UNIT- 1

Q. What do you mean by Time Series?

⇒ A set of ordered observations of a quantitative variables taken at successive points in time is known as time series. In otherwords, A time series may be defined as a collection of readings belonging to different time periods, or some economic variables.

Arrangement of statistical data in chronological order:-

Mathematically, a Time series is defined by the functions(al) relationship ($\boxed{y_t = f(t)}$) where y_t is the value of the phenomena (or variable) under consideration at time t.

For examples:-

(i) The population y_t of a country or a place in different years.
(ii) The numbers of births & deaths (y_t) in different months.
(iii) The temperature y_t of a place of different days (t) of the week.
(iv) weekly Circulation of bank notes.
(v) Growth of bacterial population per minute.
(vi) Rainfall in different years.
(vii) The weight of animals recorded at various stage of growth.

Thus, if the values of a phenomenon or variable at times $t_1, t_2, \ldots t_n$ are $y_1, y_2, \ldots, y_n$ respectively then the series

$$t : t_1, t_2, t_3, \ldots, t_n$$
$$y_t : y_1, y_2, y_3, \ldots, y_n.$$

constitute a time series.

Requirements of Time Series:-

- The data must be homogeneous.
- Timegap b/w different data values must be equal.
- Data must be available for a long time period.

Applications (or uses) of Time Series:-

The following applications may be discussed:-

① Helpful in Planning:- A time series analysis is help directly in business planning. This may help in it making sales projection for the next few years and plan.

(2) To understand past behaviours :- The analysis of time series enable us to understand past behavior or performance, we can know how the data have changed over time to find out the probable reasons responsible for such changes.

(3) To Analyse Meaningful comparison :- for eg :- Growth in consumptions at the national level can be compared with that in national income over a specified period. Such comparisons are of considerable importance to business & industry.

(4) In Financial Decision Making.

Components of Time Series :-

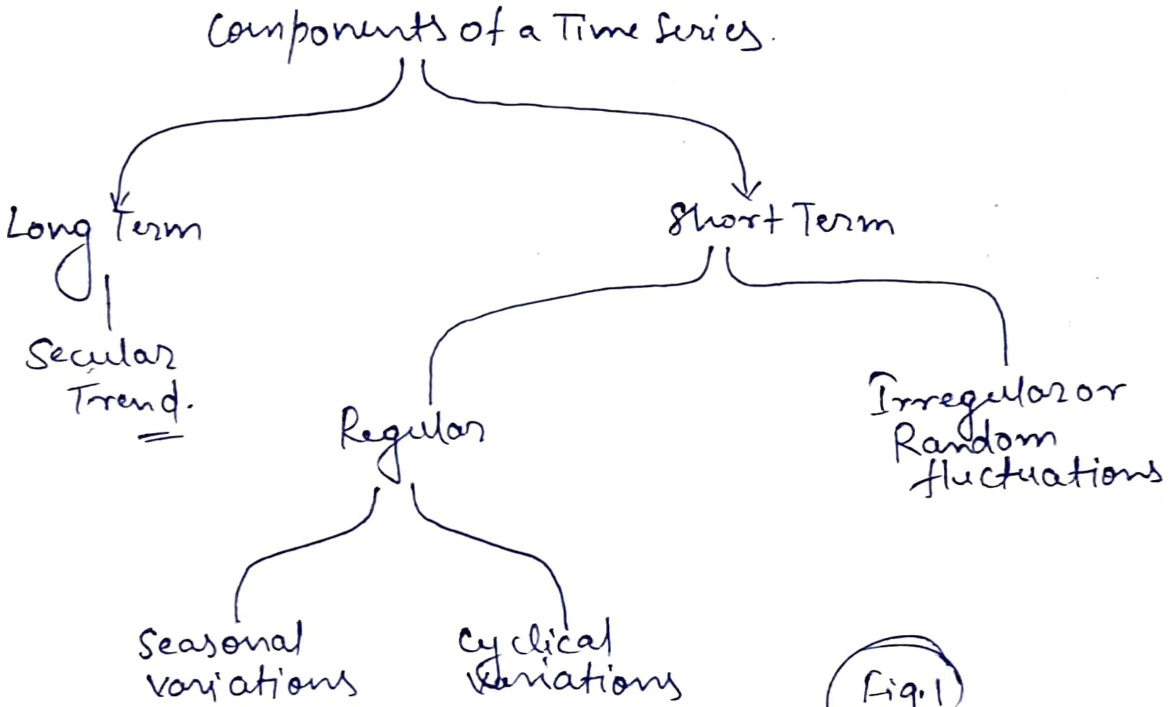


Fig. 1

The various forces at work, affecting the values of a phenomenon in a time series. A time series in its forward motion follows a certain force that represents the net effects of the interaction of the several forces pulling it up & down.

Hence, the main problem in Time - series analysis are :-

(i) what are the forces or components, the net effect of interaction is expressed by the movement of time series & (ii) how they can be separated & studied independently? The forces at work affecting the time series can be broadly classified into the following categories :-

(A) Long Term Components (B) Short Term Components.

and it is shown in fig (1)

Defn Secular Trend (T_t) :-

Trend is the general tendency of the data that means by trend, we mean smooth & irregular long term movement of the data. It may be defined as the tendency of data to increase or decrease or upward or downward or may remain constant over a period of time.
It generally happens with the time series of economic & business data.
The term 'long period' cannot be defined precisely. In some cases, the periods may be small as weeks or may be long as 20 years.
It is denoted by T_t.

