

* Aim :-

To check the unbiasedness of the estimators.

* Experiment :-

(a) A random experiment of size 3 is drawn from a population with mean μ . Then find the value of k such that the estimator $T = 2x_1 + 3x_2 - kx_3$ is unbiased for μ .

(b) Find the unbiased estimator of σ^2 from $ax^2 + bx + c$ for $(1+\sigma)(2+\sigma)$ given x follow poisson distribution with parameter σ .

* Theory & Formula used :-

Unbiased estimator : An estimator $T_n = T(x_1, x_2, \dots, x_n)$ is said to be unbiased estimator of θ if

$$E(T_n) = \theta \quad \forall \theta \in \Phi \quad \Phi = \text{Parameter Space}$$

* Result :-

$$(a) k = 4$$

$$(b) \text{ Unbiased Estimator} : x^2 + 2x + 2$$

* Calculations :-

(a) T is an unbiased estimator of μ
 $E(T) = \mu$

$$E(2x_1 + 3x_2 - kx_3) = \mu$$

$$2E(x_1) + 3E(x_2) - kE(x_3) = \mu$$

$$2\mu + 3\mu - k\mu = \mu$$

$$\begin{cases} E(ax+by) \\ = aE(x)+bE(y) \\ E(\mu) = \mu \end{cases}$$

$$k = 4$$

(b) Since X follows poisson distribution with parameter θ so

$$V(x) = \theta \quad \& \quad E(X^2) = V(x) + (E(x))^2$$

$$E(x^2) = \theta + \theta^2$$

Now, let $ax^2 + bx + c$ be an unbiased estimator of $(1+\theta)(2+\theta)$

$$\text{so } E(ax^2 + bx + c) = (1+\theta)(2+\theta)$$

$$aE(x^2) + b.E(x) + c = \theta^2 + 3\theta + 2$$

$$a(\theta^2 + \theta(a+b)) + c = \theta^2 + 3\theta + 2$$

$$a\theta^2 + \theta(a+b) + c = \theta^2 + 3\theta + 2$$

$$a = 1, \quad a+b = 3, \quad c = 2$$

$$a = 1, \quad b = 2, \quad c = 2$$

so Estimator is $x^2 + 2x + 2$

* Aim :-

To check whether the given estimators are unbiased or not & find the best estimator among them.

* Experiment :-

A random sample x_1, x_2, \dots, x_5 of size 5 is drawn from a population (normal) with unknown mean μ . Consider the following estimators to estimate μ .

$$(i) \quad f_1 = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$$

$$(ii) \quad f_2 = x_1 + x_2 + \cancel{x_3} + x_3$$

$$(iii) \quad f_3 = \frac{2x_1 + x_2 + \lambda x_3}{3}$$

where λ is such that f_3 is unbiased for μ .

Find λ . Are f_1 & f_2 unbiased? State given reason.
Select the best estimator.

* Theory & Formula used :

Unbiasedness :- An estimator $T_n = T(x_1, x_2, \dots, x_n)$ is unbiased for θ if

$$E(T_n) = \theta \quad \forall \theta \in \Phi$$

Efficiency :- The concept of efficiency refers to the sampling variability of the estimators. If two estimators are unbiased then one with smaller variance is said to be relatively more efficient.

Best estimator is some kind of estimator that fulfills all the four property of estimation like unbiasedness, consistency, efficiency, sufficiency.

* Result :-

$$\lambda = 4 \quad , \quad f_1 \rightarrow \text{unbiased} \quad , \quad f_2 \rightarrow \text{biased}$$

~~f_1~~ and ~~f_2~~ ~~are~~ f_1 and f_3 are unbiased but $V(f_1) < V(f_3)$ so f_1 is the most efficient estimator. So f_1 is the best estimator

* Calculations :

lets calculate Expected value & variance of the estimators.

