Time Series

Definition - The arrangement of statistical data in chronological order i.e accordance with occurence of time is known as Time series.

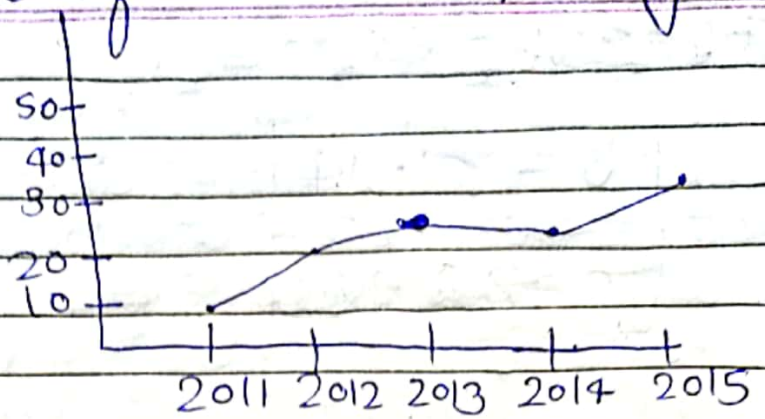
Or

Time series is a sequence of observation made on variable at regular time intervals over a specific period of time.

Examples

1. Monthly, Quartely or Yearly production of an Industrial product.
2. Yearly GDP
3. Monthly Sales in a departmental ~~stat~~ stores.

Components of Time series-The systematic part of variations or fluctuations of Time series may be referred to as component of Time series



(Long Term)

(i) Secular Trend :- The general tendency of the data either to increase, decrease or to remain constant during a long time period of time is called Secular Trend. It is smooth, long-term movement of data. The changes in the values are gradually & continuous.

Eg. Yearly population
Yearly death rates.

(ii) Seasonal Trend (Short Term, Seasonal Variation) The regular, seasonal changes in the time series of a specified period of time but less than a year are called Seasonal Trend. For example:-
demand of umbrella, raincoat during raining seasons.

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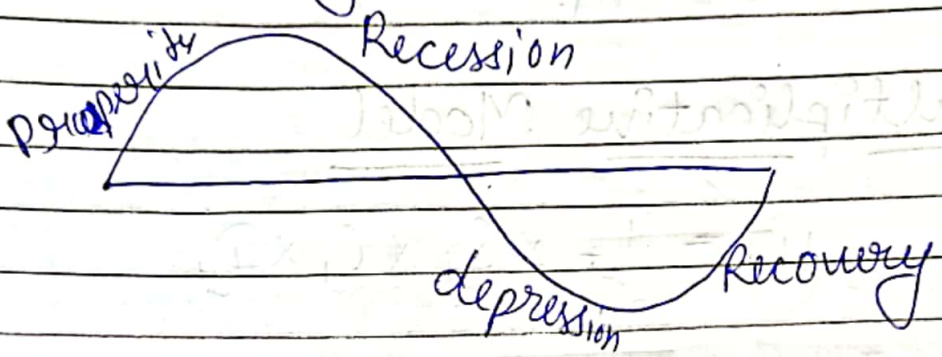
(iii) Cyclical Trend

These are changes in time series occuring over a period which is more than one year. They are recurring, periodic in nature. One complete period is called as cycle. These cycles are called as business cycle.



(iv) Irregular Trend / Random Variation (Short Term)

The change in the time series which cannot be predicted and are erractic in Nature are called Irregular variation. usually. Irregular variation caused by factors such as floods, famines, strikes, war

Mathematical Models of Time Series

(a) Additive Model - According to additive model a time series can be expressed as

$$y_t = T_t + S_t + C_t + I_t$$

In this model it is assumed that components are non-Interactive or Independent. This assumption is not realistic. Hence, this model is not used for most of economic data.

(b) Multiplicative Model

$$y_t = T_t \times S_t \times C_t \times I_t$$

Only T_t is expressed in actual value and the remaining components are expressed in terms of percentage of trend.