Seasonal Variations (S_t):-
Another important component in time series is seasonal variation.
They are those periodic movements where the period of movement is no longer than 1 year. According to the name, weather plays an important role in such type of variations.
Following are the two causes responsible for the seasonal variation:-

1. Those resulting from natural forces.
2. Those resulting from man-made conventions.

For eg:- The production of goods depends on season example- eggs.
On the other hand, profit in departmental stores go up considerably during marriages & festivals like diwali etc.
Seasonal variation are periodic & Regular. They are definite & precise and can be predicted with greater degree of accuracy.
The main objective of the measurement of the seasonal variation is to isolate them from the trend & study their effects. A study of seasonal patterns is extremely useful to businessman, producer, sales managers etc in planning future operations.

Time period - Min 4 months
Max 12 month.

Cyclic Variations (C_t):- The oscillatory movements in a time series with period of oscillation more than one year are termed as cyclic fluctuations.
One complete period is called cyclic that's why cyclic moveme-nts are generally attributed to the so called business cycle.
Business cycle is 4 phase cycle:-
Prosperity → Recession → (Depression) → Recovery → Normal
?
?

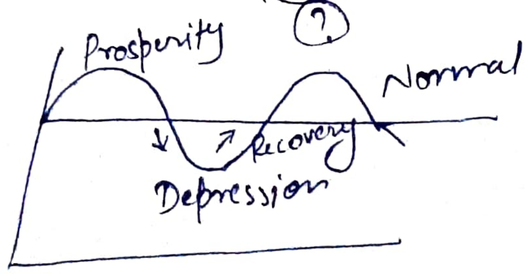


Fig - 2

4

Most of the economic & commercial series. for eg:- series related to prices, production & wages etc. are affected by these business cycles.
The seasonal variation occurs within a period of 12 months while the cyclic variation occurs within interval ranging 7 to 11 years. The period of cycle is rather uncertain as such cyclic variation cannot be predicted with accuracy.
An important example of cyclic movement is so called business cycle representing intervals of prosperity, recession, depression), Recovery & Normal.
These phases are shown in fig 2.
Ask ma'am about Recession & Depression.

Irregular or Random Component (I_t):-
or Residual

These fluctuations are purely random, erratic, unforseen, unpredictable and due to numerous, non-recurring and irregular circumstances which are beyond the human control but at the same time are a part of system such as earthquakes, wars, floods etc.
These irregular but powerfull fluctuations due to some causes like floods, revolution, famines etc. are called episodic fluctuations.

Analysis of Time Series :- A time series is the result of the combined effect of all the above four components. Using classical approach, it is assumed that there is always a good relationship between these four components & this is represented in the form of multiplicative models.

$$U_t = T_t \times S_t \times C_t \times I_t$$

where U_t = the time series value at time 't'.
T_t = The trend value.
C_t & I_t respectively are seasonal, cyclic & Irregular components of time series at time t.

Another approach is additive method:-

$$U_t = T_t + S_t + C_t + I_t$$

This method is not used since it is considered inappropri-ate for most economic data.
If U_t represents logarithms of original variable, then this model may be used because multiplicative (model) decomposition of time series is as same as additive decomposition of logarithmic values of original time series.

The concept of trend values is very useful in the study of economic events. Trend values indicates the trend of the given data and give us an idea of the different types of fluctuations. They indicate whether the fluctuations are regular or irregular.
We know that if trend values are subtracted from the original series, the remainder is the short term components.
Following are the methods for measuring trend are:-

1. Graphic Method or by Inspection ✓ ✓
2. Method of Semi- Averages ✓
3. Method of Moving Averages.
4. Method of Curve fitting by principle of least Square (Linear, Quadratic & Exponential). ✓

In syllabus (Ticked ones.)

Graphic Method or By Inspection

- A free hand smooth curve obtained on plotting the values yt against 't' enables us to form an idea about the general trend of series.
- This method does not involve any complex mathematical techniques & can be used to describe all types of trend, linear and Non-linear.

Thus simplicity & flexibility are the strong points of this method.

Method of Semi-Averages.

In this method, the entire data is divided in two parts & the value of each part added an average. These two averages are the two points corresponding to the middle years on the graph. The line obtained by joining these two points is the required trend line & may be extended downwards & upwards with a view to get intermediate values to predict future values.
When the number of years is an odd figure, the middle year is left out.

