LINEAR BLOCK CODING

CONTENTS

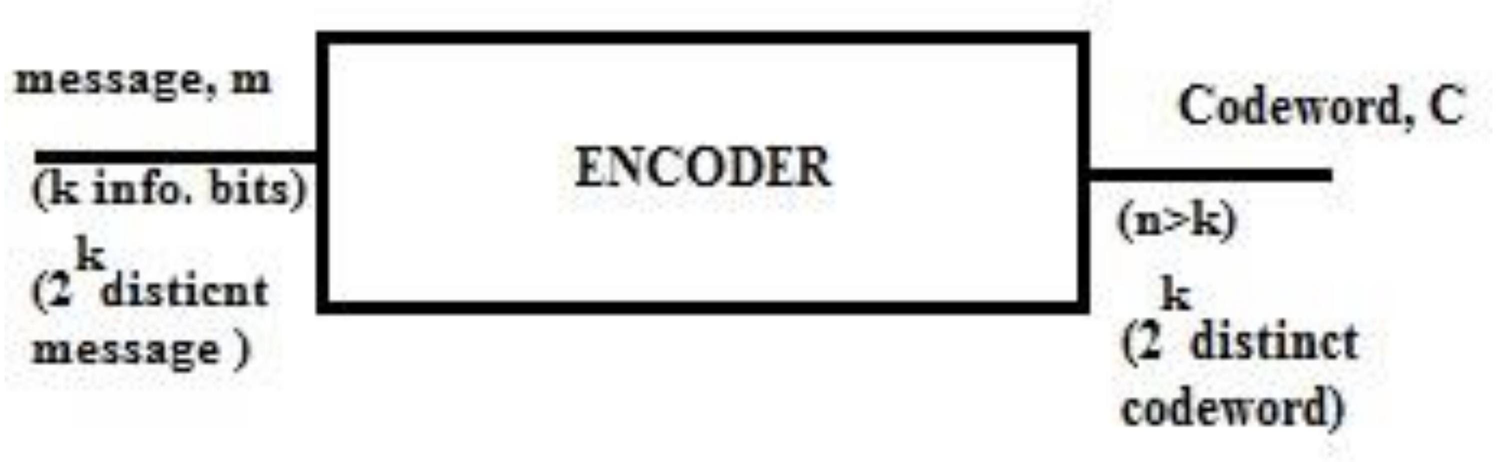
INTRODUCTION

The purpose of error control coding is to enable the receiver to detect or even correct the errors by introducing some redundancies in to the data to be transmitted.

There are basically two mechanisms for adding redundancy:

1. Block coding
2. Convolutional coding

LINEAR BLOCK CODES

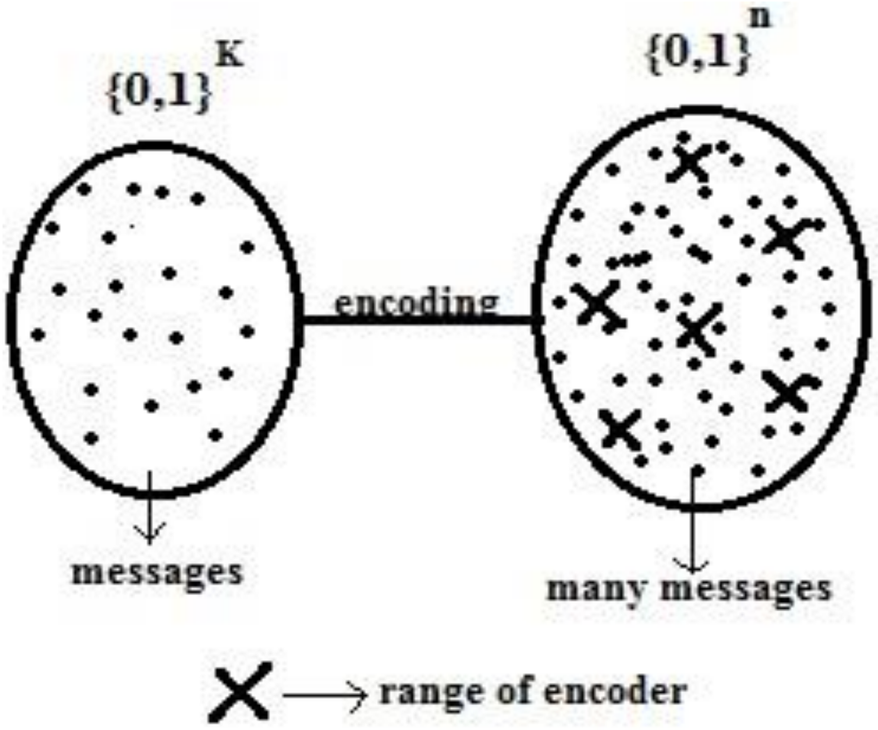


The encoder generates a block of n coded bits from k information bits and we call this as (n, k) block codes.