(c) Mixed Model

$$y_t = C_t \times T_t \times S_t \times R_t$$

$$y_t = T_t \times C_t \times S_t \times R_t$$

Uses of Time series

- It helps you know the real behaviour of past data.
- It helps in predicting the future behaviour like demand,

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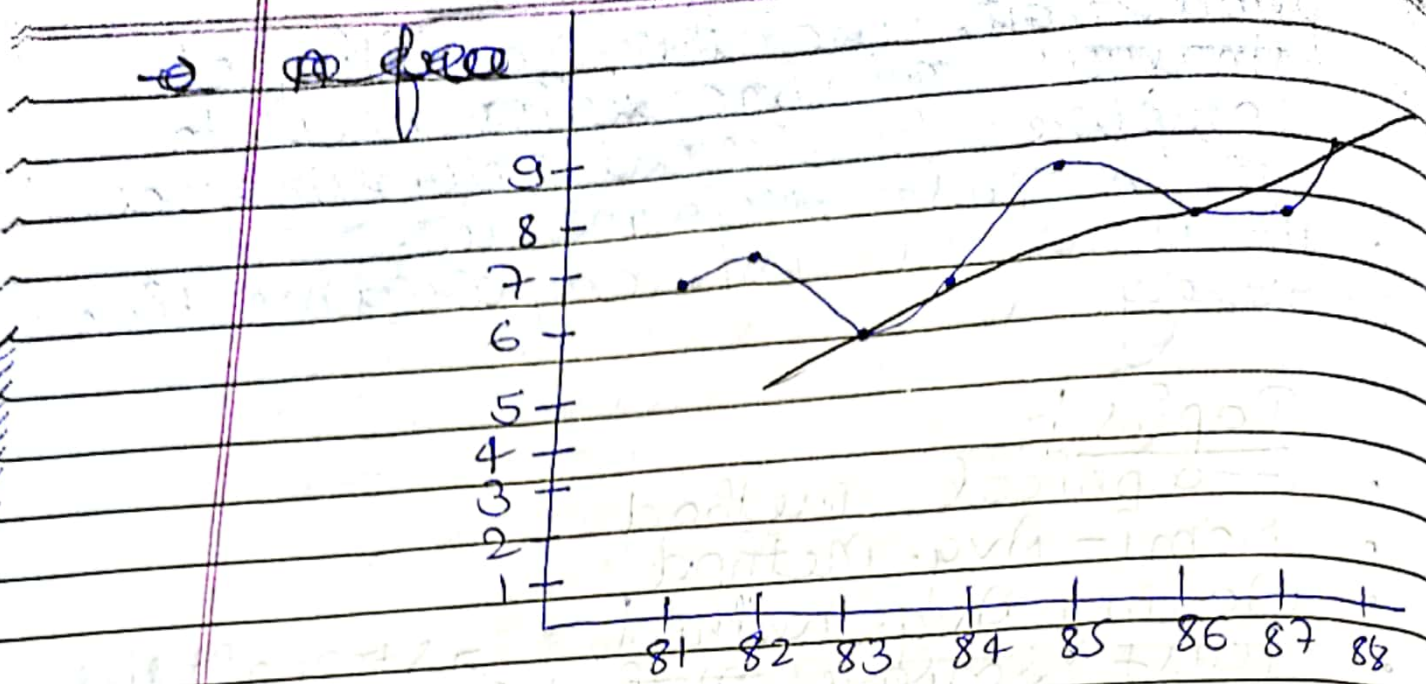
production, weather condition etc.
- Analysis of Time series helps to compare present performance with past performance.
- It is useful in comparative time study.

Topics :-
- Graphical method
- Semi-Avg. Method
- Moving Avg. method
- Least Square Method → Straight line
  → Parabolic Trend.
- Exponential Smoothening Method
- Simple Average Method
- Ratio to moving Average Method
- Ratio To Trend Method

- Graphical Method (Free hand Curve)

Years	Production
1981	6.6
1982	6.9
1983	5.6
1984	6.3
1985	8.4
1986	7.2
1987	7.2
1988	8.5



→ A free-hand smooth curve on plotting the values "y t" against t.

Merits

- This method is very simple and flexible.
- It is used to describe all types of trend.

Demerits

- It is very subjective. The bias of the person handling the data plays an important role such as different trend curves will obtain by different person.
- It does not enable us to

measure trend.

