COMPUTER GRAPHICS

KHUSHI

CHAPTER-1

A Survey of Computer Graphics

The CAD stand for computer-aided design CAD is a computer-based software which is used in design processes. CAD software is frequently used by diff types of engineers & designers. This software we can be used to create two-dimensional 2-D-drawing or 3-D models.

The purpose of CAD is to increase productivity, improve the quality & level of design, improve documentation communications etc.

Advantages of CAD:-

1. Lower production costs for designs
2. Higher quality designs.
3. Quick project completion.
4. Easy editing
5. Higher accuracy
6. Clearer design.

Applications of CAD:-

1. Architects
2. Product designers
3. Industrial designers
4. Manufacturers
5. fashion designers
6. Graphics designers
7. City planners.

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Commands used in CAD :-

1. line 2. Rectang 3. circle
4. U: short for undo 5. selecting objects.
6. Erase 7. Move. 8. Trim 9. Extend.

uses wire frame outline form.
Som CAD S/w :- QCAD, NX, OR CATIA VS.

II Presentation Graphics

→ An image designed to visually enhance a presentation.
→ can be used in electronic slide shows, as well as in printed reports.

→ Used to produce illustrations for reports or generate slides for use with projectors.

→ Commonly used to summarize financial, statistical, mathematical, scientific, economic & customer information Bulletins.

Ex:- Bar charts, line Graphs, pie charts, surface graph.

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3. Computer ART :-

Used in fine art & commercial art

- Includes artist's paintbrush programs, paint packages, CAD packages & animation packages.

- These packages provides facilities for designing object shapes & specifying object motions.

E.g :- cartoon drawing, logo design.

Electronic painting :-
→ picture painted electronically on a graphics tablet (digitizer) using a stylus
→ Cordless, Pressure sensitive stylus

Morphing :- A graphics method in which one object is transformed into another.

4. Entertainment :-
Movie Industry
→ used in motion pictures, music, videos & television shows.
→ used in making cartoon animation films

Computer Graphics is about animation (films)

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• Game industry

→ focus on interactivity
→ cost effective solutions
→ Avoiding computations & other tricks.

5. Education & training :-

→ Computer generated models of physical, financial & economic systems are used as educational aids.

→ Models of physical systems, physiological systems, population trends, or equipment such as color-coded diagram help trainees understand the operation of the system.

Specialized systems used for training applications :-

Simulators for practice sessions or training of ship captains.

Aircrafts pilots.
heavy equipment operators
air traffic-control personnel.

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6. Visualizations:—

Scientific Visualization :-

→ Producing graphical representations for Scientific, engineering, & medical data Sets.

→ Business visualization is used in connection with data sets related to commerce, industry & other non-scientific areas.

→ Techniques used - color coding, contour plots, graphs, charts & visualizations of volume interiors.

→ Image processing techniques are combined with computer graphics to produce many of the data visualizations.

7. Image Processing :-

- CG - Computer is used to create a picture
- Image Processing - applies techniques to modify or Interpret existing pictures Such as photographs & TV Scans.

- Medical applications
→ Picture enhancement
→ Tomography
→ ultrasound & nuclear medical Scanners

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8.

- 2 applications of image processing

Improving picture Quality
Machine Perception of visual Information (Robotics)

- To apply image processing methods

→ Digitize a photograph into an image file.
→ Apply digital methods to rearrange picture parts to
→ enhance color seperations
→ Improve quality of shading.

→ Tomography:- technique of X-ray photography that allows cross-sectional views of Phsiological systems to be displayed.

→ Computer X-ray tomography (CT):- and position emmision tomography (PET) use projection methods to reconstruct cross sections from digital era.

→ Computer-Aided Surgery:- is a medical appication technique to model & study physical fxn to design artificial limbs & to plan & practice surgery

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8. Graphical User Interfaces:-

→ Major Component:- window manager (multiple window areas)
→ To make a particular window active, click in that window (using an interactive pointing device)
→ Interfaces display - menus & icons.
→ Icons:- graphical symbol designed to look like the processing option it represents.
→ Advantages of icons:- less screen space, easily understood.
→ Menus contain lists of textual descriptions & icons.

Graphics packages:-

→ A set of libraries that provide programmatically access to some kind of graphics 2D functions.

• Types:
GKS - Graphics kernel system - First graphics package - accepted by ISO & ANSI.
PHIGS:-
Silicon Graphics.
open GL
postscript interpreters.
painting, drawing, design packages.

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CHAPTER-2

Overview of Graphics Systems.

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The Primary output Device of the Graphics system?.

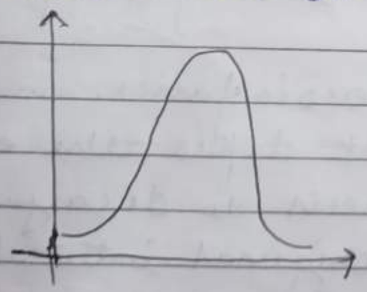
How does this operate??

Characteristics of CRT

- Intensity Distribution.
- Resolution.
- Aspect Ratio
- Persistence.

Intensity Distribution

The intensity is greatest at the center of the spot, & decrease with Gaussian distribution out to the edges of the spot



Resolution :- The maximum no. of points displayed without overlap on a CRT is referred to as the resolution.

A more precise definition of resolution is the no. of points per centimeter that can be plotted horizontally & vertically, although it is often simply stated as the total no. of points in each directions.

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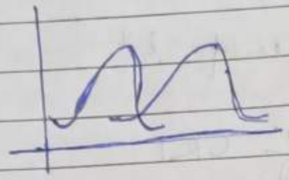
This depends on the type of phosphor used & the focusing & deflection system & the intensity to be displayed.



Aspect Ratio :- This no. gives the ratio of vertical points to horizontal points necessary to produce equal-length lines in both directions on the screen.

- An aspect ratio of 3/4 means that a vertical line plotted with three points has the same length as a horizontal line plotted with four points.

Persistance :- Persistance is defined as time ~~taken~~ it takes the emitted light from screen to decay to one-tenth of its original intensity.

Color CRT Monitors :- These monitors displays color pictures by using combination of phosphorus that emit diff colored light.

There are two techniques for producing color displays on CRT. They are

1. Beam Penetration Method
2. Shadow Mask Method.

Beam Penetration Method

Similar to simple CRT but contains different colored phosphor layers (Red & Green) coated onto inside of the screen.

The displayed color depends on how far the electron beam penetrates into the layers.

The speed of electrons – beam acceleration voltage.

Slow Electrons → Red color

fast Electrons → Green color

Intermediate Speed Electrons → Yellow and orange colors.

Quality of picture is not so good & is inexpensive.

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Shadow Mask Method

Works on the principle of combining the basic colors - Red, green and Blue - in suitable proportions to get a combination of colors.

Uses 3 electron gun placed one by the side of the other to form a triangle or a "Delta".

Each pixel point on the screen is also made up of 3 types of phosphors to produce red, blue & green colors. Before the phosphor screen is a metal screen called a "shadow mask".

This plate has holes placed strategically so that the beams from the three electron guns are focused on a particular pixel.

when the beam passes through the mask they form a dot triangle.

Another configuration is inline arrangement of electron guns & RGB dots on screen are aligned on one scan line.

By varying, we get diff colors (white, Grey, Yellow, Cyan)

where used

color

color depends

cost

Resolution.

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	Beam penetration	Shadow Mask
where used	It is used with Random Scan System to display color.	It is used with Raster Scan Systems to display color.
color	It can displays only four colors i.e. Red Green, orange & Yellow	It can displays millions of colors.
color dependency	Less color are available because the colors in Beam peneration depends on the speed of the electron beam.	Millions of colors are available because the colors in shadow Mask depends on the type of the ray.
cost	It is less Expensive as compared to shadow mask.	It is more Expensive than other methods.
Resolution.	It gives low Resolution	It gives High Resolution.

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Direct View Storage Tube

An alternative method for maintaining a screen image is to store the picture information inside the CRT instead of refreshing the screen.

A direct-view storage tube (DVST) stores the picture information as a charge distribution just behind the phosphor-coated screen.

Two electron guns are used in a DVST. one, the primary gun, is used to store the picture pattern & the second the flood gun, maintains the picture display.

A DVST monitor has both disadvantages & advantages compared to the refresh CRT. Because no refreshing is needed, very complex pictures can be displayed at very high resolutions without flicker.

Disadvantages:- are that they ordinarily do not display color and that selected parts of a picture cannot be erased. To eliminate a picture section, the entire screen must be erased & the modified picture redrawn. The erasing & redrawing process can take several seconds for a complex picture.

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Raster Scan Display

Most common method.

Beam is swept across the screen one line at a time, pixel by pixel.

Uses frame Buffer (an array, on & off state)

Screen is refreshed 60 times/sec -
Interlacing (odd numbered & even numbered lines).

Progressive ~~sa~~ scanning :- alternate to interlacing, reduces flickers - refreshes every line on screen.

Random Scan Display:

Also called vector/stroke/calligraphic Displays.
Beam is swept across the screen one line at a time, only intended point.
Picture Definition :- Display file (set of line drawing commands).
Only area where picture is drawn is refreshed.

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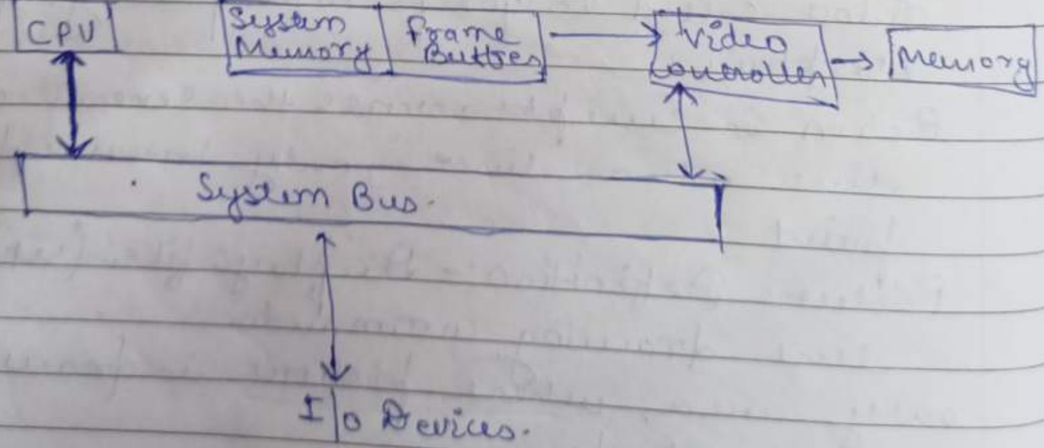
Rastere Scan System.

Interactive raster-graphics systems typically employ several processing units:

- CPU
- Special purpose registers - video/Display controller.
- System Memory.

How the Interactive Graphics Display works:-

1. frame Buffer.
2. Monitor like a TV set without the tuning and recieving electronises.
3. Display Controller:- It passes the contents of the frame buffer to the monitor.



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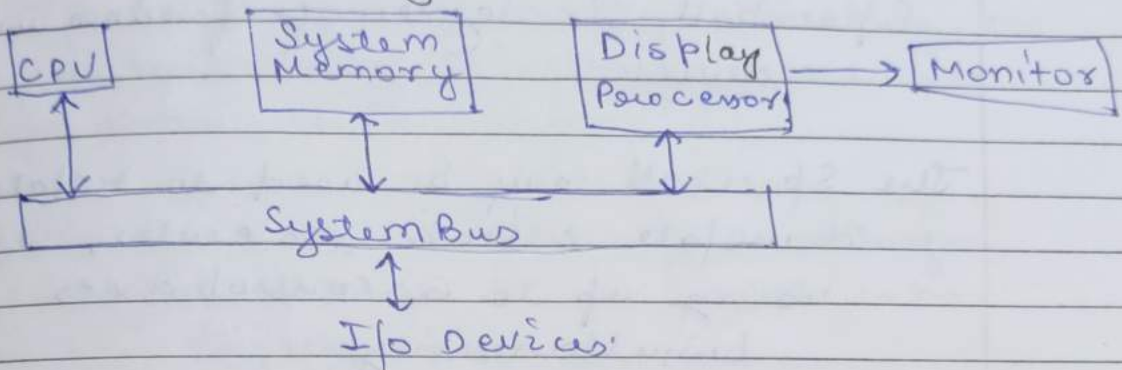
Architecture of Simple Random Scan System



Application programs are stored in system memory. Graphics commands in the program are translated by the graphics package into a display file stored in the system memory. This display file is accessed by the display processor to refresh the Screen. Display processor in a random scan System is refered to as a display processing unit or graphics controller.

Input Devices:-

→ It is a ball that is a ball that can be rotated with fingers to produce cursor movements.

→ Potentiometers attached to the ball, measure the amount & direction of rotation.

→ Normally mounted on KB.

→ 2d positioning device.

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Spaceball

Spaceball six-degree-of freedom input devices.

The Spaceball may be used to rotate, translate & Scale molecules, using up to 6 control axes simultaneously.

The spaceball can be used independently & simultaneously with the mouse.

Spaceball are used for 3D positioning & selection operations in virtual reality systems, modeling, animation CAD, & other application.

Joystick

A joystick is a input device consisting of a stick that pivots on a base & reports its angle or direction to the device it is controlling.

