Computer Graphics

Assignment 2

Q1. Show that a 2D reflection through the x axis, followed by a 2D reflection through the line $y=x$, is equivalent to a pure rotation about the origin.

A1. Step 1 : Reflection through the x axis :

$$[T_1] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Step 2 : Reflection through the line $y=x$

$$[T_2] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

When applied successively, the following transformation matrix is obtained :

$$[T_1][T_2] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Step 3 : Now, the transformation matrix for rotation about origin by an angle $\theta = 90^\circ$ is :

$$[T_3] = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos 90^\circ & \sin 90^\circ \\ -\sin 90^\circ & \cos 90^\circ \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

As matrices $[T_1][T_2]$ and $[T_3]$ yield the same result, 2D reflection about x axis, then $y=x$ is equivalent to pure rotation about the origin by an arbitrary angle.

Q2. Prove that parallel lines remain parallel after transformation.

Consider a line between $[A] = [x_1 \quad y_1]$ and $[B] = [x_2 \quad y_2]$ and a line parallel to AB between E and F.

Now, since $AB \parallel EF$, their slopes can be given by :

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Transforming the end points of AB using a general 2×2 transformation yields A^*B^* :

$$\begin{bmatrix} A \\ B \end{bmatrix} [T] = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= \begin{bmatrix} ax_1 + cy_1 & bx_1 + dy_1 \\ ax_2 + cy_2 & bx_2 + dy_2 \end{bmatrix}$$

$$= \begin{bmatrix} x_1^* & y_1^* \\ x_2^* & y_2^* \end{bmatrix} = \begin{bmatrix} A^* \\ B^* \end{bmatrix}$$

Calculating the slope of A^*B^* :

$$m^* = \frac{y_2^* - y_1^*}{x_2^* - x_1^*}$$

$$= \frac{(bx_2 + dy_2) - (bx_1 + dy_1)}{(ax_2 + cy_2) - (ax_1 + cy_1)}$$

$$= \frac{b(x_2 - x_1) + d(y_2 - y_1)}{a(x_2 - x_1) + c(y_2 - y_1)}$$

$$= \frac{b + d\left(\frac{y_2 - y_1}{x_2 - x_1}\right)}{a + c\left(\frac{y_2 - y_1}{x_2 - x_1}\right)}$$

$$= \frac{b + dm}{a + cm}$$

Since the slope m^* is independent of x_2, x_1, y_2, x_1 and since m, a, b, c, d are the same for EF and AB, it follows that m^* is the same for both A^*B^* and E^*F^*.

$\therefore$ Parallel lines remain parallel after transformation.

Q3. Show that the composition of two rotations is additive.
i.e. $R(a) * R(b) = R(a+b)$ Paper 1957 Q9(a)

A3. Rotation matrix $R(a)$ can be given by:

$$R(a) = \begin{bmatrix} \cos a & \sin a \\ -\sin a & \cos a \end{bmatrix}$$

Rotation matrix $R(b)$ can be given by:

$$R(b) = \begin{bmatrix} \cos b & \sin b \\ -\sin b & \cos b \end{bmatrix}$$

Calculating $R(a) * R(b)$:

$$R(a) * R(b) = \begin{bmatrix} \cos a & \sin a \\ -\sin a & \cos a \end{bmatrix} \begin{bmatrix} \cos b & \sin b \\ -\sin b & \cos b \end{bmatrix}$$

$$= \begin{bmatrix} \cos a \cos b - \sin a \sin b & \cos a \sin b + \sin a \cos b \\ -\sin a \cos b - \cos a \sin b & -\sin a \sin b + \cos a \cos b \end{bmatrix}$$

We know that:
$\cos(a+b) = \cos a \cos b - \sin a \sin b$
and $\sin(a+b) = \cos a \sin b + \sin a \cos b$

$$= \begin{bmatrix} \cos(a+b) & \sin(a+b) \\ -\sin(a+b) & \cos(a+b) \end{bmatrix} \quad \text{— ①}$$

Rotation matrix $R(a+b)$ can be given by:

$$R(a+b) : \begin{bmatrix} \cos(a+b) & \sin(a+b) \\ -\sin(a+b) & \cos(a+b \end{bmatrix} \quad \text{— ②}$$

eq ① = eq ②

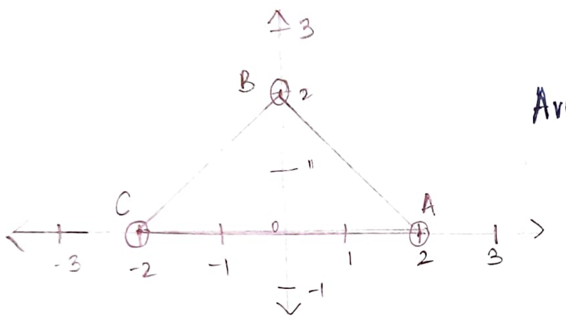
$\therefore$ $\boxed{R(a) * R(b) = R(a+b)}$

Q4. A triangle is defined by vertices (2,0), (0,2), (-2,0). It is transformed by the 2x2 transformation matrix as given below. Find the area of the transformed triangle:

$$T = \begin{bmatrix} 6 & 4 \\ 2 & 4 \end{bmatrix}$$

A4. Plotting the triangle ABC with vertices:

