9·1. INTRODUCTION

Vital Statistics is defined as that branch of Biometry which deals with data and the laws of human mortality, morbidity and demography. The term vital statistics refers to the numerical data or the techniques used in the analysis of the data pertaining to *vital events* occuring in the given section of the population. By vital events we mean such events of human life as fertility and mortality (births and deaths), marriage, divorce, separation, adoptions, legitimations, etc. Sir **Arthur Newsholme*** defined Vital Statistics as *"the whole study of mankind as affected by heredity or environment in so far as the results of this study can be arithmetically stated."* According to him *"Vital Statistics forms perhaps the most important branch of Statistics as it deals with mankind in the aggregate. It is the science of numbers applied to the life history of communities and nations."*

The following definition due to **Benjamin** amply explains the utility of vital statistics to planners, various operating agencies, medical sciences, policy makers etc.

"Vital statistics are conventional numerical records of marriages, births, sickness and deaths by which the health and growth of a community may be studied."

9·1·1. Uses of Vital Statistics. Vital statistics are being extensively used in almost all the spheres of human activity. We outline below some of the important applications of vital statistics.

1. *Study of Population Trend.* The study of births (fertility) and deaths (mortality) gives us an idea of the population trend of any region, community or country.

If 'Birth Rate > Death Rate', there is an increasing trend.

If 'Birth Rate < Death Rate', there is a decreasing trend.

The division of the population of different regions (or races) by birth and death rates enables us to form some idea about the population trend of the regions or countries and the general standard of living and virility of the races.

The *Table 9·1* gives the population of the whole world divided into three different regions together with the birth and death rates per thousand per annum and the rate of natural increase per thousand which is defined as the difference between birth rate and death rate.

TABLE 9·1 : WORLD POPULATION – BIRTH AND DEATH RATES

Region	*Countries*	*World Population (approximately)*	*Birth Rate per thousand*	*Death Rate per thousand*	*Rate of Natural Increase per thousand*
1.	Northwest, Central and Southern Europe, North America and Oceania.	20%	22	12	10
2.	Latin America, Japan, Eastern Europe and Asiatic U.S.S.R.	22%	28 to 48	15 to 17	15
3.	Africa, Near East, South Central Asia and Far East	58%	40 to 45	25 to 35	12

* Arthur Newsholme – *The Elements of Vital Statistics*

In underdeveloped countries, the birth rate is fairly high but at the same time it is accompanied by high infant mortality rate showing thereby the lack of medical facilities, poor hygienic conditions, malnutrition and low standard of living.

Various types of fertility rates discussed in § 9·7·1 to § 9·7·4 enable us to form some idea if the population of the region has a tendency to increase, decrease or remain stable over time. For instance, net reproduction rate § 9·8·3 gives the rate of replenishment of the population.

2. *Use in Public Administration.* The study of population movement, *i.e.,* population estimation, population projections and other allied studies together with birth and death statistics according to age and sex distributions provides any administration with fundamental tools which are indispensable for the overall planning and evaluation of economic and social development programmes.

3. Mortality and natality statistics also provide *guide spots* for use by the researchers in medical and pharmaceutical profession.

4. *Use to Operating Agencies.* The facts and figures relating to births, deaths and marriages are of extreme importance to various official agencies for a variety of administrative purposes. Mortality statistics serve as a guide to the health authorities for sanitary improvements, improved medical facilities and public cleanliness. The data on the incidence of diseases, together with the number of deaths by age and nature of diseases are of paramount importance to health authorities in taking appropriate remedial action to prevent or control the spread of the disease. For example, to control the spread of an epidemic, arrangements can be made for inoculation or vaccination through municipal and district local board agencies.

5. The whole of actuarial science, including life insurance is based on the mortality or life tables. The vital records concerning all possible factors contributing to deaths in various ages are indispensable tool in numerous life insurance schemes (For detailed discussion, see life table, § 9·5).

9·1·2. Methods of Obtaining Vital Statistics. The vital statistics data are usually obtained by the following methods.

1. ***Registration Method.*** The most important source of obtaining vital statistics data is the registration method which consists in continuous and permanent recording of vital events pertaining to births, deaths, marriages, migration, etc. These data, in addition to their statistical utility, also have their value as legal documents. Registration of births provide information on place of birth, sex, age and religion of the parents, legitimacy, number of previous issues and their sexes, father's occupation and birth place of parents. Similarly, death registration furnishes information on place of death, sex, age, marital status, number of issues, birth place, occupation and cause of death. Similar information is also obtained with respect to marriages and migrants.

Many countries require compulsory registration of births and deaths under the law. For example, every new birth has to be reported to the authorities along with the information as given above. Similarly, the death of a person is automatically recorded since the disposal of the body requires an appropriate death certificate from the authorities.

Remark. Data on births and deaths can also be obtained from hospital records.

Shortcomings of the Registration Method. In India, in rural areas there is no legislation which makes the registration of vital events (births, deaths, marriages) and reporting of epidemics compulsory and the requisite information is collected by village Patils under the administrative orders of the government. Consequently, a number of births are likely to remain unregistered especially in scattered rural areas. Even in municipal areas where registration is compulsory, the laws in respect of registration vary from State to State. Thus in India, the statistics of births suffer from the error of underestimation as pointed out in *Census of India paper 6, 1954,* published by Registrar General of India. *"The registration of births is non-existent in some parts of the country and incomplete in varying degrees in all parts of the country. Hence statistics of births suffer from errors of under estimation".* Similarly in registration of deaths, the data regarding age at death, cause of the death, etc., are usually unreliable. Due to non-availability of qualified doctors in villages and interior rural areas, quite often the disease and consequently the cause of death remains undiagnosed. Moreover, people have a general tendency to withhold information regarding their diseases particularly in respect of infectious or contagious diseases.

Moreover, in our country there are no proper records about the ages of mothers at the time of marriage, at the birth of first child and of subsequent children. The religious customs do not require the compulsory registration of marriages in Hindus and Muslims. Hence we do not get any reliable data in respect of marriages for whole of the country.

In order to ensure a continuous permanent recording of vital events suitable legislation, uniform all over the country, should be introduced, making the registration of births, deaths, marriages, etc., compulsory. Such legislation should also provide sanctions for the enforcement of the obligation. Separate organisation should be set up to collect this data more completely and systematically.

2. *Census Method.* Almost in all the countries, all over the world, population census is conducted at regular intervals of time, usually ten years. Census consists of complete enumeration of the population of the particular area under study and collecting information from individuals regarding age, sex, marital status, occupation, religion and other economic and social characteristics.

The main drawback of the census method is that it provides vital statistics only for the census years and fails to give any information about the vital events in the intercensal period.

9·2. MEASUREMENT OF POPULATION

In order to determine the population at any time 't' after the census or between two censuses, a number of methods have been devised. Here we discuss a suitable method which makes use of births, deaths and migration statistics. If we assume that

(*i*) the census data gives us the total size of the population of region or community together with age and sex distribution and

(*ii*) the birth, death and migration statistics during different periods are obtained from registers, then the population P_t at any time t is given by the relation :

$$P_t = P_0 + (B - D) + (I - E), \qquad \ldots(9\cdot1)$$

where P_0 is the total population at the last census, B and D give respectively the total number of births and deaths in the intervening period t and I is the total number of immigrants into the region and E is the total number of emigrants from the region during the period t.

The sex-wise population of India for the years, 1951, 1961, 1971, ..., 2001 is given in *Table 9·2* and is exhibited graphically in the *Fig. 9·1.* The age and sex distribution of the population of India for 1991 census is given *Table 9·2A*.

TABLE 9·2. POPULATION OF INDIA

Year	*Male*	*Female*	*Total*
1951	185,528,462	175,559,628	361,088,090
1961	226,293,201	212,941,570	439,234,771
1971	284,049, 276	264,110,376	548,159,652
1981	353,374,460	329,954,637	683,329,097
1991	439,230,458	407,072,230	846,302,688
2001	531,277,078	495,738,169	1,027,015,247

Source : Office of the Registrar General, India

Notes : The population of India for 2001 includes the estimated population of entire Kachchh district, Morvi, Maliya-Miyana and Wankaner talukas of Rajkot district, Jodiya taluka of Jamnagar district of Gujarat State and entire Kinnaur district of Himachal Pradesh where population enumeration of Census of India 2001 could not be conducted due to natural calamities.

TABLE 9·2A: DISTRIBUTION OF POPULATION OF INDIA BY AGE-GROUP AND SEX : 1991 CENSUS

(*in* '000)

Age-Group	*Male*	*Female*	*Total*	*% age*
0—4	52361	50017	102378	12.2
5—9	57419	53876	111295	13.3
10—14	51948	46744	98692	11.8
15—19	42231	36804	79035	9.4
20—24	37514	36959	74473	8.9
25—29	34546	34693	69239	8.2
30—34	29918	28486	58404	7.0
35—39	27558	24841	52399	6.2
40—44	22842	19714	42556	5.1
45—49	18955	17179	36134	4.3
50—54	16905	14209	31114	3.7
55—59	10942	10531	21473	2.6
60—64	11907	10842	22749	2.7
65—69	6493	6365	12858	1.5
70+	10963	10111	21074	2.5
Age not stated	2706	1990	4695	0.6
All ages	435208	403360	838568	100.0

Source : Office of the Registrar General, India

Notes 1. Excludes Jammu and Kashmir where census could not be held owing to disturbed conditions prevailing there at the time of 1991 census.

2. Excludes population of area under unlawful occupation of Pakistan and China.

9·3. RATES AND RATIOS OF VITAL EVENTS

Generally the vital statistics are available in the form of frequencies of the vital events, *e.g.*, n_1 persons die, n_2 persons are born, n_3 persons get married during a given time 't' in any community or region or country. In order that these figures are of some utility to a statistician, from statistical analysis point of view, these are generally transformed by rates or ratios.

Rate of a vital event is defined as the ratio of the total number of occurrences of the event to the total number of persons exposed to the risk of occurrence of that event.

***Remarks* 1. Sex Ratio.** *The sex ratio is defined as the total number of females per thousand males.*

$$\text{Sex Ratio} = \frac{\text{Female Population}}{\text{Male Population}} \times 1000 = \frac{f_P}{m_P} \times 100 \qquad \ldots(9\cdot2)$$

The *Table 9·3* gives the sex ratio of the Indian population for the years 1901 to 2001 for rural, urban and combined India.

TABLE 9·3 : SEX RATIO OF INDIAN POPULATION

Year	Sex - Ratio (No. of females per 1000 males)		Combined
	Rural	Urban	
1901	979	910	972
1911	975	872	964
1921	970	846	955
1931	966	838	950
1941	965	831	945
1951	965	860	946
1961	963	845	941
1971	949	858	930
1981*	951	879	934+
1991 @	938	894	927
2001#	946	901	933

Source : Office of the Registrar General, India Part II B (*i*) 1991

* : Including Assam

@ : The 1991 census was not held in J & K. For working out Sex–Ratio for India and J&K, the population figures for J&K as projected by the Standing Committee on Population Projection (Oct–1989) have been taken.

\+ : The 1981 Census could not be held in Assam. For working out sex-ratio for India & Assam, the population figures for 1981 for Assam worked out by interpolation have been taken.

\# : The total, rural and urban population of India includes the estimated total, rural and urban population of entire Kachchh district, Morvi, Maliya-Miyana and Wankaner taluks of Rajkot district, Hodiya taluka of Jamnagar district of Gujarat state and estimated total and rural population of entire Kinnaur district of Himachal Pradesh where population enumeration of Census of India, 2001 could not be conducted to due natural calamities.

2. Usually the population P_t is not constant and keeps on changing from time to time . An estimate of the population in the middle of the periods $[t_1, t_2]$ is given by the mean of the population at the extremities *i.e.*,

$$P_{(t_1+t_2)/2} = \frac{1}{2}\left[P_{t_1} + P_{t_2}\right] \qquad \ldots(9{\cdot}3)$$

9·4. MEASUREMENT OF MORTALITY

The following are the principal rates used in measuring mortality.

9·4·1. Crude Death Rate (C.D.R.). This is the simplest of all the indices of mortality and is defined as the number of deaths (from all causes) per k persons in the population of any given region or community during a given period. Thus, in particular, the *annual crude death rate* (*C.D.R.*) denoted by m for any given region or community is given by :

$$m = \frac{\text{Annual deaths}}{\text{Annual mean population}} \times k, \qquad \ldots(9{\cdot}4)$$

where k = 1000, usually.

The crude death rate for any period gives the rate at which the population is depleted through deaths over the course of the period.

Merits **1.** It is simple to understand and calculate.

2. *C.D.R.* is perhaps the most widely used of any vital statistics rates. As an index of mortality, it is used in numerous demographic and public health problems.

3. Since the entire population of the region is exposed to the risk of mortality, *C.D.R.* defined in (9·4) is a probability rate giving the probability that a person belonging to the given population will die in the given period.

Demerits. Most serious drawback of crude death rate is that it completely ignores the age and sex distribution of the population. Experience shows that mortality is different in different segments of the population. Children in the early ages of their life, and the older generation are exposed to higher risk of mortality as compared to younger people. Moreover, mortality rate is also different for females irrespective of age groups, than their male counterparts. C.D.R. is not suitable for comparing the mortality in two places or same place in two periods unless

(*i*) the populations of the places being compared have more or less the same age and sex distribution, or

(*ii*) two periods are not too distant, since in a stable large community, age-sex structure of the population shows very little change.

Remark. We can compute the crude death rate for males and females separately. For example, crude death rate for males in the given region during the given period is given by the formula :

$$\text{C.D.R. for males} = \frac{\text{Male deaths}}{\text{Male population}} \times 1{,}000 = \frac{{}^mD}{{}^mP} \times 1{,}000 \qquad \ldots(9{\cdot}5)$$

where ${}^mD$ is the number of male deaths and ${}^mP$ the male population in the given region during the given period. Similarly

$$\text{C.D.R. for females} = \frac{\text{Female deaths}}{\text{Female population}} \times 1{,}000 = \frac{{}^fD}{{}^fP} \times 1{,}000 \qquad \ldots(9{\cdot}5a)$$

C.D.R. usually lies between 8 and 30 per thousand and female C.D.R. is generally less than male C.D.R.

9·4·2. Specific Death Rates (S.D.R.) In order to arrive at a more useful figure than C.D.R., we must take into account the fact that the mortality pattern is different in different segments of the population. Various segments may be age, sex, occupation, religion, community, social status, etc. For example, the people engaged in infant or child welfare work would be interested to know the mortality condition in the age groups below 1 year, 1–4 years, 5–9 years, etc. ; those engaged in maternal health programmes would like to know the number of deaths occurring among women in the reproductive period (usually 15 to 49 years); insurance authorities would be interested in the mortality pattern at different ages of the population.

Death rate computed for a particular specified section of the population is termed as specific death rate (S.D.R.). S.D.R. for given geographical region during a given period is defined as :

$$\text{S.D.R.} = \left[\frac{\text{Total number of deaths in the \textit{specified} section of the population in the given period}}{\text{Total population of the specified section in the same period}}\right] \times k \qquad \ldots(9{\cdot}6)$$

where k = 1,000 usually. Usually S.D.R. is computed specific to (*i*) age and (*ii*) sex.