Advantages & Disadvantages:- This method is simple & trend line will not vary from person to person as ways in the case of graphic method. However, this method assume that there is only a straight line relationship between these two points. This is because with these two points, we draw simply a straight line and plot a curve. Also the effects of cyclic fluctuations are eliminated in this method.

not

(Plot?)
Not

Q. Determine straight line trend by the method of Semi-averages exports in crores:

Years	Crore
1998	19.5
1999	22.5
2000	20.1
2001	24.9
2002	30.8
2003	28.7
2004	27.5

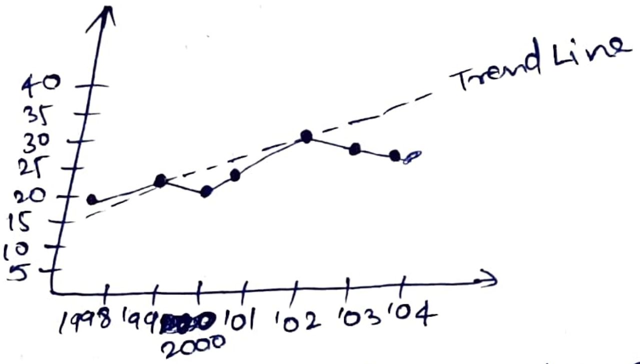
Here, since $n=7$ (odd), the two parts consists of 1998-2000 & 2002-2004

$\bar{X}_1$ = Average sales for first part

$= \frac{62.1}{3} = 20.7$ (Rs crores)

$\bar{X}_2$ = Average sales for second part

$= \frac{87}{3} = 29$ (Rs crores).



Method of Curve fitting by Principle of Least Squares

This is the best & most objective method of determining trend. In this case appropriate mathematical function $f(t)$ is first selected and then constants involved in function are obtained on the basis of data available. The selection of the appropriate function is decided by plotting the original time series. Examination of the plotting data often provides an adequate basis for deciding upon the types of trend to use. The various types of curve that may be used to describe a given type of data are:-

1. $y = a+bx$ Linear Trend (straight line).
2. $y = a+bx+cx^2$ Quadratic Trend (Second degree Parabola)
3. $y = ab^x$ Exponential Trend (Exponential Curve).

Fitting of Linear Trend:- When a straight line trend is the best fit of the original data. The mathematical device of least squares may be used for trend-fitting. This method consists of finding the values of the constant a & b in the straight line eqn.

$$y = a + bx \quad \text{—} \; (1)$$

The constants a and b are also called the parameters of the straight line 'a' represents the value of the line or the amount by which the trend increases or decreases for each unit of time. The values of 'a' & 'b' are obtained by the following two normal equations

$$\Sigma y = Na + b\Sigma x \quad - (2)$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 - (3)$$

Solving the above equations, the values of 'a' & 'b' are obtained as follows:-

$$b = \frac{\Sigma xy - \bar{x} \cdot \Sigma y}{\Sigma x^2 - \bar{x}\,\Sigma x}$$

$$a = \bar{y} - b\bar{x}$$

If a series consists of odd number of years (time) considerable arithmetic can be reduced by taking the middle year of the time series as origin & deviations of various years are calculated from that point. In this case constants 'a' and 'b' are evaluated by

$$a = \frac{\Sigma y}{N}, \quad b = \frac{\Sigma xy}{\Sigma x^2}$$ It is called short cut method.

(Q.4) Below are given the figures of production 'thousand quintals' of a sugar factory.

year	1996	1997	1998	1999	2000	2001	2002
Production	80	90	92	83	94	99	92

fill a straight line trend to the given figures

(i) If 1996 is taken as origin.

(ii) If middle year (1999) of the series is taken as origin.

(i) If 1996 is taken as origin

year	Production (y)	Deviation from X	XY	X^2	Trend values $y_t = 84 + 2X$
1996	80	0	0	0	84
1997	90	1	90	1	86
1998	92	2	184	4	88
1999	83	3	249	9	90
2000	94	4	376	16	92
2001	99	5	495	25	94
2002	92	6	552	36	96
N=7	$\Sigma Y = 630$	$\Sigma X = 21$	$\Sigma XY = 1946$	91 ΣX^2	

The values of 'a' & 'b' are given by :—

$$b = \frac{\Sigma xy - \bar{x}\Sigma y}{\Sigma x^2 - \bar{x}\Sigma x} = \frac{1946 - 3\times 630}{93 - 3\times 21} = \frac{1946 - 1890}{91 - 63}$$

$$= \frac{56}{28} = 2$$

$a = \bar{y} - b\bar{x} = 90 - 2\times 3 = 84$ where $\bar{x} = \frac{21}{7} = 3$, $\bar{y} = \frac{630}{7} = 90$

The fitted trend Eqn is $y_t = 84 + 2x$.
from this equation the trend values for various values of X have been found & shown in the last columns of the above table. It is clear that their graph will give a straight line.