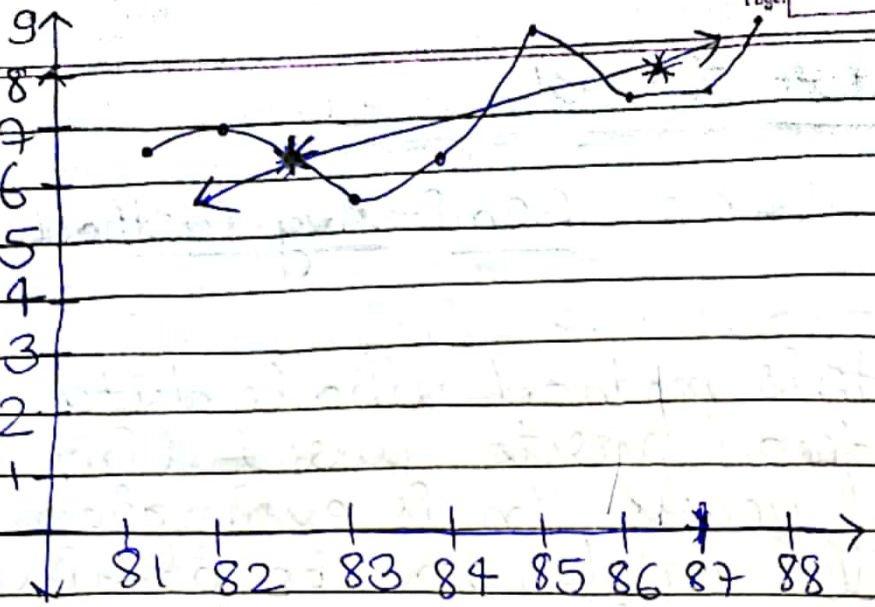
Semi-Avg Method

In this method whole data is divided into two parts w.r.t time. If the no. of years "n" is even then two equall parts can be obtained but if n is odd then two parts are obtained by obtaining the value corresponding to middle year.

Merits :- As compared with the graphical method the obvious advantage of this method is its objectivity.

Demerits- This ~~meth~~ method assumes linear relationship between plotted points which may be exists. Also the limitation of AM also stands in it way.

years	Production	
1981	6.6	
1982	6.9	6.35
1983	5.6	→
1984	6.3	
1985	8.4	
1986	7.2	→ 7.8
1987	7.2	
1988	8.5	



Moving Average Method

- This method consisting measurements of trends by smoothing out the fluctuation of data.

Year(t)	Production y_t	3-yearly moving total	3 yearly moving Avg Trend value
1976	66.6	—	—
1977	84.9	240.1	80.03
1978	88.6	251.5	83.83
1979	78.0	263.4	87.8
1980	96.8	280	93.33
1981	105.2	295.2	98.4
1982	93.2	310	103.33
1983	116.6	293.1	97.7
1984	88.3	316.9	105.63
1985	117.0	320.5	106.83
1986	115.2	—	—

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Merits

1. It is easily to understand and simple to calculate
2. Regular cyclical variation can be completely elemenated by selection of period
3. It does not involve any element of subjectivity.

Demerits

- It cannot be used for predicting future values.
- Selection of period of AM is very important.
- Greatly affected by extreme values.

Four-Yearly Moving average (trend value)

Year	y(t)	4-yearly Moving total	4-yearly moving avg.	2 unit Moving avg.	4-yearly centre moving average
1987	3.6				
1988	4.3	15.6	3.89		
1989	4.3	16.4	4.1	8	4
1990	3.4	17.3	4.325	8.425	4.2125
1991	4.4	16.8	4.2	8.525	4.2625
1992	5.2	18.3	4.575	8.775	4.3875
1993	3.8	19.3	4.825	9.4	4.7
1994	4.9				
1995	5.4				

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Least Square Method

Linear Trend (n is odd)

The principle of least square is minimizing the sum of square of deviation b/w given y_t & there estimation we find value of a & b.

$$y_t = a + bt$$

$$\Sigma y_t = na + b\Sigma t$$

$$\Sigma t y_t = a\Sigma t + b\Sigma t^2$$

Year	Production y_t	$t' = t - 1967$	t'^2	$t' y_t$
1962	6.7	-5	25	-33.5
1963	7.0	-4	16	-28.0
1964	7.9	-3	9	-23.7
1965	7.4	-2	4	-14.8
1966	10.8	-1	1	-10.8
1967	9.2	0	0	0
1968	10.5	1	1	10.5
1969	15.5	2	4	31.0
1970	13.7	3	9	41.1
1971	16.7	4	16	66.8
1972	15.0	5	25	75.0