Age-Specific Death Rate (Age-S.D.R.). To formulate ideas mathematically, let

$${}_nD_x = \text{Number of deaths in the age-group } (x, x+n)$$

i.e., number of deaths among the persons with age x or more but less than $x+n$, in a given region during a given period, t (say).

$${}_nP_x = \text{Total population of the age-group } x \text{ to } (x+n).$$

Then the age-specific death rate for the age-group x to $x+n$, usually denoted by ${}_nm_x$ is given by :

$${}_nm_x = \frac{{}_nD_x}{{}_nP_x} \times 1{,}000 \qquad \ldots(9·7)$$

Taking $n = 1$, we get the annual age- S.D.R. given by :

$$m_x = \frac{D_x}{P_x} \times 1{,}000 \qquad \ldots(9·8)$$

To be more specific, the *age–S.D.R.* for males is given by :

$${}^m_nm_x = \frac{{}^m_nD_x}{{}^m_nP_x} \times 1{,}000 \qquad \ldots(9·9)$$

where ${}^m_nP_x$ is the number of males in the population in the age-group x to $x+n$ and ${}^m_nD_x$ is the number of deaths amongst this population.

Similarly, the *age – S.D.R.* for females is given by the formula :

$${}^f_nm_x = \frac{{}^f_nD_x}{{}^f_nP_x} \times 1{,}000 \qquad \ldots(9·19a)$$

Formulae (9·9) and (9·9*a*) give the death rates specific to both age and sex.

Remarks **1.** Specific death rates reveal more glaring facts about various segments of a population than the CDR. If the death rate is high in a particular age-group, except old age, preventive measures can be taken to improve upon the situation. The SDR's are extremely helpful in planning and research.

2. Notations. Regarding symbols used in Section 9·4·2 and the following Sections, the *lower suffix* denotes the beginning of the particular age-interval and the *lower prefix*, the width of the interval, the *upper prefix*, if any, denotes the sex and *upper suffix*, if any, the particular community or place or period under consideration.

3. The symbol ${}_nP_x$ may be interpreted as the total number of persons between ages x and $x+n-1$ on last birthday (*l. b. d*). In particular P_x would mean the population with age x on *l.b.d.*. Similar interpretation may be given to other symbols.

4. The age-specific mortality rates for whole of India (rural and urban combined) for years 1971 to 1996 are given in *Table 9.4, on page 9·9*. From the table we observe that with the passage of time, there is a steady decline in the mortality rates for almost all the age-groups up to 55—59. This seems to be due to advanced medical technology, better medical facilities and general awareness about health consciousness among the people of India.

Merits **1.** The death rates specific to age and sex overcome the drawback of C.D.R, since they are computed by taking into consideration the age and sex composition of the population. By eliminating the variation in the death rates due to age-sex distribution of the population S.D.R's provide more appropriate measures of the relative mortality situation in the regions.

2. For general analytical purposes, the death rate specific for age and sex is one of the most important and widely applicable type of death rates. It also supplies one of the essential components required for computation of net reproduction rate (*c.f.* § 9·8·3) and construction of life table (*c.f.* § 9·5).

TABLE 9·4 : AGE SPECIFIC MOPTALITY RATES – ALL INDIA (RURAL AND URBAN COMBINED)

Age-Group	*1971**	*1976**	*1981*	*1986*	*1987*	*1988*	*1989*	*1990*	*1991 @*	*1992 @*	*1993 @*	*1994 @*	*1995 @*	*1996 @*
0–4	51.9	51.0	41.2	36.6	35.2	33.3	29.9	26.3	26.5	26.5	23.7	23.9	24.2	23.9
5–9	4.7	4.8	4.0	3.3	3.3	3.2	2.8	2.5	2.7	2.9	2.2	2.1	2.5	2.3
10–14	2.0	2.4	1.7	1.6	1.5	1.5	1.5	1.4	1.4	1.4	1.4	1.2	1.4	1.3
15–19	2.4	2.7	2.4	2.3	2.2	2.2	2.1	2.1	2.1	2.2	1.9	1.8	1.8	1.6
20–24	3.6	3.4	3.1	2.9	2.8	2.8	2.9	2.7	2.8	2.8	2.6	2.5	2.4	2.3
25–29	3.7	3.9	3.2	3.0	2.7	2.6	2.8	2.6	3.1	2.7	2.7	2.7	26.6	2.5
30–34	4.6	4.5	4.0	3.3	3.2	3.0	2.9	3.0	3.1	3.2	2.8	3.2	2.9	2.9
35–39	5.7	4.8	4.4	4.2	3.7	4.0	3.6	3.6	3.9	3.8	3.4	3.5	3.4	3.9
40–44	6.7	7.2	5.8	5.6	5.3	5.4	5.0	5.1	4.8	5.1	4.5	4.8	4.7	4.9
45–49	9.5	9.5	8.5	7.8	7.6	7.8	7.4	7.7	7.4	7.5	6.7	7.2	6.8	6.7
50–54	16.8	16.2	13.2	12.6	11.8	12.2	11.0	11.2	11.3	11.5	11.2	11.0	10.3	10.9
55–59	21.2	23.6	20.6	17.8	17.9	18.4	17.0	17.8	17.6	17.8	17.6	16.0	14.7	15.7
60–64	34.9	40.3	33.0	31.3	30.7	29.7	27.7	25.9	28.5	28.6	27.5	27.0	24.7	26.7
65–69	48.4	51.4	46.4	44.0	42.3	45.0	42.6	42.5	41.6	43.8	40.3	38.1	35.9	38.9
70–74	109.3	99.5	97.4	91.0	89.4	93.8	85.4	85.1	91.4	91.5	87.6	87.7	57.2	61.5
75–79	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	80.9	89.5
80–84	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	119.9	120.4
85+	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	208	182.2
All ages	14.9	15.0	12.5	11.1	10.9	11.0	10.3	9.7	9.8	10.1	9.3	9.3	9.0	9.0

Source : Office of the Registrar Genera, India, Sample Registration System
* : Excludes Bihar and West Bengal
@ : Excludes Jammu & Kashmir
NA : Not available
From 1995 onwards, age group extended.

Demerits **1.** However, S.D.R.'s are not of much utility for overall comparison of mortality conditions prevailing in two different regions, say, A and B. For example, it might happen that for certain age-groups the mortality pattern for region A is greater than that for B but for the others the case may be opposite. Hence it will not be possible to draw general conclusion regarding the overall mortality pattern in region A as compared to the region B. In order to draw some valid conclusions, the different age or/and sex specific death rates must be combined to give a single figure, reflecting the true picture of mortality in the region.

2. Moreover, in addition to age and sex distribution of the population social, occupational and topographical factors come into operation causing what is called *differential mortality*. S.D.R.'s completely ignore these factors. In order to eliminate such spurious effects, *standardised death rates* are computed.

9·4·3. Infant Mortality Rate (I.M.R.). *The infant mortality rate is defined as the chance of dying of a newly born infant within a year of its life, under the given mortality conditions.*

Notations :

D_0^z : Number of deaths (*excluding foetal deaths*) among the children between the age-group 0–1 *i.e.*, the number of deaths among the children of age 0 on last birthday (*l.b.d.*) among the residents of a region during the calendar year z.

B_0^z : Total number of *live births* reported in the same region within the same calendar year z.

The infant mortality rate during the calendar year z, usually denoted by i_m^z, is given by :

$$\text{I.M.R.} = i_m^z = \frac{D_0^z}{B_0^z} \qquad \ldots(9·10)$$

***Remarks.* 1. Age - SDR at age 0 *l.b.d.* Vs. I.M.R.** The I.M.R. and the age - SDR for age 0 *l.b.d.* (m_0), both have the same numerator. However, the denominator in age-SDR at age 0 *l.b.d.* includes all the infants upto the age below 1 year. Usually, the estimates of the population by age are not available annually. If available, it is a well known fact that the total number of infants below 1 year are very much under enumerated. Hence, the age-SDR for age 0 *l.b.d.* (m_0) is highly overstated. Accordingly, I.M.R. is regarded as a more useful measure of infant mortality as compared with m_0.

2. An obvious advantage of I.M.R. is that it does not require the data of population censuses or estimates. The I.M.R. can be obtained for any population, any region (area) and for any time period, provided the data for infant deaths (D_0) and the live births (B_0) for that period are available. However, the same is not true of age -SDR for age. 0 *l.b.d.* because the estimates of the population 0 *l.b.d.* may not be available for small area.

3. Strictly speaking *I.M.R. is not a probability rate*. For instance, for the infants born in any month, say, November of the calendar year ($z - 1$) and dying in the month, say , May of the calendar year z, the death is recorded in the numerator but birth is not recorded in the denominator of (9·10). Similarly , for the infants born in any month, say August of the calendar year z and dying in, say, March of the year (z +1), the death is not recorded in the numerator but the birth is recorded in the denominator of (9·10). Alternatively, we can say that a child born on 1st February of calendar year z is exposed to the risk of death in that year for 11 months, while a child born on 1st November of the same year z is exposed to this risk for only two months.

4. In most of the countries all over the world, it is observed that I.M.R. (the risk of death under 1 year of age) is higher than the mortality rates for any other age groups of the life span, except at the very old ages. Moreover, unlike the old-age deaths, the infant deaths are very responsive to the improvements in the environmental and medical conditions. Thus, the I.M.R. is regarded as a very sensitive index of the health conditions of a community or country and reflects any changes in its health standards.

5. A serious drawback of I.M.R. is the under-registration of live births. Also the definitions of live births and still births vary from country to country and for the same country from time to time. Quite often, the infants who are born alive but die immediately after birth are usually recorded as 'dead'. This results in under-registration of live births and over-registration of infant deaths, leading to over-statement of I.M.R. than its actual value. Thus, by improving the birth registration system, we can lower the I.M.R. without saving a single life.

Table 9·5 gives the infant mortality rates for rural, urban and combined India for some years from 1971 to 1999.

Table 9·5(A) gives the IMR for India for 1998-1999 according to the sex of the child and the educational background of the mother.

Table 9·5(B) gives the IMR for SAARC Countries for the years 1996 and 97 for comparative studies.

From *Table 9·5*, we observe that there is a steady decline in the I.M.R. over the years 19971 to 1999, reflecting upon the improvement in the environmental and medical conditions in India. *Table 9·5(A)* suggests that the education of the mothers plays an important role in reducing the IMR. This is perhaps due to the general awareness about the child care and health, among the educated mothers.

TABLE 9·5 : INFANT MORTALITY RATES BY RURAL AND URBAN AREAS – 1999 (PROVISIONAL – INDIA)

Year	*Rural*	*Urban*	*Combined*
1971	138	82	129
1976	139	80	129
1977	140	81	130
1978	137	74	127
1979	130	72	120
1981	119	62	110
1982	114	65	105
1983	114	66	105
1984	113	66	104
1985	107	59	97
1986	105	62	96
1987	104	61	95
1988	102	62	94
1989	98	58	91
1990	86	50	80
1991*	87	53	80
1992 *	85	53	79
1993*	82	45	74
1994*	80	52	74
1995*	80	48	74
1996	77	46	72
1997	77	45	71
1998$	77	45	72
1999	75	44	70

Source : Office of the Registrar General, India, (Sample Registration system)

* : Excludes Jammu & Kashmir and Mizoram

$: Estimate at the national level excludes Nagaland (Rural) due to part–receipt of returns.

TABLE 9.5(A) : INFANT MORTALITY RATE BY BACKGROUND CHARACTERISTIC 1998–99 – INDIA

Background Characteristic	*Infant Mortality Rate*
Sex of Child	
Male	74.8
Female	71.1
Mother's education	
illiterate	86.5
Lierate < Middle complete	58.5
Middle school complete	48.1
High school complete and above	32.8

Source : National Family Health Survey 1998-99.International Institute for Population Sciences, Mumbai.

9·5 (B) INFANT MORTALITY RATE FOR SAARC COUNTRIES

SAARC Countries	*Infant Mortality Rate (Per 1,000 live births)*	
	1996	*1997*
India	73	71
Bangladesh	83	81
Bhutan	90	87
Maldive	54	53
Nepal	82	75
Pakistan	95	95
Sri Lanka	17	17

Source : Human Development Report, 1998 (UNDP)

9·4·4. Standardised Death Rates. Using (9·4) and (9·8), the crude death rates in terms of age-specific death rates for two regions A and B are given respectively by :

$$m^a = \frac{D^a}{P^a} \times 1{,}000 = \frac{\sum_x m_x^a P_x^a}{\sum_x P_x^a} \qquad \ldots(9\cdot11)$$

and

$$m^b = \frac{D^b}{P^b} \times 1{,}000 = \frac{\sum_x m_x^b P_x^b}{\sum_x P_x^b} \qquad \ldots(9\cdot11a)$$

The expressions in (9·11) and (9·11a) are nothing but the weighted arithmetic means of the age-S.D.R., the weights being the corresponding populations in the age-groups. We observe that even if age-S.D.R.'s are same, *i.e.*,

$$m_x^a = m_x^b \;\forall\, x, \qquad m^a \neq m_b, \text{ since in general, } \quad \frac{P_x^a}{\sum_x P_x^a} \neq \frac{P_x^b}{\sum_x P_x^b}$$

i.e., since the age-distributions of the populations in the two regions A and B are not identical. This drawback is removed if the same set of weights is used in (9·11) and (9·11 a) for computing the weighted average of the age-S.D.R's. This is what is done in standardised death rates ($STDR$) or adjusted death rates, used with a prefix to identify the basis of adjustment as, for example, age-adjusted death rates and so on. We discuss below the two methods of age-adjustments, which are in common use.

Direct Method of standardisation. This method consists in weighting the age specific death rates *not* by the corresponding population of the area to which they refer (as is done in C.D.R.) but by the population distribution of another region chosen as a *standard*. Thus, if P_x^s is the number of persons in the age-group x to $x+1$ in the standard population, then the standardised deate rates for the regions A and B are given respectively by :

$$(STDR)_A = \frac{\sum\limits_x m_x^a P_x^s}{\sum\limits_x P_x^s} \quad \text{and} \quad (STDR)_B = \frac{\sum\limits_x m_x^b P_x^s}{\sum\limits_x P_x^s} \qquad \text{...(9·12)}$$

These-age adjusted death rates for regions A and B respectively are nothing but the crude death rates that would be observed in the standard population if it were subject to the age-S.D.R. of the regions A and B

Remarks **1.** The death rate may similarly be adjusted for other factors also such as sex, rate, etc., and may be interpreted accordingly.

2. *Standardisation of age and sex together.* Let ${}^m m_x^a$ and ${}^f m_x^a$ be the age S.D.R.s at age x (*l.b.d.*) corresponding to male and female population respectively in the region A and let ${}^m P_x^s$ and ${}^f P_x^s$ be the corresponding male and female populations in the standard population. Then the *age-sex adjusted (standaridised) death rate for the region A* is given by the formula :

$$\frac{\sum\limits_x [{}^m m_x^a \times {}^m P_x^s + {}^f m_x^a \times {}^f P_x^s]}{\sum\limits_x ({}^m P_x^s + {}^f P_x^s)} \qquad \text{...(912b)}$$

Similarly we can obtain STDRs adjusted for any other two factors, *e.g.*, age-race, sex-race and so on.

Merits **1.** Standardised death rates are readily comprehensible and easy to calculate.

2. Age-adjusted death rates are comparable since they eliminate the difference caused by the different distributions of the age-specific population for regions A and B . The difference in these death rates gives a true picture of the difference in mortality in the two regions.

Demerits. The main drawback of the method is the choice of a '*standard population*'. The choice of this 'standard' is bound to affect the magnitude of the resulting adjusted rates and may change their relative positions with respect to each other. This bias may, however, be eliminated by taking the standard population as actual population (or life-table stationary population) of a bigger region of which A and B are subsets. For instance, to compare the mortality pattern in the States of Punjab and Rajasthan, the standard chosen may be the population of Northern India or whole of India. However, there is no generally accepted standard population for international comparisons.