(ii) If middle year (1999) of the series is taken as origin

Year	Production (y)	Deviation from X (1999)	xy	x^2	Trend values $y_t = 90 + 2x$
1996	80	-3	-240	9	84
1997	90	-2	-180	4	86
1998	92	-1	-192	1	88
1999	83	0	0	0	90
2000	94	1	94	1	92
2001	99	2	198	4	94
2002	92	3	276	9	96
N = 7	$\Sigma y = 630$	$\Sigma x = 0$	$\Sigma xy = 56$	$\Sigma x^2 = 28$	

$\bar{x} = 0$, $\bar{y} = \frac{630}{7} = 90$

$$b = \frac{\Sigma xy - \bar{x}\Sigma y}{\Sigma x^2 - \bar{x}\Sigma \bar{y}} = \frac{56 - 0\times 630}{28 - 0\times 0} = \frac{52}{26} = 2.$$

Now $a = \bar{y} - b\bar{x} = 90 - 2\times 0 = 90 - 0 = 90$

The fitted trend eqn is $\boxed{y_t = 90 + 2x}$

Fitting of Quadratic Trend :-

It is noted that for a short period of time, a straight line may provide a reasonably good description of the trend of a series but that for longer periods a curved line of some sort may be called for apart from the two constants 'a' & 'b' and third constant 'c' is added to the eqn for the second degree curve added to the eqn for the second degree curve, the slope of the curve is continuously changing. If a sufficient numbers of x values are included the second degree curve will have a positive slope in one portion & a negative slope in another. The second degree curve (parabola) is little more complicated than a straight line because it involves the addition of cx^2 to the eqn of straight line. Thus the eqn of second degree curve is given by

$$\boxed{y = a + bx + cx^2} \quad - (1)$$

Since, there are 3 constants unknowns, the following three normal eqn are required i.e

$$\Sigma y = Na + b\Sigma x + c\Sigma x^2 \quad - (2)$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + \Sigma x^3 \quad - (3)$$

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 \quad - (4)$$

The origin may be taken at the middle year or between the two middle years as taken in the case of straight line.

Q: Fit a parabolic curve & find the trend values from the given data.

Year	1992	1993	1994	1995	1996	1997	1998
Value of y	6	10	15	39	25	20	15

Year	Value of y	x	x^2	x^3	x^4	xy	x^2y	Trend Value
1992	6	-3	9	-27	81	-18	54	
1993	10	-2	4	-8	16	-20	40	
1994	15	-1	1	-1	1	-15	15	
1995	39	0	0	0	0	0	0	
1996	25	1	1	1	1	25	25	
1997	20	2	4	8	16	40	80	
1998	15	3	9	27	81	45	135	
N=7	130	0	28	0	196	57	349	

$N=7$, $\Sigma y = 130$, $\Sigma x = 0$, $\Sigma x^2 = 28$, $\Sigma x^3 = 0$, $\Sigma x^4 = 196$

$\Sigma xy = 57$, $\Sigma x^2 y = 349$

$$\Sigma y_t = Na + b\Sigma x + c\Sigma x^2$$

$$130 = 7a + 0 + 28c$$

$$7a + 28c = 130 \quad - (1)$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + \Sigma x^3$$

$$57 = 0 + 28b + 0$$

$$28b = 57 \quad - (2)$$

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$$

$$349 = 28a + 0 + 196c \quad -③$$

from ②, $b = \frac{57}{28} = 2.03$

$$\boxed{b = 2.03}$$

Multiplying ① by 4 & substracting from eqn ③ we get

$$28a + 112c = 520$$
$$28a + 196c = 349$$
$$-84c = 171$$
$$c = -\frac{171}{84} = -2.03$$

$$\boxed{c = -2.03}$$

Putting in eqn-①:

$$7a - 56.84 = 130$$
$$7a = 186.84$$
$$\boxed{a = 26.6}$$

Trend → $\boxed{y_t = 26.6 + 2.03X - 2.03X^2}$

Fitting of Exponential Trend :-

Many economic and business data tend to gain a constant percentage each year. Their progress is geometric. In such a case, a straight line given by the eqn. $y_t = ab^x$ is used. This is the eqn of the exponential curve compound-interest curve describes such a trend best. It is not possible to fit the exponential curve directly to the y values by least squares. We can however make a least square fit to the logarithms of the original data & their result is minimising the squared directions of the logarithms of the observed values from the logarithmic trend values.

Putting the exponential eqn. in logarithmic from we get

$$y = ab^x$$
$$\log y = \log[ab^x] \quad \{\text{taking log from both sides}\}$$

$$\boxed{\log y = \log a + X \log b} \quad -(1)$$

which is a straight line in terms of x & log y. logarithms are substituted for the actual data when a curve is fitted by the method of each square.

Measurement of Seasonal Variation:-

Seasonal patterns are exhibited by most of the business phenomenon & their study is necessary for the following reasons:-

(i) To isolate the seasonal variation that means to determine the effect of seasonal swings on the size of the variable & to eliminate them.

(ii) The determination of seasonal effects is of paramount importance in planning.

(a) Business Efficiency or (b) a production programme.

For, the study of seasonal variations, the data must be given for the parts of the years viz. months, weeks, quarter, day, hours.

Different Method for measuring seasonal variation are:-

1. Method of simple Average.
2. Ratio to moving average method.
3. Ratio to trend Method. ->(Course)
4. Link Relative Method.

Ratio to trend Method:- This method is an improvement over the simple average method and is based on the assumption that seasonal variation for any given month is constant factor of the trend. The measurement of the seasonal variation by this method consists of the following steps.

(i) First of all the trend values are calculated by the method least square by fitting a straight line or any suitable curve to the given data.

(ii) The original data are expressed as the % of trend values if the time series is of multiplicative model, the percentage will contain season, cyclic, irregular component.