Data Glove

A glove equipped with sensors that sense the movements of the hand & interfaces those movements with a Computer.

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Digitizers:-

A device for drawing, painting, or interactively selecting coordinate positions on an object.

conversion of a Typically analog object, image or a signal into digital form.

Graphics Tablet:- It is a type of Digitizer. It enables a user to hand-draw images, animations & graphics, similar to the way a person draws images with a pencil & paper.

Image Scanners:- Is a device that optically scans images, printed text, handwritting, or an object, & converts it to a digital image.

Types:-

Drum, flat Bed, film, HandHeld

Touch Panels

Touch Panels allow displayed objects or screen positions to be selected with touch of a finger.

Types of Touch Panels:-
1. Optical touch panels
2. Resistive film touch panels
3. ~~Sao~~ capacitive touch panels.

Light Pens :- It is in the form of a light-sensitive wand used in conjuction with a computer's CRT display.
It allows the user to point to displayed objects, or draw on the screen but with greater positional accuracy.

A light pen can work with any CRT-based display, but not with LCD displays, Projectors

Voice System

A device in which speech is used to input data or system commands directly into a system.

Such equipment involves the use of Speech recognition processes & can replace or supplement other input devices.

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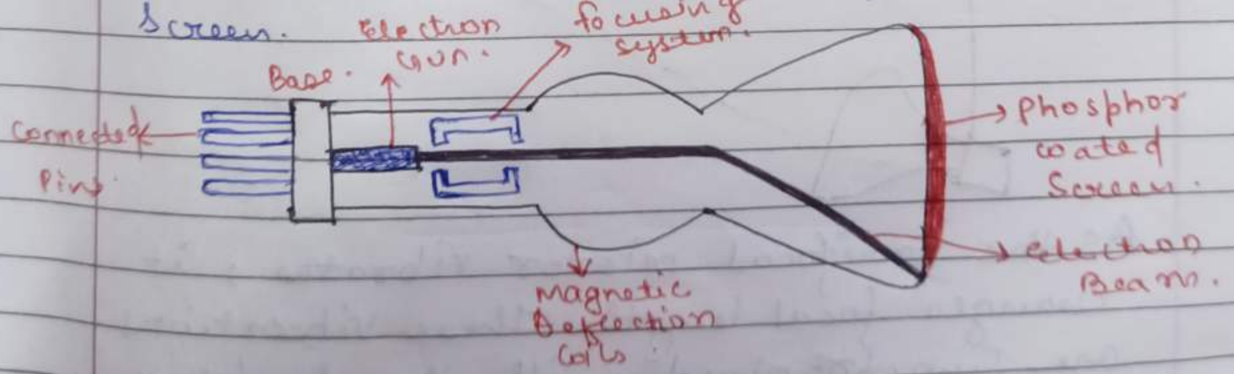
Video Display Devices :-

→ Refresh cathode-Ray Tubes (CRT's)
→ Raster Scan Displays
→ Random scan Displays
→ color CRT Monitors.
→ flat - panel Displays.

Refresh cathode-Ray Tubes (CRT)

→ A beam of electrons emitted by an electron gun, passes through focusing & deflection systems that direct the beam toward specified positions on the phosphor-coated screen.

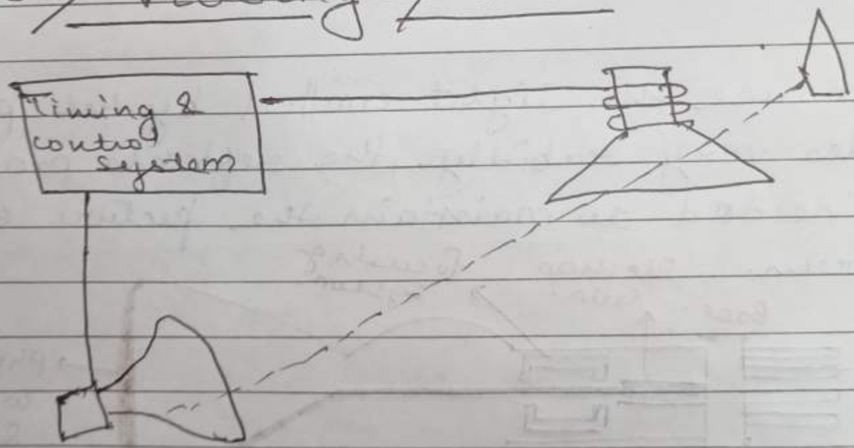
→ Because the light emitted by the phosphor fades very rapidly, the refresh process is needed to maintain the picture on the screen.



Flat Panel Displays

Thin Screen Displays found with all portable computers & becoming the new standard with desktop computers. Instead of utilizing the cathode-ray tube technology flat-panel displays use liquid-crystal Display (LCD) technology or other alternative making them much lighter & thinner when compared with a traditional monitor

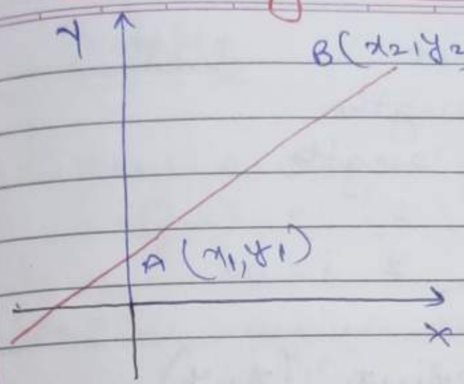
3D viewing Devices



As the varifocal mirror vibrates, it changes focal length. These vibrations are synchronized with the display of an object on a CRT so that each point on the object is reflected from the mirror into a spatial position corresponding to the distance of that point from a specified viewing position.

CHAPTER-3

Drawing Primitives:-



Equation of line

$y = mx + c$

$m = \Delta y / \Delta x$

$\Delta x = x_2 - x_1$

$\Delta y = y_2 - y_1$

Line can be draw by Plotting pixels in Sequence.

what is DDA?

Digital Differential Analyzer (DDA) algorithm is an incremental scan conversion method of line drawing. DDA is used for linear interpolation of variables over an interval between start and end point. DDA is used for rasterization of lines, triangles and polygons.

Algorithm:-

Step1 Read two end points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$

Step2: find approximate length of the line i.e

if (ABS$(x_2 - x_1)$ > ABS$(y_2 - y_1)$) then
length = ABS $(x_2 - x_1)$
otherwise
length = ABS$(y_2 - y_1)$

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Step 3:- find raster unit
$dx = (x_2 - x_1) / length$.
$dy = (y_2 - y_1) / length$.

Step 4: Set $x = x_1$, $y = y_1$ & $i = 0$.

Step 5: Now plot the point (x, y)
$x = x + dx$
$y = y + dy$.

Step 6: Repeat step 5 until $i <= length$.

Step 7: Stop

Advantage:-

1. It is simplest algo. & easy to implement
2. DDA is faster method than Direct method of line drawing.
3). DDA does not use multiplication theorem.

Disadvantage:-

1. Floating point ~~algo~~ arithmetic in DDA algo is still time consuming

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Example

Draw a line with end points (0,0) & (8,8) using DDA algo.

Step 1 receive two end points (0,0) & (8,8)

i.e $x_1=0$ $y_1=0$ $x_2=8$ $y_2=8$

step-2 $abs(x_2-x_1) = (8-0) = 8$

$abs(y_2-y_1) = (8-0) = 8$ = length.

Step 3 :- $dx = 1$ $\frac{(x_2-x_1)}{length}$

$dy = 1.$ $\left(\frac{y_2-y_1}{length}\right)$

Step 4: set initial point (x, y)

$x=0$

$y=0$

Step 5 execute until $i <= length$

i	x	y	Ploting points.
0	0	0	P_1 (0,0).
1	1	1	P_2 (1,1)
2	2	2	P_3 (2,2)
3	3	3	P_4 (3,3)
4	4	4	P_5 (4,4)
5	5	5	P_6 (5,5)
6	6	6	P_7 (6,6)
7	7	7	P_8 (7,7)
8	8	8.	P_9 (8,8)

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Bresenham's Line Drawing Algorithm

This algorithm is accurate and efficient line generating algorithm. It was developed by Jack Elton Bresenham in 1962 at IBM. It scan converts line using only incremental integer calculations

Algorithm

Step1: Read two end points $P_1(x_1, y_1)$ & $P_2(x_2, y_2)$

Step2: Calculate
$$dx = x_2 - x_1$$
$$dy = y_2 - y_1$$

Step3: Calculate descision parameter P
$$P = 2dy - dx.$$

Step4: Set initial point
$$x = x_1 \quad , y = y_1$$
$$\& \; i = 0.$$

Step5: Now plot the point (x, y)
if $P < 0$ then
$x = x + 1$
$P = P + 2dy$
Otherwise, $x = x + 1$
$y = y + 1$
$P = P + 2dy - 2dx$

Repeat step 5 until $i < dx$.

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Advantages:-

→ It is simple because it involves only integer arithmetic

→ It avoids the generation of duplicate points

→ It is faster than DDA because it doesn't involve floating point calculations.

Disadvantages:-

→ This algorithm is for basic line drawing.
→ It always assume that $x_1 < x_2$
→ we can not draw vertical line using Bresenham's line drawing algo.

Q) Draw a line with end points (20,10) & (30,18) using Bresenham's algo.

Step1 Points (20,10) & (30,18)
$x_1 = 20$ $y_1 = 10$ $x_2 = 30$ $y_2 = 18$.

Step2
$dx = x_2 - x_1 = 10$
$dy = y_2 - y_1 = 8$.

Step3: calculate decision parameter.
$P = 2dy - dx$
$= 6.$

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Step 4: Set initial point (x, y)

$x = x_1 = 20$

$y = y_1 = 10$

& $i = 0$.

Step 5. Execute until $i <= dx$.

i	P	Plotting points
		(20,10)
0		
1	6	(21,11)
2	2	(22,12)
3	-2	(23,12)
4	14	(24,13)
5	10	(25,14)
6	6	(26,15)
7	2	(27,16)
8	-2	(28,16)
9	14	(29,17)
10	10	(30,18).

Difference b/w DDA & Bresenham

DDA	Bresenham
→ DDA algo use floating point calculations.	→ Bresenham's line algo use integer arithmetic calculations.
→ DDA also use multiplication & division in its operations	→ Bresenham's line algo use addition & subtraction in its operation.

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→ DDA algorithm takes time to draw line because of floating point operations.	→ Bresenham's line algo draw line faster than DDA algo
→ DDA algo is less accurate & efficient	→ more accurate & efficient.

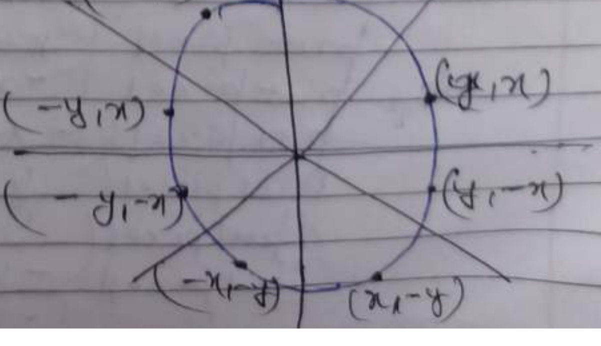
Scan Conversion a Circle

A circle is defined as the set of points that are all at a given distance (r) from a center position (xc, yc).

The circle is divided into 4 quadrant, each quadrant has two octant. So circle is also known as Eight-way symmetric fig.

For drawing we consider that circle centered at origin (0, 0). If a point (x, y) lies in one octant then seven point are:-

(x, -y)
(-x, -y)
(-x, y)
(y, x)
(-y, -x)
(-y, x)



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Bresenham's Circle Drawing

This algorithms selects nearest pixel position to complete arc. It is very significant algorithm & faster than other circle drawing algo because it uses only integer arithmetic.

Algorithm :-

Step 1 input center of the circle (p, q)

Step 2:- Input radius of the circle r

Step 3:- Calculate $d = 3 - 2r$

Step 4:- Initialize $x = 0$
$y = r$

Step 5: Now, center is at (p, q) & current pixel is (x, y) so plot eight points by using eight way symmetry concept

putpixel (x + p, y + q)
put pixel (x + p, - y + q)
put pixel (- x + p, - y + q)
put pixel (- x + p, y + q)
put pixel (y + p, x + q)
put pixel (y + p, - x + q)
put pixel (- y + p, - x + q)
put pixel (- y + p, x + q)

Step 6

Step 7 :

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Step 6: find next pixel location

if d <= 0 then
$x = x+1$
$d = d + 4x + 6$

Otherwise
$x = x+1$
$y = y-1$
$d = d + 4(x-y) + 10$

Step 7: if $x <= y$ then
go to setep 5
otherwise
Stop algo.

Advantages.
→ It is simple algo
→ It can be implement easily

→ There is the prblm of accuracy while generating points.
→ This algo is not suitable for the complex & high graphic images.