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \\ -2 & 0 \end{bmatrix}$$



Area of this triangle $= \frac{1}{2}(\text{base})(\text{height})$

$$A_i = \frac{1}{2}(4)(2) = 4$$

Given transformation matrix:

$$[T] = \begin{bmatrix} 6 & 4 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Area of transformed triangle $A^*B^*C^*$:

$$A_t = A_i(ad - bc)$$
$$= 4(24-8)$$
$$= 4(16) = \boxed{64 \text{ unit}^2}$$

Vertices of triangle $A^*B^*C^*$:

$$[X][T] = \begin{bmatrix} 2 & 0 \\ 0 & 2 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} 6 & 4 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 12 & 8 \\ 4 & 8 \\ -12 & -8 \end{bmatrix}$$

~~Plotting this triangle~~:

Area of this triangle:

$$\frac{1}{2}(12-4)(8-(-8))$$
$$= \frac{1}{2}(8)(16)$$
$$= \boxed{64 \text{ unit}^2}$$

Hence verified.

Q5. Magnify the triangle with vertices A(0,0), B(1,1) and C(5,2) to thrice its size while keeping B(1,1) fixed. Use homogeneous coordinates.

Paper 1957 Q9 (b)

A5. Steps :

1. Translate the point B(1,1) so that it passes through the origin
2. Perform scaling by a factor of 3
3. Apply inverse translation ~~back~~ to translate B back to original position

$\therefore$ Combined matrix : $[T] = [T'][S][T']^{-1}$

$$[T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ -3 & -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ -2 & -2 & 1 \end{bmatrix}$$

New coordinates :

$$[X][T] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ -2 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -2 & 1 \\ 1 & 1 & 1 \\ 13 & 4 & 1 \end{bmatrix}$$

$\therefore$ New points are : $A'(-2,-2)$
$B'(1,1)$
$C'(13,4)$

while keeping B(1,1) fixed.

Q6. What are rigid body transformations? Discuss the property of transformation matrix which should give rigid body transformation.

Paper 1957 ~~Q10~~ Q9(C)

Pure rotations and reflections are called rigid body transformations.

The corresponding transformation matrix is given by:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} a^2+b^2 & ac+bd \\ ac+bd & c^2+d^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

It has the following properties and results:

1) Determinant is +1.
2) Angles b/w intersecting lines are preserved by pure rotation
3) In case of reflective transformations (det. = −1), the magnitude of vectors is preserved but not the angle. But perpendicular lines still transform as perpen. lines.
4) Uniform scaling also preserve the angle b/w intersecting lines but not magnitude of transformed vectors.

Q7. How are vanishing points obtained in perspective projection?

There are two methods. First simply calculates the intersection point of a pair of transformed projected parallel lines. Second one is more complex but numerically more accurate. Here, an object with sides originally parallel to the principal axes is transformed to desired position and orientation. Single point perspective projection is applied. The final concatenated transformation matrix is then used to transform the points at infinity on the principal axes. The resulting ordinary coordinates are the principal vanishing points for that object.

Paper 1957 Q10 (a)

Q8. Give the transformation matrix for each of the following transformations:

(i) Scale in x dimension by 2 and y dimension by 3 with respect to fixed point (4, 2).

Ans :

$$[T] = [T'][S][T']^{-1} \qquad (m=4,\ n=2)$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & -2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ -8 & -6 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$

$$[T] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ -4 & -4 & 1 \end{bmatrix} \rightarrow \text{final}$$

(ii) Rotate by -30° about the point $(-2, 3)$

Ans :

$$[T] = [T'][R][T']^{-1}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} \cos(-30) & \sin(-30) & 0 \\ -\sin(-30) & \cos(-30) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 3 & 1 \end{bmatrix}$$

(As $\cos(-\theta) = \cos\theta$, $\sin(-\theta) = -\sin\theta$)

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ \sqrt{3} - 3/2 & -1-3\sqrt{3}/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 3 & 1 \end{bmatrix}$$

$$[T] = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ \sqrt{3} - 7/2 & 2 - \frac{3\sqrt{3}}{2} & 1 \end{bmatrix} = \begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ -1.768 & 0.598 & 1 \end{bmatrix}$$

$\rightarrow$ final

(iii) Reflect about the line $y = -5$.

Ans : $[T] = [T'][R_o][R_e][R_o]^{-1}[T']^{-1}$

Given line L i.e. $y = -5$ will pass through the origin by translating it 5 units in the y direction.

$$[T'] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix}$$

Now the line will be coincident with the x axis

∴ Rotation is not required.

Reflection through the x axis :

$$[R_e] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

∴ Required transformation matrix will be :

$$[T] = [T'][R_e][T']^{-1}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & -10 & 1 \end{bmatrix} \rightarrow \text{final}$$

Q9. Give the 3D homogeneous matrix for each of the following transformations:

(i) Scale in the x dimension by 2 and y dimension by 3 with fixed point (4,2,2).