The coded bits are also called as code word symbols.

Why linear???

A code is linear if the modulo-2 sum of two code words is also a code word.



- n code word symbols can take 2^n possible values. From that we select 2^k code words to form the code.
- A block code is said to be useful when there is one to one mapping between message m and its code word c as shown above.

GENERATOR MATRIX

- All code words can be obtained as linear combination of basis vectors.
- The basis vectors can be designated as $\{g_1, g_2, g_3, \ldots.., g_k\}$
- For a linear code, there exists a k by n generator matrix such that

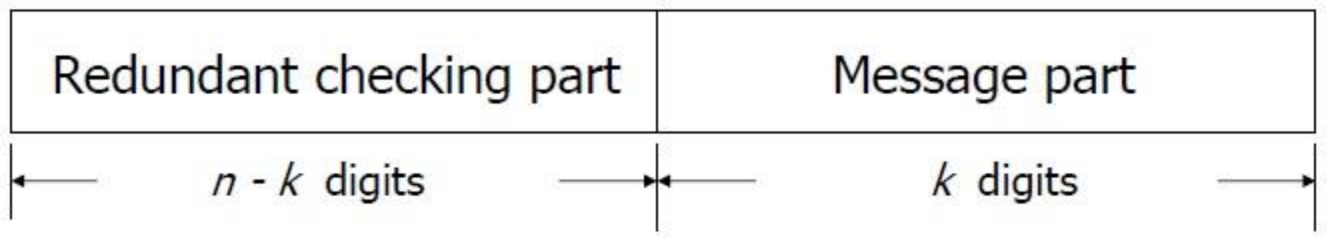
$$c_{1*n} = m_{1*k} . G_{k*n}$$

where c=$\{c_1, c_2, \ldots.., c_n\}$ and m=$\{m_1, m_2, \ldots\ldots., m_k\}$

$$G = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_k \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ g_{21} & g_{22} & \cdots & g_{2n} \\ \vdots & \vdots & & \vdots \\ g_{k1} & g_{k2} & \cdots & g_{kn} \end{bmatrix}$$

BLOCK CODES IN SYSTEMATIC FORM

- In this form, the code word consists of (n-k) parity check bits followed by k bits of the message.
- The structure of the code word in systematic form is:



- The rate or efficiency for this code R= k/n

$$G = [\, I_k \;\; P]$$

$$C = m.G = [\underbrace{m}_{\text{Message part}} \; \underbrace{mP}_{\text{Parity part}}]$$

Example:

Let us consider (7, 4) linear code where k=4 and n=7

$$m=(1110) \text{ and } G = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} 1\,1\,0\,1\,0\,0\,0 \\ 0\,1\,1\,0\,1\,0\,0 \\ 1\,1\,1\,0\,0\,1\,0 \\ 1\,0\,1\,0\,0\,0\,1 \end{bmatrix}$$

$$c = m.G = \boldsymbol{m_1 g_1 + m_2 g_2 + m_3 g_3 + m_4 g_4}$$

$$= 1.\boldsymbol{g_1} + \mathbf{1}.\,\boldsymbol{g_2} + \mathbf{1}.\,\boldsymbol{g_3} + \mathbf{0}.\,\boldsymbol{g_4}$$

c = (1101000) + (0110100) + (1110010)

= (0101110)

Another method:

Let m=(m_1, m_2, m_3, m_4) and c= $(c_1, c_2, c_3, c_4, c_5, c_6, c_7)$

$$c = m.G = (m_1, m_2, m_3, m_4)\begin{bmatrix} 1&1&0&1&0&0&0 \\ 0&1&1&0&1&0&0 \\ 1&1&1&0&0&1&0 \\ 1&0&1&0&0&0&1 \end{bmatrix}$$

By matrix multiplication we obtain :

$c_1 = m_1 + m_3 + m_4$, $c_2 = m_1 + m_2 + m_3$, $c_3 = m_2 + m_3 + m_4$, $c_4 = m_1$, $c_5 = m_2$, $c_6 = m_3$, $c_7 = m_4$

The code word corresponding to the message(1110) is (0101110) .