$$(i) \quad f_1 = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$$

$$E(f_1) = E\left(\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}\right)$$

$$E(f_1) = \frac{5\mu}{5} = \mu$$

$$V(f_1) = V\left(\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}\right)$$

$$V(f_1) = \frac{5\sigma^2}{25} = \frac{\sigma^2}{5}$$

$$(ii) \quad f_2 = \frac{x_1 + x_2}{2} + x_3$$

$$E(f_2) = \frac{E(x_1 + x_2)}{2} + E(x_3)$$

$$E(f_2) = \frac{2\mu}{2} + \mu = 2\mu$$

$$V(f_2) = V\left(\frac{x_1 + x_2}{2} + x_3\right)$$

$$V(f_2) = \frac{2\sigma^2}{4} + \sigma^2 = \frac{3}{2}\sigma^2$$

(iii) t_3 is unbiased for μ

$$\text{so } E(t_3) = \mu$$

$$E\left(\frac{2x_1 + x_2 + \lambda x_3}{3}\right) = \mu$$

$$2\mu + \mu + \lambda\mu = 3\mu$$

$$\lambda\mu = 0$$

$$\lambda = 0$$

$$\begin{aligned} V(t_3) &= \frac{V(2x_1 + x_2)}{9} \\ &= \frac{4\sigma^2 + \sigma^2}{9} = \frac{5}{9}\sigma^2 \end{aligned}$$

Now we have

$$E(T_1) = \mu, \quad V(T_1) = \frac{\sigma^2}{5}$$

$$E(T_2) = 2\mu, \quad V(T_2) = \frac{3\sigma^2}{2}$$

$$E(T_3) = \mu, \quad V(T_3) = \frac{5\sigma^2}{9}$$

$$\lambda = 0$$

* Aim :-

To construct 95% confidence interval for the given sample.

* Experiment :-

An industrial designer wants to know the average amount of time it takes an adult to assemble an "easy-to-assemble" toy. Use the following data (in min), a random sample, to construct 95% confidence interval for the mean of population sampled.

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|
| 17 | 13 | 18 | 19 | 17 | 21 | 22 | 16 | 28 | 21 |
| 26 | 23 | 24 | 20 | 8 | 17 | 21 | 32 | 18 | 25 |
| 16 | 10 | 20 | 22 | 9 | 14 | 22 | 12 | 14 | 28 |
| 15 | 29 | | | | | | | | |
| 22 | 17 | | | | | | | | |
| 11 | 30 | | | | | | | | |

* Theory & Formula used :-

Mean :- Avg. value or most common value in a collection.

$$\bar{x} = \frac{\sum f x}{\sum f} = \frac{\sum f x}{n}$$

Required Confidence Interval :

$$\bar{x} - Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Instructor's Sign

where $Z_{\alpha/2} \rightarrow$ see from table

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

n = Sample size

σ = Standard Deviation

μ = Population Mean

\bar{x} = Sample Mean

α = level of significance

* Result :-

Required Confidence Interval :

$$18.05 < \mu < 21.79$$

* Calculations :-

$$n = 36 \quad , \quad \sum f_x = 717$$

$$\text{so} \quad \bar{x} = \frac{717}{36} = 19.92$$

$$\alpha = 0.05 \Rightarrow \alpha/2 = 0.025$$

$$Z_{0.025} = 1.96 \quad \& \quad \sigma = 5.73$$

$$Z_{0.025} \cdot \frac{\sigma}{\sqrt{n}} = 1.96 \times \frac{5.73}{\sqrt{36}} = 1.87$$

Now Required Confidence Interval

$$\text{lower limit} = \bar{x} - Z_{0.025} \cdot \frac{\sigma}{\sqrt{n}}$$

$$= 19.92 - 1.87 = 18.05$$

$$\text{Upper limit} = \bar{x} + Z_{0.025} \cdot \frac{\sigma}{\sqrt{n}}$$

$$= 19.92 + 1.87 = 21.79$$

$$18.05 < \mu < 21.79$$

* Aim :-

To construct 94% confidence interval for the difference between mean of two samples.

* Experiment :-

Construct a 94% confidence interval for the difference between the means of lifetimes of two kind of light bulbs. Given that a random sample of 40 light bulbs of first kind lasted on the average 418 hrs of continuous use and 50 light bulbs of second kind on the average 402 hrs of continuous use. The population standard deviation are known to be $\sigma_1 = 26$ and $\sigma_2 = 22$

* Theory & Formula used :-

If we want to find out the difference between means of two sampled population & with known variances then we use following statistics:

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

So required confidence interval

$$(\bar{x}_1 - \bar{x}_2) - Z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + Z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

where ~~n~~ n_1, n_2 = Sample size of first & second population
 \bar{x}_1, \bar{x}_2 = Sample mean of first & second population
 μ_1, μ_2 = Population mean " "
 σ_1^2, σ_2^2 = Variance " "

* Result :-

Required Confidence Interval

$$6.31 < \mu_1 - \mu_2 < 25.69$$

Since the difference is positive that means first kind of light bulbs are superior to second kind.