Ans. $[T] = [T'][S][T']^{-1}$

$$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -4 & -2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 4 & 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -4 & -4 & 2 & 1 \end{bmatrix} \rightarrow \text{final}$$

(ii) Rotate by 30° about the x axis

$[T] = [T'][R_x]$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ l & m & n & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 30^\circ & \sin 30^\circ & 0 \\ 0 & -\sin 30^\circ & \cos 30^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

If the point (l,m,n) is at origin, then it is the same as multiplying $[R_x]$ with identity matrix i.e. multiplication by 1. Else, multiply with given ~~go~~ point (l,m,n).

$$[T] = [I] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & 1/2 & 0 \\ 0 & -1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

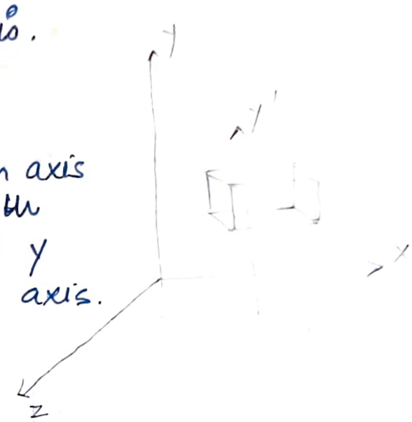
$\hookrightarrow$ final

(iii) Reflect about the y axis.

Reflection occurs through a plane, xz in this case.

$$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \text{final}$$

Q10. Derive the 3D homogeneous transformation matrix to rotate an object about a line parallel to y axis.



$$[X^*] = [X][T_r][R_y][T_r]^{-1}$$

Step 1: Translate the object so that the rotation axis coincided with the parallel y coordinate axis.

$$[T_r] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -X & 0 & -Z & 1 \end{bmatrix}$$

Step 2: Perform the specified rotation by an angle θ.

$$[R_y] = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ +\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 3: Translate object so that rotation axis is moved back to original position.

$$[T_r]^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ X & 0 & Z & 1 \end{bmatrix}$$

∴ Required transformation matrix:

$$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -X & 0 & -Z & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ +\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ X & 0 & Z & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ \cancel{2x\sin\theta} \; x(\cos\theta - 1) - z\sin\theta & 0 & z(1-\cos\theta) + x\sin\theta & 1 \end{bmatrix} \rightarrow \text{final}$$

Q11. Give a 4x4 homogeneous transformation matrix which will have the same effect as each of the following transformation :

(i) Rotate counter clockwise about x axis and then translate up by 2 units. Paper 1957 Q10 (b)

Ans. $[T] = [R_x][T]$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 2 & 2 & 2 & 1 \end{bmatrix}$$

(ii) Overall reduce the ~~fi~~ size of object to half.

Ans. Scaling the object :

~~$[X][T] = [X][S]$ where $S_x = S_y = S_z = 1/2$~~ Let $s = 2$ so that :

$$\cancel{= \begin{bmatrix} 1/2 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

$[x^* \; y^* \; z^* \; 1] = \left[\frac{x'}{s} \; \frac{y'}{s} \; \frac{z'}{s} \; 1\right]$

and

~~keep~~

$$[X][T] = [x \; y \; z \; 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} = [x' \; y' \; z' \; s]$$

$[X]$ $[T]$

where $[T]$ is the required matrix.

(iii) Apply two point perspective projection on $x=0$ plane with center of projection on x axis and y axis given as (1,0,0) and (0,-2,0).

Paper 1957 Q10 (b)

~~$p = \frac{1}{-1}, q = -\frac{1}{-2}$~~

$$[T] = [P_{pq}][P_z] = \begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Here, $p = \frac{-1}{1} = -1$ and $q = \frac{-1}{-2} = 0.5$

Ex *

$$[T] = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0.5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \text{Final}$$

Q12. Write the 3D projection matrices for each of the following

(i) Orthographic projection on $x=0$ plane

$$[P_x] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \text{Final}$$

(ii) Single point perspective projection with centre of projection on Y-axis ~~and~~ at $y = 2$.

$$r = -\frac{1}{Y_c} = \frac{-1}{2} = -0.5$$

$$\therefore [T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \text{final}$$

Q13. Consider a line AB parallel to z axis with end points A [3 2 4 1] and B [3 2 8 1]. Perform perspective projection onto z=0 plane with centre of projection at z = 2.

$$r = \frac{-1}{z} = \frac{-1}{2} = -0.5$$

$$[X^*] = [X][T]$$

$$= \begin{bmatrix} 3 & 2 & 4 & 1 \\ 3 & 2 & 8 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 & 4 & -1 \\ 3 & 2 & 8 & -3 \end{bmatrix}$$

converting to unity factor $h = 1$ by :

$$R_1 = R_1 \times -1$$

$$R_2 = \frac{R_2}{-3}$$

$$[X^*] = \begin{bmatrix} -3 & -2 & -4 & 1 \\ -1 & -0.66 & -2.67 & 1 \end{bmatrix}$$

Q14. Consider a 3D object X with the position vectors :

$$X = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

(i) Obtain a cavalier projection for X, choosing horizontal angle at 30°.

A cavalier proj. is an oblique proj. with $\beta = 45°$ i.e. $f = 1$. and horizontal angle $\alpha = 30°$ (given).

$$\therefore [T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -f\cos\alpha & -f\sin\alpha & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -0.866 & -0.5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[X^*] = [X][T] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -0.866 & -0.5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[X^*] = \begin{bmatrix} -0.866 & -0.5 & 0 & 1 \\ 0.134 & -0.5 & 0 & 1 \\ 0.134 & -0.5 & 0 & 1 \\ -0.866 & -0.5 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

(ii) Obtain a single point perspective transformation for x with centre of projection $X_c = (-10, 0, 0)$ projected onto $y = 0$ plane. Also indicate the vanishing point.

