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Unit-3 Statistical Quality Control

S.Q.C. refers to the systematic control of those variables encountered in a manufacturing process which affect the excellence of the end product. Such variables are from the application of materials, men, machines and manufacturing conditions.

S.Q.C. is one of the most important application of the statistical techniques in industry. These techniques are based on the theory of probability and sampling, and are being extensively used in almost all industries.

- What is 'Quality'?

Quality means an attribute of the product that determines its fitness for use. It is a measure of how closely an item conforms to the standards. The range of these attributes is wide - physical, chemical, aesthetic etc. A product may have several aspects of quality as well as an overall quality which is more than the sum of its individual quality aspects - as property technically known as <u>synergy</u>.

Quality depends on materials, manpower, machines and management. Quality Control covers all the factors and processes of prod^n which may be broadly classified as follows-

i) Quality of materials –
material of good quality will result in smooth processing thereby reducing the waste and increasing the output.

ii) Quality of manpower –
Trained and qualified personnel will give increased efficiency through better application of skill and also reduce prodn cost and waste.

iii) Quality of machines –
Better quality equipment will result in efficient work due to lack or scarcity of breakdowns and thus, reduce cost of defectives.

iv) Quality of management –
A good management is imperative for increased efficiency, harmony in relations and growth of business and markets.

– Basis of Statistical Q. Control / Causes of variations in quality.

The basis of SQC is the degree of 'variability' in the size ~~of~~ or the magnitude of a given characteristics of the product. Variation in the quality of manufactured product in the repetitive process in industry is inherent and inevitable. These variations are broadly classified as being due to two causes –

i) Chance causes – Some 'stable pattern of variation'

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or 'a constant cause system' is inherent in any particular scheme of production and inspection. This pattern results from many minor causes that behave in a random manner. This is usually termed as 'allowable variation'. The range of such variation is known as 'natural tolerance of the process'.

ii) Assignable Causes – This variation attributed to any ~~rando~~ prodn process is due to non-random or assignable causes and is termed as 'preventable variation'.

Some of the important factors of this type of variation are –

a) substandard or defective raw materials.
b) new techniques / operations.
c) negligence of the operators.
d) wrong / improper handling of machines.
e) faulty equipment.
f) unskilled / inexperienced technical staff.

These causes can be identified and eliminated and are to be discovered in a production process before it goes wrong i.e. prodn becomes defective.

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• Difference b/w chance and assignable causes.

Chance causes of variation	Assignable causes of variation
i) Consists of many individual causes.	Consists of just a few individual causes.
ii) Any one chance cause results in only a small amt. of variation.	Any one assignable cause can result in a large amt. of variation.
iii) It cannot be eliminated from a process.	The presence of this type of variation can be detected, and action to eliminate the causes is usually economically justified.
iv) Some typical chance causes of variation are -	Some typical assignable causes of variation are -
- slight vibration of a machine.	- negligence of operators.
- Lack of human perfection in reading instruments and setting controls.	- defective raw material
- Voltage fluctuations and variation in temperatures.	- faulty equipment
	- Improper handling of machines.

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SQC means planned collection and effective use of data for studying causes of variations in quality either as processes, procedures, materials, machines etc. or over periods of time.

The main purpose of SQC is to devise statistical techniques which would help us in separating the assignable causes from the chance causes, and thus enabling us to take remedial action whenever assignable causes are present. The elimination of assignable causes of assignable causes of erratic fluctuations is described as bringing a process under control.

- A production process is said to be in a state of statistical control, if it is governed by chance causes alone, in the absence of assignable causes of variation.

Control is two-fold – controlling the process (process control) and controlling the finished products (product control).

– Process control and Product control

The main objective of any prodn process is to control and maintain a satisfactory quality level of the manufactured product so that it conforms to specified quality standards. This is termed as 'process control' and is achieved through the technique of 'control charts' developed by W.A. Shewhart in 1924.

Product control means controlling the quality of the product by critical examination at strategic points and is achieved through **sampling inspection plans** developed by H.F. Dodge and H.C. Romig.

Techniques of SQC

- **Process Control** (by control charts)
  - Variables
    - $\bar{X}$
    - R
    - σ
  - Attributes
    - c
    - np
    - p
- Product Control (by sampling inspection plans)
  - Variables
  - Attributes

1) Control Limits —

These are limits of sampling variation of a statistical measure (eg- mean, range or fraction defective) such that if the prod^n process is under control, the value of the measure calculated from diff. rational sub-groups will lie within these limits.

Points falling outside control limits indicate that the process is not operating unde

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a system of chance causes and assignable causes of variation are present, which must be eliminated. Control limits are used in Control charts.