Indirect Standardisation. In computing the standardised death rates by formula (9·12) it is necessary to know the number of persons and the age-specific death rates for different age-groups. Quite often we have a population classified by age but the age-S.D.R.'s may not be known. However, the total number of deaths and hence C.D.R. may be known. In such case, we use the indirect method of standarisation which consists in multiplying the crude death rate of the region A, say, by adjustment factor 'C' *measuring the relative 'mortaity' proneness of the population of the region* such that the result is equal to the standaridised death rate. Thus the problem is to find C such that

$$\text{C.D.R} \times C = \text{S.T.D.R.}$$

$$\Rightarrow \quad \frac{\sum\limits_x m_x^a P_x^a}{\sum\limits_x P_x^a} \times C = \frac{\sum\limits_x m_x^a P_x^s}{\sum\limits_x P_x^s} \quad \text{or} \quad C = \left.\frac{\sum\limits_x m_x^a P_x^s}{\sum\limits_x P_x^s} \div \frac{\sum\limits_x m_x^a P_x^a}{\sum\limits_x P_x^a}\right\} \qquad \text{...(9·13)}$$

Since m_x^a are not usually known, an approximate value of C is obtained on replacing m_x^a by m_x^s , the age – S.D.Rs for the standard population, thus giving

$$\hat{C} = \frac{\sum\limits_x m_x^s P_x^s}{\sum\limits_x P_x^s} \div \frac{\sum\limits_x m_x^s P_x^a}{\sum\limits_x P_x^a} = (\text{C.D.R. for standard population}) \div \frac{\sum\limits_x m_x^s P_x^a}{\sum\limits_x P_x^a} \qquad \text{...(9·13a)}$$

Finally, the (*indirect*) *standardised* death rate for region A is given by [From (9·13)]

$$\text{STDR for } A = (\text{C.D.R. for } A) \times \hat{C} \qquad \text{...(9·13b)}$$

Remark. Actually there is no point in comparing the two methods of standardisation. We use indirect standardisation as an approximation to direct standardisation only when the necessary data for the latter is not available. The two methods will be exactly equivalent if the age-S.D.R. for the given population happen to be proportional to the SDRs of the standard population.

Both the methods of direct and indirect standarisation are subject to the obvious criticism that the mortality indices so obtained depend on the age-sex composition of the standard population used and as such the greater gains (losses) in mortality reduction obtained at younger (older) age are not adequately accounted for.

Example 9·1. *Compute the crude and standardised death rates of the two populations A and B, regarding A as standard population, from the data in Table 9·6.*

TABLE 9·6

Age-group (Years)	*A*		*B*	
	Population	*Deaths*	*Population*	*Deaths*
Under 10	*20,000*	*600*	*12,000*	*372*
10 — 20	*12,000*	*240*	*30,000*	*660*
20 — 40	*50,000*	*1250*	*62,000*	*1612*
40 — 60	*30,000*	*1050*	*15,000*	*525*
Above 60	*10,000*	*500*	*3,000*	*180*

Solution.

TABLE 9·7 : COMPUTATION OF C.D.R. AND S.T.D.R.

Age-group (Years)	*Population A*			*Population B*			$m_x^b P_x^a$
	Population P_x^a	*Deaths* D_x^a	*Death rate per 1,000* m_x^a	*Population* P_x^b	*Deaths* D_x^b	*Death rate per 1,000* m_x^b	
Under 10	20,000	600	30	12,000	372	31	6,20,000
10 – 20	12,000	240	20	30,000	660	22	2,64,000
20 – 40	50,000	1250	25	62,000	1612	26	13,00,000
40 – 60	30,000	1050	35	15,000	525	35	10,50,000
Above 60	10,000	500	50	3,000	180	60	6,00,000
Total	1,22,000	3,640		1,22,000	3,349		38,34,000

Crude Death Rates :

$$\text{C.D.R. for population } A = \frac{\sum\limits_x D_x^a}{\sum\limits_x P_x^a} \times 1{,}000 = \frac{3{,}640}{1{,}22{,}000} \times 1{,}000 = 29·8$$

$$\text{C.D.R. for population } B = \frac{\sum\limits_x D_x^b}{\sum\limits_x P_x^b} \times 1{,}000 = \frac{3{,}349}{1{,}22{,}000} \times 1{,}000 = 27·4$$

Standard Death Rates :

Since population A is taken as standard population,

$$STDR \text{ for } A = \text{C.D.R. for } A = 29{\cdot}8$$

$$STDR \text{ for } B = \frac{\sum_x m_x{}^b P_x{}^a}{\sum_x P_x{}^a} = \frac{38{,}34{,}000}{1{,}22{,}000} = 31{\cdot}4$$

We, thus conclude that death rate in population B is greater than in population A.

Example 9·2. *Estimate the standardised death rates for the two countries from the data in Table 9·8.*

TABLE 9·8

Age group (in years)	*Death Rate per 1000*		*Standardised*
	Country A	*Country B*	*Population (in lakhs)*
0 – 4	*20·00*	*5·00*	*100*
5 –14	*1·00*	*0·50*	*200*
15 – 24	*1·40*	*1·00*	*190*
25 – 34	*2·00*	*1·00*	*180*
35 – 44	*3·30*	*2·00*	*120*
45 – 54	*7·00*	*5·00*	*100*
55 – 64	*15·00*	*12·00*	*70*
65 – 74	*40·00*	*35·00*	*30*
75 and above	*120·00*	*110·00*	*10*

Solution.

TABLE 9·8A : CALCULATIONS FOR STANDARDISED DEATH RATES

Age-group (Years)	*Death rate per 1,000*		*Standardised Population*	$m_x{}^a P_x{}^s$	$m_x{}^b P_x{}^s$
	Country A $m_x{}^a$	*Country B* $m_x{}^b$			
0– 14	20·0	5·0	100	2,000·0	500·0
5– 14	1·0	0·5	200	200·0	100·0
15 – 24	1·4	1·0	190	266·0	190·0
25 – 34	3·0	1·0	180	360·0	180·0
35 – 44	3·3	2·0	120	396·0	240·0
45 – 54	7·0	5·0	100	700·0	500·0
55 – 64	15·0	12·0	70	1,050·0	840·0
65 – 74	40·0	35·0	30	1,200·0	1,050·0
75 and above	120·0	110·0	10	1,200·0	1,100·0
Total			1000	7,372	4,700

Standardised Death Rate for Country A $= (STDR)_A = \dfrac{\sum m_x{}^a P_x{}^s}{\sum P_x{}^s} = \dfrac{7372}{1000} = 7{\cdot}372$

Standardised Death Rate for Country B $= (STDR)_B = \dfrac{\sum m_x{}^b P_x{}^s}{\sum P_x{}^s} = \dfrac{4700}{1000} = 4{\cdot}7$

Example 9·3. ***Find the standardised death rate by Direct and Indirect methods for the data given in Table 9·9.***

TABLE 9·9

Age	*Standard Population*		*Population A*	
	Population (in '000)	*Specific Death Rate*	*Population (in' 000)*	*Specific Death Rate*
0 – 5	*8*	*50*	*12*	*48*
5 – 15	*10*	*15*	*13*	*14*
15 – 50	*27*	*10*	*15*	*9*
50 and above	*5*	*60*	*10*	*59*

Solution.

TABLE : 9·9(*a*) : COMPUTATION OF STDR BY DIRECT AND INDIRECT METHODS

Age	*Standard Population*			*Population A*			$m_x{}^a P_x{}^s$	$m_x{}^s P_x{}^a$
	$P_x{}^s$	$m_x{}^s$	$P_x{}^s m_x{}^s$	$P_x{}^a$	$m_x{}^a$	$P_x{}^a m_x{}^a$		
0 – 5	8,000	50	400,000	12,000	48	576,000	384,000	600,000
5 – 15	10,000	15	150,000	13,000	14	182,000	140,000	195,000
15 – 50	27,000	10	270,000	15,000	9	135,000	243,000	150,000
50 and above	5,000	60	300,000	10,000	59	590,000	295,000	600,000
Total	50,000		11,20,000	50,000		14,83,000	10,62,000	15,45,000

Direct Method :

$$(STDR)_A = \frac{\sum m_x{}^a P_x{}^s}{\sum P_x{}^s} = \frac{10{,}62{,}000}{50{,}000} = 21{\cdot}24$$

Indirect Method :

$$(CDR)_A = \frac{\sum m_x{}^a P_x{}^a}{\sum P_x{}^a} = \frac{14{,}83{,}000}{50{,}000} = 29.66$$

Adjustment Factor :

$$\hat{C} = \frac{\sum m_x{}^s P_x{}^s}{\sum P_x{}^s} \times \frac{\sum P_x{}^a}{\sum m_x{}^s P_x{}^a} = \frac{11{,}20{,}000}{50{,}000} \times \frac{50{,}000}{15{,}45{,}000} = 0{\cdot}7249$$

$$(STDR)_A = \hat{C} \times (CDR)_A = 0{\cdot}7249 \times 29{\cdot}66 = 21{\cdot}5005.$$

9·5. MORTALITY TABLE OR LIFE TABLE

The life table gives the life history of a *hypothetical group* or *cohort* as it is gradually diminished by deaths. It is a conventional method of expressing the most fundamental and essential facts about the age distribution of mortality in a tabular form and is a powerful tool for measuring the probability of life and death of various age sectors. A life table provides answers to the following questions :

(*i*) How will a group of infants all born at the same time and experiencing unchanging mortality conditions throughout the life time, gradually die out ?

(*ii*) When in the course of time all these infants die, what would be the average longevity per person ?

(*iii*) What is the probability that persons of specified age will survive a specified number of years ?

(*iv*) How many persons, out of selected number of persons living at some initial age, survive on the average to each attained age ?

The life table thus gives a summary of the mortality experience of any population group during a given period and is a very effective and comprehensive method for providing concise measures of the longevity of that population.

The data for constructing a life table are the census data and death registration data. Life tables are generally constructed for various sections of the people which, as experience shows, have sharply different patterns of mortality. Thus there are life tables constructed for different races, occupational groups and sex. Life tables are as well constructed on regional basis and other factors accounting differential mortality.

We give below a detailed discussion of the various terms and factors that enter in the construction of the *Life Tables.*

Notations and Terminology :

l_x is the number of persons living at any specified age x in any year out of an assumed number of births, say, l_0 usually called the *cohort* or *radix* of the life table.

d_x is the number of persons among the l_x persons (attaining a precise age x) who die before reaching the age $(x+1)$. Obviously, we have

$$d_x = l_x - l_{x+1} = -\Delta l_x \qquad \dots (9{\cdot}14)$$

where Δ is the difference operator.

${}_np_x$ is the probability that a person aged x survives up to age $x+n$. Thus if l_{x+n} is the number of persons living at age $(x+n)$ in any year, then

$${}_np_x = \frac{l_{x+n}}{l_x} \Rightarrow l_{x+n} = l_x\,{}_np_x \qquad \dots (9{\cdot}15)$$

In particular, if we take $n=1$, we have

$$p_x = \frac{l_{x+1}}{l_x} \qquad \dots (9{\cdot}15\,a)$$

which gives the probability that a person aged x will survive till his next birthday.

$q_x = 1 - p_x$, is the complementary probability of survival, *i.e.*, q_x, is the probability that a person of exact age x will die within one year following the attainment of that age. Thus, we have

$$q_x = \frac{d_x}{l_x} \qquad \dots (9{\cdot}16)$$

Note. Also, by definition

$$l_{x+1} = l_x - d_x = l_x(1-q_x) \qquad \text{[From (9·16)]}$$

$$\Rightarrow \qquad l_{x+1} = l_x\,p_x \qquad \dots (9{\cdot}16\,a)$$

L_x is the number of years lived in the aggregate by the cohort of l_0 persons between age x and $(x+1)$ or L_x may be interpreted as the average size of the cohort between ages x and $(x+1)$. Thus, *if deaths are assumed to be uniformly distributed over the whole year* or equivalently, if we assume the linearity of l_{x+t} for $t \in [0, 1]$, then we get

$$L_x = \int_0^1 l_{x+t}\,dt \quad \text{and} \quad l_{x+t} = l_x - t d_x \qquad \dots (*)$$

$$\therefore \qquad L_x = \int_0^1 (l_x - t\,d_x)\,dt$$

$$= l_x \mid t \mid_0^1 - d_x \mid t^2/2 \mid_0^1$$

$$= l_x - \frac{1}{2}d_x = l_x - \frac{1}{2}(l_x - l_{x+1}) \qquad \dots (9{\cdot}17)$$

$$= \frac{1}{2}(l_x + l_{x+1}) \qquad \dots (9{\cdot}17a)$$

From (9·17), on using (*), we also get

$$L_x = l_{x+(1/2)} \qquad \dots (9{\cdot}17\,b)$$

T_x is the total number of years lived by the cohort l_0 after attaining the age x, *i.e.*, T_x is the total future life time of the l_x persons who reach age x. Thus, we have

$$T_x = L_x + L_{x+1} + L_{x+2} + \dots \qquad \dots (9{\cdot}18)$$

Thus if w is the highest age at which any survivors are recorded in the mortality table, *i.e.*, if $l_w = 0$, then

$$T_x = \sum_{t=0}^{w-x-1} L_{x+t} \qquad \dots (9{\cdot}18\,a)$$

We also observe from (9·18) that

$$T_x = L_x + T_{x+1} \qquad \dots (9{\cdot}18b)$$

The following *theorems* establish the *relationship* between the various quantities defined above.

Theorem 1. $\quad {}_np_x = p_x \cdot p_{x+1} \cdots p_{x+n-1} \qquad \dots (9{\cdot}19)$

Proof. We have by *def.*,

$${}_np_x = \frac{l_{x+n}}{l_x}$$

$$= \frac{l_{x+1}}{l_x} \cdot \frac{l_{x+2}}{l_{x+1}} \cdots \frac{l_{x+n}}{l_{x+n-1}}$$

$$= p_x \cdot p_{x+1} \cdots p_{x+n-1}$$

Theorem 2. $\quad {}_nq_x = \dfrac{d_{x+n-1}}{l_x} \qquad \dots (9{\cdot}20)$

Proof. ${}_nq_x$ = Probability that a person aged x dies in the nth year after attaining that age.

= Prob. that person aged x survives till age $(x+n-1)$ but dies in age period $(x+n-1, x+n)$.

= P [A person aged x survives for $(n-1)$ years]

$\times P$ [A person aged $x+n-1$ dies within one year]

(By compound probability theorem)

$$= \frac{l_{x+n-1}}{l_x} \times \frac{d_{x+n-1}}{l_{x+n-1}} = \frac{d_{x+n-1}}{l_x}$$

Cor. We have

$${}_np_x - {}_{n+1}p_x = \frac{l_{x+n}}{l_x} - \frac{l_{x+n+1}}{l_x}$$

$$= \frac{l_{x+n} - l_{x+n+1}}{l_x} = \frac{d_{x+n}}{l_x}$$

$$= {}_{n+1}q_x \qquad \text{[From 9·20]}$$

Theorem 3. *If* ω *is the last age at which* l_x *vanishes, i.e., if* $l_\omega = 0$, *then*

$$l_x = \sum_{i=x}^{\omega-1} d_i \qquad \ldots(9{\cdot}21)$$

Proof.

$$\sum_{i=x}^{\omega-1} d_i = d_x + d_{x+1} + \ldots + d_{\omega-1}$$

$$= (l_x - l_{x+1}) + (l_{x+1} - l_{x+2}) + \ldots + (l_{\omega-2} - l_{\omega-1}) + (l_{\omega-1} - l_\omega)$$

$$= l_x \qquad (\because \; l_\omega = 0)$$

Theorem 4.

$$T_x = \frac{1}{2} l_x + l_{x+1} + l_{x+2} + \ldots \qquad \ldots(9{\cdot}22)$$

Proof. By def.,

$$T_x = \sum_{t=0}^{\infty} L_{x+t} = \sum_{t=0}^{\infty} \left[\frac{1}{2}(l_{x+t} + l_{x+t+1})\right] \qquad \text{[From (9·17 a)]}$$

$$= \frac{1}{2} l_x + \sum_{t=1}^{\infty} l_{x+t}$$

Expectation of Life. The *Curate Expectation* of life, usually denoted by e_x gives the average number of complete years of life lived by the cohort l_0 after age x by each of l_x persons attaining that age. The *complete Expectation of life,* usually denoted by e_x^0 measures the average number of years a person of given age x can be expected to live under the prevailing mortality conditions. It gives the number of years of life entirely completed and includes the fraction of the year survived in the year in which death occurs, which on the average can be taken to be (1/2) year. Thus we have

$$e_x^0 = e_x + \frac{1}{2} \qquad \ldots(9{\cdot}23)$$

Since total number of years lived by l_x persons of age x is given by :

$$T_x = \int_0^\infty l_{x+t}\, dt,$$

the complete expectation of life of a person attaining age x is obtained from the following relation :

$$e_x^0 = \frac{T_x}{l_x} \qquad \ldots(9{\cdot}23a)$$

e_0^0, the expectation of life at age 0, is the average age at death or the average longevity of a person belonging to a given community.