$p = -10$

Centre of projection : $[-1/p \quad 0 \quad 0 \quad 1]$

$= [-1/10 \quad 0 \quad 0 \quad 1]$

$= [+0.1 \quad 0 \quad 0 \quad 1]$

$\therefore$ Vanishing point will be located at :

$[1/p \quad 0 \quad 0 \quad 1]$

$= [-0.1 \quad 0 \quad 0 \quad 1]$

Matrix for projection onto $y = 0$ plane :

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$[T] = [T_p][P_y]$

$$= \begin{bmatrix} 1 & 0 & 0 & -0.1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[T] = \begin{bmatrix} 1 & 0 & 0 & -0.1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[X^*] = [X][T]$$

$$= \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -0.1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0.9 \\ 1 & 0 & 1 & 0.9 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0.9 \\ 1 & 0 & 0 & 0.9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Converting to unity factor $h=1$

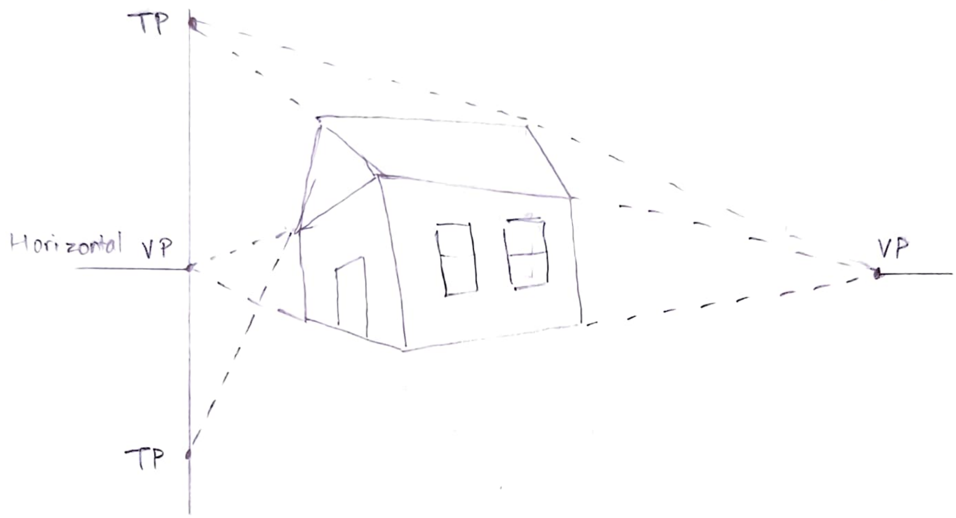
by dividing:

$R_2/0.9$, $R_3/0.9$, $R_6/0.9$, $R_7/0.9$

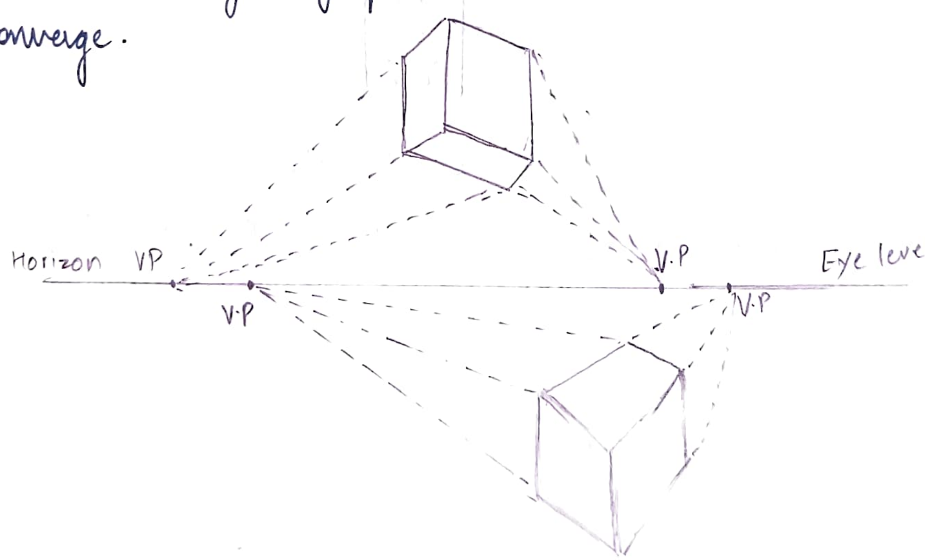
$$[X^*] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1.11 & 0 & 1.11 & 1 \\ 1.11 & 0 & 1.11 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1.11 & 0 & 0 & 1 \\ 1.11 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q15. What are trace points? How are they different from principal vanishing points?

For planes of an object which are tilted relative to the untransformed principal axes, the vanishing points fall above or below the horizontal reference line. These are called trace points.



Principal vanishing points: Points on the horizontal reference line at which lines originally parallel to the untransformed principal axes converge.



Difference:
Vanishing points are a point on the image plane of a perspective sketch whereas trace points a kind of break point that perform a custom action.

Q16. Obtain the blending function for Heeminte curve.

Step 1 : Defining G_{H_x}, the x component of the Hermite geometry matrix :

$$G_{H_x} = \begin{bmatrix} P_1 \\ P_4 \\ R_1 \\ R_4 \end{bmatrix}_x$$

where P_1, P_4 are end points of curve and R_1, R_4 are trangent vectors at P_1 and P_2 respectively.