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Digital Differential Analyzer (DDA)

→ (1,1) and (3,3)

Slope $(m) = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{3-1}{3-1} = \frac{2}{2} = 1.$

II Find Δx and Δy

$\Delta x = \frac{\Delta y}{m} = \Delta y \frac{x_2 - x_1}{y_2 - y_1}$

$\Delta x = 2$

$\Delta y = m \cdot \Delta x = \frac{y_2 - y_1}{x_2 - x_1} \cdot \Delta x$

$\Delta y = 2$

III If $|\Delta x| \geq |\Delta y|$

Assign $\Delta x = 1$

$x_{i+1} = x_i + \Delta x$

$= x_i + 1$

$y_{i+1} = y_i + \Delta y = y_i + m \cdot \Delta x = y_i + m$

• if $|\Delta x| < |\Delta y|$

Then assign $\Delta y = 1$

$x_{i+1} = x_i + \Delta x = x_i + \frac{\Delta y}{m} = x_i + \frac{1}{m}$

$y_{i+1} = y_i + \Delta y = y_i + 1.$

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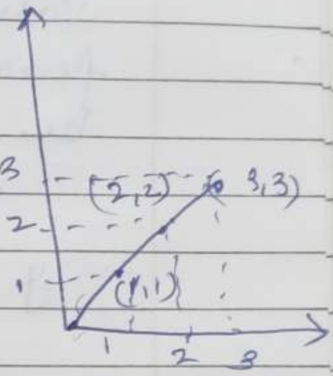
3. $|\Delta x| \geq |\Delta y|$, so assign $\Delta x = 1$.

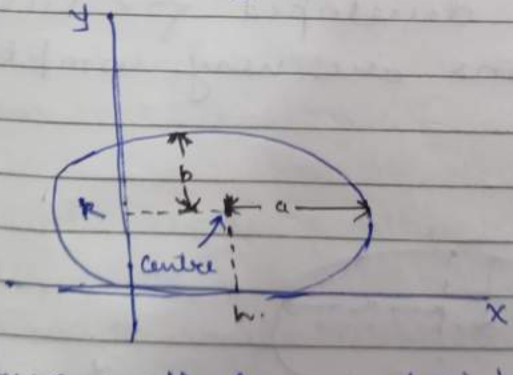
$x_{i+1} = 2$ and $y_{i+1} = 1+1 = 2$
$x_{i+1} = 2+1$ and $y_{i+1} = 2+1 = 3$.

x_i	y_i	x_{i+1}	y_{i+1}
1	1	2	2
2	2	3	3
3	3		



Scan Converting Eclipse

The ellipse is also a symmetric figure like a circle but is four-way symmetry rather than eight-way.



- There two methods of defining an Ellipse:

1. Polynomial method of defining an ellipse.
2. Trigonometric method of definining an Ellipse.

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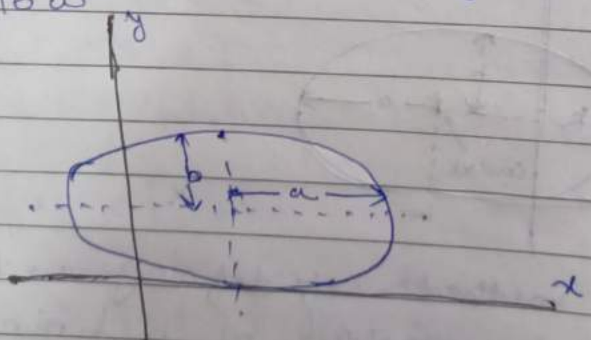
The ellipse has a major and minor axis. If a_1 and b_1 are major and minor axis respectively. The center of ellipse is (i, j). The value of x will be incremented from i to a, & value of y will be calculated using the following formula:

$$y = b_1 \sqrt{1 - \frac{x-i}{a_1^2}} + j$$

Drawback of Polynomial Method:-

1. It requires squaring of values. So floating point calculation is required

2. Routines developed for such calculations are very complex & slow.



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$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Major axis $= 2a = 2r_x$
Minor axis $= 2b = 2r_y$
Semi-major axis $= a = r_x$
Semi-minor axis $= b = r_y$

$$\frac{x^2 b^2 + y^2 a^2}{a^2 b^2} = 1.$$

$\Rightarrow b^2 x^2 + a^2 y^2 = a^2 b^2$
$\Rightarrow b^2 x^2 + a^2 y^2 - a^2 b^2 = 0.$

$r_y^2 x^2 + r_x^2 y^2 - r_x^2 r_y^2 = 0$ — (1)

If we put any point in eq^n (1)

$= 0$	< 0	> 0
Point lies on ellipse	Inside the ellipse	Outside ellipse

Difference b/w circle & Ellipse.

Circle has 8 way symmetry
Ellipse has 4-way symmetry.

In circle we need to plot only (1) octant, of any quadrant, but in ellipse we need to plot (2) octants i.e. 1 complete quadrant i.e. plot entire ellipse.

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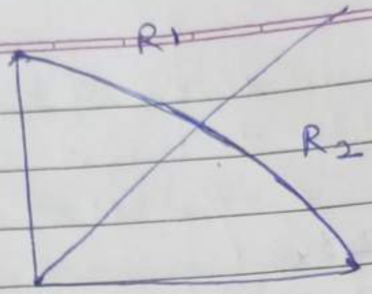

$x = a * \cos(\theta) + h$ and
$y = b * \sin(\theta) + k$

where (x, y) = the current coordinates
a = length of major axis
b = length of minor axis
θ = current angle
(h, k) = ellipse center

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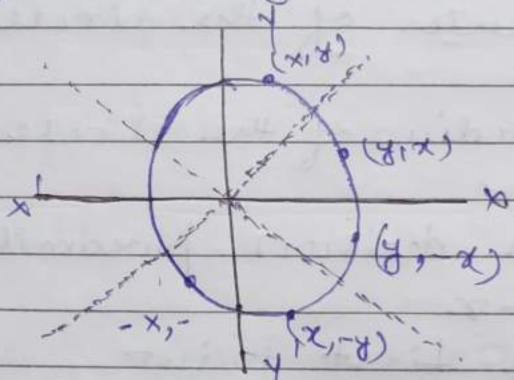
Midpoint Circle Drawing Algorithm

The Midpoint circle drawing algorithm use to determine the points needed for rasterizing a circle.
The mid point algorithm calculates all the points in the first octant of the circle & then prints them along with their mirror points in the other seven octants [because circle is a symmetric figure].



Mid-point circle algo based on the equation of the circle $x^2 + y^2 = r^2$

$$P = f(x, y) = x^2 + y^2 - r^2$$

First point is $(x, y) = (0, r)$
Next point of first octant will be the mid-point of $(x+1, y)$ and $(x+1, y-1)$

i.e. $(x+1, y-0.5)$
Put this point into equation of circle :-

$$P = (x+1)^2 + (y-0.5)^2 - r^2$$

To define decision parameter put $(0, r)$

$$P = 1 - r$$

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if $P < 0$ then
next point $(x+1, y)$
$P = P + 2x + 3$

if $P >= 0$ then
next point $(x+1, y-1)$
$P = P + 2(x-y) + 5$.

Algorithm :-

Step 1 Input center of the circle (xc, yc)

Step 2 Input radius of the circle r.

Step 3 Calculate decision parameter.
$P = 1 - r$

Step 4 Initialize first point
$x = 0$
$y = r$

Step 5: Now center is at (xc, yc) and current pixel is (x, y). So plot eight points by using eight way symmetry concept.

Putpixel $(x + xc, y + yc)$
" $(x + xc, -y + yc)$
" $(-x + xc, -y + yc)$
" $(-x + xc, y + yc)$
" $(y + xc, x + yc)$
" $(y + xc, -x + yc)$
" $(-y + xc, -x + yc)$
" $(-y + xc, x + yc)$

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Step 6: Find next pixel.

if $P<0$ then

$x = x+1$

$P = P+2x+3$.

otherwise

$x = x+1$

$y = y-1$

$P = P+2(x-y)+5$.

Step 7: if $x <= y$ then

go to step 5.

otherwise

stop algo.

Advantages :-

1. It is very effective & efficient algo.
2. The algo is based on the Eq. of circle $x^2+y^2=r^2$
3. It can be implement Easily

Disadvantage:-

1. There is a problem of accuracy while generating points.
2. This algo is time consuming

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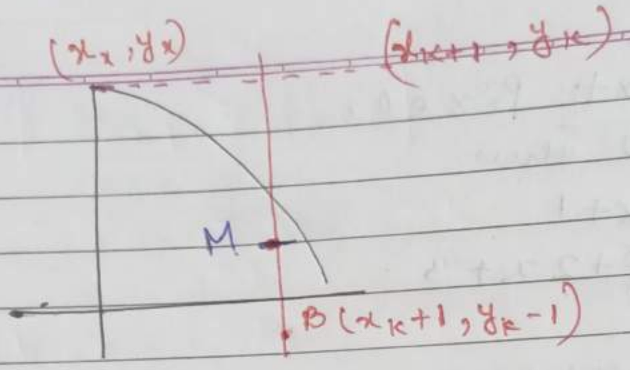

$$x^2 + y^2 = r^2$$
$$x^2 + y^2 - r^2 = 0$$

$(x_m, y_m) \rightarrow$ coordinates of M.

$$(x_m)^2 + (y_m)^2 - r^2$$

M $\rightarrow$ 0 : Point lies on circle.
A $\rightarrow$ <0 : point lies inside
B $\rightarrow$ >0 : Point lies Outside.

Mid point coordinates :- $\left[\left(\frac{x_k+1 \; x_k+1}{2}\right), \frac{(y_k + y_k - 1)}{2}\right]$

$$= \left[x_k + 1 \;,\; y_k - \frac{1}{2}\right]$$

$$d_k = (x_k+1)^2 + \left(y_k - \frac{1}{2}\right)^2 - r^2$$

$$d_{k+1} = (x_{k+1}+1)^2 + \left(y_{k+1} - \frac{1}{2}\right)^2 - r^2$$

$$d_{k+1} - d_k.$$

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$d_{k+1} = d_k + 2x_k + 3 + (y_k+1)^2 - y_{k+1} - (y_k)^2 + y_k$.

If $d_k < 0$ then $y_{k+1} = y_k$.

$d_{k+1} = d_k + 2x_k + 3$.

If $d_k \geq 0$ then $y_{k+1} = y_k - 1$

$d_{k+1} = d_k + 2x_k - 2y_k + 5$.

$(0, r)$ IDP(d_0)

$d_0 = (0+1)^2 + \left(r - \frac{1}{2}\right)^2 - r^2$

$= \frac{5}{4} - r$.

Numerical Based Algorithm (MPA)

coordinate $(0,0)$

Radius (10)

1. $x_k = 0$ $y_k = 10$

2. $d_0 = 1 - R$
   $= 1 - 10$
   $= -9$.

3. $d_0 < 0$
   $x_{k+1} = x_k + 1 = 0 + 1 = 1$.

   $y_{k+1} = y_k = 10$

   $d_{k+1} = -9 + 2 \times 0 + 3$

   $= -6$

(4) $\boxed{x_{k+1} \geq y_{k+1}}$

Ques

-9	-6	-1	6	-3	8
-6	-1	6	-3	8	5
(1,10)	(2,10)	(3,10)	(4,9)	(5,9)	(6,8)

$x_{k+1} = x_k + 1 = 7$
$y_{k+1} = y_k - 1 = 7.$

$d_k + 1 = d_k + 2x_k + 2y_k + 5$

(x,y)	(-x,y)	(-x,-y)	(x,-y)
(0,10)	(0,10)	(10,-10)	(0,-10)
(1,10)	(-1,10)	(-1,-10)	(1,-10)
(2,10)	(-2,10)	(-2,-10)	(2,-10)
(3,10)	(-3,10)	(-3,-10)	(3,-10)
(4,9)	(-4,9)	(-4,-9)	(4,-9)
(5,9)	(-5,9)	(-5,-9)	(5,-9)
(6,8)	(-6,8)	(-6,-8)	(6,-8)

x
y

(8,6)	-8,6	-8,-6	(8,-6)
(9,5)	8,6		-9+2
(9,4)	9,5		
(10,3)			
(10,2)			
(10,1)			
(10,0)			

$y_k = y_k + 1$
$\Rightarrow 10$

$x_k = x_{k+1} + 1$
$\Rightarrow 2 + 1$
$= 3.$

$d_{k+1} = d_k + 2x_k + 3$
$\Rightarrow -9 + 2 \times 3 + 3$
$\Rightarrow -9 + 9$
$\Rightarrow 0.$

$\Rightarrow -6 + 2 \times 2 + 3$
$\Rightarrow -6 + 4 + 3$
$\Rightarrow 1$

Ques $(5, 5)$ (x_c, y_c)

$p = 10$.

$x_k = 5$ $y_k = 5 + 10 = 15$.

$d_0 = 1 - R$
$= 1 - 5$
$= -4$

$x' = x + x_c$
$y' = y + y_c$.

$d_0 < 0$

$x_{k+1} = x_k + 1 = 5 + 1 = 6$.