(iii) Arrange these percentages according to years & months for monthly data (or according to years & quarters for quaterly data).

(iv) Find the monthly (quarterly for quarterly data) averages by considering mathematical mean or median according to data.

(v) Find overall average of these monthly averages. If the overall average is 100 then the monthly average will stand for the seasonal indices. However if the overall average differ from 100 then we express the monthy (quarterly) as percentage of the overall averages to get the seasonal indices.

Merits & Demerits :–

Merit:- Since, this method attempts at ironing out the cyclical or irregular components by the process of averaging, the purpose will be accomplished only if the cyclical variations are known to be absent or they are not so pronaun-ced even if present.

Demerit:- If the series exhibits pronounced cyclical swings, the trend values obtained by the least square method can never follow the actual data as closely as 12-month moving average & as such seasonal indices obtained by 'ratio to trend ~~moving average~~' method are liable to be more biased than those obtained by 'ratio to moving average' method.

UNIT-2

Index Numbers:- An index numbers is a statistical device for measuring the magnitude of relative changes in series of data. It is an indicator of changing tendency in variables. According to "Croxtons" & "Cowden":- "Index number are device for measuring differences in the magnitude of a group of related variables i.e. An index number is a relative number which express relationship between two figures. Actually, Index numbers are special type of average in which the commodities are expressed in different units like wheat is expressed in kilograms, cloth in metres, kerosene in litres and so on.

Here the price pattern may be different for different commodities. For some, it may be upward but for some, it may be downward. Our main aim is to get a complete expression of this type of complete variations, thus an Index number is a numbers which summarise the movement of such heterogeneous but associated elements. For comparative purposes, Index numbers are represented in the form of percentages with some period taken as random.

Uses of Index Numbers:- Index number have become an important tool to measure the changes in economic & business activities and thus it is sometimes called 'Economic Barometer'.

Some uses are shown pointwise below:-

(i) Help in studying trend:- On the basis of index number, the trends in industrial production, international trade, National income can be studied. Also, the index number are useful in future forecasting, helps in policy making. Index number are used by government association or an individual business organisation for efficiency planning and decisions. For examples - the employee uses, cost of living index, Allowances for the service etc.

(ii) Help in defining various values:- (defines deflate)
Index number are effectively used in defining national income on the basis of price of base year to enable us to find out whether there is any change in income of the people.

(iii) See book for other uses and just go through it.
Helps in Policy Formation →

Basic Problems Involved in the Construction of Index Number:-

(i) The purpose of Index Number:- The first major problem is that, the object of the index number must be very clear that means for what kind of change, we're going to construct an index number. The main reason is that, there is no single index number that can be used for all the purpose and this is why "Retail Price Index Number" is used for studying the probems related to income and salary and "General Price Index Number" is used for studying.

(ii) The selection of base year:- Index numbers are always constructed with reference to some base period. The point of reference is called the base. while selection of base period, we should be careful that the base period should not be too distant in the past and that period must be

a normal period. If the base period or reference period is fixed for all current period of comparison, it is called "fixed base period".
In Chain base method, the change is the level of phenomena for any given period is compared with the level of phenomena in the preceding period and not in base period.

(iii) Selection of representative items:- There are many number of items which are sold and purchased. But all the items are not taken into consideration, only those items are taken which are relevant and serve the purpose of the index. Another point which play an important role is the number of items. We know that small number of items will not provide a representative index and a large number of items will not be economically viable. Thus, a fairly large number of items may be taken.

(iv) Selection of representative prices:- After the items are decided, the next step is to collect the price quotations of these items. But it is a fact that the prices of a number of items vary from place to place even shop to shop. In such situations, a selection must be made of representative places and persons. These places must be well known for trading. After the decision of well known place, a reliable person or an agency must be appointed to collect price quotations. Further, the price quotations in the market are divided into two parts, the retail prices and wholesale prices. The choice would depend upon the purpose of the index. If a consumer price index is to be considered (constructed), the retail prices will be suitable not the wholesale prices.

(v) Choice of a suitable Average: To summarise a large amount of information, the selection of a suitable average is necessary. This average plays an important role in computing index numbers. We know that index numbers are special type of averages and thus the choice of a suitable average is an important task. Arithmetic mean, median and geometric means are the commonly used averages in index numbers each having its own limitations. For example, arithmetic mean is unduly affected by the extreme items. Median also is not satisfactory average as it completely ignores the extreme observations and it cannot measure relative changes which are essentially required for the construction of index numbers. The geometric mean is the most suitable average for construction of index numbers. It has the merit of measuring changes (relative) and also not affected by the extreme observations.

(vi) Selection of Proper Weights: In almost every collection of

numerical data. It may be observed that the items are of varying importance. So weighing is very necessary in the construction of index numbers. If weights are not assigned to these items, the importance given to each commodity to constructing an index number, shall be a matter of chance. The only solution to this problem is to assign suitable weights to each item included in the index. This makes an index number truly representative of the population under study.
weight may be either fixed or fluctuating. But as the relative importance of the commodities keep changing, it is always advisable that the weights should be fluctuating from period to period.