Step 2: Rewriting $x(t)$:

$$x(t) = a_x t^3 + b_x t^2 + c_x t + d_x$$

$$= T \cdot C_x$$

$$= T \cdot M_H \cdot G_{H_x}$$

where $x(t)$ is one the three cubic polynomials that define a curve segment $Q(t)$ $0 \leq t \leq 1$ with $T = [t^3 \ t^2 \ t \ 1]$ and matrix of coefficients :

$$C_x = \begin{bmatrix} a_x \\ b_x \\ c_x \\ d_x \end{bmatrix}$$

Now, $x(t) = [t^3 \ t^2 \ t \ 1] \cdot M_H \cdot G_{H_x}$ —— (1)

For the curve end points P_1 and P_4, values of t are :

$t = 0$ at P_1

and $t = 1$ at P_4

Step 3 : The constraints $x(0)$ and $x(1)$ are found by direct substitution :

$x(0) = P_{1x} = [0 \ 0 \ 0 \ 1] \, M_H \cdot G_{H_x}$ —— (2)

$x(1) = P_{4x} = [1 \ 1 \ 1 \ 1] \, M_H \cdot G_{H_x}$ —— (3)

Step 4 : Differentiating equations ~~(2) and (3)~~ (1) :

$$x'(t) = [3t^2 \quad 2t \quad 1 \quad 0] M_H \cdot G_{H_x} \quad \text{—— (4)}$$

Step 5 : Putting $t=0$ (R_1) and $t=1$ (R_4) in (4)

$\therefore$ The tangent vector constraint equations can be written as :

$$x'(0) = R_{1x} = [0 \quad 0 \quad 1 \quad 0] M_H \cdot G_{H_x} \quad \text{—— (5)}$$

$$x'(1) = R_{4x} = [3 \quad 2 \quad 1 \quad 0] M_H \cdot G_{H_x} \quad \text{—— (6)}$$

Now, the constraints (2), (3), (5) and (6) can be written in matrix form as :

$$\begin{bmatrix} P_1 \\ P_4 \\ R_1 \\ R_4 \end{bmatrix}_x = G_{H_x} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 3 & 2 & 1 & 0 \end{bmatrix} M_H \cdot G_{H_x} \quad \text{—— (7)}$$

Step 6 : For this equation (7) ~~and~~ to be satisfied, M_H must be the inverse of the 4x4 matrix.

$$\therefore M_H = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 3 & 2 & 1 & 0 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Basis matrix

(similarly for $y(t)$ and $z(t)$)

Step 7: Expanding the product $T \cdot M_H$ in $Q(t) = T \cdot M_H \cdot G_H$ gives the Hermite blending function B_H

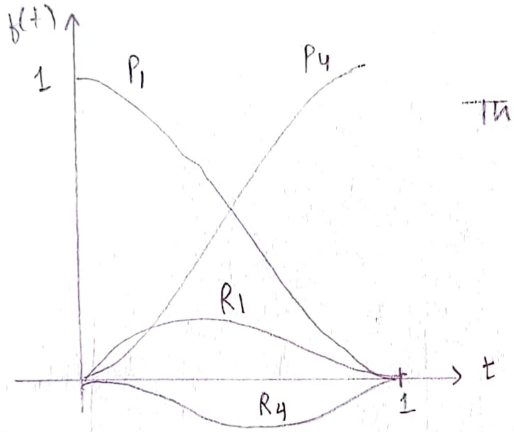
$$\therefore Q(t) = \underbrace{T \cdot M_H}_{\downarrow} \cdot G_H$$

$$= B_H \cdot G_H$$

$$= [t^3 \quad t^2 \quad t \quad 1] \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_1 \\ P_4 \\ R_1 \\ R_4 \end{bmatrix}$$

$$= (2t^3 - 3t^2 + 1) P_1 + (-2t^3 + 3t^2) P_4 + (t^3 - 2t^2 + t) R_1 + (t^3 - t^2) R_4$$ parametric eq^n

As $0 \leq t \leq 1$:



The hermite blending functions.

Q17. Derive the basis matrix for Bezier curve.

Paper 1957 Q11 (a)

Step 1 : Bezier curve :

$$Q(t) = \sum_{i=0}^{n} P_i B_{i,n}(t) \quad \text{—— (1)}$$

where $0 \leq t \leq 1$

P_i = control points

$B_{i,n}(t)$ = Basis function

Step 2 : Expanding the basis function,

$$B_{i,n}(t) = {}^nC_i t^i (1-t)^{n-i} \quad \text{—— (2)}$$

where $${}^nC_i = \frac{n!}{i!(n-i)!}$$

Step 3 : For a bezier curve of degree $n = 3$, four control points are required to specify a cubic bezier curve segment.