Theorem 5.

$$e_x = \left(\sum_{n=1}^{\infty} l_{x+n}\right) \div l_x \qquad \ldots(9{\cdot}24)$$

Proof. In the usual notations, l_x is the number of persons of age x and d_x is the number of persons who die before attaining age $(x+1)$, *i.e.*, d_x is number of persons dying in the first year without completing one year of life at age x.

$\therefore$ Total number of years lived by d_x individuals $= 0 \times d_x = 0$

Similarly d_{x+1} is the number of individuals who die between the age period $(x+1, x+2)$, *i.e.*, the number of persons who die in the second year after completing 1 year at age x.

$\therefore$ Total number of years lived by d_{x+1} persons $= 1 \times d_{x+1} = d_{x+1}$

In general d_{x+i} is the number of persons dying in the age period $(x+i, x+i+1)$, *i.e.*, dying in the $(i+1)$ th year after completing i years at age x. Thus, the total number of years lived by d_{x+i} individuals is given by :

$$i \times d_{x+i}, \; (i = 0, 1, 2, \ldots)$$

Thus

$$e_x = \text{Average number of years lived by persons of the given age } x$$

$$= \left(\sum_{i=0}^{\infty} i\, d_{x+i} / l_x\right)$$

$$= (d_{x+1} + 2\, d_{x+2} + 3 d_{x+3} + \ldots)/l_x$$

$$= \frac{1}{l_x}\left[(l_{x+1} - l_{x+2}) + 2(l_{x+2} - l_{x+2}) + 3\,(l_{x+2} - l_{x+4}) + \ldots\right]$$

$$= \frac{1}{l_x}\left[l_{x+1} + l_{x+2} + l_{x+3} + \ldots\right]$$

$$= \left(\sum_{n=1}^{\infty} l_{x+n}\right) / \, l_x$$

Cor 1. We have

$$e_x^0 = e_x + \frac{1}{2} = \left[\frac{l_{x+1} + l_{x+2} + l_{x+3} + \ldots}{l_x}\right] + \frac{1}{2}$$

$$= \left(\frac{1}{2} l_x + l_{x+1} + l_{x+2} + \ldots\right) / l_x$$

$$\Rightarrow \qquad e_x^0 = \frac{T_x}{l_x}, \qquad \text{(From 9·22)} \qquad \ldots(9{\cdot}25)$$

a result, which is already given in (9·23*a*).

Cor 2. From (9·24), we have

$$l_x e_x = l_{x+1} + l_{x+2} + l_{x+3} + \ldots \quad \text{and} \quad l_{x+1} e_{x+1} = l_{x+2} + l_{x+3} + l_{x+4} + \ldots$$

Subtracting, we get $l_x e_x - l_{x+1} e_{x+1} = l_{x+1} \;\Rightarrow\; l_x e_x = l_{x+1}(1 + e_{x+1})$

$$\therefore \quad \frac{l_{x+1}}{l_x} = \frac{e_x}{1 + e_{x+1}} \;\Rightarrow\; p_x = \frac{e_x}{1 + e_{x+1}} \qquad \ldots(9{\cdot}26)$$

Also,

$$q_x = 1 - p_x = \frac{1 - (e_x - e_{x+1})}{1 + e_{x+1}} \qquad \ldots(9{\cdot}26\,a)$$

We give in *Tables 9·8, 9·8 (A) and 9·8 (B)*, the expectation of life at birth for India, some selected developed countries and SAARC countries, for comparative study.

TABLE 9.10 : EXPECTATION OF LIFE AT BIRTH BY SEX - INDIA

Year	Male	Female	Person
1901–11	22.6	23.3	22.9
1911–21	19.4	20.9	20.1
1921–31	26.9	26.6	26.8
1931–41	32.1	31.4	31.8
1941–51	32.4	31.7	32.1
1951–61	41.9	40.6	41.3
1961–71	46.4	44.7	45.6
1971–75+	50.5	49.0	49.7
1976–80+	52.5	52.1	52.3
1981–85+	55.4	55.7	55.4
1986–90+	57.7	58.1	57.7
1987–91*	58.1	58.6	58.3
1988–92*	58.6	59.0	58.7
1989–93	59.0	59.7	59.4
1990–94	59.4	60.4	60.0
1991–95	59.7	60.9	60.3
1992–96#	60.1	61.4	60.7
1993–97#	60.4	61.8	61.1

Source: Office of the Registrar General, India

1 Census Actuarial Reports

2 Sample Registration System based abridged life tables 1986-90 (Occasional paper no. 1 of 1994)

\+ Based on Sample Registration System 1971 to 1992

\# Unpublished (under printing)

* Excludes Jammu & Kashmir

Note. Figures for 1901–11 to 1961–71 are based on Census Actuarial Reports and for 1970–75 onwards on the basis of estimates from Sample Registration System

TABLE 9.10(A) : EXPECTATION OF LIFE AT BIRTH OF SELECTED DEVELOPED AND SAARC COUNTRIES FOR 1998

Name of the Country	*Life expectancy at birth (years)*	
	Female	*Male*
Developed Countries		
Canada	81.9	76.2
USA	80.2	73.5
Japan	83.0	76.9
U.K.	80.0	74.7
Australia	81.2	75.6
Germany	80.3	74.1
Russian Federation	72.9	60.7
SAARC Countries		
India	63.3	62.5
Bangladesh	58.7	58.6
Bhutan	62.5	60.0
Maldives	63.8	66.1
Nepal	57.6	58.1
Pakistan	65.6	63.3
Sri Lanka	75.6	71.1
World	69.1	64.9

Source : Human Development Report, 2000 (UNDP)

TABLE 9.10(B): EXPECTATION OF LIFE AT BIRTH – SAARC COUNTRIES

Countries	*Life expectancy at birth years*		
	1960	*1995*	*1997*
India	44.0	61.6	62.6
Bangladesh	39.6	56.9	58.1
Bhutan	37.3	52.0	60.7
Maldive	43.6	63.3	64.5
Nepal	38.3	55.9	57.3
Pakistan	43.6	62.8	64.0
Sri Lanka	62.0	72.5	73.1
World	50.2	63.6	66.7

Source : Human Development Report, 1998 (UNDP)

9·5·1. Stationary Population.

A population is said to be *stationary* if it is of constant size, and constant age and sex composition over time.

Such a population may be conceived of under the following conditions.

(*i*) if every year, the number of births is exactly l_0 (say) and is equal to the number of deaths and these are distributed uniformaly throughout the year, and

(*ii*) if the population is not affected by emigration or immigration.

Under the above conditions, in the long run, the population will be of the same size from year to year and will have the same age-distribution so that the number of persons between the ages x and $(x+1)$ denoted by L_x will always be the same. Thus the columns L_x and T_x of the life table may be interpreted as giving respectively the age-distribution and the number of persons with age x or more in a stationary population.

9·5·2. Stable Population. Concept of a *stable population* is due to **A.J. Lotka** and is very much akin to that of stationary population.

A population is said to be *stable*

(*i*) if it has a fixed age and sex distribution,

(*ii*) if constant mortality and fertility rates are experienced at each age, and

(*iii*) if the population is closed to emigration or immigration.

In other words, *for a stable population* the overall rates of births and deaths remain constant and consequently such a population increases at a constant rate, thus supporting the *Malthus Law* (*Compound Interest Law*) of population growth.

In a stable population , mortality and fertility rates are constant but need not be equal. In particular, in a stable population, if the constant overall birth and death rates are equal, then the population size remains fixed and in this case stable population becomes a stationary population.

Remark. *A stationary population is always stable but a stable population need not be stationary.* In a stationary population, the rate of overall change in the population is zero whereas in a stable population, the rate of overall change in the population is constant (not necessarily zero).

Lotka and Dublin's Model for Stable Population

Assumptions. Lotka and Dublin's stable population analysis is based on the following assumptions :

(*i*) The fertility (birth) rates are independent of time (t)

(*ii*) The mortality (death) rates are independent of time (t)

(*iii*) The age distribution between the ages x to $(x + \delta x)$ is independent of t.

(*iv*) The population is closed to migration.

(*v*) The analysis is done with respect to female population (cohorts) only.

Notations :

$P(t)$: Size of the population at any time t

$C(x,t)\,\delta x$: The proportion of population in age interval $(x, x+\delta x)$ at time t.

$B(t)$: Total number of births at time t

$p(x)$: Probability that a female child (born alive) will survive upto age x under the given mortality conditions.

$i(x)\,\delta x$: Probability that a woman aged x will give birth to a female child in the age interval $(x, x+\delta x)$ under the given fertility conditions, (independent of time).

Then

$P(t)\,C(x,t)\,\delta x =$ Population in the age group x to $(x+\delta x)$ at time t ...(*)

and $B(t-x)\,p(x)\,\delta x =$ A group of persons born $(t-x)$ years ago will survive upto age x or age interval $(x, x+\delta x)$ at time t

$=$ Number of persons or population in the age group $(x, x+\delta x)$ at time t. ... (**)

$$= \frac{\sum_{j=0}^{\infty} \frac{(-r)^j}{j!} R_{j+1}}{\sum_{j=0}^{\infty} \frac{(-r)^j}{j!} R_j} = \frac{R_1 - r R_2 + \frac{r^2}{2!} R_3 - \frac{r^3}{3!} R_4 + \dots}{R_0 - r R_1 + \frac{r^2}{2!} R_2 - \frac{r^3}{3!} R_3 + \dots},$$

where $$R_j = \int_0^{\infty} x^j \phi(x)\, dx \qquad \dots(9.41)$$

$$\therefore \quad A(r) = \frac{R_1 \left[1 - r\frac{R_2}{R_1} + \frac{r^2}{2!} \cdot \frac{R_3}{R_1} - \frac{r^3}{3!} \cdot \frac{R_4}{R_1} + \dots \right]}{R_0 \left[1 - r\frac{R_1}{R_0} + \frac{r^2}{2!} \cdot \frac{R_2}{R_0} - \frac{r^3}{3!} \cdot \frac{R_3}{R_0} + \dots \right]}$$

$$= \frac{R_1}{R_0} \left[1 + \left(\frac{R_1}{R_0} - \frac{R_2}{R_1} \right) r + \frac{1}{2} \left\{ \frac{R_3}{R_1} - \frac{3R_2}{R_0} + 2 \left(\frac{R_1}{R_0} \right)^2 \right\} r^2 + \dots \right]$$

(By long division)

$$= \frac{R_1}{R_0} + \left[\left(\frac{R_1}{R_0} \right)^2 - \frac{R_2}{R_0} \right] r + \frac{1}{2} \left[\frac{R_3}{R_0} - \frac{3R_1R_2}{R_0} + 2 \left(\frac{R_1}{R_0} \right)^3 \right] r^2 + \dots$$

$$= \alpha + \beta r + \gamma r^2 + \dots, \qquad \dots(9.42)$$

where

$$\left.\begin{aligned} \alpha &= \frac{R_1}{R_0}\ ; \quad \beta = \left(\frac{R_1}{R_0} \right)^2 - \frac{R_2}{R_0} \\ \gamma &= \frac{1}{2} \left[\frac{R_3}{R_0} - \frac{3R_1R_2}{R_0} + 2 \left(\frac{R_1}{R_0} \right)^3 \right] \end{aligned}\right\} \qquad \dots(9.43)$$

Neglecting terms involving r^2 and higher powers of r, we have

$$A(r) \simeq \alpha + \beta r \;\Rightarrow\; \int A(r)\, dr \simeq \alpha r + \frac{\beta r^2}{2} \quad i.e., \quad \log_e R_0 \simeq \alpha r + \frac{\beta r^2}{2} \qquad \text{[From (9·40)]}$$

$\therefore \quad \beta r^2 + 2\alpha r - 2\log_e R_0 = 0$, which is quadratic equation in r.

$$\Rightarrow \quad r = \frac{-2\alpha \pm \sqrt{4\alpha^2 + 8\beta \log_e R_0}}{2\beta} = \frac{-\alpha \pm \sqrt{\alpha^2 + 2\beta \log_e R_0}}{\beta} \qquad \dots(9.44)$$

Substituting for α and β from (9·43) in (9·44), we have

$$r = \frac{-\frac{R_1}{R_0} \pm \sqrt{\left(\frac{R_1}{R_0} \right)^2 + 2 \left\{ \left(\frac{R_1}{R_0} \right)^2 - \frac{R_2}{R_0} \right\} \log_e R_0}}{\left(\frac{R_1}{R_0} \right)^2 - \frac{R_2}{R_0}} \qquad \dots(9.45)$$

where R_0, R_1 and R_2 are estimated as :

$$\left.\begin{aligned} \hat{R}_0 &= NRR = \sum_x p(x)\, i(x) = \sum_x \phi(x) \\ \hat{R}_1 &= \frac{\sum_x x\, p(x)\, i(x)}{\sum_x p(x)\, i(x)} = \frac{\sum x\, \phi(x)}{\sum \phi(x)} = \text{Mean age of child bearing period} \\ \hat{R}_2 &= \frac{\sum x^2\, p(x)\, i(x)}{\sum p(x)\, i(x)} = \frac{\sum x\, \phi(x)}{\sum \phi(x)} \end{aligned}\right\} \dots(9.46)$$

Substituting these estimated values of R_0, R_1 and R_2 in (9·45), we get two real values of r, one of which will be positive and the other will be negative. The value of r obtained in (9·45) gives the growth (rate) parameter of the stable population.