$$y_t = a + bt'$$

$$\Sigma y_t = na + b\Sigma t'$$

$$\Sigma t' y_t = a\Sigma t' + b\Sigma t'^2$$

$$n = 11$$

$$\Sigma y_t = 120.4$$

$$\Sigma t' = 0$$

$$\Sigma t' y_t = 113.6$$

$$\Sigma t'^2 = 110$$

$$120.4 = 11a + b(0) \quad \text{—— (1)}$$

$$113.6 = a \times 0 + b \times 110 \quad \text{— (2)}$$

from (1)

$$120.4 = 11a$$

$$a = \frac{120.4}{11}$$

$$\boxed{a = 10.9455}$$

$$113.6 = 110b$$

$$b = \frac{113.6}{110}$$

$$b = 1.0327$$

$$y_t = 10.9455 + (1.0327)t'$$

$1973 - 1967$

$= 6$

$y_{1973} = 10.9455 + 1.0327 \times 6$

$=$

Merits

- Since it is based on Algebraic method of calculating the trend it is free from any biased subjectivity.
- This method helps to compute trend values.

Demerits

- It is quite tedious method and involve more calculation
- The trend can be estimate a value for immediate future not for distant future.
- Limitation is the determination of type of the trend value.

~~2020~~
2056.5
1452.5

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Linear Trend (n is even)

Year	y_t	$t' = (t - 1980.5)$	t'^2	$t' y_t$
1976	115	-4.5	20.25	-517.5
1977	130	-3.5	12.25	-455
1978	137	-2.5	6.25	-342.5
1979	135	-1.5	2.25	-202.5
1980	130	-0.5	0.25	-65
1981	140	0.5	0.25	70
1982	148	1.5	2.25	222
1983	155	2.5	6.25	387.5
1984	162	3.5	12.25	567
1985	180	4.5	20.25	810

$n = 10$

$\Sigma Y_t = 1432$

$\Sigma t' = 0$

$\Sigma t' y_t = 604$

$\Sigma t'^2 = 82.5$

$\Sigma Y_t = na + b \Sigma t'$

$1432 = 10a + b \times 0$

$a = \frac{1432}{10}$

$\boxed{a = 143.2}$

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$6040 = 82.56$

$b = 7.32$

$$y_t = 143.2 + 7.32t$$

Parabolic Trend

Year	y(t)	t = Year - 2001	t^2	t^3	t^4	ty_t	t^2y_t
1998	15	-3	9	-27	81	-45	135
1999	14	-2	4	-8	16	-28	56
2000	18	-1	1	-1	1	+18	18
2001	20	0	0	0	0	0	0
2002	17	1	1	1	1	17	17
2003	24	2	4	8	16	48	96
2004	27	3	9	27	81	81	243

$$\Sigma y_t = na + b\Sigma t + c\Sigma t^2 \quad -(1)$$
$$\Sigma t y_t = a\Sigma t + b\Sigma t^2 + c\Sigma t^3 \quad -(2)$$
$$\Sigma t^2 y_t = a\Sigma t^2 + b\Sigma t^3 + c\Sigma t^4 \quad -(3)$$

$\Sigma y_t = 135$
$n = 7$
$\Sigma t = 0$
$\Sigma t^2 = 28$

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$\Sigma t^3 = 0$

$\Sigma t^4 = 196$

$\Sigma t y_t = 55$

$\Sigma t^2 y_t = 565$

By equ. ①

$135 = 7a + b(0) + c(28)$

$7a + 28c = 135$ — ④

$55 = a(0) + b(28) + c(0)$

$b = \frac{55}{28} = 1.96$

$565 = a28 + b(0) + c(196)$

$28a + 196c = 565$ — ⑤

By equ. ① ④ and ⑤

$7a + 28c = 135$

$28a + 196c = 565$

$28a + 112c = 540$

$28a + 196c = 565$

$(-) \quad (-) \quad (-)$

$84c = 25$

$c = \frac{25}{84} = 0.297$

$$7a + 28 \times \frac{25}{84} = 540$$

$$7a + 8.33 = 540$$

$$7a = 540 - 8.33$$

$$7a = 531.67$$

$$a = \frac{531.67}{7}$$

$$28a + 196 \times \frac{25}{84} \times 0.297 = 565$$

$$28a + 58.21 = 565$$

$$28a = 506.79$$

$$\boxed{a = 18.09}$$

$$\boxed{y_t = 18.09 + 1.96t + 0.297t^2}$$

Exponential smooth

Define "α"