$\therefore$ from eq (1) :

$$Q(t) = \sum_{i=0}^{3} P_i B_{i,3}(t) \qquad 0 \leq t \leq 1$$

$= P_0 B_{0,3}(t) + P_1 B_{1,3}(t) + P_2 B_{2,3}(t) + P_3 B_{3,3}(t)$

from eq (2) :

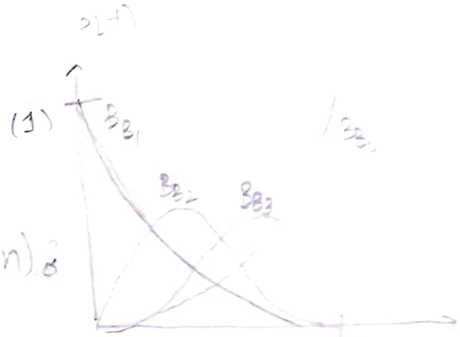
$$B_{0,3}(t) = \frac{3!}{0!\,3!}(t^0)(1-t)^3$$
$$= (1-t)^3$$

$$B_{1,3}(t) = \frac{3!}{2!\,1!}(t)^1(1-t)^2$$
$$= 3t(1-t)^2$$

$$B_{2,3}(t) = \frac{3!}{2!\,1!}(t)^2(1-t)^1$$
$$= 3t^2(1-t)$$

$$B_{3,3}(t) = \frac{3!}{0!\,3!}(t)^3(1-t)^0$$
$$= t^3$$

(Blending fun) :



Step 4 : $\therefore Q(t) = (1-t)^3 P_0 + 3t(1-t)^2 P_1 + 3t^2(1-t)P_2 + t^3 P_3$
$(0 \le t \le 1)$

$$= (1-3t+3t^2-t^3)P_0 + 3t(1+t^2-2t)P_1 + (3t^2-3t^3)P_2 + t^3 P_3$$

$$= (-t^3+3t^2-3t+1)P_0 + (3t^3-6t^2+3t)P_1 + (-3t^3+3t^2)P_2 + t^3 P_3 \quad \text{——— (3)}$$

This is the Bezier Blending functions

Step 5 : In matrix form :

$C(t) = T \cdot M_B \cdot G_B$ where $G_B = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$

$[T] = [t^3 \ \ t^2 \ \ t \ \ 1]$

and from eq (3) :

$$M_B = \begin{bmatrix} -1 & 3 & -3 & 1 \\ +3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$ Similarly for $y(t)$ and $z(t)$.

This is ~~which~~ is the required Bezier basis matrix M_B.

Q19. Find the equation of the Bezier curve that passes through $(0,0)$ and $(4,2)$ and controlled through $(14,10)$ and $(4,0)$.

P_1 P_4 P_2 P_3

As the curve passes through $(0,0)$ and $(4,2)$, these will be the end points of the curve.

$\therefore$ $P_0 = (0,0)$ and $P_3 = (4,2)$

$P_1(14,10)$ and $P_2(4,0)$ are Intermediate points

As there are four control points, degree $= 3$

Now

$$Q(t) = T \cdot M_B \cdot G_B$$

$$= [t^3 \ \ t^2 \ \ t \ \ 1] \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

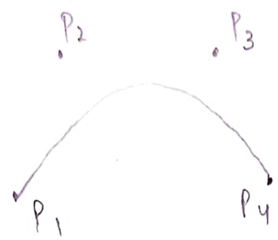
$$= [t^3 \ \ t^2 \ \ t \ \ 1] \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 14 & 10 \\ 4 & 0 \\ 4 & 2 \end{bmatrix}$$

$$= [t^3 \ \ t^2 \ \ t \ \ 1] \begin{bmatrix} 34 & 32 \\ -72 & -60 \\ 42 & 30 \\ 0 & 0 \end{bmatrix}$$

$$= \left[34t^3 - 72t^2 + 42t \qquad 32t^3 - 60t^2 + 30t\right]$$

$$\therefore \boxed{\begin{aligned} x(t) &= 34t^3 - 72t^2 + 42t \\ y(t) &= 32t^3 - 60t^2 + 30t \end{aligned}}$$

Q20. Given the following Bezier curve defined by four control points P_1, P_2, P_3 and P_4, divide it into curve segments in the ratio 1:1.



Q21. Write the pseudo code for z-buffer visible surface determination algorithm. What are its advantages and disadvantages?

```
void zBuffer (void)
{
    int x, y;
    for ( y = 0; y < YMAX; y++) {        /* Clear frame buffer and z-buffer */
        for (x = 0; x < XMAX ; x++) {
            WritePixel (x, y, BACKGROUND_VALUE);
            WriteZ (x, y, 0)
        }
    }
    for (each polygon) {                 /* Draw polygons */
        for ( each pixel in polygon's projection) {
            double pz = polygon's z value at pixel coords
             (x, y);
            if ( pz >= ReadZ (x, y)) {   /* New point is not farther */
                WriteZ (x, y, pz);
                WritePixel (x, y, polygon's color at
                 pixel coords (x, y));
            }
        }
    }
} /* zBuffer */
```

Advantages :

1) It can be used to render any object if a shade and a z-value can be determined for each point in its projection. 2) It is not constrained to polygons and no explicit intersection algorithms are needed.

3) ~~4)~~ It is easy to implement and lacks additional data structures, thus decreasing the memory costs.

3) Since ~~it~~ z-buffer is the only data structure used by the visible surface
11) algorithm proper, it can be saved along with the image and later used to merge in other objects whose z can be computed.

~~Disadvantages~~ :

5) ~~4)~~ The algorithm performs radix sorts in x and y, requiring no comparisons, and its z sort takes only one comparison per pixel for each polygon containing that pixel.