Remark $\qquad \phi(x) = p(x)\, i(x), \qquad \dots(*)$

is called *'Net Maternity Function'* or child bearing period and the probability distribution of net maternity function or child bearing period (X) is given by :

$$\psi(x) = \frac{p(x)\, i(x)}{\int_0^{\infty} p(x)\, i(x)\, dx} = \frac{\phi(x)}{\int_0^{\infty} \phi(x)\, dx} \qquad \dots(9.47)$$

We have $$R_k = \int_0^{\infty} x^k\, p(x)\, i(x)\, dx = \int_0^{\infty} x^k\, \phi(x)\, dx \qquad \dots(9.46a)$$

The mean age of child bearing period is given by :

$$E(X) = \int_0^{\infty} x\, \psi(x)\, dx = \frac{\int_0^{\infty} x\, \phi(x)\, dx}{\int_0^{\infty} \phi(x)\, dx} = \frac{R_1}{R_0} = \alpha \qquad \dots(9.48)$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2 = \int_0^{\infty} x^2\, \psi(x)\, dx - \left(\frac{R_1}{R_0} \right)^2$$

$$= \frac{\int_0^{\infty} x^2\, \phi(x)\, dx}{\int_0^{\infty} \phi(x)\, dx} - \left(\frac{R_1}{R_0} \right)^2 = \frac{R_2}{R_0} - \left(\frac{R_1}{R_0} \right)^2 \qquad \dots(9.48\,a)$$

Hence, the mean and variance of the child bearing period are given by : [on using (9·43)]

$$\text{Mean} = E(X) = \frac{R_1}{R_0} = \alpha \quad \text{and} \quad \text{Var}(X) = \frac{R_2}{R_0} - \left(\frac{R_1}{R_0} \right)^2 = -\beta \qquad \dots(9.49)$$

9·5·3. Central Mortality Rate. *The central mortality or death rate* is the probability that a person whose exact age is not known but lies in between x and $(x + 1)$ will die within one year following the attainment of that age. It is denoted by m_x and is given by the expression :

$$m_x = \frac{\text{Number of deaths within age-interval } x \text{ to } (x+1)}{\text{Average } l_x \text{ of the cohort in that interval}}$$

$$\Rightarrow \quad m_x = \frac{d_x}{L_x} \qquad \dots(9.50)$$

$$= \frac{d_x}{l_x - \frac{1}{2} d_x} = \frac{2\frac{d_x}{l_x}}{2 - \frac{d_x}{l_x}} \qquad \text{[On using (9·17)]}$$

$$\therefore \quad m_x = \frac{2q_x}{2 - q_x} \qquad \dots(9.50a)$$

Solving (9·50 a) for q_x, we get

$$q_x = \frac{2m_x}{2 + m_x} \qquad \dots(9.51)$$

Remark. The results in (9·50 a) and (9·51) have been obtained under the assumption that deaths are uniformly distributed over the age interval $(x, x+1)$ [*c.f.* assumptions in obtaining (9·17)]. This assumption is not satisfied for early ages especially for $x = 0$, since the mortality rate is generally very high in the first few weeks after birth and then declines sharply. Moreover, due to the defects in the census records, *viz.*, registration of births and deaths, the values of m_x computed from (9·50) are not much reliable for the early years of life. Consequently there is a need for more accurate formula than formula (9·51) for q_x for $x = 0, 1, 2$ say. Kuczynski *R.R.* has obtained formula for q_x under the assumption that the population is closed to migration, an assumption which is quite valid for $x = 0$. Some other accurate formulae for q_x for early ages have been given in the book *'Construction of Mortality and other Tables'* by Anderson J.L. and Dow. J.B.

9·5·4. Force of Mortality. So far we have confined ourselves to the values of l_x for integral values of x. But since deaths occur at all ages and at every fraction of time of the year, l_x is a continuous function of x. At any age x, the rate of decrease in l_x is given by the expression :

$$\lim_{t \to 0} \frac{l_x - l_{x+t}}{t} = -\lim_{t \to 0} \frac{l_{x+t} - l_x}{t} = -\frac{dl_x}{dx}$$

where $\dfrac{dl_x}{dx}$ is the differential coefficient of l_x *w.r.t.* x.

The *force of mortality* at age x is *defined as* the ratio of *instantaneous rate of decrease in l_x to the value of l_x*. It is denoted by μ_x and is given by the expression :

$$\mu_x = -\frac{1}{l_x} \cdot \frac{dl_x}{dx} = -\frac{d}{dx}(\log l_x) \qquad \ldots(9·52)$$

It gives *'nominal annual rate of mortality'*, *i.e.*, the probability of a person of age x exactly dying within the year if the risk of dying is same at every moment of the year as it is during the moment following the attainment of age x.

Theorem. $\mu_{x+\frac{1}{2}} = m_x$ ...(9·53)

Proof. By *def.*, we have

$$L_x = \int_0^1 l_{x+t}\, dt$$

$$\therefore \quad \frac{d}{dx} L_x = \frac{d}{dx}\int_0^1 l_{x+t}\, dt = \int_0^1 \frac{d}{dx}(l_{x+t})\, dt.$$

(assuming the validity of differentiation under the integral sign)

$$= \int_0^1 \frac{d}{dt}(l_{x+t})\, dt,$$

this being possible, since l_{x+t} is continuous both in x and t.

$$\therefore \quad \frac{d}{dx} L_x = \Big| l_{x+t} \Big|_{t=0}^{t=1} = l_{x+1} - l_x = -d_x$$

$$\Rightarrow \quad \frac{d_x}{L_x} = -\frac{1}{L_x} \cdot \frac{d}{dx} L_x$$

$$\Rightarrow \quad m_x = -\frac{1}{l_{x+(1/2)}} \frac{d}{dx}\left[l_{x+(1/2)}\right] \qquad \text{[On using (9·50) and (9·17}b\text{)]}$$

$$= \mu_{x+(1/2)} \qquad \text{[From (9·52)]}$$

***Remarks* 1.** This result is obtained under the assumption that deaths are uniformly distributed over the interval x to $x+1$.

2. Since l_x is monotonically decreasing function of x, $\dfrac{dl_x}{dx} \le 0 \quad \Rightarrow \quad \mu_x \ge 0$

Thus μ_x is an index of relative rate of growth, the growth being $-\dfrac{dl_x}{dx}$ relative to l_x.

3. *Approximate Expression for μ_x.* Explicit expression for μ_x can be obtained only if the mathematical form of l_x is known. Usually form of the function l_x is not known and we endeavour to obtain approximate expression for μ_x.

Assuming that l_x is capable of being expanded as a Taylor's series, we get

$$l_{x+h} = l_x + hl_x' + \frac{h^2}{2!}l_x'' + \frac{h^3}{3!}l_x''' + \ldots \quad \text{and} \quad l_{x-h} = l_x - hl_x' + \frac{h^2}{2!}l_x'' - \frac{h^3}{3!}l_x''' + \ldots$$

where l_x^r is the r th differential coefficient of l_x *w.r.t.* x.

$$\therefore \quad l_{x+h} - l_{x-h} = 2hl_x' + \frac{h^3}{3}l_x''' + \frac{h^5}{60}\, l_x^v + \ldots \qquad \ldots(*)$$

Assuming that l_x''' and higher order differential coefficients are negligible, on putting $h = 1$ in (*), we get

$$l_{x+1} - l_{x-1} = 2l_x'$$

$$\Rightarrow \quad \mu_x = -\frac{l_x'}{l_x} = \frac{l_{x-1} - l_{x+1}}{2l_x} = \frac{(l_{x-1} - l_x) + (l_x - l_{x+1})}{2l_x} \qquad \ldots(9·54)$$

$$\therefore \quad \mu_x = \frac{d_{x-1} + d_x}{2\, l_x}, \; x \ge 1 \qquad \ldots(9·54\ a)$$

A better approximation to μ_x is obtained on retaining terms up to fourth order differential coefficient of l_x and neglecting higher order differential coefficients. Thus on putting $h = 1$ and $h = 2$ respectively in (*), we get correct to l_x^{iv}.

$$l_{x+1} - l_{x-1} = 2l_x' + \frac{1}{3}l_x''' \quad \text{and} \quad l_{x+2} - l_{x-2} = 4l_x' + \frac{8}{3}l_x'''$$

~~Eliminating l_x''' between these equations, we get~~

$$8(l_{x+1} - l_{x-1}) - (l_{x+2} - l_{x-2}) = 12\, l_x'$$

$$\therefore \quad \mu_x = -\frac{l_x'}{l_x} = \frac{8(l_{x-1} - l_{x+1}) - (l_{x-2} - l_{x+2})}{12\, l_x} \qquad \ldots(9·55)$$

4. *Estimation of μ_x from Mortality Table.* Without proof, we state the following formula .

$$p_x = \text{Probability of survival in the age group } x \text{ to } (x+1)$$

$$= \exp\left[-\int_0^1 \mu_{x+t}\, dt\right]$$

$$\Rightarrow \quad \int_0^1 \mu_{x+t}\, dt = -\log_e p(x) \qquad \ldots(9·56)$$

The integral on the left hand side represents mean value of μ_x in the interval $(x, x+1)$. If we assume that the mean value of μ_x in $(x, x+1)$ is $\mu_{x+(1/2)}$, then as an approximation from (9·56), we get

$$\mu_{x+(1/2)} \doteq -\log_e(p_x) \qquad \ldots(9·57)$$

Alternatively, let us consider

${}_2p_{x-1}$ = Probability of survival in the age group $(x-1)$ to $(x-1+2)$ *i.e.*, $(x-1, (x+1)$

$$\Rightarrow \quad {}_2p_{x-1} = \exp\left[-\int_{-1}^1 \mu_{x+t}\, dt\right] \qquad \text{[Using (9·56)]}$$

$$\Rightarrow \quad -\int_{-1}^1 \mu_{x+t}\, dt = -\log_e({}_2p_{x-1}) = -\log(p_{x-1}\, p_x) \qquad [\because\ {}_np_x = p_x \cdot p_{x+1} \cdots p_{x+n-1}]$$

Approximating the integral on the left side by $2\mu_x$, we get

$$\mu_x = -\tfrac{1}{2}\log_e(p_{x-1} \cdot p_x) = -\tfrac{1}{2}\left[\log_e(p_{x-1}) + \log_e(p_x)\right] \qquad \ldots(9·58)$$

Hence, we may use any one of the expressions (9·56) or (9·58) to estimate the value of μ_x in the life table.

9·5·5. Assumptions, Description and Construction of Life Tables.

Assumptions. The following are a few simplified assumptions which are used in the construction of the life tables.

(*i*) The cohort is closed for emigration or immigration. In other words, ***there is no change in the census except the losses due to deaths.***

(*ii*) Individuals die at each age according to pre-determined schedule which is fixed and does not change.

(*iii*) The cohort originates from some standard number of births, say 10,000 or 1,00,000 which is called the *radix* of the table.

(*iv*) The deaths are distributed uniformly over the period $(x, x+1)$ for each x (except for first few years). In other words, deaths are uniformly distributed between one birthday and the next.

Description of a Life Table. A typical life table has generally the following columns :

TABLE 9·11 : COLUMNS OF A LIFE TABLE

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
x	l_x	d_x	q_x	p_x	m_x	μ_x	L_x	T_x	e_x^0	e_x

The various symbols entering in this table have already been defined in the preceding sections. Here we shall very briefly outline the various steps required for completing the table, starting with the second column. Of course, these relations have been obtained under the assumptions given above. For $x = 0, 1, 2, \ldots$, we have

1. $d_x = l_x - l_{x+1}$ 2. $q_x = \dfrac{d_x}{l_x}$ 3. $p_x = 1 - q_x = \dfrac{l_{x+1}}{l_x}$

4. $m_x = \dfrac{2q_x}{2 - q_x}$ 5. $\mu_{x+(1/2)} = m_x$ 6. $L_x = l_x - \frac{1}{2}d_x = \frac{1}{2}(l_x + l_{x+1})$

7. $T_x = L_x + L_{x+1} + L_{x+2} + \ldots \Rightarrow T_{x+1} = T_x - L_x$ 8. $e_x^0 = \dfrac{T_x}{l_x}$ 9. $e_x = e_x^0 - \dfrac{1}{2}$

Remark. The columns (5), (6) and (7) do not occur in all the life tables. However, the remaining columns are a must in any life table.

Construction of Life Table. It will be seen, as discussed below, that the complete life table can be constructed if we can compute the quantities q_x (or p_x) for all $x > 0$. The only other data which is needed is the *radix* l_0. The q_x column is thus called the *pivotal column* of the life table. Starting with radix l_0 and q_x, $(x = 0, 1, 2, \ldots)$, we have

$$d_0 = l_0 q_0 \Rightarrow l_1 = l_0 - d_0 \quad ; \quad d_1 = l_1 q_1 \quad \text{or} \quad l_2 = l_1 - d_1$$

and so on. Form these values of l_x, $(x = 0, 1, 2, \ldots)$ the columns L_x, T_x and e_x^0 of the table can now be completed by using the relations : $L_x = \frac{1}{2}(l_x + l_{x+1})$, $T_x = \sum_{i=x}^{\infty} L_i$, $e_x^0 = (T_x / l_x)$.

Remarks **1.** We have seen above that the complete life table can be constructed if we know the values of l_0 and q_x $(x = 0, 1, 2, \ldots)$. The values of q_x are obtained from (9·54), where the corresponding values $m_x = d_x / L_x$ are computed on the basis of census records and death registration data. It should be borne in mind that the construction of the life tables from the death registers as outlined above will yield reliable results only if the population has been stationary over a period at least equal to the age of the oldest survivor.

2. For reference, we reproduce in Tables 9·10 and 9·11, the All India Life Tables of Males and Females separately for the decade 1961— 1970, as prepared by Registrar-General of India. It may be observed that in these tables $L_x \neq \frac{1}{2}(l_x + l_{x+1})$ for initial values of x, since the assumption that deaths are uniformly distributed over the years is not satisfied for $x = 0, 1, 2, 3$. The values in the tables have been obtained by a more complicated formula.

TABLE 9·12 : LIFE TABLES INDIA, MALES 1961–1970

Age	*Number living at age x*	*Survival ratio*	*Mortality rate*	*Number living between ages x and x + 1*	*Number living above age x*	*Expectation of life*
x	l_x	p_x	q_x	L_x	T_x	e_x^0
1	2	3	4	5	6	7
0	100000	0.86500	0.13500	89175	4707539	47.1
1	86500	0.94944	0.04056	84430	4617664	53.3
2	82992	0.98744	0.01056	82440	4533234	54.6
3	81950	0.99315	0.00685	81658	4450794	54.3
4	81388	0.99422	0.00578	81149	4369136	53.7
5	80918	0.99509	0.00491	80720	4287987	53.0
6	80521	0.99577	0.00423	80351	4207267	52.3
7	80181	0.99627	0.00373	80032	4126916	51.5
8	79882	0.99659	0.00341	79746	4046884	50.7
9	79609	0.99672	0.00328	79479	3967138	49.8
10	79348	0.99698	0.00302	79229	3887659	49.0
11	79109	0.99736	0.00264	79004	3808430	48.1
12	78900	0.99755	0.00245	78803	3729426	47.3
13	78706	0.99756	0.00244	78611	3650623	46.4
14	78514	0.99739	0.00260	78411	3572012	45.5
15	78309	0.99724	0.00276	78201	3493601	44.6
16	78092	0.99722	0.00278	77984	3415400	43.7
17	77875	0.99715	0.00258	77764	3337416	42.9
18	77653	0.99703	0.00297	77538	3259652	42.0
19	77422	0.99688	0.00312	77301	3112114	41.1
20	77181	0.99686	0.00314	77060	3104813	40.2
21	76938	0.99676	0.00324	76813	3027753	39.4
22	76689	0.99667	0.00333	76561	2950940	38.5
23	76433	0.99627	0.00373	76065	2874379	37.6
24	76148	0.99566	0.00434	75983	2708314	36.7
25	75818	0.99507	0.00493	75632	2722331	35.9
26	75445	0.99462	0.00538	75242	2646699	35.1
27	75039	0.99414	0.00586	74820	2571457	34.3
28	74599	0.99364	0.00639	74463	2496637	33.5
29	74125	0.99311	0.00689	73871	2422274	32.7
30	73615	0.99307	0.00693	73360	2348403	31.9
31	73105	0.99225	0.00775	72822	2275043	31.1
32	72538	0.99149	0.00851	72230	2202221	30.4
33	71921	0.99080	0.00920	71591	2129991	29.6
34	71260	0.99017	0.00983	70910	2058400	28.9
35	70559	0.98949	0.01051	70189	1987490	28.2
36	69818	0.98874	0.01126	69425	1917301	27.5
37	69032	0.98812	0.01188	68622	1847876	26.8
38	68212	0.98755	0.01245	67787	1779254	26.1
39	67362	0.98705	0.01295	66926	1711467	25.4