2) Specification Limits –

When an article is supposed to be manufactured, the max^m and min^m allowable dimensions of some quality characteristics have to be decided so that the product can be gainfully utilised for which it is intended.
If the dimensions are beyond these limits, the product is treated as defective. These max^m and min^m limits of variation of individual items, as mentioned in the product design, are known as 'specification limits'.

3) Tolerance Limits –
These limits are limits of variation of a quality measure of the product between which atleast a specified proportion of the product is expected to lie (with a given probability), provided the process is in a state of SQC.
These limits are also known as 'statistical tolerance limits'.

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- Control Charts

Control chart, as devised by Shewhart, is a simple pictorial device for detecting unnatural patterns of variations in the data resulting from repetitive processes.

In other words, control charts provide criteria for ~~ale~~ detecting lack of statistical control. They simple and easy to construct and interpret.

They are used to tell if a sample point lies within 3- σ control limits or not.

A typical control chart consists of 3 horizontal lines -

i) A central line (C.L.), indicating desired standard of the process.

ii) Upper Control Limit (U.C.L.), indicating the upper limit of tolerance.

iii) Lower Control Limit (L.C.L), indicating the lower limit of tolerance.

- Major Parts of a Control Chart -

a control chart, generally includes the foll. 4 parts -

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1) Quality Scale –
This is a vertical scale. The scale is marked according to quality characteristics (either var. or att.) of each sample.

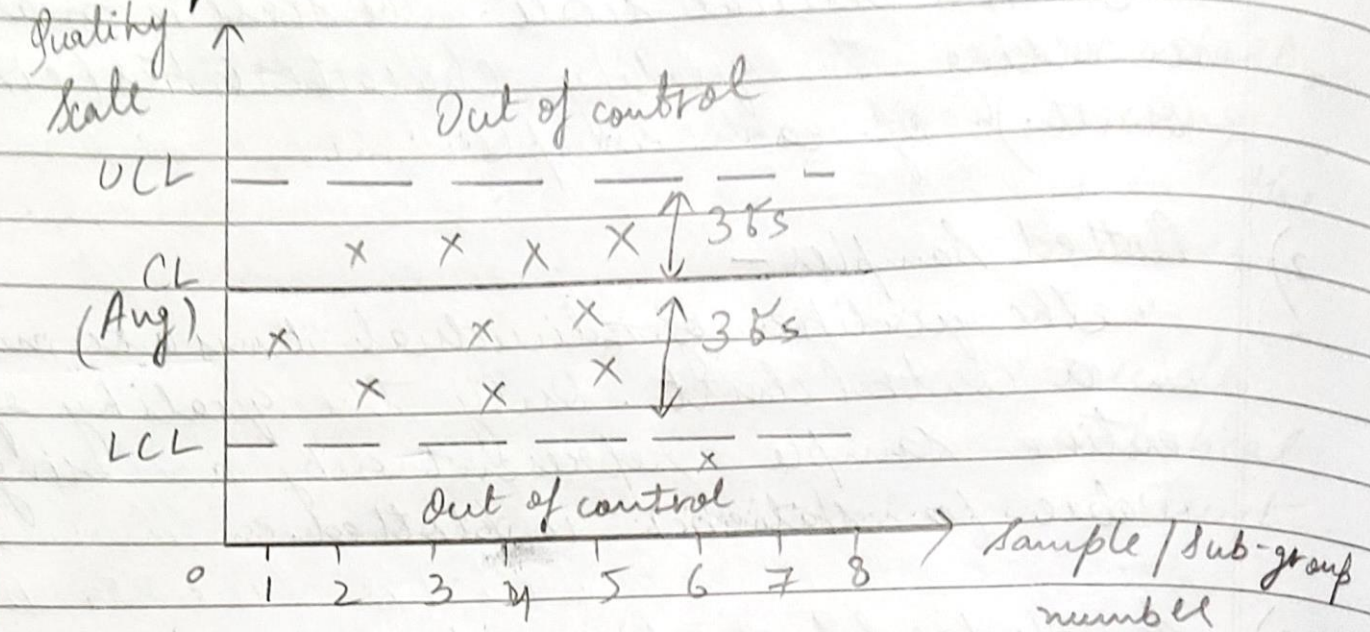
2) Plotted Samples –
The qualities of individual items are not shown on a control chart. Only the quality of the entire sample represented by a single value (a statistic) is plotted.

3) Sample (or Sub-group) numbers –
The samples plotted on a control chart (C.C.) are numbered individually and consecutively on a horizontal line. The line is usually placed at the bottom of the chart. Generally, 25 sub-groups are used in ~~cost~~ constructing a control chart.