(vii) Selection of Suitable Formula :- The selection of a formula depend on the information available and the accuracy desired, which again depends on the purpose of constructing a particular index number. There are various formulae for calculating Index Number such as aggregate method or the average of relatives method weighted aggregate index numbers, Fisher's Ideal formula and so on.

Types of Index Numbers:-
In economics and business, the index numbers are classified in four heads:

1. Price Index Numbers
2. Quantity Index Numbers
3. Value Index Numbers
4. Special Purpose Index Numbers.

1.) Price Index Numbers:- When the relative change in price is studied, it is called price index number. Such index numbers are further classified as (i) wholesale price index numbers and (ii) Retail or cost of living price index numbers.
Price relative is the simplest example of an index number. It is defined as the ratio of the price of a single commodity in the current year to its price in the base year. Thus, if P_1 is the current year price of a (single) commodity and P_0 is the base year price of the commodity then,

$$\text{Price Index Number} = \frac{P_1}{P_0} \times 100$$

2.) Quantity Index Numbers:- When the relative change in the quantity is studied, it is called quantity index number. By quantity, we mean the volume of quantity such as quantity

of production, consumption etc.
Quantity relative is also the simplest example of an index number. It is defined as the ratio of the quantity of a single commodity in the current year to its quantity in the base year. Thus, if q_1 is the quantity of current year and q_0 is the base year quantity, then

$$\text{Quantity index number} = \frac{q_1}{q_0} \times 100$$

(3) **Value Index Numbers:–** The value is defined as the product of price and quantity i.e.

$$\text{Value} = \text{Price} \times \text{Quantity}$$

These are intended to study the change in the total value in current year in comparison to base year. Thus, if

Value of current period $= P_1 q_1$

Value of base period $= P_0 q_0$

$$\text{Then, value index number} = \frac{P_1 q_1}{P_0 q_0} \times 100$$

(4) **Special Purpose Index Numbers:–** If index numbers are prepared for some specific purposes then they are called special purpose index numbers such as index numbers of national income, productivity, growth rate etc.

Method of Constructing Index Number:– The method of constructing an index number are broadly classified into two categories, one based on price relatives and the other on aggregative. As calculated above, a price relative is simply the current price of a commodity divided by the base price. Like, the index itself, it is expressed as a percentage. Thus, the price relatives of different commodities are averaged in some way to get index numbers. The other method is to consider the aggregate of goods e.g. the goods actually bought or sold in a given year and compare their value at base year price. To study these methods in greater details, we may represent them by the following chart.

Index Number
- Unweighted (Simple)
  - Simple Aggregate
  - Average of Relatives
- Weighted
  - Simple Aggregate
  - Average of Relatives.

Unweighted (Simple) Index Number:-

① **Simple Aggregate Method:-** This is the simplest method of construction of index number. In this method, we express the total of commodity prices in the given year as a percentage of the total commodity prices in the base year. Thus:

Simple aggregative price index $P_{01} = \frac{\Sigma p_1}{\Sigma p_0} \times 100$

where, P_{01} = Index number of the current year

ΣP_1 = Sum of all commodity prices in current year.

ΣP_0 = Sum of all commodity prices in base year.

Q. Obtain Index numbers for 2000 taking 1998 as the base year by using simple aggregate method.

Commodities	Prices 1998	Prices 2000
A	100	140
B	80	120
C	160	180
D	220	240
E	40	40
	$\Sigma P_0 = 600$	$\Sigma P_1 = 720$

Computation of Index Number (Simple Aggregate Method)

∴ Price Index Number of 1998

$$= P_{01} = \frac{\Sigma P_1}{\Sigma P_0} \times 100 = \frac{720}{600} \times 100 = 120$$

Q. you can do some more questions given in books of your preference on this particular topic.

② **Average of Price Relative Method:-** This method is based on price relatives for all the commodities included in the index number. We know that price relative is a ratio of the price of a single commodity in a given period to its price in base period and it is denoted by p_1/p_0. Thus price relatives for different commodities are averaged by using any measure of central tendency (A.M., G.M., or median). If arithmetic mean is used, the formula for computing index number is:

$$P_{01} = \frac{\Sigma \frac{P_1}{P_0} \times 100}{N}$$

, Here $\Sigma \frac{P_1}{P_0}$ is the sum of price relatives of N commodities. If geometric mean is used, the formula for computing index number is:

6

$$= \text{Antilog}\left\{\frac{\sum \log P}{N}\right\}, P = \frac{P_1}{P_0} \times 100$$

Weighted Index Numbers:- These are those kind of index numbers in which comparative or relative importance is assigned to different items. As such these index numbers are considered more rational and logical. These index numbers are also classified as:

1.) Weighted Average Index Numbers.
2.) Weighted Average of Relatives.