Disadvantages :

1) It requires a large amount of space for the z buffer
2) As it operates in image precision, it is subject to aliasing
3) 16 bits z-buffer do not enough precision to represent environments in which objects defined with millimeter detail are positioned a kilometer apart.
4) If a perspective projection is used, the compression of distant z values resulting from the perspective divide has a serious effect on the depth ordering and intersection of different objects.
5) Some of the pixels, generated from scan conversion algorithms, shared by the rendered edges may be assigned slightly different z values because of numerical inaccuracies in performing the z interpolation.

Q22. Discuss depth sorting method for hidden surface elimination. What tests are performed when there is depth overlap?

Depth sort algorithm is used to paint the polygons into the frame buffer in order of decreasing distance from viewpoint. Three conceptual steps are performed :

1. Sort all polygons according to the smallest (farthest) z coordinate of each.
2. Resolve any ambiguities. This may cause when the polygon's z extents overlap, splitting polygons if necessary.
3. Scan convert each polygon in ascending order of smallest z coordinate (i.e. back to front).

The simplified version of this algorithm is called painter's algorithm. To resolve the ambiguities of step 2, up to five tests are performed, in order of increasing complexity:

1) Do the polygons' x extents not overlap?
2) Do the polygons' y extents not overlap?
3) Is P entirely on the opposite side of Q's plane from the viewpoint?
4) Is Q entirely on the same side of P's plane as the viewpoint?
5) Do the projections of the polygons onto the (x, y) plane not overlap?

(Here, P = polygon currently at the far end of the sorted list of polygons

Q = each polygon whose z extent overlaps that of P.)

Q23. What is dithering? What are its advantage over halftoning?

Primarily, dithering refers to techniques for approximating halftones without reducing resolution, as pixel grid patterns do. It is also applied to halftone-approximation methods using pixel grids and sometimes used to refer to color halftone approximations only. Advantage over halftoning:

1) Dithering can reduce the effects of pixel-to-pixel errors in the flatfield or spatially varying detector sensitivity. The errors are distributed among image pixels.

Q24. Explain intensity interpolation scheme for polygon. What is its drawback? Paper 1957 Q 4 (b)

Intensity interpolation scheme / ~~Gouraud~~ Gouraud shading renders a polygon surface by linearly interpolating intensity values across the surface. ~~Each~~ The following calculations are performed:

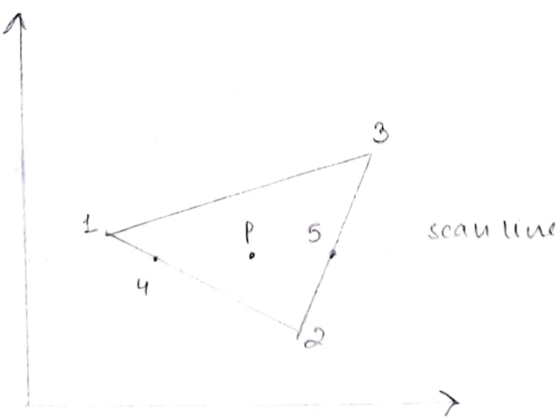
1) Determine the average unit normal vector at each polygon vertex
2) Apply an illumination model to each vertex to calculate the vertex intensity.
3) Linearly interpolate the vertex intensities over the polygon surface.

For any vertex position V, unit ~~vector~~ vertex normal can be given by:

$$N_V = \frac{\sum_{k=1}^{n} N_k}{\left|\sum_{k=1}^{n} N_k\right|}$$

Once we have the vertex normals, we can determine the intensity at the vertices from a lightening model.

Next, we interpolate intensities along the polygon edges. For each scan line, the intensity at the intersection of the scan line with a polygon edge is linearly interpolated from the intensities at the edge endpoints.



$$I_4 = \frac{y_4 - y_2}{y_1 - y_2} I_1 + \frac{y_1 - y_4}{y_1 - y_2} I_2$$

$$I_p = \frac{x_5 - x_p}{x_5 - x_4} I_4 + \frac{x_p - x_4}{x_5 - x_4} I_5$$

Drawbacks :

1) Highlights on the surface are sometimes displayed with anomalous shapes.

2) Linear intensity interpolation can cause bright or dark intensity streaks to appear on the surface.

Q25. Derive the illumination using Phong specular reflection model. Include the contribution of diffuse, ambient and specular reflection.

Using the spectral-reflection function $W(\theta)$, Phong specular-reflection model can be written as :

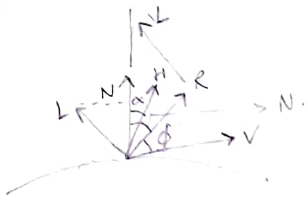
$$I_{spec} = W(\theta) I_l \cos^{n_s}\phi$$

where I_l is the intensity of the light source
ϕ is the viewing angle relative to the specular-reflection direction R.

Now, since V and R are unit vectors in the viewing specular-reflection directions, $\cos\phi$ can be calculated as dot product V·R.
Assuming the specular-reflection coefficient is a constant, the intensity of specular-reflection at surface point is :

$$I_{spec} = k_s I_l (V \cdot R)^{n_s}$$

Now, vector R in this expression can be caculated in terms of vectors L and N.



$$R + L = (2N \cdot L) N$$

$$R = (2N \cdot L) N - L$$

For a single light point source, combined diffuse and specular reflections from a point on an illuminated surface can be given as :

$$I = I_{diff} + I_{spec}$$

$$= \underbrace{k_a I_a}_{I_{amb}} + \underbrace{k_d I_l (N \cdot L)}_{I_{diff}} + \underbrace{k_s I_l (N \cdot H)^{n_s}}_{I_{spec}}$$

where H = halfway vector along bisector of angle b/w L and V.