4) The horizontal lines –
consists of the CL, UCL and LCL.

CL - represents avg. quality of the samples plotted on the chart.
UCL - obtained by adding 3 σ's to the avg.
= Mean + 3σ.
LCL = obtained by deducting 3σ's from the avg.
= Mean − 3σ.

· Outline of a control chart —



- Tools for SQC

1) Shewhart's CC for variables -
i.e, for a characteristic that can be measured quantitively. Such variables are of continuous type and are said to follow normal probability law.

For Q.C. of such data, two types of CCs are used —

i) Charts for $\bar{X}$ (mean) and R (range)

ii) Charts for $\bar{X}$ (") and σ (s.d.)

2) Shewhart's CC for fraction defective or p-chart —
This chart is used, when dealing with attributes, in which case quality characteristics of the product are not amenable to measurement but can be identified by their absence or presence from the product or by classifying

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the product as defective or non-defective.

3) Shewart's CC. for the 'no. of defects' per unit or C-chart -

This is used with advantage when the characteristic representing the quality of a product is a discrete variable.

Eg - i) no. of defective rivets in an aircraft wing.

ii) no. of surface defects in a sheet of photographic film.

4) The portion of sampling theory which deals with the quality protection given by any specified sampling acceptance procedure.

- Control Charts for variables -

In order to control a measurable characteristic, we have to exercise control on the measure of location, as well as the measure of dispersion.

- $\overline{X}$ and R charts

The control limits in the $\overline{X}$ and R charts are so placed that they reveal the presence or absence of assignable causes of variation in the -

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a) average – mostly related to machine setting

b) range – mostly " " negligence on the part of the operator.

• Steps

1) Measurement –

Any method of measurement has its own inherent variability. Errors in measurement must be minimised so as to draw conclusions from CCs that are valid.

2) Selection of samples/sub-groups –

The choice of the sample size size n and the frequency of sampling i.e., the time b/w the selection of two groups, depends upon the process. Usually, n is taken to 4 or 5 while the frequency of sampling depends on the state of control being exercised.

3) Calculation of $\bar{x}$ and R for each sub-group.

Let x_{ij} ; $j = 1, 2, 3, \ldots n$ be the measurements on the ith sample $(i = 1, 2, 3, \ldots, k)$. Then,

$$\bar{x}_i = \frac{1}{n} \sum_j x_{ij} \; , \; R_i = \max x_{ij} - \min x_{ij}$$

$$s_i^2 = \frac{1}{n} \sum_j (x_{ij} - \bar{x}_i)^2 \qquad (i = 1, 2, \ldots, k)$$

for sample no. i.

Next, $\bar{\bar{X}}$, $\bar{R}$ and $\bar{S}$, the averages of sample means, ranges and s.d., are to be found.

$$\bar{\bar{X}} = \frac{1}{k} \sum_i \bar{X}_i, \quad \bar{R} = \frac{1}{k} \sum_i R_i, \quad \bar{S} = \frac{1}{k} \sum_i S_i$$

4) Setting of Control Limits.

An estimate of σ can be obtained from $\bar{R}$ by the relation -

$$\bar{R} = d_2 \cdot \sigma$$

$$\Rightarrow \hat{\sigma} = \bar{R}/d_2$$

where, d_2 is a constant depending on the sample size n.

Also, $\bar{\bar{X}}$ gives an unbiased estimate of the pop^n mean, μ since,

$$E(\bar{\bar{X}}) = \frac{1}{k} \sum_{i=1}^{k} E(\bar{X}_i) = \frac{1}{k} \sum_{i=1}^{k} \mu = \mu$$

- Control limits for $\bar{X}$ chart -

Case I - when standards are given, i.e; both μ and σ are known.

The 3-σ control limits for $\bar{X}$ chart are given by -

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$$E(\bar{X}) \pm 3\, S.E(\bar{X})$$

$$= \mu \pm 3\frac{\sigma}{\sqrt{n}}$$

$$= \mu + A\sigma \quad \text{where, } A = 3/\sqrt{n}$$

$$UCL = \mu + \underbrace{\frac{3\sigma}{\sqrt{n}}}_{A}, \quad LCL = \mu - \underbrace{\frac{3\sigma}{\sqrt{n}}}_{A}$$

Find A's value from table.