$(0 < \alpha < 1)$ as the smoothening constant and assume that the time series points for past "t" periods are $y_1, y_2 - - - y_t$ The estimates for period y_{t+1} is computed as

$$\boxed{y_{t+1} = \alpha y_t + (1-\alpha) y_t^{\wedge}}$$

Month	Demand	y_t	$y_t \phi$
1	46		—
2	56		46 → 0.1×46 + 0.9×46
3	54		47 → 0.1×56 + 0.9×46
4	43		47.7 → 0.1×54 + 0.9×47
5	57		47.23 → 0.1×43 + 0.9×47.7
6	56		48.207 → 0.1×57 + 0.9×47.23
7	67		48.9863 → 0.1×56 + 0.9×48.207
8	62		50.787 → 0.1×67 + 0.9×48.98
9	60		51.9083 → 0.1×62 + 0.9×50.78
10	56		52.717 → 0.1×60 + 0.9×51.90
11	—		53.0453 → 0.1×56 + 0.9×52.77

☆ Measurement of Seasonal Variation

- This study is very useful for business to plan their production in an efficient way.
- There are different methods to measure this
  - → ~~Simple~~ Method of simple Averages
  - → Ratio to Moving Average Method.
  - → Ratio to Trend Method.
  - → Link Relative Method.

Method of Simple Average

Years	I	II	III	IV
2004	3.7	4.1	3.3	3.5
2005	3.7	3.9	3.6	3.6
2006	4.0	4.1	3.3	3.1
2007	3.3	4.4	4.0	4.0
Total	14.7	16.5	14.2	14.2
Avg.	3.675	4.125	3.55	3.55

Avg. of Avg. $= \frac{3.675 + 4.125 + 3.55 + 3.55}{4}$

$= 4$

$S.I = \frac{\text{Quartely Avg.}}{\text{General Avg.}} \times 100$

S.I for first Quarter $= \frac{3.675}{3.725} \times 100$

$= 98.66$

SI for second Quarter $= \frac{4.125}{3.725} \times 100$

$= 110.74$

SI for third Quarter $= \frac{3.55}{3.785} \times 100$

$= 95.30$

S.I for fourth Quarter $= 95.30$

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Advantages

- Pt is easiest method to compute SI
- Pt gives good result if trend & cyclic variation have negligible effect on time series.

Disadvantages

- Pt is based on assumptions that data do not contain any trend & cyclic components
- Since most of the economic and business time series have trends and as such this method through simple is not of much practical use.

Ratio to moving average Method

Year	Quarter	Data	Moving Total	2-figure Total	4 year mov avg.	% of mov avg
2005	I					
	II	254				
	III	251				
	IV	247				
2006	I	252				
	II	250				
	III	253				
	IV	25				

Year	Quarter	Data	Moving Total	2-fig. Total	4-year moving avg.	% of moving avg.
2005	I	68				
	II	62	254			
	III	61	251	505	63.125	96.63
	IV	63	247	498	62.25	101.20
2006	I	65	252	499	62.375	104.21
	II	58	250	502	62.75	92.43
	III	66	253	503	62.875	104.97
	IV	61	258	511	63.875	95.50
2007	I	68	255	513	64.125	106.04
	II	63	261	516	64.5	97.67
	III	63				
	IV	67				

$$\% \text{ of moving} = \frac{61}{63.125} \times 100$$

$$= \frac{0}{\text{T}} \times 100$$

Year	I	II	III	IV
2005	—	—	96.63	101.20
2006	104.21	92.43	104.97	95.50
2007	106.04	97.67	—	—
Total	210.25	190.10	201.60	196.70
Avg.	105.13	95.05	100.80	98.35

$$\text{Avg of Averages} = \frac{105.125 + 95.05 + 100.80 + 98.35}{4}$$

$$= 99.83$$

S.I of I Avg. $= \frac{105.125}{99.83} \times 100$

$= 105.30$

" " II Avg. $= \frac{95.05}{99.83} \times 100$

$= 95.21$

" " III Avg. $= \frac{100.80}{99.83} \times 100$

$= 100.97$

" " IV Avg. $= \frac{98.35}{99.83} \times 100$

$= 98.52$

Advantages

- It is one of the most satisfactory widely used method.
- This method gives better result as compared to other method.