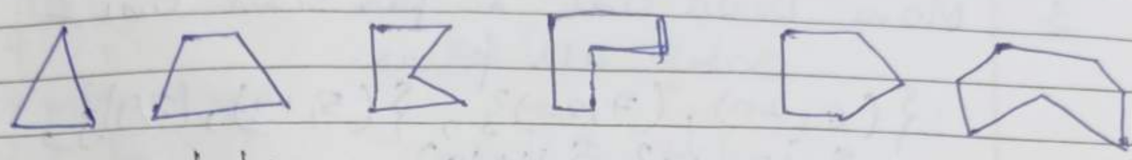
$y_{k+1} = y_k = 15$.

x	5	6	7	8	9	10	11
y	15	15.	15	15.	14	14	13

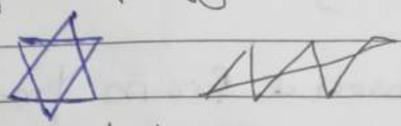
$d_{k+1} = d_k + 2x_k + 3$
$= -4 + 2 \times 5 + 3$
$= -4 + 10 + 3$
$= \underline{9}$.

$y_{k+1} = y_k - 1$
$= 15 - 1$
$= 4$.

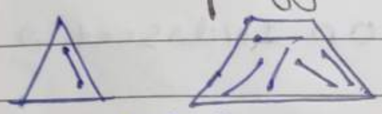
Polygon



complex polygon.

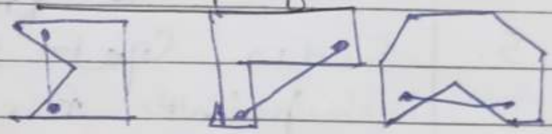


Concave polygon :-



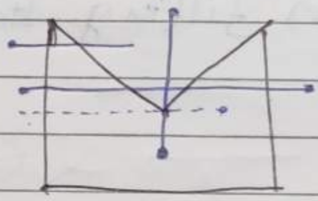
<180°

Convex polyon :-



>180'

Odd-even :-
Odd - Inside.
Even - Outside.

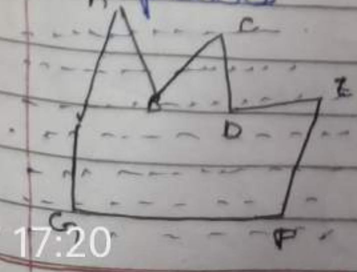


Same-even.(2)
not on same side (odd)

Polygon Area Filling :- Scan line Algorithm

Steps :-

1. Locate the intersection of the scan line with the polygon Edges.
2. pairing Inter section points.
3. Move Down as per Scan line & sort all pairs.



3. Move Down side as per Scan line & sort all pairs.

{(6,20), (7,20)}, {(5,19), (8,9)},
{12,19} {14,19}

4. All pairs are sorted from Y_{max} to Y_{min}.

5. Sides Get sorted on Intersection point Basis.

6. Area filling starts now.

1)
2)
3)
4)

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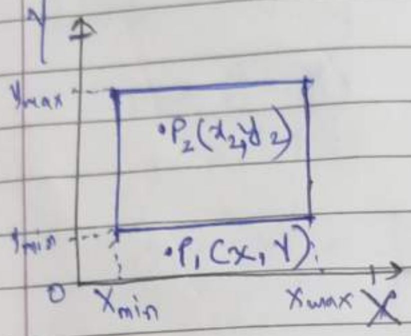
Points Clipping



BL(2, 3)
↓ X_{min} ↓ y_{min}

UR(9, 11)
↓ X_{max} ↓ y_{max}.

1). $X_1 \geq X_{min}$.
2). $X_1 \leq X_{max}$.
3). $y \geq y_{min}$
4). $y \leq y_{max}$

satisfying all conditions we can say the point is in the plane

(5, 1)
x_1, y_1

$5 \leq 9$
$1 \leq 3$ X
$1 > 9$ X

Mi.
(2, 3)

(9, 11)

Cohen Sutherland Algorithm

I/P.

O/P.

1001	1000	1010
0001		0010
0101	0100	0110

4 bit region code.

T B R L.
0 0 0 0. (center).

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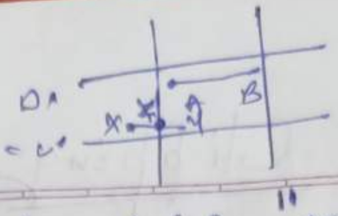


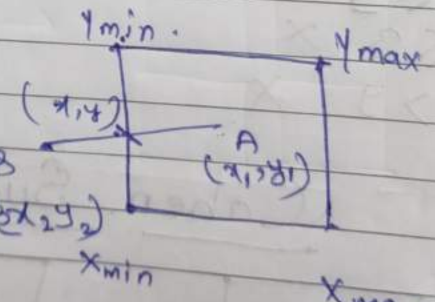
1. Line inside A & B "0000"

2. Outside Reject.
C 0001
d 0001
Logic AND 0001 $\boxed{\text{NOT } 0000}$

3. Partially. (Mehat)
X 0001
Y. 0000
0000

X'Y → Accept.
X−X' → Reject.

Algo



→ left: $x = x_{min}$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{y - y_1}{x - x_1} = \frac{y - y_1}{x_{min} - x_1}$$

$$y - y_1 = m(x_{min} - x_1)$$
$$y = m(x_{min} - x_1) + y_1$$

→ Right: $x = x_{max}$.
$$y = y_1 + m(x_{max} - x_1)$$

Exam

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Top :- $y = y_{max}$

$$m = \frac{y - y_1}{x - x_1} = \frac{y_{max} - y_1}{x - x_1}$$

$$x = x_1 + \frac{(y_{max} - y_1)}{m}$$

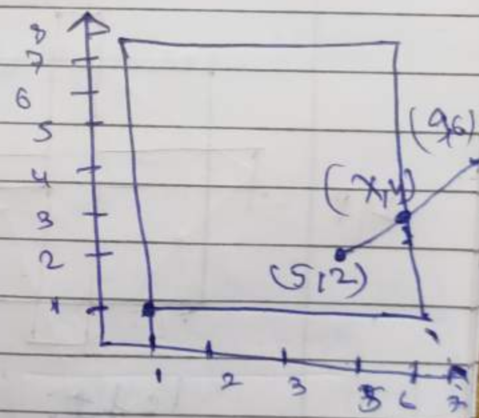
Bottom

$y = y_{min}$

$$x = x_1 + \frac{(y_{min} - y_1)}{m}$$

Examples

BL : (1,1)
UR : (7,8)



$x_{min} = 1$ $\quad$ $x_{max} = 7$

$y_{min} = 1$ $\quad$ $y_{max} = 8$

line :- (5,2) $\quad$ (9,6)

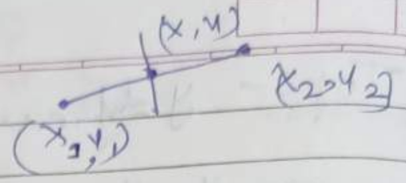
$\cancel{x_1 y_1}$ $\quad$ $\cancel{x_2 y_2}$

$x = x_{max}$

$y = y_1 + m(x_{max} - x_1)$

$\rightarrow 2 + m($

T	B	R	L
0	0	0	0
0	0	1	0
0	0	0	0



$y = y_1 + m(x_{max} - x_1)$

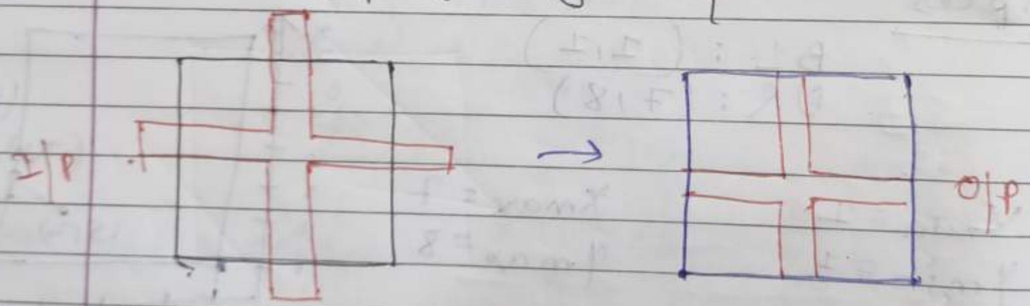
$\Rightarrow = 2 + 1(7 - 5)$ $\quad$ $\underline{\underline{(7, 4)}}.$

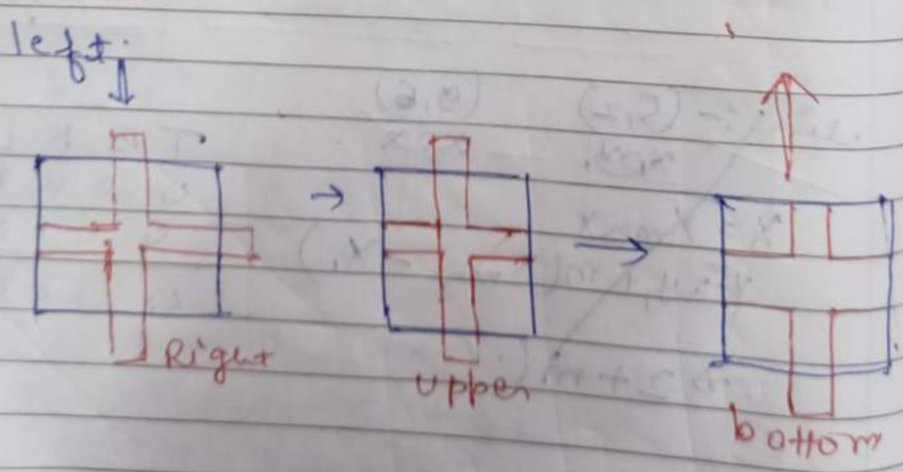
$m = \frac{y_2 - y_1}{x_2 - x_1}$

$\Rightarrow 2 + 2\times 1$
$= 4$

$\Rightarrow \frac{6-2}{9-5} = \frac{4}{4} \Rightarrow 1$

Sutherland Hodgeman Polygon Clipping Algorithm





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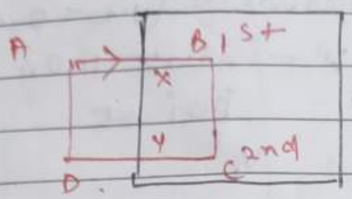


1). O → I (X B)

2) I → I 2nd c

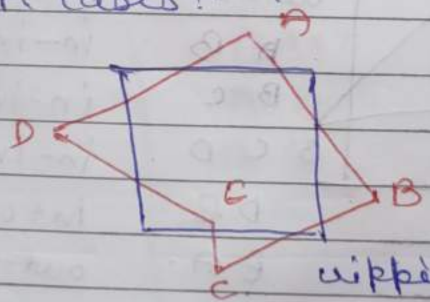
3). I → O(Y)

4). O → O (ignore).

- It clips the region of the polygon lying outside the window.

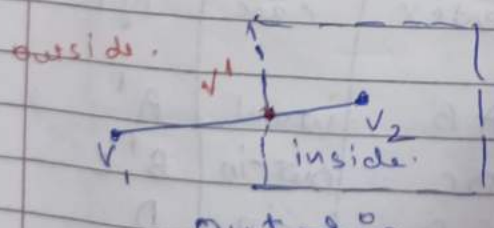
→ clip against each edge of window & obtain new set of vertices.

For finding new sequence of vertices, there are 4 cases:-



clipping window.

(i)



out → in

O/P → intersect point + destination point

→ V_1' V_2

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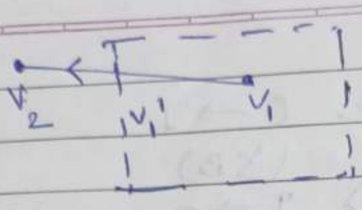
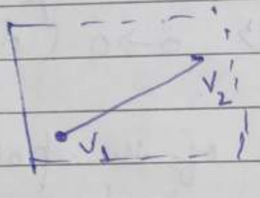
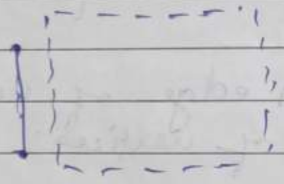
(ii) 

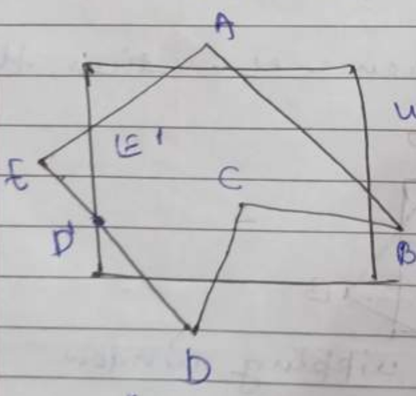
Movement (in - out)
o/p - intersection point
V_1'

(iii) 

Movement in to in
o/p → distination
$= V_2$

(iv) 

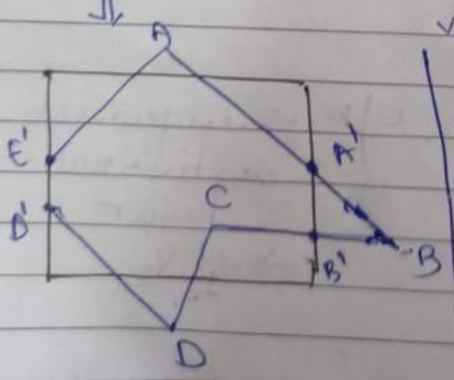
→ NULL



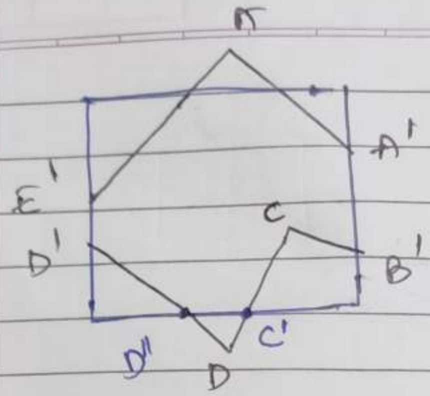
clipping:

vertex	cas	out
A B	in-in.	B
B C	in-in	C
C D	in-in	D.
D E	in+out	D'
E A.	out-in	E'A

