2 & 1 & 0 \end{bmatrix} \cdot M_H \cdot G_{Hx} \quad —⑦ $$

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Step 6 : For this equation (7) to be satisfied, M_H must be the inverse of the 4x4 matrix

$$\therefore M_H = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 3 & 2 & 1 & 0 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Step 7 : Expanding the product $T \cdot M_H$ in $Q(t) = T \cdot M_H \cdot G_H$ gives Hermite Blending f^n and the parametric eq^n :

$$Q(t) = T \cdot M_H \cdot G_H$$

$$= B_H \cdot G_H$$

$$= [t^3 \; t^2 \; t \; 1] \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_1 \\ P_4 \\ R_1 \\ R_4 \end{bmatrix}$$

$$= (2t^3 - 3t^2 + 1) P_1 + (-2t^3 + 3t^2) P_4 + (t^3 - 2t^2 + t) R_1 + (t^3 - t^2) R_4$$

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Q12: (a) Phong-specular reflection model

It is an empirical model for calculating the specular-reflection range. It sets the intensity of specular reflection proportional to $\cos^{n_s}\phi$. Angle ϕ can be assigned values in the range $0°-90°$ so that $\cos\phi$ varies from 0 to 1. The value assigned to specular reflection parameter n_s is determined by the type of surface to be displaced. A very shiny surface is modeled with large n_s value and smaller n_s values for duller surface are used. for a perfect reflector, n_s is 1. The intensity of specular reflection depends on material properties of the surface, angle of incidence, polarization and color of incident light. $W(\theta)$ (Specular reflection coefficient) tends to increase ~~over the range $\theta=0$ to $\theta=90°$~~ ~~for a few~~ as the angle of incidence increases.

The Phong specular-reflection model can be written as:

$$I_{spec} = W(\theta) I_l \cos^{n_s}\phi$$

where

I_l = intensity of light source

ϕ = viewing angle relative to specular-reflection direction R.

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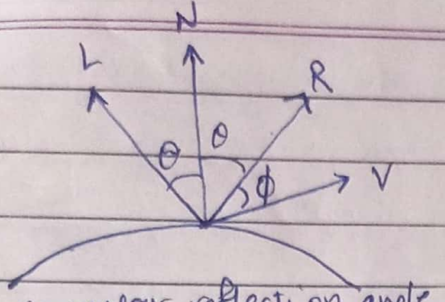
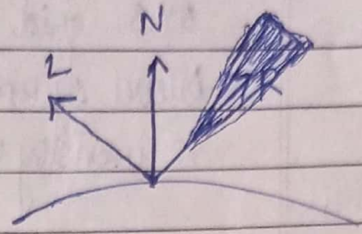
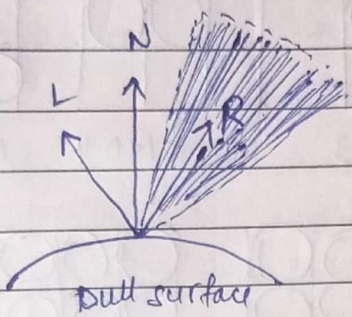


specular reflection angle equals angle of incidence θ



shiny surface (large n_s)



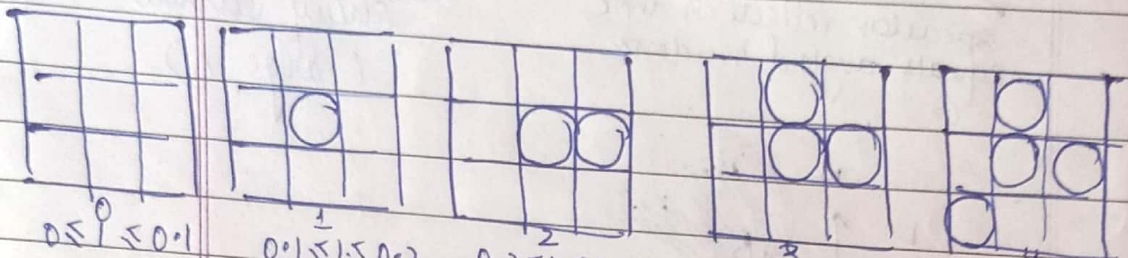
Dull surface (small n_s)

Assuming the specular refl. coefficient is constant, the intensity of specular refl. at the surface point:

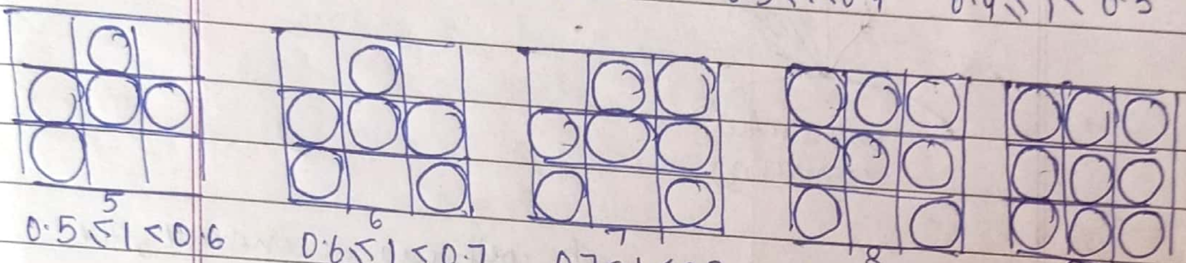
$$I_{spec} = k_s I_l (V \cdot R)^{n_s}$$

(b) continuous tone photographs are reproduced for magazines, newspaper etc. using ~~that~~ printing technique called halftoning. For a black-and white photo, each intensity area is reproduced as a series of black circles on a white background. The diameter of each circle is proportional to the darkness required ~~at~~ for than intensity region. Darker region are printed with larger circles and lighter regions with smaller circles. Coloured halftones are printed using dots of various ~~sizes~~ sizes & colours.