* Calculations :-

$$d = 0.06 \quad \& \quad Z_{\alpha/2} = Z_{0.03} = 1.88$$

| I | II |
|-------------------|-------------------|
| $n_1 = 40$ | $n_2 = 50$ |
| $\bar{x}_1 = 418$ | $\bar{x}_2 = 402$ |
| $\sigma_1 = 26$ | $\sigma_2 = 22$ |

$$\text{So } Z_{\alpha/2} \cdot \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = 1.88 \sqrt{\frac{576}{40} + \frac{484}{50}} \\ = 9.69$$

So required confidence interval

$$\text{lower limit} = (\bar{x}_1 - \bar{x}_2) - Z_{\alpha/2} \cdot \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \\ = 16 - 9.69 = 6.31$$

$$\text{Upper limit} = (\bar{x}_1 - \bar{x}_2) + Z_{\alpha/2} \cdot \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \\ = 16 + 9.69 = 25.69$$

$$\therefore 6.31 < \mu_1 - \mu_2 < 25.69$$

* Aim :-

To test the hypothesis for the parameters of a normal distribution.

* Experiment :-

(a) In 12 test runs over a marked course, a newly designed motorboat averaged 33.6 sec with a standard deviation of 2.3 sec. Assuming that it is reasonable to treat the data as a random sample from a normal population. Test the null hypothesis $\mu = 35$ against $\mu < 35$ at 0.05 level of significance.

(b) A study of the no. of business launches that executives in the insurance and banking industry claim as deductible expenses per month was based on random samples and yields the following result

$$n_1 = 40, \bar{x}_1 = 9.1, s_1 = 1.9$$

$$n_2 = 50, \bar{x}_2 = 8.0, s_2 = 2.1$$

Test the null hypothesis $\mu_1 - \mu_2 = 0$ against the alternative $\mu_1 - \mu_2 \neq 0$ with 0.05 level of significance.

* Theory & Formula used :-

Test of significance is a

statistical test where we check the validity of H_0 (null hypothesis) against H_1 .

(alternative hypothesis) on the basis of available evidences.

Test of Significance

One tail test

$$H_0 : \mu = 0$$

$$H_1 : \mu > 0 \text{ or } \mu < 0$$

Two tail test

$$H_0 : \mu = 0$$

$$H_1 : \mu \neq 0$$

Depends on sample size (n) & population variance (σ^2), we use Z and t statistics and check the validity of H_0 with the help of critical region.

In this question we will use

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} \quad \text{where } \bar{x} = \text{sample mean}$$

μ_0 = population mean

s = sample standard deviation

n = sample size

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

* Result :-

- (a) H_0 must be rejected because $-2.1086 < -1.96$
- (b) H_0 must be rejected because $2.604 > 1.96$

* Calculations :-

(a) Given : $n = 12$, $\bar{x} = 33.6$,
 $s = 2.3$, $\mu_0 = 35$, $\alpha = 0.05$

Hypothesis : $H_0 : \mu = 35$
 $H_1 : \mu < 35$ (left tail test)

Test Statistic :

$$t = \frac{33.6 - 35}{2.3 / \sqrt{12}} = -2.1086$$

$$t = -2.1086$$

$$t_{0.05, 11} = -1.796$$

So $t < t_{0.05, 11}$

H_0 must be rejected. So we can say that
the average of newly motorboat is less than
35 sec.

$$(b) \text{ Given : } n_1 = 40 \quad \bar{x}_1 = 9.1 \quad s_1 = 1.9 \\ n_2 = 50 \quad \bar{x}_2 = 8.0 \quad s_2 = 2.1 \\ \alpha = 0.05$$

Hypothesis:

$$H_0 : \mu_1 - \mu_2 = 0 \\ H_1 : \mu_1 - \mu_2 \neq 0 \quad (\text{two tail test})$$

Test statistic:

$$Z = \frac{9.1 - 8.0}{\sqrt{\frac{(1.9)^2}{40} + \frac{(2.1)^2}{50}}}$$

$$Z = 2.604$$

$$Z_{0.025} = 1.96$$

$$\text{So } Z > Z_{0.025}$$

H_0 must be rejected.