It is obtained as :

$$H = \frac{L+V}{|L+V|}$$

where V·R in the phong model is replaced with the dot product N·H. and $\cos\phi$ is now $\cos\alpha$.

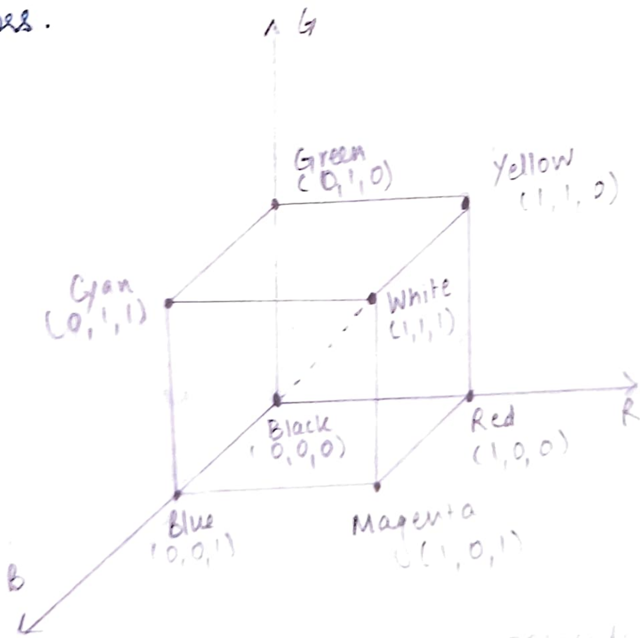
Q26. Differentiate between Phong and Gouraud shading models.

Phong shading	Gouraud shading
1) Interpolates normal vectors to render a polygon surface	Linearly interpolates intensity values across the surface to render a polygon
2) Requires considerably more calculation and thus greatly increases the cost of shading steeply.	Requires less calculation and thus decreases the cost of shading steeply.
3) Greatly reduces the Mach band effect	Linear intensity interpolation can cause Mach bands to appear on the surface
4) Displays more realistic highlights on a surface	Highlights on the surface are sometimes displayed with anamalous shapes.
5) Incremental methods are used to evaluate normals b/w and along individual scan lines	Incremental calculations are used to obtain successive edge intensity values b/w and along scan lines.

Q28. What is RGB color model? How RGB model is represented?

Our eyes perceive color through the stimulation of three visual pigments in the ~~cones~~ cones of the retina. These have a peak sensitivity at wavelengths of about 630 nm (red), 530 nm (green) and 450 nm (blue). By comparing intensities in a light source, we perceive the color of light. This theory is the basis for displaying color output on a video monitor using the three color primaries red, green and blue, referred to as the RGB color model.

It can be represented with the unit cube defined on R, G and B axes. The origin represents black and the vertex with coordinates (1,1,1) is white. Vertices of the cube on the axes represent the primary colors. and the remaining vertices represent the complementary color for each of the primary colors.



Q29. Write steps to design an animation sequence.

An animation sequence is designed with the following steps :

1) Storyboard layout

→ Storyboard is an outline of the action. ~~which~~ It defines the motion sequence as a set of basic events that are to take place.

2) Object definitions

→ It is given for each participant in the action.

3) Key frame specification

→ A key frame is a detailed drawing of the scene at a certain time in the animation sequence.

4) Generation of in-between frames

→ In-betweens are the intermediate frames between the key frames.

Q30. How do we simulate acceleration in any animation? Describe how the frame spacing controls the motion simulated in an animation scene.

incomplete ?

Given the vertex positions at the key frames, we can fit the positions with linear or non linear paths. This determines the trajectories for the in-betweens. To simulate accelerations, we can adjust the time spacing for the in betweens.

derivation ?

For instance, for zero acceleration, we use equal interval time spacing for the in-betweens.

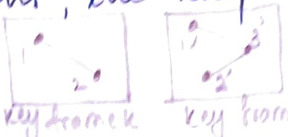
Some key frames are chosen at extreme positions in the action; others are spaced so that the time interval b/w key frames is not too great. ~~Moo~~ If the motion is not too complicated, the key frames are spaced farther apart and vice versa.

Discuss the characteristics of key frame animation. Paper 1957 Q 2 (b)

→ Motion paths can be given with a kinematic description as set of spline curves, or they can be physically based by specifying the the forces acting on the objects to be animated.

→ For complex scenes, the frames can be seperated into individual components called ~~obs~~ cels.

→ With complex object transformations, the shapes of objects may change over time.



→ ~~If all st~~ The total number of line segments can be different in different frames.

Q34. Briefly explain any one basic method to draw thick primitives with its advantages and disadvantages. Paper 1957 Q7 (c)

There are four basic methods for drawing thick primitives. One of them is filling areas between boundaries. This method involves ~~constr~~ constructing the primitive's inner and outer boundary at a distance t/2 on either side of the ideal (single-pixel) primitive trajectory.

Advantage :

1) This filling technique has the advantage of handling both odd and even thickness.

2) It does not increase the extent of a primitive when the primitive is thickened.

Disadvantage :

An area-defining primitive effectively "shrinks" a bit and that its "center-line", the original 1 pixel outline, appears to shift.