Case 2 – when standards are not given –

If both μ and σ are unknown, we use their estimates, and the control limits for $\bar{X}$ chart are given by –

$$\bar{\bar{X}} \pm 3\frac{\bar{R}}{d_2} \cdot \frac{1}{\sqrt{n}}$$

$$= \bar{\bar{X}} \pm \left(\frac{3}{d_2\sqrt{n}}\right) \cdot \bar{R}$$

$$= \bar{\bar{X}} \pm A_2 \cdot \bar{R} \qquad \text{where, } A_2 = \left(\frac{3}{d_2\sqrt{n}}\right)$$

$$\therefore UCL = \bar{\bar{X}} + A_2\bar{R}$$

$$LCL = \bar{\bar{X}} - A_2\bar{R}$$

Find d_2 values from table.

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- Control Limits for R-chart

1) Compute the range, $R_i = \max X_{ij} - \min X_{ij}$, $(i = 1, 2, \cdots n)$

for each sample.

2) Compute the mean of the sample ranges –

$$\overline{R} = \frac{1}{k} \sum_{i=1}^{k} R_i = \frac{1}{k}(R_1 + R_2 + \cdots + R_k)$$

3) Computation of control limits –

The 3-σ control limits for R-chart are –

$$E(\overline{R}) \pm 3\sigma_R$$

$E(R)$ is estimated by $\overline{R}$ and σ_R is estimated by the relation –

$$\sigma_R = d_3 \hat{\sigma} = d_3 \cdot \frac{\overline{R}}{d_2}$$

Case I – σ is unknown

where d_2, d_3 are constants depending on n.

$$UCL_R = E(R) + 3\sigma_R$$

$$= \overline{R} + \frac{3 d_3 \overline{R}}{d_2}$$

$$= \left(1 + \frac{3 d_3}{d_2}\right)\overline{R} = D_4 \overline{R}$$

$$LCR_R = \left(1 - \frac{3 d_3}{d_2}\right)\overline{R} = D_3 \overline{R}$$

for D_3, D_4 – use table.

Case 2 - σ is known

$$UCL_R = E(R) + 3\sigma_R$$

$$= d_2 \sigma + 3 d_3 \sigma$$

$$= (d_2 + 3d_3)\sigma = D_2 \sigma$$

$$LCL_R = E(R) - 3\sigma_R$$

$$= d_2 \sigma - 3 d_3 \sigma$$

$$= (d_2 - 3d_3)\sigma$$

$$= D_1 \sigma$$

In each case, central line is given by -

$$CL_R = E(R) = \bar{R}$$

Since range can never be -ve, LCL_R must be greater than or equal to 0. In case, it comes out to be negative, it is taken as 0.

4) Construction of control charts for $\bar{X}$ and R.

- For $\bar{X}$,
  - central line is drawn as a solid horizontal line at $\bar{\bar{X}}$ and $UCL_{\bar{X}}$ and $LCL_{\bar{X}}$ are drawn at the computed values as dotted lines.

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- For $\bar{R}$ chart,

- the central line is drawn as a solid horizontal line at $\bar{R}$ and UCL_R is drawn at the computed value as a dotted - line.

- If $n \geq 7$, $LCLR$ is drawn as a dotted - line at the computed value, and if $n < 7$, LCL_R is taken as 0.

- Interpretation of $\bar{x}$ and R charts -

In order to judge if a process is in control, $\bar{x}$ and $\bar{R}$-charts should be examined together and the process should be deemed in statistical control if both the charts show a state of control.

Read table from book. Pg - 1.17 PDF pg - 9.

- Control Chart for attributes.

As an alternative to $\bar{x}$ and R charts, we have the CC for attributes (attri.) which can be used for quality characteristics -

i) which can only be observed as attri. by classifying an item as defective / non-defective i.e. conforming to specifications or not.

ii) which are observed as attri. even though

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they could be measured as variables.

eg - go and no-go gauge test results.

1) Control Chart for Fraction Defective (p-chart).

While dealing w/ attri, a process will be adjudged in statistical control, if all the samples / sub-groups are ascertained to have the same pop^n proportion P.

If 'd' is the no. of defectives in a sample size, n, then the sample proportion defective is $p = d/n$. Hence, d is a binomial variate with parameters n and P.