x	l_x	p_x	q_x	L_x	T_x	e_x^0
1	2	3	4	5	6	7
40	66490	0.98673	0.01327	66049	1644541	24.7
41	65607	0.98635	0.01365	65159	1578492	24.1
42	64711	0.98572	0.01428	64249	1513333	23.3
43	63787	0.98485	0.01515	63304	1449084	22.7
44	62821	0.98373	0.01627	62310	1385780	22.1
45	61799	0.98276	0.01724	61266	1323470	21.4
46	60733	0.98196	0.01804	60185	1262204	20.8
47	59637	0.98093	0.01907	59068	1202049	20.2
48	58500	0.97968	0.02032	57905	1142951	19.5
49	57311	0.97820	0.02180	56682	1085046	18.9
50	56062	0.97682	0.02318	55413	1028364	18.3
51	54763	0.97560	0.02440	54095	972951	17.8
52	53426	0.97422	0.02578	52737	918856	17.2
53	52049	0.97270	0.02730	51338	866119	16.6
54	50627	0.97103	0.02897	49894	814781	16.1
55	49161	0.97001	0.02999	48424	764887	15.6
56	47687	0.96908	0.03092	46950	716463	15.0
57	46212	0.96720	0.03280	45455	669513	14.5
58	44697	0.96473	0.03563	43901	624058	14.0
59	43105	0.96059	0.03941	42255	580157	13.5
60	4406	0.96584	0.04316	40512	537902	13.0
61	39618	0.95374	0.04626	38703	497390	12.6
62	37786	0.95062	0.04938	368533	454687	12.1
63	35920	0.94748	0.05252	34977	421834	11.7
64	34034	0.94432	0.05568	33087	386857	11.4
65	32139	0.94079	0.05921	31188	353770	11.0
66	30236	0.93728	0.06272	29288	322582	10.7
67	28340	0.93437	0.06563	2410	293294	10.3
68	26480	0.93205	0.06795	25580	265884	10.0
69	24680	0.93030	0.06970	23281	240304	9.7
70	22960	0.92879	0.07121	22142	216483	9.4
71	21325	0.92690	0.07310	20546	194341	9.1
72	19766	0.92467	0.07533	19022	173795	8.8
73	18277	0.92212	0.07788	17566	154773	8.5
74	16854	0.91924	0.08076	16174	137207	8.1
75	15494	0.91658	0.08342	14848	121033	7.8
76	14201	0.91416	0.08584	13593	106185	7.5
77	12983	0.91142	0.08858	12408	92592	7.1
78	11832	0.90835	0.09165	11290	80184	6.8
79	10747	0.90497	0.09503	10237	68894	6.4
80	9726	0.90182	0.09818	9249	58657	6.0
81	8771	0.89890	0.10110	8328	49408	5.6
82	7884	0.89566	0.10434	7473	41080	5.2
83	7062	0.89212	0.10788	6682	33607	4.8
84	6300	0.88826	0.11174	5948	26925	4.3
85+	5597	—	—	—	20977	3.7

Source: Life Tables, 1961 – 70 Census of India, 1971 Census, Registrar General, India

TABLE 9·12A : LIFE TABLES, FEMALES 1961–1970

Age	*Number living at age x*	*Survival ratio*	*Mortality rate*	*Number living between ages x and x + 1*	*Number living above age x*	*Expectation of life*
x	l_x	p_x	q_x	L_x	T_x	e_x^0
1	2	3	4	5	6	7
0	100000	0.87000	0.13000	90250	4564177	45.6
1	87000	0.95174	0.04826	84523	4473297	51.4
2	82801	0.98281	0.01719	82047	4389404	53.0
3	81378	0.99100	0.00900	80997	4307357	52.9
4	80646	0.99230	0.00700	80329	4226360	52.4
5	80025	0.99338	0.00662	79760	4146031	51.8
6	79495	0.99423	0.00577	79266	4066271	51.2
7	79036	0.99485	0.00515	78833	3987005	50.4
8	78629	0.99537	0.00463	78446	3908172	49.7
9	78265	0.99597	0.00403	78108	3829725	48.9
10	77950	0.99656	0.00344	77816	3751617	48.1
11	77682	0.99673	0.00327	77555	3673801	47.3
12	77428	0.99680	0.00320	77304	3596246	46.4
13	77180	0.99676	0.00324	77055	3518942	45.6
14	76930	0.99660	0.00340	76799	3441887	44.7
15	76668	0.99650	0.00350	76534	3365087	43.9
16	76400	0.99647	0.00353	76265	3288554	43.0
17	76130	0.99636	0.00364	75992	3212289	42.2
18	75853	0.99616	0.00384	75708	3136297	41.3
19	75562	0.99588	0.00412	75407	3060589	40.5
20	75251	0.99565	0.00435	75088	2985182	39.7
21	74924	0.99548	0.00452	74755	2910094	38.8
22	74585	0.99523	0.00477	74407	2835339	38.0
23	74229	0.99491	0.00509	74040	2760932	37.2
24	73851	0.99451	0.00549	73649	2686892	36.4
25	73446	0.99412	0.00588	73230	2613243	35.6
26	73014	0.99379	0.00621	72788	2540013	34.8
27	72561	0.99343	0.00657	72323	2467225	34.0
28	72084	0.99306	0.00694	71834	2394903	33.2
29	71584	0.99267	0.00733	71322	2323068	32.4
30	71059	0.99253	0.00747	70794	2251746	31.7
31	70528	0.99241	0.00759	70260	2180952	30.9
32	69993	0.99190	0.00810	69709	2110692	30.2
33	69425	0.99102	0.00898	69114	2040983	29.4
34	68802	0.98977	0.01023	68450	1971869	28.7
35	68098	0.98845	0.01155	67705	1903419	28.0
36	67311	0.98738	0.01262	66887	1836714	27.3
37	66462	0.98642	0.01358	66011	1768827	26.6
38	65559	0.98555	0.01445	65086	1702816	26.0
39	64612	0.98479	0.01521	64121	1037730	25.3
40	63629	0.98390	0.01610	63117	1673609	24.7

1	2	3	4	5	6	7
41	62605	0.98295	0.01705	62072	1510492	24.1
42	61538	0.98218	0.01782	60990	1448420	23.0
43	60441	0.98161	0.01839	59385	1387430	23.0
44	59329	0.98123	0.01877	58772	1327545	22.4
45	58215	0.98079	0.01921	57656	1268773	21.8
46	57097	0.98017	0.01983	56531	1211117	21.2
47	55965	0.97957	0.02043	56394	1154486	20.6
49	53667	0.97825	0.02175	53084	1044947	19.5
50	52500	0.97746	0.02254	51909	991863	18.9
51	51317	0.97663	0.02337	50713	939954	18.3
52	50118	0.97580	0.02420	49512	889236	17.7
53	48905	0.97485	0.02515	48290	839724	17.2
54	47675	0.97368	0.02632	47049	791434	16.6
55	46420	0.97218	0.02782	45775	744386	16.0
56	45129	0.97030	0.02970	44459	698611	15.5
57	43739	0.96812	0.03188	43091	654152	14.9
58	12393	0.96570	0.03430	41666	611061	14.4
59	40939	0.96312	0.03688	40184	569395	13.9
60	39429	0.96044	0.03956	38649	529211	13.4
61	33869	0.95758	0.04242	37066	490562	13.0
62	36263	0.95450	0.04550	35438	453496	12.5
63	34613	0.95132	0.04868	33376	418058	12.1
64	32928	0.94815	0.05185	32074	384282	11.7
65	31221	0.94512	0.05482	30365	352208	11.3
66	29508	0.94223	0.05777	28656	321832	10.9
67	27803	0.93940	0.06060	26961	292187	10.6
68	26118	0.93663	0.06337	25291	266226	10.2
69	24403	0.93389	0.06611	23655	240935	9.8
70	22846	0.93118	0.06882	22060	217280	9.5
71	21274	0.92845	0.07146	20514	195220	9.2
72	19754	0.92599	0.07401	19023	174706	8.5
73	18292	0.92344	0.07656	17392	155683	8.5
74	16892	0.92094	0.07906	16225	138091	9.2
75	15557	0.91810	0.08190	14920	121866	7.8
76	14283	0.91527	0.08472	13678	106946	7.5
77	13073	0.91241	0.08759	12501	93286	7.1
78	11928	0.90944	0.09056	11388	80767	7.1
79	10848	0.90371	0.09371	10340	69379	6.4
80	9831	0.90132	0.09868	9346	59039	6.0
81	8861	0.89818	0.10182	8410	49693	5.6
82	7959	0.89448	0.10552	7539	41283	5.2
83	7119	0.89023	0.10977	6729	33744	4.7
84	6338	0.88543	0.11457	5975	27015	4.3

Source : Life tables, 1961-70, Census of India, 1971 Census, Registrar-General, India

9·5·6. Uses of Life Tables. Although the basic objective of life tables is to give a clear picture of the age distribution of mortality in a given population group, it has been used widely in a large number of spheres. Today life table is widely accepted as important basic material in demographic and public health studies. In the words of **William Farr**, life table is the 'Biometer' of the population. We enumerate below some of important *applications* of life tables.

1. ***For Use by Actuaries in Insurance.*** Life tables are indispensable for the solution of all questions concerning the duration of human life. These tables, based on the scientific use of statistical methods, are the key stone or the pivot on which the whole science of life assurance hinges. Life tables form the basis for determining the rates of premiums to be paid by persons of different age groups, for various amount of life assurance. Life tables provide the actuarial science with a sound foundation, converting the insurance business from a mere gambling in human lives to the ability to offer well calculated safeguard in the event of death.

2. ***For Population Projections.*** Life tables are used by demographers to devise measures such as '*Net Reproduction Rate*' to study the rate of growth of population . They have also been used in preparation of population projections by age and sex, *i.e.*, in estimating what the size of the population will be at some future date.

3. ***For Comparison of Different Populations.*** Life tables for two or more different groups of population may be used for the relative comparison of various measures of mortality such as death rate, expectation of life at various ages, etc. Of particular interest is the comparison of e_x^0, the average longevity for members of a population.

4. Life tables are as well used by the government and the private establishments for determining the rates of retirement benefits to be given to its employees or for formulating various programmes for retired persons.

5. Since a life table depicts the distribution of the people according to age and sex, it is extremely useful in planning in respect of education and for predicting the school going population in connection with school building programmes.

6. Life tables are also used :

(*i*) For making policies and programmes relating to public health, by the government and public administration.

(*ii*) To evaluate the impact of family welfare programmes on the population growth.

(*iii*) For estimating the probable number of future widows and orphans in a community, and

(*iv*) For computing the approximate size of future labour force and military forces, etc.

Example 9·4. *In the usual notations, prove that*

(*i*) $\dfrac{dL_x}{dx} = -d_x$, (*ii*) $\dfrac{dT_x}{dx} = -l_x$, *and* *iii*) $\dfrac{d}{dx}\left(\overset{0}{e}_x\right) = \left(-1 + \mu_x \overset{0}{e}_x\right)$

Solution. By *def.*, we have

$$L_x = \int_0^1 l_{x+t}\,dt$$

$$\therefore \quad \frac{dL_x}{dx} = \frac{d}{dx}\int_0^1 l_{x+t}\,dt = \int_0^1 \frac{d}{dx}(l_{x+t})\,dt$$

(Assuming the validity of differentiation under the integral sign)

$$= \int_0^1 \frac{d}{dt}(l_{x+t})\,dt \qquad \text{(Since } l_{x+t} \text{ is continuous function of } x \text{ and } t\text{)}$$

$$= \Big| l_{x+t} \Big|_0^1 = l_{x+1} - l_x = -(l_x - l_{x+1})$$

$$= -d_x$$

(ii) $$T_x = \int_0^\infty l_{x+t}\,dt$$

$$\therefore \quad \frac{dT_x}{dx} = \frac{d}{dx}\int_0^\infty l_{x+t}\,dt = \int_0^\infty \frac{d}{dx}(l_{x+t})\,dt$$

$$= \int_0^\infty \frac{d}{dt}(l_{x+t})\,dt = \Big| l_{x+t} \Big|_0^\infty = l_\infty - l_x = -l_x \qquad (\because\ l_\infty = 0)$$

(iii) $$e_x^0 = \frac{T_x}{l_x}$$

$$\therefore \quad \frac{de_x^0}{dx} = \frac{d}{dx}\left(\frac{T_x}{l_x}\right) = \frac{l_x \dfrac{dT_x}{dx} - T_x \dfrac{dl_x}{dx}}{l_x^2} \qquad \left[\because\ \mu_x = \frac{-\dfrac{d}{dx}(l_x)}{l_x}\right]$$

$$= -1 + \mu_x (T_x / l_x)$$

$$= -1 + \mu_x e_x^0.$$

Example 9·5. *Given the following table for l_x, the number of rabbits living at age x, complete the life table for rabbits.*

x...	0	1	2	3	4	5	6
l_x...	100	90	80	75	60	30	0

X,Y,Z are three rabbits of age 1, 2 and 3 years respectively. Find the probability that :

(i) *at least one of them will be alive for one year more,*

(ii) *X, Y, Z will be alive for two years time,*

(iii) *exactly one of the three is alive in two years, and*

(iv) *all will be dead in two years time.*

Solution. The complete life table for the above data is given in Table 9·13.

TABLE 9·13

Age, x	l_x	$d_x = l_x - l_{x+1}$	$q_x = \dfrac{d_x}{l_x}$	$L_x = \dfrac{l_x + l_{x+1}}{2}$	$T_x = \sum_{i=x} L_i$	$e_x^0 = \dfrac{T_x}{l_x}$
1	2	3	4	5	6	7
0	100	10	0·10	95	385	3·85
1	90	10	0·11	85	290	3·22
2	80	5	0·06	77·5	205	2·56
3	75	15	0·20	67·5	127·5	170
4	60	30	0·50	45	60	10
5	30	30	1·00	15	15	0·5
6	0	—	—	—	—	—

(i) Let p_1 = Probability that the rabbit X will die in one year $= \dfrac{d_1}{l_1} = \dfrac{10}{90}$

p_2 = Probability that the rabbit Y will die in one year $= \dfrac{d_2}{l_2} = \dfrac{5}{80}$

p_3 = Probability that the rabbit Z will die in one year $= \dfrac{d_3}{l_3} = \dfrac{15}{75}$

Hence the probability that all will die in one year is given by the compound probability theorem by the expression :

$$p = p_1 \times p_2 \times p_3 = \frac{10}{90} \times \frac{5}{80} \times \frac{15}{75} = \frac{1}{720}$$

$\therefore$ P(at least one rabbit will survive for one year more)

$= 1 - P$ (none of the rabbits will survive for one year more)

$$= 1 - p = \frac{719}{720}$$

(ii) Let E_1, E_2 and E_3 denote the events that the rabbits X,Y and Z survive for two years more respectively. Then

$$P(E_1) = \frac{l_3}{l_1} = \frac{75}{90} = \frac{5}{6} \Rightarrow P(\bar{E}_1) = 1 - \frac{5}{6} = \frac{1}{6};\ P(E_2) = \frac{l_4}{l_2} = \frac{60}{80} = \frac{3}{4} \Rightarrow P(\bar{E}_2) = 1 - \frac{3}{4} = \frac{1}{4}$$

$$P(E_3) = \frac{l_5}{l_3} = \frac{30}{75} = \frac{2}{5} \Rightarrow P(\bar{E}_3) = 1 - \frac{2}{5} = \frac{3}{5}$$

Hence the required probability that all the three rabbits will survive for two years is given by the compound probability theorem by the expression :

$$P(E_1 \cap E_2 \cap E_3) = P(E_1)\,P(E_2)\,P(E_3) = \frac{5}{6} \times \frac{3}{4} \times \frac{2}{5} = 0·25$$

(iii) Let E denote the event that exactly one of the three rabbits is alive for two years more. Then the event E can materialise in the following mutually exclusive ways :

(a) $E_1 \cap \bar{E}_2 \cap \bar{E}_3$ happens, (b) $\bar{E}_1 \cap E_2 \cap \bar{E}_3$ happens, and (c) $\bar{E}_1 \cap \bar{E}_2 \cap E_3$ happens.