PARITY CHECK MATRIX (H)

- When G is systematic, it is easy to determine the parity check matrix H as:

$$H = [I_{n-k} \;\; P^T]$$

- The parity check matrix H of a generator matrix is an (n-k)-by-n matrix satisfying:

$$H_{(n-k)*n} G^{\mathrm{T}}_{n*k} = 0$$

- Then the code words should satisfy (n-k) parity check equations

$$c_{1*n} H^{\mathrm{T}}_{n*(n-k)} = m_{1*k} G_{k*n} H^{\mathrm{T}}_{n*(n-k)} = 0$$

Example:

Consider generator matrix of (7, 4) linear block code

$$H = [I_{n-k} \ \ P^T] \qquad \text{and} \qquad G = [\, P \ \ I_k]$$

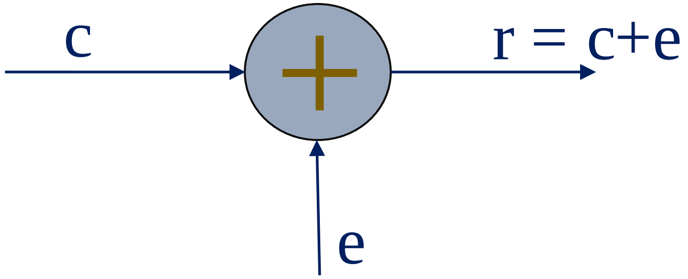
The corresponding parity check matrix is:

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$$G.\, H^T = \begin{bmatrix} 1\,1\,0\,1\,0\,0\,0 \\ 0\,1\,1\,0\,1\,0\,0 \\ 1\,1\,1\,0\,0\,1\,0 \\ 1\,0\,1\,0\,0\,0\,1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} = 0$$

SYNDROME AND ERROR DETECTION

- For a code word c, transmitted over a noisy channel, let r be the received vector at the output of the channel with error e



$$e_i = \begin{cases} 1, \text{ if } r \neq c \\ 0, \text{ if } r = c \end{cases}$$

Syndrome of received vector r is given by:

$$s = r.H^T = (s_1, s_2, s_3, \ldots \ldots \ldots, s_{n-k})$$

Properties of syndrome:

- The syndrome depends only on the error pattern and not on the transmitted word.

$$s = (c+e).H^T = c.H^T + e.H^T = e.H^T$$

- All the error pattern differ by atleast a code word have the same syndrome s.

Example:

Let C=(0101110) be the transmitted code and r=(0001110) be the received vector.

s=r. H^T=(s_1, s_2, s_3)

$$=(r_1,r_2,r_3,r_4,r_5,r_6,r_7)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

The syndrome digits are:

$$s_1 = r_1 + r_4 + r_6 + r_7 = 0$$
$$s_2 = r_2 + r_4 + r_5 + r_6 = 1$$
$$s_3 = r_3 + r_5 + r_6 + r_7 = 0$$

The error vector, e=($e_1, e_2, e_3, e_4, e_5, e_6, e_7$)=(0100000)

$C^* = r + e$

$= (0001110)+(0100000)$

$= (0101110)$

where C^* is the actual transmitted code word

MINIMUM DISTANCE OF A BLOCK CODE

- **Hamming weight w(c) :** It is defined as the number of non-zero components of c.

  For ex: The hamming weight of c=(11000110) is 4
- **Hamming distance d(c, x):** It is defined as the number of places where they differ .

  The hamming distance between c=(11000110) and x=(00100100) is 4
- The hamming distance satisfies the triangle inequality

$$d(c, x)+d(x, y) \geq d(c, y)$$

- The hamming distance between two n-tuple c and x is equal to the hamming weight of the sum of c and x

$$d(c, x) = w(c+ x)$$

For ex: The hamming distance between c=(11000110) and x=(00100100) is 4 and the weight of c + x = (11100010) is 4.

- **Minimum hamming distance d_{min} :** It is defined as the smallest distance between any pair of code vectors in the code.

  For a given block code C, d_{min} is defined as:

  $$d_{min} = \min\{ d(c, x): c, x € C, c \neq x\}$$

- The Hamming distance between two code vectors in C is equal to the Hamming weight of a third code vector in C.

  $$d_{min} = \min\{w(c+x):c, x € C, c\neq x\}$$
  $$= \min\{w(y):y € C, y\neq 0\}$$
  $$= w_{min}$$

APPLICATIONS

- Communications:
  - Satellite and deep space communications.
  - Digital audio and video transmissions.
- Storage:
  - Computer memory (RAM).
  - Single error correcting and double error detecting code.

ADVANTAGES

- It is the easiest and simplest technique to detect and correct errors.
- Error probability is reduced.

DISADVANTAGES

- Transmission bandwidth requirement is more.
- Extra bits reduces bit rate of transmitter and also reduces its power.