Disadvantages

- This method does not completely utilize the data.
- For eg., In case of 12 months moving average SI cannot be obtained for first 6 months & last 6 months.

Ratio to Trend Method

Year	Q_1	Q_2	Q_3	Q_4
2003	30	45	36	34
2004	34	52	50	44
2005	40	58	54	48
2006	54	76	68	62
2007	80	92	86	82

Year	Year Total	Yearly avg $y(t)$	$t = Y - 2005$	ty_t	t^2	Trend Values
2003	140	35	−2	−70	4	32
2004	180	45	−1	−45	1	44
2005	~~150~~ 200	50	0	0	0	56
2006	260	65	1	65	1	68
2007	340	85	2	170	4	80
N=5		$\Sigma Y_t = 280$	$\Sigma t = 0$	$\Sigma ty_t = 120$	$\Sigma t^2 = 10$	

$$y_t = a + bt$$

$$a = \frac{\Sigma y_t}{N}$$

$$a = \frac{280}{5}$$

$$\boxed{a = 56}$$

$$b = \frac{\Sigma t y_t}{\Sigma t^2}$$

$$b = \frac{120}{10} \qquad \boxed{b = 12}$$

$$y_t = a + bt$$

$$= 56 + 12t$$

Quartely Increament $= \frac{12}{4}$

$= 3$

Year	Q_1	Q_2	Q_3	Q_4
2003	27.5	30.5	32.5	36.5
2004	39.5	42.5	45.5	48.5
2005	51.5	54.5	57.5	60.5
2006	63.5	66.5	69.5	72.5
2007	75.5	78.5	81.5	84.5

Ratio to Trend $= \frac{\text{Original value}}{\text{Trend value}} \times 100$

Year	Q_1	Q_2	Q_3	Q_4
2003	109.09	131.15	107.96	93.15
2004	86.08	122.35	109.89	90.72
2005	77.67	106.92	93.91	79.34
2006	85.04	114.29	97.84	85.52
2007	105.96	117.20	105.52	97.04
Total	463.84	591.41	514.62	495.77
Average	92.77	118.28	102.92	89.46
SI adjusted	92.05	117.36	102.12	88.46

$96.77 + 118.28 + 102.92 + 89.15$

$= 403.12$

Q Average SI adjusted $= 92.77 \times \frac{400}{403.12}$

$= 92.05$

Merits

- It is easy to compute & easy to understand.
- It has an advantage over ratio to moving average that in this method we obtain ratio to trend values for each period for which data are available which is not possible for ratio to moving average method.

Demerits

The main defect is that there are cyclical swings in the series the trend whether a straight line or curve can never follow the actual data as closely as 12-mothly moving average does. So a seasonal index computed by ratio to moving average may be less biased than one calculated by ratio to trend method.

Link Relative Method

Quarter	2003	2004	2005	2006	2007
I	6.0	5.4	6.8	7.2	6.6
II	6.5	7.9	6.5	5.8	7.3
III	7.8	8.4	4.3	7.5	8.0
IV	8.7	7.3	6.4	8.5	7.1

Years	Q_1	Q_2	Q_3	Q_4	
2003	—	108.3	120.0	111.5	
2004	62.5	146.3	106.3	86.9	
2005	93.2	95.6	143.1	68.8	
2006	112.5	80.6	129.3	113.3	
2007	77.6	110.6	109.6	88.8	
Avg	86.35	108.28	121.66	93.86	
CR	100	108.28	131.73	123.64	
Corrected SI	100	106.605	128.38	118.615	113.4
	88.18	94.01	113.21	104.60	

$$LR = \frac{\text{Current period}}{\text{previous period}} \times 100$$

$$= \frac{6.5}{6.0} \times 100$$

$$CR = \frac{\text{Avg LR} \times \text{LR for/of Previous}}{100}$$

$$= \frac{108.28 \times 100}{100}$$

$$= 108.28$$

Corrected CR

OR

CR for $Q_1 = \frac{86.35 \times 123.64}{100}$

$= 106.7$

$d = \frac{1}{4}(106.7 - 100)$

$d = \frac{6.7}{4}$

$d = 1.675$

$$SI = \frac{\text{Corrected CR}}{113.4} \times 100$$

$SI = \frac{100}{113.4} \times 100$

Merits

- As compared to method of moving average link Relative uses more data.

Demerits

→ The link relative methods needs extensive calculation.