* Aim :-

To perform chi-square test of association.

* Experiment :-

A T.V. channel program manager wants to know whether there are any significant differences among male & female viewers between the type of program they watch. A survey conducted for the purpose given the following results. Calculate chi-square statistic and determine whether type of T.V. program is independent of viewers' sex. Take 0.1 significance level and critical value as 2.706.

* Theory & Formula used :-

Contingency Table :- It's a two-way table for attributes.

Different levels of 2 attributes are considered and the table gives frequencies according corresponding to i^{th} level of one attribute and j^{th} level of another attribute $i=1, 2, \dots, m$ and $j=1, 2, \dots, n$.

This type of table is called contingency table of $m \times n$.

$$\text{Expected Frequency} = \frac{\text{Row total} \times \text{column total}}{\text{Grand Total}}$$

$$\text{Test statistic: } \chi^2 = \sum_{i=1}^m \sum_{j=1}^n \left\{ \frac{(O_{ij} - E_{ij})^2}{E_{ij}} \right\}$$

where O_{ij} = observed frequency for (i, j) in contingency table.

If $\chi^2 > \chi^2_{\alpha, n-1}$ we reject H_0 otherwise not.

* Result :-

$$16.66 > 2.706$$

i.e. $\chi^2 > \chi^2_{0.1, 1}$ so H_0 is rejected

Hence, there is no any relation between gender of viewers and T.V. program watched by them.

* Calculations :-

| Type of T.V. Programs | Viewers' Sex | | Total |
|--------------------------|--------------|--------|-------|
| | Male | Female | |
| News | 30 | 10 | 40 |
| Serial | 20 | 40 | 60 |
| Total | 50 | 50 | 100 |

H_0 : There is no relation between gender and program.

H_1 : Both are independent.

| O_{ij} | e_{ij} | $(O_{ij} - e_{ij})^2$ | $\frac{(O_{ij} - e_{ij})^2}{e_{ij}}$ |
|----------|----------|-----------------------|--------------------------------------|
| 30 | 20 | 100 | 5 |
| 20 | 30 | 100 | 10/3 |
| 10 | 20 | 100 | 5 |
| 40 | 30 | 100 | 3.33 |
| | | | Total = 16.33 |

$$\chi^2 = 16.33 \quad \text{and} \quad \chi^2_{0.001, 1} = 2.706$$

$$\chi^2 > \chi^2_{0.1, 1}$$

H_0 is rejected

* Aim :-

To perform chi-square test of goodness of fit.

* Experiment :-

A survey of 320 families with 5 children each revealed the following distribution:-

| | | | | | | |
|---------------|---|---|---|---|---|---|
| No. of boys : | 5 | 4 | 3 | 2 | 1 | 0 |
|---------------|---|---|---|---|---|---|

| | | | | | | |
|----------------|---|---|---|---|---|---|
| No. of girls : | 0 | 1 | 2 | 3 | 4 | 5 |
|----------------|---|---|---|---|---|---|

| | | | | | | |
|-------------------|----|----|-----|----|----|----|
| No. of families : | 14 | 56 | 110 | 88 | 40 | 12 |
|-------------------|----|----|-----|----|----|----|

Is this result consisted with the hypothesis that male and female births are equally probable?

* Theory & Formula used :-

$$\text{Expected Frequency} = f(r) = N \cdot p(r) \cdot N \cdot P(r)$$

Where $p(r) = \text{Probability of male birth in a family of } 5$

$$p(r) = {}^5C_r p^r q^{5-r}$$

Our test statistic will be

$$\chi^2 = \sum \left[\frac{(O - e)^2}{e} \right]$$

where $O = \text{observed frequency}$

~~reject~~ $e = \text{expected frequency}$

Reject H_0 if $\chi^2 > \chi^2_{0.05, 4}$

* Result :-

$$11.07 > 7.16$$

$$\chi^2_{0.05, 4} > \chi^2 \quad H_0 \text{ can't be rejected}$$

Hence the data are consistent with the hypothesis of equal probability for male & Female births.