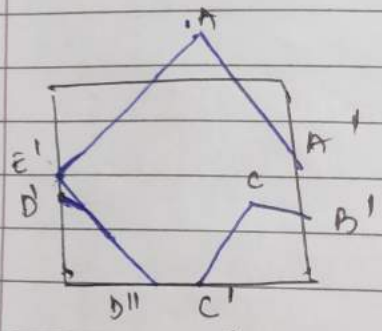
⇓



vertex	case	o/p
A B	in to out	A'
B C	out -in	B' C
C D	in-in	D
D D'	in-in	D'
D'E'	in-in	E'
E'A	in-in	A



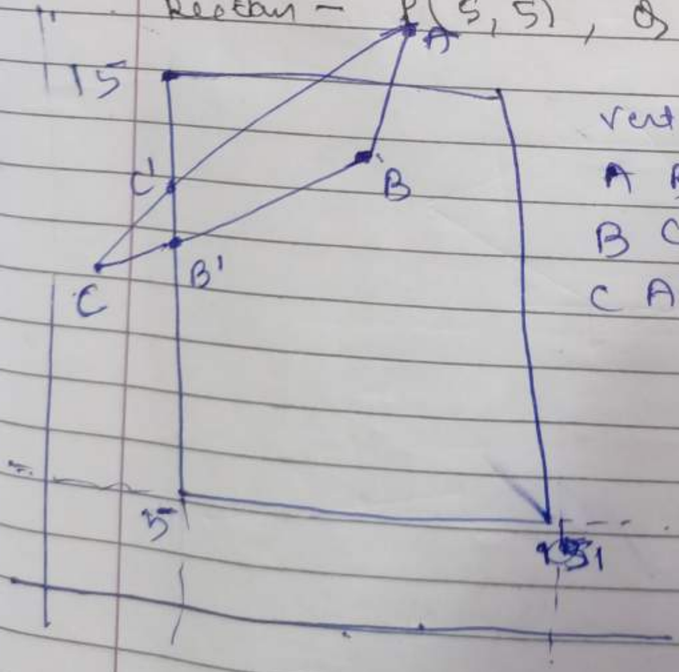
vertex		
A A'	in-in.	A'
A' B'	in-in	B'
B' C	in-in.	C
C D	in-out	C'
D D'	out-in	D'' D'
D' E'	in-in	E'
E' A	in-in.	A



A A'	out in.
A' B'	in-in
B' C'	
C C'	
C' D''	

Ques. A(10, 17), B(13,12) & C(3,8)

" Rectan – P(5, 5), Q(15,5), R(15,15) S(5,15)



vertex	case	out.
A B	in-in	B
B C	in-out	B'
C A	out-in	C' A.

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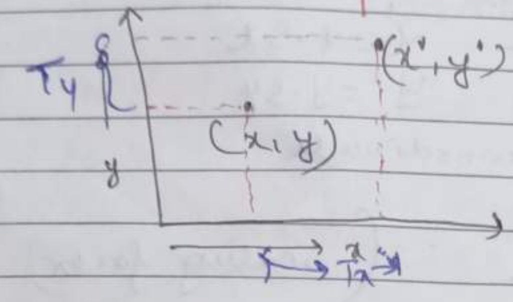
CHAPTER - 2 Unit - 4

Viewing And Transformation



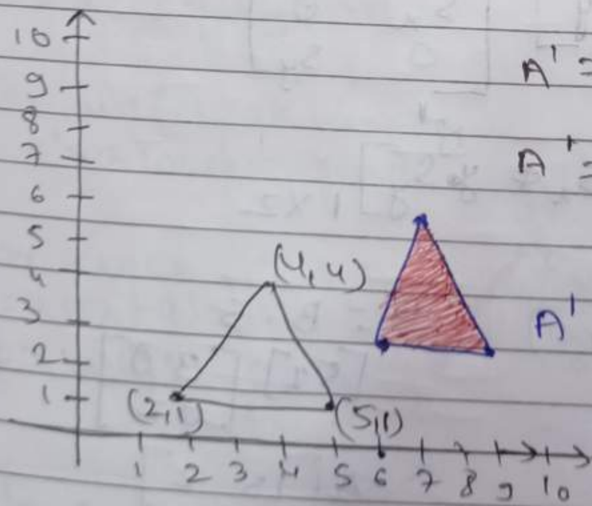
$$x' = x + T_x$$
$$y' = y + T_y$$

$$\boxed{A' = A + T}$$

$$A' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$A = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \quad T = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$A' = \begin{bmatrix} 3 \\ 4 \end{bmatrix} + \begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 7 \\ 9 \end{bmatrix}$$



$$A' = A + T$$

$$A' = \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$A' = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

$$B = B + T$$
$$\Rightarrow \begin{bmatrix} 5 \\ 1 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \end{bmatrix}$$

$$C = C + T$$
$$= \begin{bmatrix} 4 \\ 4 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

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Scaling :-

$x' = x \cdot S_x$
$y' = y \cdot S_y$

△ = △
old coordinate   new coordinate

S_x S_y (scaling factor)

$S_x, S_y > 1$ small → big
$S_x, S_y < 1$ big - small

$A' = A \cdot S$ $A = x\ y$
$B' = B \cdot S$ $S = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$
$C' = C \cdot S$

$$A' = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

$$A' = \begin{bmatrix} \overset{x'}{x S_x} & \overset{y'}{y S_y} \end{bmatrix}_{1 \times 2}$$

Examples

$A = (2, 1)$
$B (5, 1)$
$C (4, 4)$

$B' = B \cdot S$
$\begin{bmatrix} 5 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$
$\rightarrow \begin{bmatrix} 15 & 2 \end{bmatrix}$

3 units in 'x'
2 units in 'y'

$C' = \begin{bmatrix} 4 & 4 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$
$\rightarrow \begin{bmatrix} 12 & 8 \end{bmatrix}$

$A = A \cdot S$
$= \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} S_x{=}3 & 0 \\ 0 & S_y{=}2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$

$\begin{bmatrix} 6 & 2 \end{bmatrix}$ $A' = \overset{(x, y)}{6, 2}$

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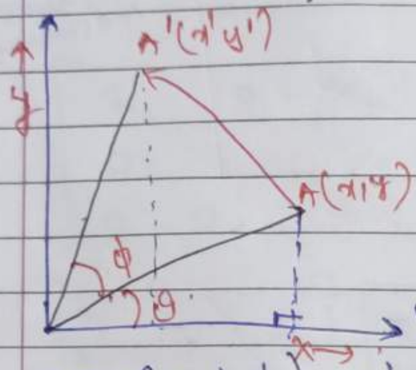
2) Rotation



$\cos\theta = \frac{x}{r}$

$x = r\cos\theta$

$\sin\theta = \frac{y}{r}$

$y = r\sin\theta$

$\cos(\theta + \phi) = \frac{x'}{r}$ $\sin(\theta+\phi) = y/r$

$x' = r\cos(\theta + \phi)$ $y' = r\sin(\theta+\phi)$

$= r[\cos\theta\cos\phi - \sin\theta\sin\phi]$ $= r[\sin\theta\cos\phi + \cos\theta\sin\phi]$

$= r\sin\theta\cos\phi + r\cos\theta\sin\phi$

$= r\cos\theta\cos\phi - r\sin\theta\sin\phi$

$x' = x\cos\phi - y\sin\phi$ $y' = \cancel{y\cos\phi + x\sin\phi}$

$y' = x\sin\phi + y\cos\phi$

for anticlockwise

$x' = x\cos\phi - y\sin\phi$ $y' = \cancel{x\sin\phi}\, x\sin\phi + y\cos\phi$

for clockwise

$x' = x\cos(-\phi) - y\sin(-\phi)$ $\cancel{x\sin(-\phi) + x\sin}$

$x = x\cos(\phi) + y\sin\phi$ $y' = x\sin(-\phi) + y\cos(-\phi)$

$= -x\sin\phi + y\cos\phi$

$A' = A \cdot R$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$

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$0 \quad \frac{1}{2} \quad \frac{1}{\sqrt{2}} \quad \frac{\sqrt{3}}{2} \quad 1$

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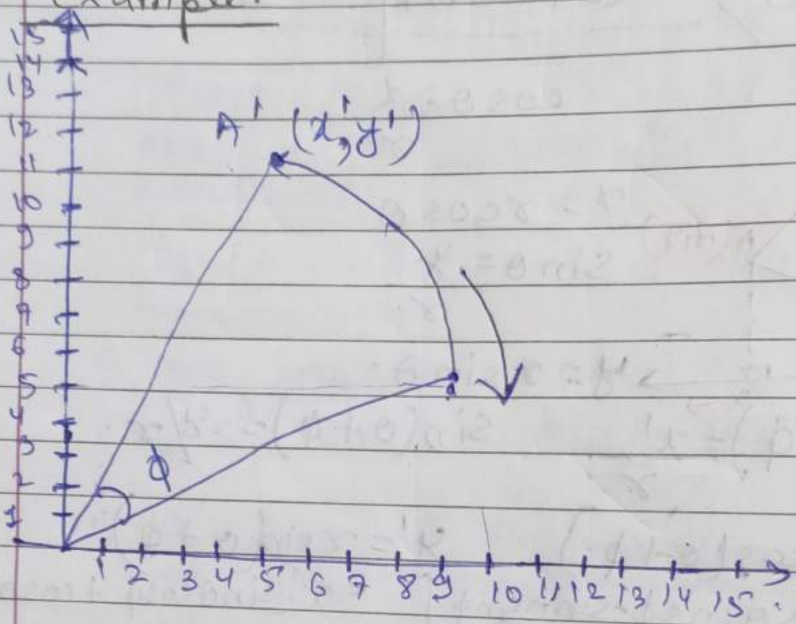
Example:-



$A' = A \cdot R$

$$x'y' = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix}$$

$$\begin{bmatrix} 10 & 5 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\begin{bmatrix} \frac{10}{\sqrt{2}} - \frac{5}{\sqrt{2}} & \frac{10}{\sqrt{2}} + \frac{5}{\sqrt{2}} \end{bmatrix}$$

anti°

$$\begin{bmatrix} \frac{5}{\sqrt{2}} & \frac{15}{\sqrt{2}} \end{bmatrix}$$

clock

$$\begin{bmatrix} \frac{15}{\sqrt{2}} & \frac{-5}{\sqrt{2}} \end{bmatrix}$$

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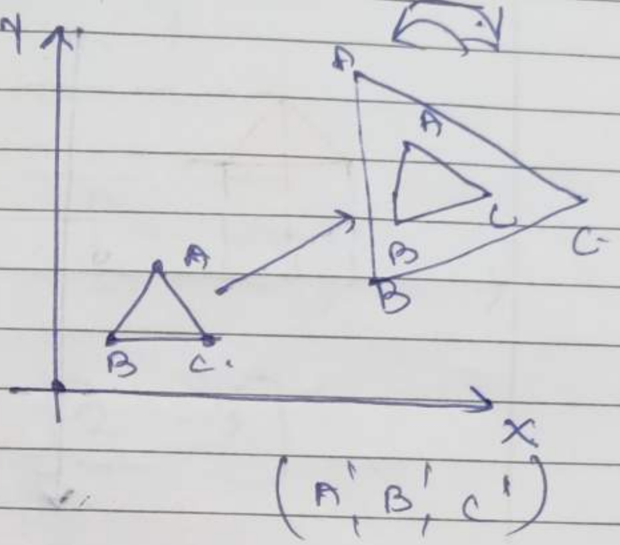
Scaling

T S R Rotation.