① **Weighted Aggregate Index Numbers:-**

$$\text{Weighted Aggregative Price Index} = P_{01} = \frac{\sum P_1 W}{\sum P_0 W} \times 100$$

By using different type of weights, we have different formulae for index numbers. These weights may be price or quantity of base year or current year or both. Following are the different formulae for index numbers:

① **Laspeyre's Index Numbers:-** In this method quantities of the base ~~current~~ year i.e. q_0 is taken as weight. The formula is

$$P_{01} = \frac{\sum P_1 q_0}{\sum P_0 q_0} \times 100$$

③ **Marshall Edgeworth's Index Number:-** This formula was suggested by Fisher and was supposed by Marshall Edgeworth. In this method, average quantity of current year and base year is taken as weights. The formula is : 2

$$P_{01} = \frac{\sum (q_0 + q_1) P_1}{\sum (q_0 + q_1) P_0} \times 100$$

④ **Fisher's Ideal Index Number:-** In this method, the geometric mean of Laspeyre's and Paasche's formulae is taken. The formulae is

$$P_{01} = \sqrt{\left[\frac{\sum P_1 q_0}{\sum P_0 q_0} \times \frac{\sum P_1 q_1}{\sum P_0 q_1}\right]} \times 100$$

② **Paasche's Index Number:-** In this method quantities of the current year i.e. q_1 is taken as weight. The formula is

$$P_{01} = \frac{\sum P_1 q_1}{\sum P_0 q_1} \times 100$$

(5) Dorbish and Bowley's Index Number:- In this method, the arithmetic mean of Laspeyre's and Paasche's formulae is taken. The formulae is

$$P_{01} = \frac{1}{2}\left[\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} + \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1}\right] \times 100$$

(6) Walsh Index Number:- In this method, the geometric mean of the current and base year quantities is taken as weight. The formula is

$$P_{01} = \frac{\Sigma \sqrt{q_1 q_0}\,\frac{P_1}{P_0}}{\Sigma \sqrt{q_1 q_0}} \times 100$$

(2) Weighted Average of Relatives Method :- We've already discussed average of relatives method. This method is used to overcome the limitations of the simple average of relatives method. In this method, the price relatives for the current year are calculated on the basis of the prices of base year $\frac{P_1}{P_0} \times 100$. These price relatives are assigned some weights W. The weights are multiplied by the price relatives and these product are added up and are divided by the sum of the weights.

Symbolically:

$$\text{Weighted Index Number} = P_{01} = \frac{\Sigma\left(\frac{P_1}{P_0} \times 100\right) W}{\Sigma W} = \frac{\Sigma PW}{\Sigma W}$$

where $\frac{P_1}{P_0} \times 100 = P$

Similarly, the index number based on geometric mean of price relatives is given by:

$$P_{01} = \text{Antilog}\left(\frac{\Sigma W \log P}{\Sigma W}\right)$$

Test for Index Numbers :- Prof. Fisher made a careful study of different formulae which are used to compute index numbers. He suggested that a good formula should satisfy the following tests:

1. Time Reversal Test
2. Factor Reversal Test
3. Circular Test.

(1) Time Reversal Test :- Fisher said "The formula for calculating an index number should be such that it will give the same ratio as between one point of comparison and the other, no matter

which of the two is taken as base". That is product of two index numbers must be equal to one. Since two index number should be reciprocal of each other, their product should always be equal to 1.

Hence: $P_{01} = 1/P_{10}$ or $P_{01} \times P_{10} = 1$

where, P_{01} = Price change in current year

P_{10} = Price change in base year (based on current year)

Considering fisher's formula

$$P_{01} = \sqrt{\left[\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1}\right]}$$

and thus $$P_{10} = \sqrt{\left[\frac{\Sigma P_0 q_1}{\Sigma P_1 q_1} \times \frac{\Sigma P_0 q_0}{\Sigma P_1 q_0}\right]}$$

<u>Now</u> $P_{01} \times P_{10} = 1$

Therefore, Fisher's Index satisfies time reversal test.

Similarly, Marshall-Edgeworth's and Walsh's index number satisfy time reversal tests.

while Laspeyre's formula, Paasche's index do not.

(2) Factor Reversal Test:- Fisher said "Just as a formula should permit interchange of two items without giving inconsistent results, so it ought to permit interchanging the prices and quantities without giving inconsistent results (i.e) the two results multiplied together should give the true value ratio."

This means that as a formula should permit interchange of two item it should also permit interchange of price and quantities without causing any change in the ratio of their true value. In its simplest form, we conclude that a change in price (denoted by P_{01}) is always followed by a change in quantity denoted by Q_{01}

Symbolically, the factor reversal test is satisfied if

$$P_{01} \times Q_{01} = \frac{\Sigma P_1 q_1}{\Sigma P_0 q_0} = \frac{\text{Value in the given year (1)}}{\text{Value in the given year (0)}}$$

* Laspeyre's Index doesn't satisfy factor reversal test
* Fisher's Index also satisfies the factor reversal test. Since, it satisfies both the test, it's called "Fisher's ideal index number".

Circular Test:- It is often desired to shift the base. Hence, the formula for the construction of an index number should be such that it ensures that new index number which may be obtained by the shifting process should be equal to the original index number. Thus,

$$P_{01} \times P_{02} \times P_{12} = 1$$

where P_{01} = Price change of the current year on the base year.

P_{02} = Price change of the current year on the second base year.

P_{12} = Price change of the base year on some other base.

This test of base shifting is called "Circular Test". It is a sort of extension of Time Reversal Test.