$\therefore E(d) = nP \quad , \quad Var(d) = nPQ \quad , \quad Q = 1 - P.$

Thus, $E(p) = E(d/n) = \frac{1}{n} E(d) = P$

$Var(p) = Var(d/n) = \frac{1}{n^2} Var(d) = \frac{PQ}{n}$

- The 3-σ control limits of p-chart are -

$E(p) \pm 3 \, S.E.(p)$

$= \quad P \pm 3 \cdot \sqrt{PQ/n} \qquad = P \pm A\sqrt{PQ}$, where $A = \frac{3}{\sqrt{n}}$

* value of A in table

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Case 1 – standards are given –

If P' is the known value of P, then,

$$UCL_p = P' + A\sqrt{P'(1-P')}$$

$$LCL_p = P' - A\sqrt{P'(1-P')}$$

$$CL_p = P'$$

Case 2 – standards are not given –

Let d_i be the no. of defectives and p_i the fraction defective for the ith sample $(i=1,2,\ldots,k)$ of size n_i. Then, the popn proportion P is estimated by the statistic $\bar{p}$ given by:

$$\bar{p} = \frac{\sum d_i}{\sum n_i} = \frac{\sum n_i p_i}{\sum n_i}$$

Note → $\bar{p}$ is an unbiased estimator P.

$$E(\bar{p}) = E\left(\frac{\sum d_i}{\sum n_i}\right)$$

$$= \left[\sum(n_i P) / \sum n_i\right] = P.$$

In this case,

$$UCL_p = \bar{p} + A\sqrt{\bar{p}(1-\bar{p})}$$

$$LCL_p = \bar{p} - A\sqrt{\bar{p}(1-\bar{p})}$$

$$CL_p = \bar{p}$$

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Note —

a) If sample size remains constant for each sample i.e. if $n_1 = n_2 = \cdots = n_k$ (say)

then an estimate of the pop^n proportion is given by —

$$\hat{p} = \bar{p} = \frac{\sum_{i=1}^{k} d_i}{\sum n_i}$$

$$= \frac{\sum_{i=1}^{k} d_i}{nk}$$

$$= \frac{n \sum_{i=1}^{k} p_i}{nk}$$

$$= \frac{1}{k} \sum_{i=1}^{k} p_i$$

In this case, same set of control limits can be used for all the samples inspected and it is immaterial if one uses p-chart or d-chart.

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- Interpretation of p- chart -

Read from book : pg - 1.32 PDF pg - 16

- CC for no. of defects per unit (c-chart)

c-chart applies to the no. of defects per unit. Sample size for c-chart may be a single unit or a group of units or it may be a unit of fixed time, length, area etc.

- Control limits for c-chart.

The 3-σ control limits are based on the Poisson distribution. Since for a poisson dist, mean and variance are equal, if we assume that c is a Poisson variate with parameter, λ, we get.

$$E(c) = \lambda, \quad Var(c) = \lambda$$

Thus, 3-σ control limits are —

$$UCL_c = E(c) + 3\sqrt{Var(c)} = \lambda + 3\sqrt{\lambda}$$

$$LCL_c = E(c) - 3\sqrt{Var(c)} = \lambda - 3\sqrt{\lambda}$$

$$CL_c = E(c) = \lambda$$

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Case 1 - Standards are given/specified.

If λ' is the specified value of λ, then

$$UCL_c = \lambda' + 3\sqrt{\lambda'}$$
$$LCL_c = \lambda' - 3\sqrt{\lambda'}$$
$$CL_c = \lambda'$$

Case 2 - Standards are not given/specified.

If value of λ is not known, then it is ~~specified~~ estimated by the mean no. of defects per unit. Thus, if c_i is the no. of defects observed on the ith $(i = 1, 2, \ldots, k)$ inspected unit, then -

$$\hat{\lambda} = \bar{c} = \sum_{i=1}^{k} c_i/k$$

$\bar{c}$ is an unbiased estimate of λ.

Thus,
$$UCL_c = \bar{c} + 3\sqrt{\bar{c}}$$
$$LCL_c = \bar{c} - 3\sqrt{\bar{c}}$$
$$CL_c = \bar{c}$$

Since, c can't be negative, if LCL comes out to be -ve, it is taken as 0.

The interepretations are similar to those of p-chart.

Read application from book pg. 14.31 PDF page-20

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- Natural Tolerance Limits

A process in statistical control implies that the control charts for both the mean and range show complete homogeneity and in such a case, a measure of variation of the individual products is given by the std-dev. (σ), estimate by $\bar{R}/d_2$ or $\bar{s}/c_2$ ~~and~~ from control data.

If μ and σ are the process average and s.d. respectively, then the limits $\mu \pm 3\sigma$ are called natural tolerance limits.

The ~~witth~~ width '6σ' which is the inherent variability of the process is called natural tolerance.

If μ and σ are not known, then $\hat{\mu} \pm 3\hat{\sigma}$ are the estimates of natural tolerance limits, where,

$$\hat{\mu} = \bar{\bar{X}} \text{ and } \hat{\sigma} = \bar{R}/d_2 .$$