Trans formation

Homogeneous Coordinate

$$\begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$



$$\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta & (-\sin\theta) & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

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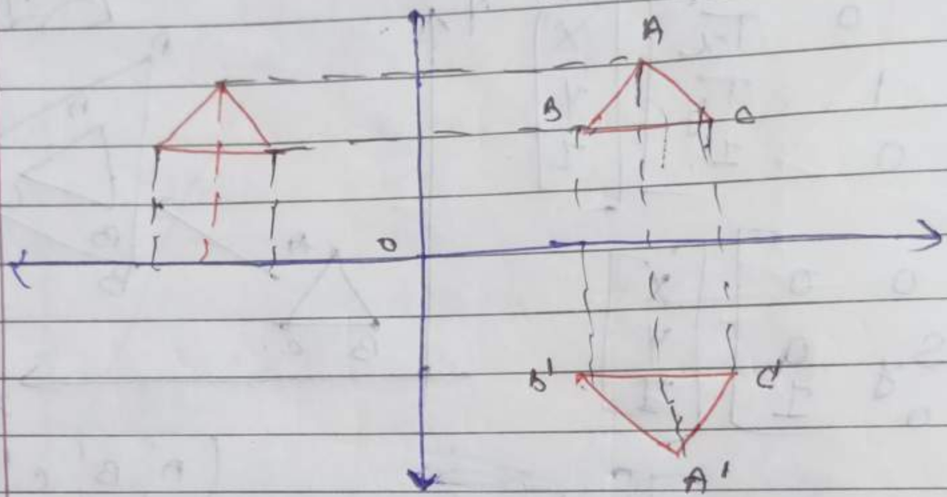
Reflection



Reflection on X axis

$x' = x$
$y' = -y$

$A' = R \cdot A$

$$\begin{matrix} x' \\ y' \end{matrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Reflection on Y axis:—

$x' = -x$
$y' = y$

$$\begin{matrix} x' \\ y' \end{matrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

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Examples

$A(3,4)$ $\qquad x = x'$

$B(2,3)$ $\qquad y = -y$

$C(4,3)$

$$A' = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 3 & -4 \end{bmatrix}$$

$$B' = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & -3 \end{bmatrix}$$

$$C' = \begin{bmatrix} 4 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -3 \end{bmatrix}$$

$x' = -x$ $\qquad \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} -3 & 4 \end{bmatrix}$

$y = y$

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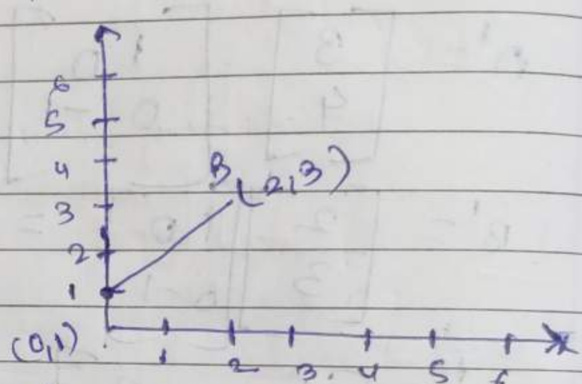
Transformation of Straight line

Straight line defined by two position vectors - end points A and B.

$A = [0, 1]$

$B = [2, 3]$

$$T = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2}$$



new points A^* & B^*

$$[A][T] = [0\ 1]\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} = [3\ \ 1]$$

$$[B][T] = [2\ 3]\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} = [11\ \ 7]$$

2+6

$$[L][T] = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 11 & 7 \end{bmatrix}$$

7

6+1

2+9

11

4+3

Mid-point Transformation

2×2 Matrix.

Points on second line have a one to one correspondance with points on 1^{st} line.

To confirm this - transformation of mid-point.

Let:

$A = [x_1 \quad y_1]$

$B = [x_2 \quad y_2]$

$$T = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

After transformation of line

$$\begin{bmatrix} A^* \\ B^* \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix} [T] = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ax_1 + cy_1 & bx_1 + dy_1 \\ ax_2 + cy_2 & bx_2 + dy_2 \end{bmatrix} \quad (1)$$

$L = \begin{matrix} A^* \\ B^* \end{matrix}$

Hence, the endpoints of transformed line $A^* B^*$ are

$$[A^*] = [(ax_1 + cy_1) \quad (bx_1 + dy_1)] = [x_1^* \quad y_1^*] \quad (2)$$

$$[B^*] = [(ax_2 + cy_2) \quad (bx_2 + dy_2)] = [x_2^* \quad y_2^*]$$

(x_2, y_2)

(x_1, y_1)

Now we calculate midpoint of transformed line.

$$x_m^* \quad y_m^* = \begin{bmatrix} \frac{x_1^* + y_1^*}{2} & \frac{y_1^* + y_2^*}{2} \end{bmatrix} \quad (3)$$

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$$= \left[\frac{(ax_1 + cy_1 + ax_2 + cy_2)}{2} \quad \frac{(bx_1 + dy_1 + bx_2 + dy_2)}{2} \right]$$

$$[X_m^* \;\; Y_m^*] = \left[\frac{a(x_1+x_2) + c(y_1+y_2)}{2} \quad \frac{b(x_1+x_2) + d(y_1+y_2)}{2} \right] \quad \text{(4)}$$

mid-point of transformed line.

So, mid point of original line AB,

$$[x_m \;\; y_m] = \left[\frac{x_1+x_2}{2} \quad \frac{y_1+y_2}{2} \right] \quad \text{— (5)}$$

using transformation matrix [T] we transform mid point.

$$[x_m \;\; y_m][T] = \left[\frac{x_1 \, x_2}{2} \quad \frac{y_1+y_2}{2} \right] \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= \left[\frac{a(x_1+x_2)}{2} + \frac{c(x_1+x_2)}{2} \quad \frac{b(x_1+x_2)}{2} + \frac{d(y_1+y_2)}{2} \right] \quad \text{(6)}$$

So, Here Equation of (4) & (6) both are same.

the means of transformed line = mid point of original line

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Example of Mid-point

Consider the line AB with end points $[A] = [0 \quad 1]$ and $[B] = [2 \quad 3]$

$$T = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$$

After transformation of line AB. $A^* B^*$

$$\begin{bmatrix} A^* \\ B^* \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 11 & 7 \end{bmatrix}$$

Mid point of $A^* B^*$

$$[x_m^* \quad y_m^*] = \left[\frac{3+11}{2} \quad \frac{1+7}{2}\right] = [7 \quad 4] \rightarrow (1)$$

Now, mid point of original line.

$$[x_m \quad y_m] = \left(\left[\frac{0+2}{2}\right] \cdot \left[\frac{1+3}{2}\right]\right)$$

$$\Rightarrow [1 \quad 2]$$

Transforming this mid point

$$[1 \quad 2] \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} = [1+6 \quad 2+2]$$

$$\Rightarrow [7 \quad 4] \rightarrow (2)$$

Eqv (1) & (2) are same

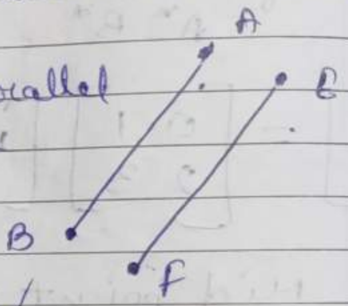
Transformation of Parallel lines

Consider a line AB with end points

$[A] = [x_1 \quad y_1]$

$[B] = [x_2 \quad y_2]$

* line AB is parallel with line EF



Since AB & EF are parallel the slope of both are same

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{—(1)}$$

Now, we transform line AB to $A^* B^*$

$$\begin{bmatrix} A \\ B \end{bmatrix} [T] = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= \begin{bmatrix} ax_1 + cy_1 & bx_1 + dy_1 \\ ax_2 + cy_2 & bx_2 + dy_2 \end{bmatrix} = \begin{bmatrix} A^* \\ B^* \end{bmatrix} = \begin{bmatrix} x_1^* & y_1^* \\ x_2^* & y_2^* \end{bmatrix}$$

using Equation —(2) calculate slope of $A^* B^*$.

$$m^* = \frac{(bx_2 + dy_2) - (bx_1 + dy_1)}{(ax_2 + cy_2) - (ax_1 + cy_1)}$$

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$$m^* = \frac{y_2^* - y_1^*}{x_2^* - x_1^*} = \frac{b(x_2 - x_1) + d(y_2 - y_1)}{a(x_2 - x_1) + c(y_2 - y_1)}$$

$$m^* = \frac{(b+d)\left(\frac{y_2 - y_1}{x_2 - x_1}\right)}{(a+c)\left(\frac{y_2 - y_1}{x_2 - x_1}\right)}$$

$$m^* = \frac{b + dm}{a + cm} \quad \text{— (3)}$$

Slope of transformed line A^*B^* is independent on (x_1, x_2, y_1, y_2) & dependent on (a, b, c, d, m).

Since,
a, b, c, d & m are same for AB & EF, it shows that m^* is same for A^*B^* & E^*F^*

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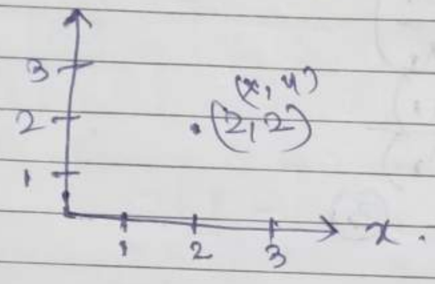
Representation of points

1-row & 2-column matrix.

2D $\begin{bmatrix} x & y \end{bmatrix}$



2D $\begin{bmatrix} x & y \end{bmatrix}$

3D $\begin{bmatrix} x & y & z \end{bmatrix}$

→ 2 rows & 1 col.

2D $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

3D = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ = .

Transformation of Matrices:- [T]

1. Addition
2. Substraction
3. Multiplication
4. Determinant.
5. Transpose.
6. Inverse.

In 2D it is [2×2] matrix.

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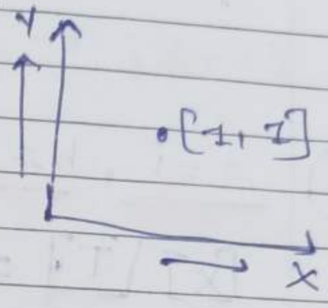
Transformation of point

Let $X = [x \quad y]$
represent a point p.



$$T = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2}$$

transformation Matrix.

$$[X][T] = [x \quad y]\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= [ax + cy \quad bx + dy]$$

$$= [x^* \quad y^*]$$

Examples

1. Case $a = d = 1$. & $c = b = 0$.

$$[X]\,[T] = [x \quad y]\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = [x^* \quad y^*]$$

(2) $d = 1,\ b = c = 0$.

$$[X]\,[T] = [x \ y]\begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix} = [(ax+0) + (0+y)]$$

$$= [ax + y]$$

$x^* \quad y^*$.

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$$[X][T] = \begin{array}{cc} x & y \\ \end{array}\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{array}{c} a \\ \end{array}\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} ax & y \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 1 \end{bmatrix}$$

(4)

(3) $a = 1$, $b = c = 0$.

$$[X]\,[T] = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 2 \end{bmatrix} = \begin{bmatrix} x^* & y^* \end{bmatrix}$$

(3) case $b = c = 0$ $a \& d \rightarrow$ value

$$[X][T] = [x \; y] \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} ax & yd \end{bmatrix} = \begin{bmatrix} x^* & y^* \end{bmatrix}$$

Scaling of both x & y coordinates of point p.

(5)

$$[X][T] = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 3 \end{bmatrix} = \begin{bmatrix} x^* & y^* \end{bmatrix}$$

If $a \neq d$ scaling is not equal.
if $a = d > 1$, Pure enlargement.
if $a = d < 1$, compression.

(4) case: a & d are -ve, b = c = 0.

Consider b = c = 0 and a = -1, d = +1.

$$[X][T] = [x \ \ y]\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow [-x \ \ y] = [x^* \ \ y^*]$$

Reflection through y-axis.

$$[X][T] = [1 \ \ 1]\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow [-1 \ \ 1] \ \ [x^* \ \ y^*]$$

(5) $$[X][T] = [1 \ \ 1]\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} \qquad d = -1$$

$$= [1 \ -1]$$ Reflection through x axis.

(5) Case if b = c = 0, a = d < 0.

$$[X][T] = [1 \ \ 1]\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$$

$$= [-1 \ -1] = [x^* \ \ y^*]$$

Reflection through origin.
diagonal points are considered.

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$a = d = 1 \quad c = 0.$

⑥ Effects of off diagonal points

$[X][T] = [x \; y] \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix}$

$= [x \quad xb+y]$

Observe $x^* = x$ = unchanged.

$y^* = bx + y$ depends linearly on both x & y original coordinates.

$[X][T] = [2 \; 1] = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$= [2 \quad 2 \times 2 + 1]$

$= [2 \quad 5]$

① case $a = d = 1 \quad b = 0.$

$[X][T] = [2 \; 1] \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$\Rightarrow [4 \quad 1]$

② $[x \quad y] = [0 \quad 0] \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad [x, y = 0]$

$\Rightarrow [0 \; 0]$

limitation Not possible to transform the origin

Homogeneous Coordinates

Rotation, Reflection, Scaling.
But the origin of the coordinate axis never change.
So for translating the origin & any other points in 2D plane we need translation factor.

$x^* = ax + cy + m.$

$$\begin{bmatrix} x^* & y^* \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$y^* = bx + dy + m$

$x^* = ax + cy + n.$

Introducing homogeneous coordinates.

$[x \quad y]$ and $[x' \quad y' \quad h]$ where.

$x' = hx$ and $y' = hy$, h is real number

E.g.

$[x \quad y] = [3 \quad 2]$

then

homogeneous coordinate $= [6 \quad 4 \quad 2]$
$h = 2$

$[9 \quad 6 \quad 3]$ when $h = 3$

$[3 \quad 2 \quad 1]$ $h = 1$

$h \neq 0.$

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The general transformation matrix is now 3X3.

$$[T] = \begin{bmatrix} a & b & 0 \\ c & d & 0 \\ m & n & 1 \end{bmatrix} \quad -(1)$$

Two-dimensional transformation is

$$[x^* \; y^* \; 1] = [x \; y \; 1] \begin{bmatrix} a & b & 0 \\ c & d & 0 \\ m & n & 1 \end{bmatrix}.$$

$$= [x+m \quad y+n \quad 1] \quad -(2)$$

1 Translation

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

2. Scaling $\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3. Rotation $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4. Rotation (anti) $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

5. R

6. R

7. R

8.