Note:- The most common formula i.e. Fisher's ideal formula fails to satisfy this test. However, index numbers obtained by simple aggregate method or by fixed weight aggregative method satisfy this test.

you can solve some questions given in book. we'll do practical also in this topic.

Quantity Index Number:- Instead of comparing the relative change in prices we may be interested in studying the relative change in quantity. In other words, if we study the changes in quantity of production or consumption of certain commodities over a given period of time, we call them quantity index number Q_{01}. Accordingly this formulae may be listed as under:

1. $Q_{01} = \frac{\Sigma q_1}{\Sigma q_0} \times 100$; Simple aggregative quantity index.

2. $Q_{01} = \frac{\Sigma q_1/q_0}{N} \times 100$, Simple average of relative method.

3. $Q_{01} = \frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} \times 100$; Laspeyre's quantity index using base year prices as ⑦.

(4) $q_{01} = \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1} \times 100$; Paasche's quantity index using given year prices as (1)

Similarly, Fisher's, Marshall, Edgeworth's formula for quantity index number can also be obtained.

Value Index Number :- The value is defined as the product of price and quantity i.e.

value = Price × quantity.

If P_0 and q_0 denote the price and quantity of a commodity during a base period while P_1 and q_1 denote the price and quantity of the commodity during a given period, the total values during these periods are given by $P_0 q_0$ and $P_1 q_1$ respectively and we define.

Value Relative $= \frac{P_1 q_1}{P_0 q_0} = \left(\frac{P_1}{P_0}\right)\left(\frac{q_1}{q_0}\right)$ = Price relative × quantity relative.

Just as we've obtained price index numbers and quantity index numbers, we can convert the above formula to finding out the value index number also. The simplest such index is:

$$\text{Value index} = \frac{\Sigma P_1 q_1}{\Sigma P_0 q_0} \times 100$$

As the values have not been weighted, the value index numbers are the simple aggregative index numbers.

Fixed based Index Number :- An index number for which the base period for the calculation is selected and remains unchanged during the lifetime of the index.

Chain based Index Number :- The index number, we've considered so far are the fixed base type, that means the base period with which we compare the other time periods remain fixed with the progress of time.

We also know that with the passage of time new commodities enter the market and old ones disappear, besides, the quality of the commodities may undergo a change. Also the relative importance of various commodities is dependent on tastes and habits of the consumer's changes. If an index number is needed for comparing successive time periods, say 0, 1, ..., n, It is not necessary to use a fixed base '0'. We use the previous period as base for comparing any time period and construct what are called "Link Relatives". There is no change in calculation, only the base period changes for each comparison and in each case, it is the previous period. The symbol used for such an index for comparing the prices of period k with those of (k-1) is $P_{k-1,k}$. Thus, we construct n link indices - $P_{01}, P_{12}, \ldots, P_{n-1,n}$. By multiplying successive links i.e. by chaining, we obtain the chain indices as;

P_{01} = First Link relative

$P_{02} = P_{01} \times P_{12}$

$P_{03} = P_{01} \times P_{12} \times P_{23}$

$\ldots$

$\ldots$

$$P_{0n} = P_{01} \times P_{12} \times P_{23} \times \ldots \times P_{n-1,n} \rightarrow (2)$$

The practical advantage of a chain index is that the sample of commodities and/or the set of weights may be kept quite up-to-date in any index number.

Relative Merits and Demerits of Chain Base and Fixed Base Method

We've seen that fixed based index numbers become more and more inaccurate as the distance between the base period and the current period increases. Fixed base index numbers are generally easier to calculate and are more easily understood by users of index numbers than chain base index numbers. A further disadvantage of the chain-base method is that any defect or abnormality in the index for one year is perpetuated in all subsequent years. The chain is only as strong as it's weakest link. Also if there is any bias in the formula employed, e.g. the A.M. of price relatives, the cumulative effect of that bias may become serious as time goes on.

Q Prepare chain base index numbers from the following data:

Year	1996	1997	1998	1999	2000
Price (Rs)	80	120	132	264	396

Soln

Year	Price (Rs)	Calculation	$\frac{P_1 \times 100}{P_0}$
1996	80	–	100
1997	120	$\frac{120 \times 100}{80}$	150
1998	132	$\frac{132 \times 100}{120}$	110
1999	264	$\frac{264 \times 100}{132}$	200
2000	396	$\frac{396 \times 100}{264}$	150

Base Conversion:- In some specific situations, we also need to convert chain base index numbers to fixed based index numbers. Base conversion may be of two types:

1. From fixed base to chain base and
2. From Chain base to fixed base.

for more details, you can go through book.

Limitation of Index Numbers:- Although index numbers are very important tools for the relative measurement of changes in different fields but in reality they also suffer from certain limitations. These limitations are:

1. Index numbers are constructed on the basis of sample information, not on the facts hence they may not be complete.
2. Index numbers just specify the direction of change but not study about the reality of facts.
3. Index numbers are constructed for quantitative changes. If changes are of quantitative type, index numbers may not express them properly.
4. Index numbers indicate only average or approximate trend of changes. Hence, they should be interpreted keeping this limitation into consideration.
5. Index numbers are not suitable for all purpose.