3X3 grid
bilevel system
10 intensity levels



0 ≤ I < 0.1 (0) | 0.1 ≤ I < 0.2 (1) | 0.2 ≤ I < 0.3 (2) | 0.3 ≤ I < 0.4 (3) | 0.4 ≤ I < 0.5 (4)



0.5 ≤ I < 0.6 (5) | 0.6 ≤ I < 0.7 (6) | 0.7 ≤ I < 0.8 (7) | 0.8 ≤ I < 0.9 (8) | 0.9 ≤ I ≤ 1.0 (9)

Mask used :

$$\begin{bmatrix} 8 & 3 & 7 \\ 5 & 1 & 2 \\ 4 & 9 & 6 \end{bmatrix}$$

Q13. Warnock's algorithm subdivides each area into four equal squares. At each stage in the recursive subdivision process, the projection of each polygon has one of four relations to the area of interest :

1. Surrounding polygons completely contain the (shaded) area of interest
2. Intersecting polygons intersect the area
3. Contained polygons are completely inside the area.

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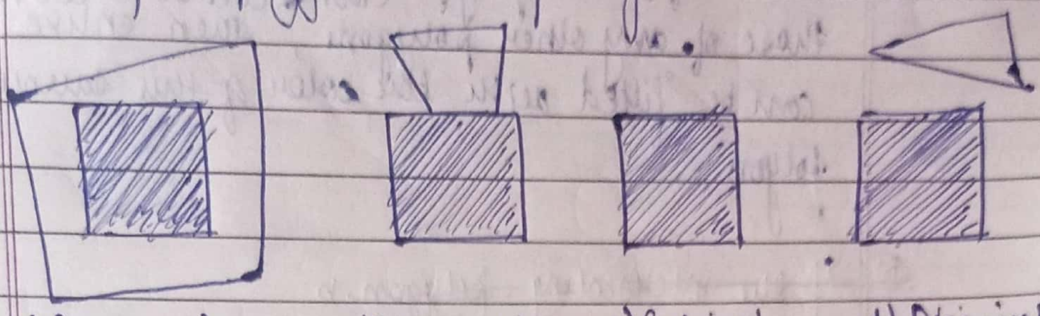
4. Disjoint polygons are completely outside the area.



Disjoint polygons and the part of intersecting polygon that is outside the area irrelevant. In four cases, decision can be made easily and so the area does not need to be divided further:

1. All polygons are disjoint from the area

The bg color can be displayed in the area

2. There is only one intersecting or only one contained polygon. The area is first filled with bg color, and then part of the polygon contained in the area is scan converted.
3. There is a single surrounding polygon, but no inters. of contained polygons. The area is filled with the color of surrounding polygon
4. More than one polygon is intersecting, contained in, or surrounding the area, but one is a surrounding polygon that is in front of all other polygons. Determining whether a surr. polygon is in front is done by computing the z coordinates of the planes of all surr., inters. and contained polygons at the four corners of the

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area. If there is a surr. polygon (four corner z coordinates) those of any other polygons, then entire area can be filled with the color of this surrounding polygon

5. ~~If the intersecting polygon is~~
When the surrounding polygon seems to be in front of the intersecting polygon, the algo. subdivides the area to simplify the problem. After subdivision, only contained and intersecting poly. needs to be reexamined. Surr. & disjoint polygons of the original area are surr. and disjoint polygons of each subdivided area.

(b)
Zero accelerations (~~cons~~ constant speed): We use equal-interval time spacing for the in-betweens. Suppose we want n in-betweens for key frames at times t_1 and t_2. The time interval b/w k.f. is then divided into $n+1$ subintervals, yielding in-bet. spacing of:

$$\Delta t = \frac{t_2 - t_1}{n+1}$$

Time for in-between: $tB_j = t_1 + j\Delta t$, $j = 1, 2 \dots n$

Positive acceleration (increasing speed): we want the time spacing b/w k.f. to increase so that greater changes in position occur as the object move faster. ~~The~~ Increasing interval size can be obtained by:
$1 - \cos\theta$, $0 < \theta < \pi/2$

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For n in-betweens, time for jth in-between :

$$t_{Bj} = t_1 + \Delta t \left[1 - \cos\left(\frac{j\pi}{2(n+1)}\right)\right], \quad j = 1, 2 \ldots n$$

where Δt = time diff. b/w two k.f.

Deacceleration (decreasing speed) : It can be modeled with $\sin\theta$ in the range $0 < \theta < \pi/2$. The position of an in-between :

$$t_{Bj} = t_1 + \Delta t \sin\left(\frac{j\pi}{2(n+1)}\right), \quad j = 1, 2 \ldots n$$