9.

Ex:-

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5. Reflection (x-axis) = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

6. Reflection against y axis : $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

7. Reflection against origin $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

8. Reflection against $y = x$ $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

9. Reflection against $y = -x$ $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Ex:- Translate the square with coordinates A [0,0] B(3,0) C (3,3) D(0,3) with translation factor 2.

$$\begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 0 & 1 \\ 3 & 3 & 1 \\ 0 & 3 & 1 \end{bmatrix}$$

Then the translation matrix T, $t_x = 2$, $t_y = 2$.

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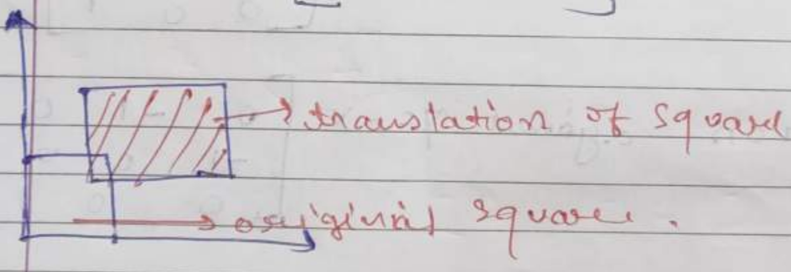
$$[T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \underset{t_x}{2} & \underset{t_y}{2} & 1 \end{bmatrix}$$

$$[X^*] = [X][T] = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 0 & 1 \\ 3 & 3 & 1 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 2 & 1 \\ 5 & 2 & 1 \\ 5 & 5 & 1 \\ 2 & 5 & 1 \end{bmatrix}$$



Ex2 Scale the Square ABCD whose coordinates A (0,0) B (0,4), C (4,4) D (4,0) with a scale factor of 1/2 uniform.

$$\begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 4 & 1 \\ 4 & 4 & 1 \\ 4 & 0 & 1 \end{bmatrix}$$

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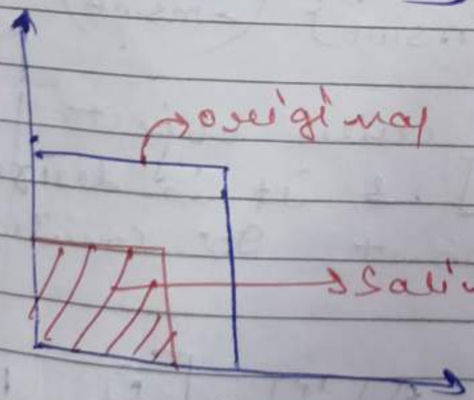
For scaling transformation matrix.

$$= \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow T = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[X^*] = [X][T] = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 1 \\ 4 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 1 \\ 2 & 2 & 1 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} A^* \\ B^* \\ C^* \\ D^* \end{bmatrix}$$



Rotation about an arbitrary point

1. Translate the point to the origin
(2) Perform the required rotation and
(3) translate the result back to the origin

So, the rotation of position vector $[x \; y \; 1]$ about the point m, n with angle θ.

$$[x^* \; y^* \; 1] = [x \; y \; 1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -m & -n & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ m & n & 1 \end{bmatrix}$$

$$= [x \; y \; 1] \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ \left\{ \begin{matrix} -m(\cos\theta - 1) \\ +n\sin\theta \end{matrix} \right\} & \left(\begin{matrix} -n(\cos\theta - 1) \\ -m\sin\theta \end{matrix} \right) & 1 \end{bmatrix}$$

Ex Suppose that the center object is at $[4, 3]$ & it is desired to rotate about $90°$ counter clockwise.

1 $T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & -3 & 1 \end{bmatrix}$ 2 $[T] = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

(3)

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$$\begin{bmatrix} & \sin\theta & 0 \\ & \cos\theta & 0 \\ d & & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ m & n & 1 \end{bmatrix}$$

(3) $$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 3 & 1 \end{bmatrix}$$

The entire operation is

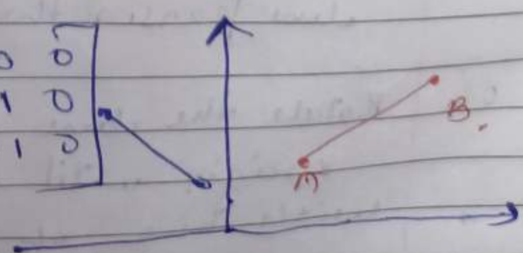
$$[x^* \ y^* \ 1] = [x \ y \ 1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & -3 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 3 & 1 \end{bmatrix}$$

$$[x^* \ y^* \ 1] = [x \ y \ 1] \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 7 & -1 & 1 \end{bmatrix}$$

Exam. Consider rotation of line AB A (2,2) & B(5,5) about point P(1,1) through 90°

$$[x^*] = [x][T] = \begin{bmatrix} 2 & 2 & 1 \\ 5 & 5 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & 1 \\ 5 & 5 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix}$$



$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 4 & 4 & 0 \end{bmatrix}$$

$$(2)\ \begin{bmatrix} 1 & 1 & 0 \\ 4 & 4 & 0 \\ -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ -4 & 4 & 0 \\ -1 & -1 & 0 \end{bmatrix}$$

$$(3)\ \begin{bmatrix} -1 & 1 & 0 \\ -4 & 4 & 0 \\ -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 \\ -3 & 5 & 0 \\ -1 & -1 & 0 \end{bmatrix}$$

Reflection through an Arbitrary Line

Reflection is the mirror of original object. In other words, we can say that it is a rotation operation with 180°.

In reflection transformation, the size of the object doesn't change.

Reflection of An object About an arbitrary line.

1. Translate the line & the object so that the line passes through origin.

2. Rotate the line & the object about the origin until the line is coincident with one of the coordinate axis.

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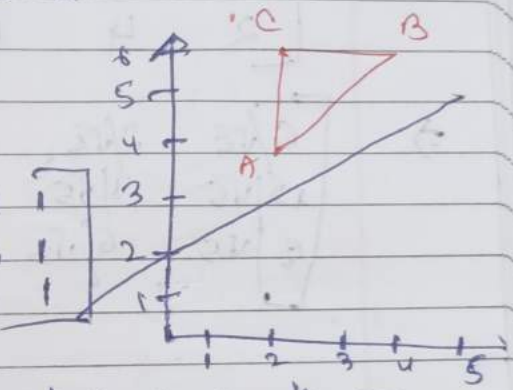
3. Reflect object through coordinate axis
4. Rotate back about the origin (Inverse Rotation)
5. Translate back to the original location. (Inverse translation)

$$[T] = [T][R][R'][R]^{-1}[T]^{-1}$$

$[T]$ = Translation Matrix.
$[R]$ = Rotation matrix about the origin.
$[R']$ = Reflection Matrix.



Example:-

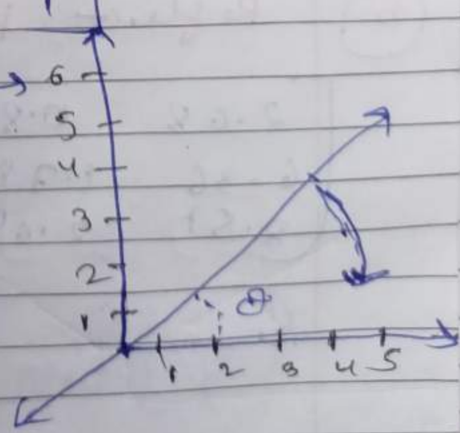
$y = \frac{1}{2}(x+4)$ & $\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 2 & 4 & 1 \\ 4 & 6 & 1 \\ 2 & 6 & 1 \end{bmatrix}$

$y = \frac{1}{2}x + 2$

(1). Translate the line & object so that it passes through origin.

$m = 0 \qquad n = -2$

$$\begin{bmatrix} 2 & 4 & 1 \\ 4 & 6 & 1 \\ 2 & 6 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 1 \\ 4 & 4 & 1 \\ 2 & 4 & 1 \end{bmatrix}$$



(2). Rotate line & object coincident with x-axis.

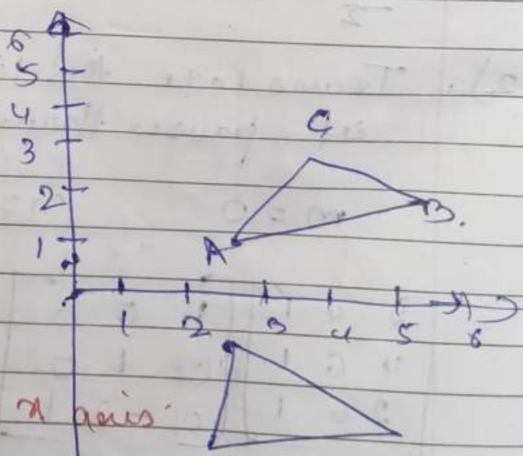
$m = \tan\theta$ → clockwise
$m = -\tan\theta$
$\theta = -\tan^{-1}(m)$
$\theta = -\tan^{-1}(1/2)$
$\Rightarrow -26.57°$

Now rotate the line & object with $\theta = -26.57°$

$$\begin{bmatrix} 2 & 2 & 1 \\ 4 & 4 & 1 \\ 2 & 4 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta(-26.57) & \sin(-26.57) & 0 \\ -\sin(-26.57) & \cos(-26.57) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 2 & 1 \\ 4 & 4 & 1 \\ 2 & 4 & 1 \end{bmatrix} \begin{bmatrix} 2/\sqrt{5} & -1/\sqrt{5} & 0 \\ 1/\sqrt{5} & 2/\sqrt{5} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 6/\sqrt{5} & 2/\sqrt{5} & 1 \\ 12/\sqrt{5} & 4/\sqrt{5} & 1 \\ 8/\sqrt{5} & 6/\sqrt{5} & 1 \end{bmatrix} = \begin{bmatrix} 2.68 & 0.89 & 1 \\ 5.36 & 1.78 & 1 \\ 3.57 & 2.68 & 1 \end{bmatrix}$$



(b) Reflect through x axis

$$\begin{bmatrix} 2.68 & 0.89 & 1 \\ 6.36 & 1.78 & 1 \\ 3.57 & 2.68 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2.68 & -0.89 & 1 \\ 5.36 & -1.78 & 1 \\ 3.57 & -2.68 & 1 \end{bmatrix}$$

[T]

(c) Apply inverse rotation

$$\theta = 26.57^\circ$$

$$\begin{bmatrix} 6/\sqrt{5} & -2/\sqrt{5} & 1 \\ 12/\sqrt{5} & -4/\sqrt{5} & 1 \\ 8/\sqrt{5} & -6/\sqrt{5} & 1 \end{bmatrix} \begin{bmatrix} 2/\sqrt{5} & 1/\sqrt{5} & 0 \\ -1/\sqrt{5} & 2/\sqrt{5} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

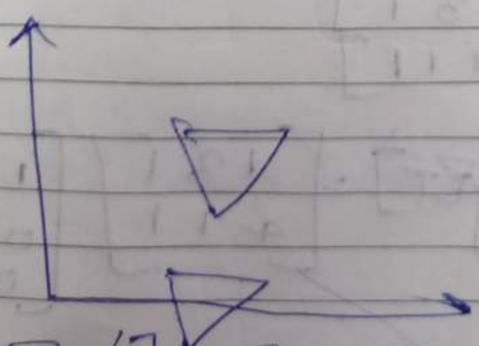
$$\begin{bmatrix} 2.8 & 0.4 & 0 \\ 5.6 & 0.8 & 0 \\ 4.4 & 0.8 & 1 \end{bmatrix}$$

(d) Reverse translation with y-direction.

$m = 0 \quad n = 2$

$$\begin{bmatrix} 2.8 & 0.4 & 0 \\ 5.6 & 0.8 & 0 \\ 4.4 & -0.8 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2.8 & 0.4 & 0 \\ 5.8 & 2.8 & 0 \\ 4.4 & 1.2 & 1 \end{bmatrix}$$



$$[T] = [T'][R][R'][R'^{-1}][R^{-1}][T]^{-1}$$

Translation Rotation Reflection x axis Inverse Rotation Inverse translation

$$T = [T'][R][R'][R^{-1}][T^{-1}]$$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 0 \end{bmatrix} \begin{bmatrix} 2/\sqrt{5} & -1/\sqrt{5} & 0 \\ 4/\sqrt{5} & 2/\sqrt{5} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2/\sqrt{5} & 1/\sqrt{5} & 0 \\ -1/\sqrt{5} & 2/\sqrt{5} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$X^* = [X][T] = \begin{bmatrix} 2 & 4 & 1 \\ 4 & 6 & 1 \\ 2 & 6 & 1 \end{bmatrix} \begin{bmatrix} 3/5 & 4/5 & 0 \\ 4/5 & -3/5 & 0 \\ -8/5 & 16/5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 14/5 & 12/5 & 1 \\ 28/5 & 14/5 & 1 \\ 22/5 & 4/5 & 1 \end{bmatrix} = \begin{bmatrix} 14/5 & 12/5 & 1 \\ 28/5 & 14/5 & 1 \\ 22/5 & 4/5 & 1 \end{bmatrix}$$

$$X^* = [X][T] \quad \begin{bmatrix} 2.8 & 2.4 & 1 \\ 5.6 & 2.8 & 1 \\ 4.4 & 1.2 & 1 \end{bmatrix}$$

Q) Projection in Homogeneous coordinate.