Hence, by addition theorem of probability, we obtain

$$P(E) = P(a) + P(b) + P(c) = P(E_1 \cap \bar{E}_2 \cap \bar{E}_3) + P(\bar{E}_1 \cap E_2 \cap \bar{E}_3) + P(\bar{E}_1 \cap \bar{E}_2 \cap E_3)$$

$$= P(E_1)\,P(\bar{E}_2)\,P(\bar{E}_3) + P(\bar{E}_1)\,P(E_2)\,P(\bar{E}_3) + P(\bar{E}_1)\,P(\bar{E}_2)\,P(E_3)$$

(By Compound Probability Theorem)

$$= \frac{5}{6} \times \frac{1}{4} \times \frac{3}{5} + \frac{1}{6} \times \frac{3}{4} \times \frac{3}{5} + \frac{1}{6} \times \frac{1}{4} \times \frac{2}{5} = \frac{26}{120} = 0·2167.$$

(iv) In the notations of part (iii) above, the required probability that all will die in two years is given by : (By Compound Probability Theorem)

$$P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3) = P(\bar{E}_1)\,P(\bar{E}_2)\,P(\bar{E}_3) = \frac{1}{6} \times \frac{1}{4} \times \frac{3}{5} = \frac{1}{40} = 0·025.$$

Example 9·6. *Complete the life table of the population of a certain types of insects, x being the age in days and* l_x *= 1,000 for x = 0 :*

x...	0	1	2	3	4	5	6	7	8
q_x...	0·120	0·005	0·010	0·050	0·100	0·500	0·800	0·900	0·950

Solution. Since we are given the values of q_x, in order to complete the life table, first of all we shall find the values of l_x, (x = 0, 1, 2, ..., 8) by using the relations :

$$q_x = \frac{d_x}{l_x} \quad \text{and} \quad l_{x+1} = l_x - d_x$$

We are given $l_0 = 1{,}000$

$\therefore\ d_0 = l_0 q_0 = 1000 \times 0{\cdot}12 = 120{\cdot}0$

$l_1 = l_0 - d_0 = 1000 - 120 = 880$

$d_1 = l_1 q_1 = 880 \times 0{\cdot}005 = 4{\cdot}0$

$l_2 = l_1 - d_1 = 880 - 4 = 876$

$d_2 = l_2 q_2,\ l_3 = l_2 - d_2$ and so on.

The values of l_x and d_x, (x = 0, 1, 2, ... 8) obtained on using (*) repeatedly are given in the *Table 9·13.*

TABLE 9·14

x	q_x	l_x	$d_x = l_x q_x$
0	0·120	1000	100 × 0·120 = 120
1	0·005	100 − 120 = 880	880 × 0·005 = 4·4 ≃ 4
2	0·010	880 − 4 = 876	876 × 0·010 = 8·76 ≃ 9
3	0·050	876 − 9 = 867	867 × 0·050 = 43·35 ≃ 43
4	0·100	867 − 43 = 824	824 × 0·100 = 82·4 ≃ 82
5	0·500	824 − 82 = 742	742 × 0·500 = 371·0
6	0·800	742 − 371 = 371	371 × 0·800 = 296·8 ≃ 297
7	0·900	371 − 297 = 74	74 × 0·900 = 66·6 ≃ 67
8	0·950	74 − 67 = 7	7 × 0·950 = 6·65 ≃ 7

The remaining columns of the life table, *viz.*, L_x, T_x and e_x^0 can now be completed as discussed in *Example 9·4.*

Example 9·7. *Fill in the blanks of the following table which are marked with question marks :*

Age, x :	l_x	d_x	q_x	p_x	L_x	e_x^0
20	6,93,435	?	?	?	?	35,081,126
21	6,90,673	—	—	—	—	?

Solution. Taking age 20 x, *i.e.*, age 21 as $x + 1$, in the usual notations, we get

$d_{20} = l_{20} - l_{21} = 693435 - 690673 = 2762$

$$q_{20} = \frac{d_{20}}{l_{20}} = \frac{2762}{693435} = 0{\cdot}00398$$

$p_{20} = 1 - q_{20} = 1 - 0{\cdot}00398 = 0{\cdot}99602$

$$L_{20} = \frac{l_{20} + l_{21}}{2} = \frac{693435 + 690673}{2} = 692054$$

$T_{21} = T_{20} - L_{20} = 35081126 - 692054 = 34389072$

$$e_{20}{}^0 = \frac{T_{20}}{l_{20}} = \frac{35081126}{693435} = 50{\cdot}59$$

$$e_{21}{}^0 = \frac{T_{21}}{l_{21}} = \frac{34389072}{690673} = 49{\cdot}79$$

Example 9·8. *Fill in the blanks in a portion of life table given below:*

Age (in years)	l_x	d_x	p_x	q_x	L_x	T_x	e_x^0
4	95,000	500	?	?	?	4,850,300	?
5	?	400	?	?	?	?	?

Solution. In the usual notations, we have

$l_5 = l_4 - d_4 = 95000 - 500 = 94500$

$$p_4 = \frac{l_5}{l_4} = \frac{94500}{95000} = 0{\cdot}9947$$

$q_4 = 1 - p_4 = 1 - 0{\cdot}9947 = 0{\cdot}0053$

$l_6 = l_5 - d_5 = 94500 - 400 = 94100$

$$p_5 = \frac{l_6}{l_5} = \frac{94100}{94500} = 0{\cdot}9958$$

$q_5 = 1 - p_5 = 1 - 0{\cdot}9958 = 0{\cdot}0042$

$q_5 = 1 - p_5 = 1 - 0{\cdot}9958 = 0{\cdot}0042$

$$L_4 = \frac{l_4 + l_5}{2} = \frac{95000 + 94500}{2} = 94750$$

$$L_5 = \frac{l_4 + l_5}{2} = \frac{94500 + 94100}{2} = 94300$$

$$e_4{}^0 = \frac{T_4}{l_4} = \frac{4850300}{95000} = 51{\cdot}06$$

$T_5 = T_4 - L_4 = 4850300 - 94750 = 4755550$

$$e_5{}^0 = \frac{T_5}{l_5} = \frac{4755550}{94500} = 50{\cdot}32.$$

Example 9·9. *Given that the complete expectation of life at ages 30 and 31 for a particular group are respectively 21·39 and 20·91 years and that the number living at age 30 is 41,176; find (i) the number that attains the age 31 and (ii) the number that will die without attaining the age 31.*

Solution. (*i*) In the usual notations we want the value of l_{31} and we are given :

$$\overset{0}{e}_{30} = 21{\cdot}39,\ \overset{0}{e}_{31} = 20{\cdot}90 \quad \text{and} \quad l_{30} = 41176$$

$$\therefore \quad e_{30} = \overset{0}{e}_{30} - \frac{1}{2} = 21{\cdot}39 - 0{\cdot}5 = 20{\cdot}89\ ; \quad \text{and} \quad e_{31} = \overset{0}{e}_{31} - \frac{1}{2} = 20{\cdot}91 - 0{\cdot}5 = 20{\cdot}41$$

Here we shall use the formulae :

$$p_x = \frac{l_{x+1}}{l_x} = \frac{e_x}{1 + e_{x+1}} \qquad ...(*)$$

Taking $x = 30$, we get

$$\frac{l_{31}}{l_{30}} = \frac{e_{30}}{1 + e_{31}} \Rightarrow l_{31} = \frac{l_{30} e_{30}}{1 + e_{31}} = \frac{41176 \times 20{\cdot}89}{20{\cdot}41} = 40176$$

(*iii*) The number of persons who die without attaining the age 31 is equal to the numbers of persons dying between the age period 30 and 31 and is given by :

$$d_{30} = l_{30} - l_{31} = 41176 - 40176 = 1000.$$

Example 9·10. *The number of persons dying at age 75 is 476 and the complete expectation of life at 75 and 76 years are 3·92 and 3·66 years. Find the numbers living at ages 75 and 76.*

Solution. In the usual notations, we are given

$$\overset{0}{e}_{75} = 3{\cdot}92,\ \overset{0}{e}_{75} = 3{\cdot}66,\ d_{75} = 476. \text{ We want } l_{75} \text{ and } l_{76}$$

We have

$$e_{75} = \overset{0}{e}_{75} - 0{\cdot}50 = 3{\cdot}42\ ; \quad e_{76} = \overset{0}{e}_{76} - 0{\cdot}50 = 3{\cdot}16$$

As in the above example, we get from (*)

$$l_{76} = \frac{l_{75} e_{75}}{1 + e_{76}} = \frac{3{\cdot}42}{4{\cdot}16} l_{75} = 0{\cdot}8221\, l_{75}$$

Also $\quad d_x = l_x - l_{x+1} \Rightarrow d_{75} = l_{75} - l_{76} = 476$ (given)

$\therefore \quad l_{75} - 0{\cdot}8221\, l_{75} = 476 \Rightarrow (1 - 0{\cdot}8221)\, l_{75} = 476$

$$\Rightarrow \quad l_{75} = \frac{476}{0{\cdot}1779} = 2675{\cdot}66 \simeq 2676$$

and $\quad l_{76} = l_{75} - d_{75} = 2676 - 476 = 2200.$

***Remark.* On simplifications in (**).** Collecting like terms in (9·78), we get

$$\textit{Coefficient of} \log p_x = \left[1 + \left(1 - \frac{1}{n}\right) + \left(1 - \frac{2}{n}\right) + \dots \left(1 - \frac{n-1}{n}\right)\right] = n - \frac{1}{n}\left[1 + 2 + \dots + (n-1)\right] = \frac{n+1}{2}.$$

$$\text{Coefficient of } \log p_{x+n} = \frac{1}{n} + \frac{1}{2n} + \dots + \frac{n-1}{n} = \frac{1}{n}\left[1 + 2 + \dots + (n-1)\right] = \frac{n-1}{2}$$

$$\text{Coefficient of } \Delta^2 \log p_x = \frac{1}{3\,!}\left[\frac{1}{n}\left\{\frac{1}{n^2} - 1\right\} + \frac{2}{n}\left\{\left(\frac{2}{n}\right)^2 - 1\right\} + \dots + \frac{n-1}{n}\left\{\left(\frac{n-1}{n}\right)^2 - 1\right\}\right]$$

$$= \frac{1}{3\,!}\sum_{r=1}^{n-1}\frac{r}{n}\left\{\left(\frac{r}{n}\right)^2 - 1\right\} = \frac{1}{3!\,n^3}\sum_{r=1}^{n-1}(r^3 - n^2.\,r) \qquad \dots(**)$$

$$= \frac{1}{3\,!\,n^3}\left[\sum_{r=1}^{n-1} r^3 - n^2 \sum_{r=1}^{n-1} r\right] = \frac{1}{3!\,n^3}\left[\left\{\frac{(n-1)n}{2}\right\}^2 - n^2 \,.\, \frac{(n-1)n}{2}\right]$$

$$= \frac{n^2}{3\,!\,n^3}\left[\frac{n-1}{2}\left\{\frac{n-1}{2} - n\right\}\right] = -\frac{1}{24n}(n-1)(n+1) = -\left(\frac{n^2-1}{24n}\right)$$

$$\text{Coefficient of } \Delta^2 \log p_{x-n} = \frac{1}{3\,!}\left[\left(1 - \frac{1}{n}\right)\left\{\left(1 - \frac{1}{n}\right)^2 - 1\right\} + \left(1 - \frac{2}{n}\right)\left\{\left(1 - \frac{2}{n}\right)^2 - 1\right\}\right.$$
$$\left. + \dots + \left(1 - \frac{n-1}{n}\right)\left\{\left(1 - \frac{n-1}{n}\right)^2 - 1\right\}\right]$$

$$= \frac{1}{3\,!}\sum_{r=1}^{n-1}\left(1 - \frac{r}{n}\right)\left\{\left(1 - \frac{r}{n}\right)^2 - 1\right\} = \frac{1}{3!\,n^3}\sum_{r=1}^{n-1}\left\{(n-r)^3 - n^2(n-r)\right\}$$

$$= \frac{1}{3\,!\,n^3}\sum_{t=1}^{n-1}(t^3 - n^2 t)\,;\ [t = n - r],$$

which is same as the expression obtained in (**).

$$\therefore \qquad \text{Coefficient of } \Delta^2 \log p_{x-n} = -\frac{n^2-1}{24n}.$$

9·7. FERTILITY

In demography, the word *fertility* is used in relation to the actual production of children or '*occurrence of births, specially live births*'. Fertility must be distinguished from *fecundity* which refers to the capacity to bear children. In fact, fecundity provides an upper bound for fertility. As a measure of the rate of growth of population various *fertility rates are computed*.

In the following sections we shall discuss briefly some of the important rates which are usually computed to have an idea about the fertility in the relevant section of the population.

9·7·1. Crude Birth Rate (C.B.R.). This is the simplest of all the measures of fertility and consists in relating the number of live births to the total population. This provides an index of the relative speed at which additions are being made through child birth. The fertility pattern of the above mentioned measure is given by *crude birth rate* (C.B.R.) defined as follows :

$$\text{Crude Birth Rate} = \frac{B^t}{P^t} \times k \qquad \dots(9{\cdot}82)$$

where B^t = Total number of live births in the given region or locality during a given period, say t.

P^t = Total population of the given region during the period t.

k = A constant, usually 1000.

Merits. It is simple, easy to calculate and readily comprehensible. It is based only on the number of births (B^t) and the total size of the population (P^t) and does not necessitate the knowledge of these figures for different sections of the community or the population.

Demerits **1.** The crude birth rate, though simple, is only a crude measure of fertility and is unreliable since it completely ignores the age and sex distribution of the population.

2. C.B.R. is not a probability ratio, since the whole population P^t cannot be regarded as exposed to the risk of producing children. In fact, only the females and only those between the child bearing age group (usually 15 to 49 years) are exposed to risk and as such whole of the male population and the female population outside the child-bearing age should be excluded from P^t, the denominator in (9·82). Moreover, even among the females who are exposed to risk, the risk varies from one age group to another, a woman under 30 is certainly under greater risk as compared to a woman over 40.

3. As a consequence of variation of climatic conditions in various countries, the child-bearing age-groups are not identical in all the countries. In tropical countries, the period starts at an apparent earlier date than in countries with cold weather. Accordingly, crude birth rate does not enable us to compare the fertility situations in different countries.

4. Crude birth rate assumes that women in all the ages have the same fertility, an assumption which is not true since younger women have, in general higher fertility than elderly women. C.B.R. thus gives us an estimate of a hetrogeneous figure and is unsuitable for comparative studies.

5. The level of crude birth rate is determined by a number of factors such as age and sex distribution of the population, fertility of the population, sex ratio, marriage rate, migration, family planning measures and so on. Thus a relatively high crude birth rate may be observed in a population with a favourable age and sex structure even though fertility is low, *i.e.*, a population with large proportion of the individuals in the age-group 15—49 years will have a high crude birth rate, other things remaining same.