* Calculations :-

H_0 : Male and Female births are equal.
 H_1 : Both are not equal.

$$P(r) = \left(\frac{1}{2}\right)^5 {}^5C_r$$

Expected Frequency of Male birth is given by

$$f(r) = N \cdot P(r) = 320 \times \left(\frac{1}{2}\right)^5 {}^5C_r$$

$$f(r) = 10 \times {}^5C_r$$

| O | e | $(O-e)^2$ | $\frac{(O-e)^2}{e}$ |
|--------------|-----|-----------|---------------------|
| 14 | 10 | 16 | 1.6 |
| 56 | 50 | 36 | 0.72 |
| 110 | 100 | 100 | 1.0 |
| 88 | 100 | 144 | 1.44 |
| 40 | 50 | 100 | 2.0 |
| 12 | 10 | 4 | 0.4 |
| Total = 7.16 | | | |

$$\chi^2 = 7.16 , \chi^2_{0.05,1} = 11.07$$

$$\chi^2 < \chi^2_{0.05,1}$$

H_0 can't be rejected

* Aim :-

To perform sign test for the given distribution.

* Experiment :-

The grade of the result of the students of a college of the mid-term and end-term examination are given below. Then by sign test, test the hypothesis that the performance of the students in both the examination is equal. Use the binomial distribution at 5% significance level.

Grade M.T. (x)

Grade E.T. (y)

B^+

C^+

A^+

A^+

B

C^+

B^+

B

A^+

B^+

C^+

C^+

D

D^+

B

B^+

C^+

B

A

A

* Theory & Formula used :-

Sign Test :- It is a statistical method to test for consistent difference between pair of observations such as weight of a person before and after treatment.

The sign test can also test if the median of collection is significantly greater or less than a specified value.

Since we have to use binomial distribution so

$$P(r \leq r_0) = \left(\frac{1}{2}\right)^n \sum_{s=0}^{r_0} {}^n C_s$$

if $P(r \leq r_0) > \alpha$, Accept H_0 .

* Result :-

$$\therefore P(r \leq 3) = 0.5 > 0.05$$

H_0 is accepted

Hence the performance of students in both exams are equal.

* Calculation :-

H_0 : The performance in both exams are equal

H_1 : Both are not equal

Grad Grade M.T.
(X)

Grade E.T.
(Y)

Y-X

| | | |
|-----|-----|---|
| B + | C + | - |
| A + | A + | 0 |
| B | C + | - |
| B + | B | - |
| A + | B + | - |
| C + | C + | 0 |
| D | D + | + |
| B | B + | + |
| C + | B | + |
| A | A | 0 |

no. of (+) = 3, no. of (-) = 4, no. of 0 = 3

$$n = 10 - 3 = 7$$

$$P(X \leq 3) = \left(\frac{1}{2}\right)^7 \sum_{x=0}^3 {}^7C_x$$

$$= \frac{64}{128} = 0.5$$

$$P(X \leq 3) = 0.5 > 0.05 (\alpha)$$

H_0 is accepted.

* Aim :-

To perform wilcoxon signed rank test.

* Experiment :-

Generally it is claimed that there is more effect of remembering by visual than audio. Use wilcoxon test to test this hypothesis.

Visual (X) 20 17 14 18 15 16 19 16 17 18

Audio (Y) 19 16 15 16 13 16 15 18 14 17

* Theory & Formula used :-

Wilcoxon Signed-rank test :- It is a non-parametric test used either to test the location of a set of samples or to compare the location of two population using a set of matched samples.

T^+ = Total no. of ranks at + diff.

T^- = Total no. of ranks at - diff.

$$\text{Mean} = \frac{n(n+1)}{4}, \quad S.D. = \sqrt{\frac{n(n+1)(2n+1)}{24}}$$

$$Z = \frac{T - \text{mean}}{S.D.}$$

$$[T = \min(T^+, T^-)]$$

If $Z \leq Z_\alpha$, Accept H_0 .

* Result :-

$$-1.66 < -1.645$$

$$Z < Z_a$$

H_0 is accepted

Hence, There is effect of remembering by visual than audio.

* Calculations :-

H₀ : There is effect of remembering by visual than audio.