$P = q = 1$

$A = [1 \ 3 \ 1]$

$B = [4 \ 1 \ 1]$

$$\begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix} [T] = \begin{bmatrix} 1 & 3 & 1 \\ 4 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 5 \\ 4 & 1 & 6 \end{bmatrix}$$

$[C] = \begin{bmatrix} 1 & 3 & 5 \end{bmatrix} \quad [D] = \begin{bmatrix} 4 & 1 & 6 \end{bmatrix}$

$h = x + y + 1$

$[C^*] = \begin{bmatrix} 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 1/5 & 3/5 & 1 \end{bmatrix}$

$[D^*] = \begin{bmatrix} 4 & 1 & 6 \end{bmatrix} = \begin{bmatrix} 2/3 & 1/6 & 1 \end{bmatrix}$

Overall Scaling

$$[x \quad y \quad h] = [x \quad y \quad 1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & s \end{bmatrix}$$

$$= [x \quad y \quad s]$$

$X = x \qquad Y = y \qquad h = s.$

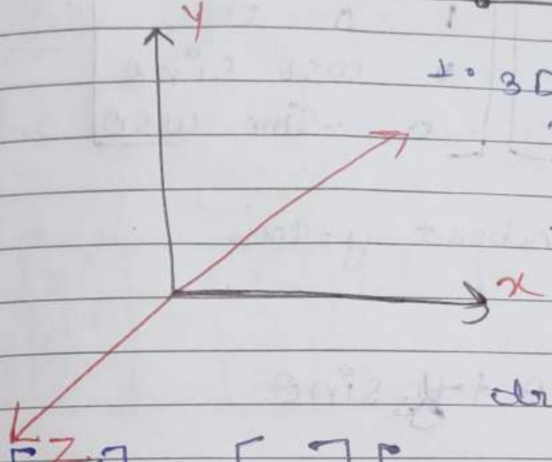
$x^* = \frac{x}{s} \qquad y^* = \frac{y}{s}.$

$$[x \quad y \quad 1][T] = \begin{bmatrix} \frac{x}{s} & \frac{y}{s} & 1 \end{bmatrix}$$

$s < 1 \rightarrow$ Expansion

$s < 1 \rightarrow$ compression.

3D Transformation



1. 3D translation :-

$x' = x + t_x$

$y' = y + t_y$

$z' = z + t_z$

where t_x t_y t_z are translation Matrix.

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & t_z \end{bmatrix}.$$

2. Rotation in 3D :- Rotation about Z-axis

Rotation about Z-axis.

$z' = z$

$x' = x\cos\theta + y\sin\theta$

$y' = -x\sin\theta + y\cos\theta$

$$\begin{array}{cc} x & y \\ \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{array}$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(ii) Rotation about x-axis :-

$x' = x$

$y' = y\cos\theta + z\sin\theta$

$z' = -y\sin\theta + z\cos\theta$

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$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$ (i)

3. Rotation about y-axis

$y' = y$

$z' = z\cos\theta + \cancel{y}\,x\sin\theta$

$x' = z\sin\theta + x\cos\theta$. (ii)

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

3D scaling

(iii)

Scaling factor are s_x, s_y & s_z

$x' = x * s_x$

$y' = y * s_y$

$z' = z * s_z$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{bmatrix}$$

(i)

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3D reflection:-

(i) Reflection through xy plane,
$x = 1, y = 1, z = -1$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

(ii) Reflection through yz plane,
$x = -1 \quad y = 1, z = 1$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(iii) Reflection through xz plane, $x = 1$
$y = -1, z = 1$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Shearing

Slants the object shape.

(i) z-axis Shearing:-

$z' = z$
$x' = x + z\,Sh_x$
$y' = y + z\,Sh_y$

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$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & Sh_x \\ 0 & 1 & Sh_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

(ii) x-axis shearing

$x' = x$
$y' = y + x Sh_y$
$z' = z + x Sh_z$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ Sh_y & 1 & 0 \\ Sh_z & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

(iii) y-axis shearing

$y' = y$
$x' = x + y Sh_x$
$z' = z + y Sh_z$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} Sh_x & 0 & 0 \\ 0 & 1 & 0 \\ Sh_z & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

3D Transformation with Homogeneous Coordinates

$x \quad y \quad z \quad h$

$[x' \quad y' \quad z' \quad h]$, $x' = h_x$, $y' = h_y$, $z' = h_z$

General Transformation matrix (4x4):

$$[T] = \begin{bmatrix} a & b & c & p \\ d & e & f & q \\ g & i & j & r \\ l & m & n & s \end{bmatrix}$$

3x3, 3x1, 1x3, 1x1

4x4 transformation matrix partition into 4 parts.

$$\begin{bmatrix} 3\times3 & 3\times1 \\ 1\times3 & 1\times1 \end{bmatrix}$$

upper left = 3x3 local scaling, shearing, rotation & reflection.

lower left (1x3) = translation

upper right = 3x1 - perspective transformation

lower right = 1x1 overall scaling.

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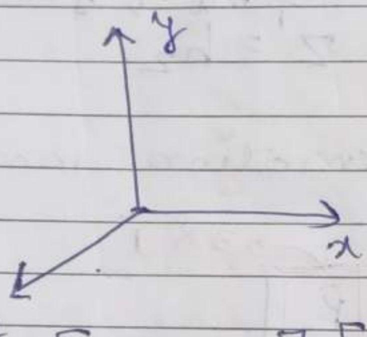
3D Translation :-

$$[x' \ y' \ z' \ 1] = [x \ y \ z \ 1]$$



$$= [x \ y \ z \ 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ l & m & n & 1 \end{bmatrix}$$

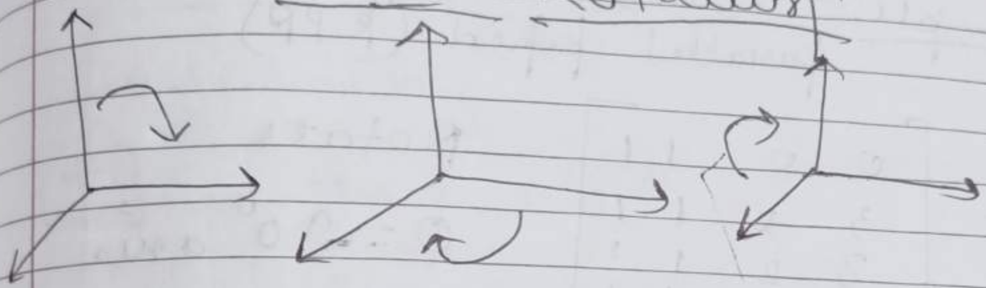
$$= [x+t_x \quad y+t_y \quad z+t_z \quad 1]$$

E.g :- A point has coordinates in x, y, z direction $(5, 6, 7)$ & $t_x = 3$, $t_y = 3$, $t_z = 2$.

$$[x' \ y' \ z' \ 1] = [5 \ 6 \ 7 \ 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 3 & 3 & 2 & 1 \end{bmatrix}$$

$$= [8 \ 9 \ 9 \ 1]$$

3D Rotation



1. $\theta \rightarrow$ angle & the axis

1. Rotation about x-axis

$$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. Rotation about y-axis

$$[T] = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Rotation about z-axis

$$[T] = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\cos 90 = 0$
$\sin\theta\ 90 = 1$.

Example:- Consider rectangular parallel piped (RPP)

$$[X] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 3 & 0 & 1 & 1 \\ 3 & 2 & 1 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 3 & 0 & 0 & 0 \\ 3 & 2 & 0 & 1 \\ 0 & 2 & 0 & 1 \end{bmatrix}$$

Rotate $\theta = -90^\circ$ $\vec{x}$ axis

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[X][T] = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 3 & 1 & 0 & 1 \\ 3 & 1 & -2 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 \\ 3 & 0 & 0 & 1 \\ 3 & 0 & -2 & 1 \\ 0 & 0 & -2 & 1 \end{bmatrix}$$

Rotation $\theta = 90^\circ$ y-axis

$$\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & -3 & 1 \\ 1 & 2 & -3 & 1 \\ 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -3 & 1 \\ 0 & 2 & -3 & 1 \\ 0 & 2 & 0 & 1 \end{bmatrix}$$

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3D Scaling :-

local scaling

$$[T] = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & j & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad [x \quad y \quad z \quad 1]$$

$$[X][T] = [ax \quad ey \quad jz \quad 1]$$

$S_x = 1/2 \quad S_y = 1/2 \quad S_z = 1$

$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 2 & 3 & 1 & 1 \\ 0 & 3 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 1 \\ 2 & 3 & 0 & 1 \\ 0 & 3 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1/2 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

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Overall Scaling

$$[X][T] = [x \; y \; z \; 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & s \end{bmatrix}$$

$$= [x \quad y \quad z \quad s]$$

The physical coordinates are

$$[x^* \; y^* \; z^* \; 1] = [x'/s \quad y'/s \quad z'/s \quad 1]$$

Ex- To double the object (uniform scaling)

$s = 1/2$

$$[X'] = [X][T] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 1 & 0.5 \\ 1 & 0 & 1 & 0.5 \\ 1 & 1 & 1 & 0.5 \\ 0 & 1 & 1 & 0.5 \\ 0 & 0 & 0 & 0.5 \\ 1 & 0 & 0 & 0.5 \\ 1 & 1 & 0 & 0.5 \\ 0 & 1 & 0 & 0.5 \end{bmatrix}$$

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Homogeneous coordinate is
$h = 0.5 \left(1/2 \right)$

each position vector must be divided by 1/2.

$$x^* = \begin{bmatrix} 0 & 0 & 2 & 1 \\ 2 & 0 & 2 & 1 \\ 2 & 2 & 2 & 1 \\ 0 & 2 & 2 & 1 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 1 \\ 2 & 2 & 0 & 1 \\ 0 & 2 & 0 & 1 \end{bmatrix}$$

if $S < 1$, Expansion occurs
$S > 1$, compression occurs.

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Shearing

Slanting : off diagonal terms upper left 3x3

$$[T] = \begin{bmatrix} 1 & b & c & 0 \\ d & 1 & f & 0 \\ g & i & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * [X]$$

$$= \begin{bmatrix} x & y & z & 1 \end{bmatrix}$$

$$[X][T] = \begin{bmatrix} x + yd + zg & bx + y + iz & cx + fy + z & 1 \end{bmatrix}$$

$$[X^*] = [X][T] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -0.85 & 0.25 & 0 \\ -0.75 & 1 & 0.7 & 0 \\ 0.5 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Multiple Transformation

→ Successive transformations can be combined or concatenated into a single 4x4.

$$[X][T] = [X][T1][T2][T3][T4]$$

$$T = [T1][T2][T3][T4]$$

where $[T_i]$ are any combination of scaling, shearing rotation, reflection.

Q x, y, z $-1, -1, -1$
$\theta = +30°$ rotation x-axis
$\theta = +45°$ rotation Y-axis.
homogeneous vector $[3 \ 2 \ 1 \ 1]$

.25 0
0
0
1

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Rotation about an Axis parallel to a coordinate axis

Single Relative Rotation :- Rotation of the object about any of the individual x', y' or z' local axis is accomplished using following procedure :

- Translate the object until the local axis is coincident with the coordinate axis in the same direction.
- Rotate about the specified axis.
- Translate the transformed object back to its original position.

$$[X] = [x][T_r][R_x][T_r]^{-1}$$

T_r = Transformation matrix
$R_x \rightarrow$ rotation matrix
T_r = inverse

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$$X = \begin{array}{c} \begin{bmatrix} 1 & 1 & 2 & 1 \\ 2 & 1 & 2 & 1 \\ 2 & 2 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 \\ 2 & 2 & 1 & 1 \\ 1 & 2 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \end{array}$$

Q $\theta = 30^\circ$

$$[x_c \quad y_c \quad z_c \quad 1] = [3/2 \quad 3/2 \quad 3/2 \quad 1]$$

$$[X^*] = [X]\,[T_r]\,[R]\,[T_r]^{-1}$$

$$T_r \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -y_c & -z_c & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -3/2 & -3/2 & 1 \end{bmatrix}$$

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.8 & 0.5 & 0 \\ 0 & -0.5 & 0.80 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and

$$[T_r^{-1}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \cancel{x_c} & \cancel{y_c} & \cancel{y_c} & z_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 3/2 & 3/2 & 1 \end{bmatrix}$$

Multiple Relative Rotation

→ Translate the origin of the local axis system to make it coincident with the global coordinate system

Ques $\theta = +30^\circ$ (x-axis) $\phi = 45^\circ$ (y axis)

$$[X^*] = [X][T] = [X][T_r][R_y][R_x][T_r^{-1}]$$

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -x_c & -y_c & -z_c & 1 \end{bmatrix} \begin{bmatrix} \cos\phi & 0 & -\sin\phi & 0 \\ 0 & 1 & 0 & 0 \\ \sin\phi & 0 & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ x_c & y_c & z_c & 1 \end{bmatrix}$$

T.

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