H₁ : No effect.

| Visual(X) | Audio(Y) | $d = X - Y$ | Rank |
|-----------|----------|-------------|------|
| 20 | 19 | 1 | 2.5 |
| 17 | 16 | 1 | 2.5 |
| 14 | 15 | -1 | 2.5 |
| 18 | 16 | 2 | 6 |
| 15 | 13 | 2 | 6 |
| 16 | 16 | 0 | 0 |
| 19 | 15 | 4 | 9 |
| 16 | 18 | -2 | 6 |
| 17 | 14 | 3 | 8 |
| 18 | 17 | 1 | 2.5 |

$$\text{Revised } n = \text{higher Rank} = 10 - 1 = 9$$

$$\text{Tie 1} = \frac{1+2+3+4}{4} = 2.5$$

$$\text{Mean} = \frac{90}{4} = 22.5$$

$$\text{Tie 2} = \frac{5+6+7}{3} = 6$$

$$\text{S.D.} = \sqrt{\frac{9 \times 10 \times 19}{24}} = 8.44$$

$$T^+ = 36.5, \quad T^- = 8.5$$

$$T = \{ \min(T^+, T^-) \} = 8.5$$

$$\text{Now } Z = \frac{8.5 - 22.5}{8.44} = -1.66$$

$$Z_{0.05} = 1.645$$

* Aim :-

To test given hypothesis using sign test. (For Median)

* Experiment :-

A random sample of size 8 taken from the population is given below:

19, 18, 11, 9, 13, 15, 17, 13

Test the hypothesis that the population mean median is 12 using sign test. ($Z_{\alpha} = 1.96$)

* Theory & Formula used :-

Sign Test :- It's a statistical test to check the consistent difference betw between pairs of observations.

The sign test can also test if the median of collection is greater than or less than specified value.

$$d = x - M \quad ; \quad d = (+, -)$$

$$Z = \frac{x - \text{Mean}}{\text{S.D.}}, \quad \text{Mean} = \frac{n}{2}, \quad \text{S.D.} = \sqrt{\frac{n}{4}} = \frac{\sqrt{n}}{2}$$

* Result :-

$$0.76 < 1.96$$

$Z < 1.96$, H_0 is accepted

Hence, Population Median = 12

* Calculations :-

$$H_0 : M = 12$$

$$H_1 : M \neq 12$$

$$M_0 = 12$$

X

19

18

11

9

13

12

17

13

A

$$d = \text{Sign}(X - M_0)$$

+

+

-

-

+

0

+

+

$$(+)\text{ sign} = 5, \quad (-)\text{ sign} = 2, \quad \gamma = 2$$

$$\text{Revised } n = 8 - 1 = 7, \quad \text{Mean} = 3.5, \quad \text{S.D.} = 1.32$$

$$\therefore \gamma < \text{Mean}$$

$$\gamma' = \gamma + 0.5 = 2.5$$

$$Z = \frac{2.5 - 3.5}{1.32} = -0.76$$

$$Z = -0.76$$

$$Z_d = 1.96$$

*** Aim :-**

To perform sign test for median.

*** Experiment :-**

To test the claim that median of mathematics faculty in state community college at least 42 year, the result from a random sample of 32 mathematics facility give the following are in years

56, 62, 61, 54, 52, 32, 24, 35, 50, 42, 52,
 49, 26, 31, 31, 54, 38, 36, 45, 53, 37, 40,
 38, 31, 29, 25, 45, 32, 49, 39, 36, 38

Use the sign test at least 5% level of significance to test the claim by critical value method.

*** Theory & Formula used :-**

Sign Test :- It's a statistical method to test the consistent differences between pair of observations.

$$d = X - M, \quad d = (+, -)$$

$$Z = \frac{Y - \text{Mean}}{\text{S.D.}}, \quad \text{Mean} = \frac{n}{2}, \quad \text{S.D.} = \sqrt{\frac{n}{2}}$$

*** Result :-**

$$-0.7184 < -1.96 \quad \text{so } Z < Z_{\alpha/2}$$

Hence, Median of mathematics faculty is atleast 42.

* Calculations :-

$$H_0 : M \geq 42$$

$$\alpha = 0.05$$

$$H_1 : M < 42$$

$$Z_{0.025} = -1.96$$

$$T = \min(T^+, T^-) = 13$$

$$n = 32 - 1 = 31, \text{ Mean} = \frac{31}{2} = 15.5$$

$\therefore T < \text{Mean}$

$$T = 13 + 0.5 = 13.5$$

$$\text{Now } Z = \frac{13.5 - 15.5}{\sqrt{\frac{31}{2}}} = -0.7184$$

$$\therefore Z < Z_{\alpha/2}$$

H_0 is accepted