***Remarks* 1.** *C.B.R.* usually lies between 10 and 55 per thousand.

2. Since it is only a live birth that augments the existing population, only live births are considered in measuring fertility, thus excluding *still* births.

3. The *Table 9·15* gives C.B.R., C.D.R. infant mortality rate (I.M.R.) for rural, urban and combined India for different years. Also see *Table 9·17A* on *Page 9·49*.

TABLE 9·15. CBR, CDR AND IMR FOR RURAL, URBAN AND COMBINED INDIA

	1970	1980	1990	1998	1999	2000	2001	2002	2003
Birth rate (Per '000 population)									
Rural	38·9	34·6	31·7	28·9	28·0	27·6	27·6	27·1	26·6
Urban	29·7	28·1	24·7	21·5	21·0	20·8	20·7	20·3	19·9
Combined	36·8	33·3	30·2	27·2	26·5	26·0	25·8	25·4	25·0
Death rate (Per '000 population)									
Rural	17·3	13·5	10·5	9·6	9·7	9·4	9·3	9·1	8·7
Urban	10·2	8·0	6·8	6·5	6·6	6·3	6·3	6·3	6·1
Combined	15·7	12·4	9·7	8·9	9·0	8·7	8·5	8·4	8·1
Infant mortality rate (Per '000 live births)									
Rural		124	86	77	77	75	74	72	69
Urban		65	50	45	45	44	44	42	40
Combined		114	80	71	72	70	68	66	64

Secondary Source : Monthly Abstract of Statistics (C.S.O.) Volume 56, Number 10, October 2003.

9·7·2. General Fertility Rate (G.F.R.) This consists in relating the total number of live births to the number of females in the *reproductive* or child bearing ages and is given by the formula.

$$\text{G.F.R.} = \frac{B^t}{\sum_{\lambda_1}^{\lambda_2} {}^fP_x} \times k \qquad \ldots(9{\cdot}83)$$

where B^t = number of live births occurring among the population of a given geographic area during a given period t,

$\sum_{\lambda_1}^{\lambda_2} {}^fP_x$ = female population in the reproductive age , in the given geographical region during the same time t,

λ_1, λ_2 = lower and upper limits of the female child bearing age, and k = a constant, usually 1,000.

Thus general fertility rate may be defined as the number of babies per k women in the reproductive age group.

Remark. Generally λ_1 and λ_2 are taken as 15 and 49. This births to females outside this range (*i.e.*, below 15 and over 49) are very rare. Such births, if any are recorded separately and are included in the age group 15 and 49 respectively.

Merits **1.** General fertility rate is a *probability* rate since the denominator in (9·83) consists of the entire female population which is exposed to the risk of producing children.

2. G.F.R. reflects the extent to which the female population in the reproductive ages increases the existing population through live births. Obviously, G.F.R. takes into account the sex distribution of the population and also the age structure to a certain extent.

Demerits. G.F.R. gives a heterogeneous figure since it overlooks the age composition of the female population in the child-bearing age. Hence it suffers from the drawback of non-comparability in respect of time and country.

As such *two populations with altogether different G.F.R's may have the same fertility in each one year age group*. This is due to the difference in the recundity of the women according to the are groups since the females belonging to different child bearing age groups are not exposed to uniform risk of giving births to babies. In India recundity of women is very low in the age-group 15—19 but it increases very rapidly in the age-group 20—24 and only signify in 25—29 after which it defines. On the other hand, in U.S.A. it reaches its peak in the age-group 20—24 and there-after declines.

Similarly, *the two populations with more or less the same G.F.R. may exhibit entirely different fertility status*, since the proportion of young females (who are exposed to a greater risk of producing children) in one may be quite different from that of the other.

9·7·3. Specific Fertility Rate (S.F.R.) The concept of specific fertility rate originated from the fact that fertility is affected by a number of factors such as age, marriage, migration, state or region, etc. The fertility rate computed with respect to any specific factor is called *specific fertility* rate (S.F.R.) and is defined as :

$$\text{S.F.R.} = \frac{\text{Number of births to the female population of the specified section in a given period}}{\text{Total number of female population in the specified section}} \times k \qquad \ldots(9{\cdot}84)$$

where k = 1,000; usually.

Age-specific Fertility Rate. In order to overcome the drawback of G.F.R. and get a better idea of the fertility situation prevailing in a community or locality it is necessary to compute the fertility rates for different age-groups of reproductive age separately. The fertility rate for different age-groups of reproductive age separately. The fertility rate so computed on the basis of specification *w.r.t.* age is called the *age-specific fertility rate*. For its computation, the reproductive span is split into different subgroups and *S.F.R.* is worked out for each sub-group.

Symbolically, the age-specific fertility rate for the age group x to $x + n$, denoted by ${}_nI_x$, is given by the formula :

$${}_nI_x = \frac{{}_nB_x}{{}_n{}^fP_x} \times k \qquad \ldots(9{\cdot}85)$$

${}_nB_x$ = Number of births to the females in age group $(x, x + n)$ *i.e.*, age $\geq x$ but less than $x + n$ in the given geographic region during a period t.

${}_n{}^fP_x$ = Average female population (*i.e.*, average number of females) of ages (x to $x + n$) in the given area during the period t, and k = 1000, usually.

In particular, if we take n = 1 in (9·85), we get the so-called *annual* age-specific fertility rate given by :

$$i_x = \frac{B_x}{{}^fP_x} \times k \qquad \ldots(9{\cdot}85a)$$

Remarks **1.** In the computation of age-specific fertility rate, the female population in the child-bearing age group is placed in small age-groups so as to put them in common with other of the child bearing capacity, As already pointed out, the grouping of women of different ages is necessary since the capacity to bear children varies from age to age, *e.g.*, the women in the age group 20 to 25 are more liable to the risk of producing children than the women in the age group 40 to 45,

2. Fertility data for different countries show that generally specific fertility starts from a low point, rises to a peak somewhere between 20 and 29 years of age and after that declines steadily. The age-specific fertility curve is, therefore, a highly positively skewed curve. The data in *Table 9·16* for rural and urban India bears testimony to the above statement.

TABLE 9·16 : AGE SPECIFIC FERTILITY RATES INDIA

Age group (Years)		1971*	1976*	1981	1996	1987	1988	1989	1990	1991 @	1992 @	1993 @	1994 @	1995 @	1996 @
	Rural	110·6	87·0	98·2	100·3	97·5	97·1	91·6	92·6	84·5	83·3	80·6	79·5	61·9	62·7
15—19	Urban	64·9	64·6	58·1	62·1	58·7	57·2	50·3	49·6	46·1	42·4	37·4	36·7	34·4	33·6
	Combined	100·8	83·0	90·4	91·1	88·3	88·2	82·0	83·1	76·1	74·4	69·6	68·1	55·2	55·3
	Rural	260·9	260·2	261·3	264·6	262·8	260·1	259·6	249·8	244·6	249·4	247·7	260·9	256·3	244·0
20—24	Urban	213·9	213·7	195·0	217·8	221·3	211·9	206·5	197·8	200·7	189·6	196·4	195·9	186·9	183·0
	Combined	250·8	249·5	246·9	252·8	252·3	248·1	264·4	237·0	234·0	235·2	234·4	244·6	238·4	229·1
	Rural	261·6	250·8	244·9	229·4	223·3	220·7	216·3	209·7	202·3	200·8	196·2	200·2	203·5	201·2
25—29	Urban	227·9	197·5	187·0	179·0	184·6	173·0	163·4	165·5	158·7	155·5	174·3	159·7	164·1	149·4
	Combined	254·8	238·8	232·1	216·4	213·5	208·5	202·5	198·5	191·3	189·6	189·7	188·9	194·2	188·1
	Rural	212·4	190·9	180·4	153·6	148·4	143·4	140·8	135·0	128·6	125·1	124·3	129·8	134·5	124·0
30—34	Urban	158·0	133·9	117·8	94·5	96·4	89·3	85·1	81·8	81·6	75·8	89·1	88·9	76·6	75·1
	Combined	202·2	179·7	167·7	139·2	135·9	130·2	127·0	121·6	117·0	113·0	114·3	119·1	119·1	112·4
	Rural	147·5	126·3	112·6	89·3	88·1	84·8	81·5	82·2	75·9	75·3	70·5	65·9	67·4	66·8
35—39	Urban	96·5	73·6	60·1	45·0	49·6	45·2	42·5	40·7	37·4	35·5	38·1	32·3	32·6	28·0
	Combined	137·8	116·1	102·5	78·6	78·8	75·3	72·2	72·5	66·8	66·0	61·1	56·8	59·2	56·0
	Rural	68·2	58·9	48·4	43·5	40·2	39·1	39·7	36·0	35·3	35·2	33·6	29·2	37·5	33·6

40—44	Urban	34·9	28·9	24·5	17·6	18·8	18·7	17·3	15·7	14·9	16·7	14·2	14·4	13·2	11·2
	Combined	62·2	53·3	44·0	37·9	35·5	34·5	34·5	31·2	30·6	30·9	28·5	25·4	31·0	28·3
	Rural	26·3	17·3	22·0	17·8	17·9	14·4	13·4	14·7	14·0	13·0	11·2	4·4	12·1	12·3
45—49	Urban	15·4	8·3	9·1	4·7	5·3	4·8	4·9	6·2	5·3	5·8	7·1	4·3	3·9	3·6
	Combined	24·4	15·7	19·6	14·9	15·1	12·4	11·6	12·9	12·1	11·4	10·1	4·4	10·3	10·2
Total fertility rate	Rural	5·4	5·0	4·8	4·5	4·4	4·3	4·2	4·1	3·9	3·9	3·8	3·8	3·9	3·7
	Urban	4·1	3·6	3·3	3·1	3·2	3·1	2·8	2·8	2·7	2·6	2·8	2·7	2·6	2·4
	Combined	5·2	4·7	4·5	4·2	4·1	4·0	3·9	3·8	3·6	3·6	3·5	3·5	3·5	3·4

Source : Office of the Registrar General, India Sample Registration System

: Excludes Bihar and West Bengal

Excludes Jammu and Kashmir

3. Age-specific fertility rate is a probability rate. It removes the drawback of G.F.R. by taking into account the age-composition of the women in the child-bearing age group and is thus suitable for comparative studies. However, the use of age-S.F.R. for comparing the fertility situations of two regions (or of the same region for two different periods) is not an easy job. Generally age-S.F.R. will be higher for certain age-groups and lower for the remaining age-groups in one region than in the other. Accordingly it is difficult to say if the fertility is higher or low in one region as compared to other.

9·7·4. Total Fertility Rate (T.F.R.). As already pointed out in Remark 3, § 9·7·3 age-specific fertility rate is not of much practical utility for comparative purposes. In order to arrive at more practical measure of the population growth, the age specific fertility rates for different groups have to be combined together to give a single quantity. A simple technique is to obtain standardised fertility rate. This leads to total fertility rate (*T.F.R.*) which is obtained on adding the annual age-specific fertility rates. Thus, symbolically.

$$T.F.R = \sum_{\lambda_1}^{\lambda_2} i_x = \sum_{\lambda_1}^{\lambda_2} \frac{B_x}{fP_x} \times k \qquad \ldots(9\cdot86)$$

where i_x is as defined in (9·85*a*) and λ_1 and λ_2 are the lower and upper limits of the female reproductive period. Thus *T.F.R.* gives the number of children born per k (= 1000, usually) females in the child bearing age divided into different age groups. Thus *T.F.R.* for a particular region during a given period may be regarded as an index of the overall fertility conditions operating in that region during the same period.

Usually $\lambda_1 = 15$ and $\lambda_2 = 49$. Thus in order to compute *T.F.R.* from (9·86), we shall have to calculate 34 age-specific fertility rates. The arithmetic may be reduced to a great extent by working with age groups, say x to $x + n$, where in general n, the width of interval may vary from one group to the other. In such a case, the *T.F.R.* is approximately given by the formula :

$$T.F.R. = \sum_x n({}_ni_x) \qquad \ldots(9\cdot87)$$

where summation is taken over different age-groups in the reproduction period.

In particular, if we deal with quinquennial age group, *i.e.*, $n = 5$ for each class then

$$T.F.R. = \sum_x 5({}_5i_x) = 5 \sum_x {}_5i_x \qquad \ldots(9\cdot88)$$

The calculation of *T.F.R.* based on quinquennial age group [*c.f.* formula (9·88)] requires only 1/5th of the arithmetic as compared to *T.F.R.* based on single age-groups [*c.f.* formula (9·86)] and from practical point of view it is almost as accurate.

Remarks **1.** T.F.R. is a hypothetical figure giving the number of children born to a cohort of k = 1000 females (all born at the same time) assuming that :

(*i*) none of them dies before reaching the end of the child bearing age, *i.e.*, all of them live till at least the age of 50 years, and

(*ii*) at each age-group (in the child bearing ages) they are subject to the fertility condition given by the observed age-specific fertility rate.

2. If the number of women at each age is decreasing and the age-specific fertility rate is increasing then the total fertility rate obtained from the data expressed in groups of five years is lower than the actual value and if the fertility rate is decreasing, the age-specific resulting value of T.F.R. will be higher than the actual one. This is also true for the gross reproduction rate (G.R.R.) and net reproduction rate (N.R.R.) discussed in § 9·8·2 and § 9·8·3 respectively.

3. ${}_ni_x$, the age-specific fertility rate for the age-group x to $x + n$ implies that k females exactly aged x would by the time they reached $x + n$ have born $({}_ni_x) \times n$ children. It is necessary to multiply by n [*c.f.* formula (9·60)] since the *specific fertility rate is a rate per annum* and by the time they reach the age $x + n$ they will have spent n years in the age-group x to $x + n$.

The *Table 9·17.* gives the total fertility rates for rural and urban India by background characteristics for the year 1998-99.

TABLE 9·17 : FERTILITY RATES BY BACKGROUND CHARACTERISTICS, 1998-99 FOR INDIA

Background Characteristics	*Total Fertility Rate*		*Mean number of children ever born to ever married women aged 40-49 years*	
	1992-93*	1998-99	1992-93*	1998-99
Residence				
Urban	2·7	2·27	4·16	3·78
Rural	3·67	3·07	5·13	4·73
Education				
Illiterate	4·03	3·47	5·26	4·98
Literate < middle school complete	3·01	2·64	4·50	4·06
Middle school complete	2·49	2·26	3·71	3·41
High School complete and above	2·15	1·99	2·80	2·66
Caste/Tribe				
Scheduled Caste	3·92	3·15	5·40	4·85
Scheduled Tribe	3·55	3·06	4·81	4·74
Other	3·3	2·66	4·76	4·20
Total	3·39	2·85	4·84	4·45

Source : National Family Health Survey, 1998-99.

Note : Total fertility rate for the three years preceding the Survey, and mean number of children by selected background characteristics, India.

* Rate for women age 15-48 years

9·8. MEASUREMENT OF POPULATION GROWTH

Having obtained the measures of mortality and fertility, our next objective is to find out if the given population has a tendency to increase, decrease or remain stable. *Fertility rates are inadequate to give us any idea about the rate of population growth since they ignore the sex of the newly born children and their mortality.* Obviously the population increases through female births. Thus if a majority of births are those of girls, the population is bound to increase while it will have a downward trend if the majority of births are boys. Similarly if we ignore the mortality of the newly born children we can not form a correct idea of the rate of growth of the population, since it is possible that a number of female children may die before reaching the reproductive age. In the following sections we shall study some measures of the growth of population under the assumption that in future also it is subject to the current fertility and mortality rates.

9·8·1. Crude Rate of Natural Increase and Pearle's Vital Index. The simplest measure of the population growth known as *crude rate of natural increase* is defined as *the difference between the crude birth rate (per thousand) and the crude death rate (per thousand)* and is given by :

$$\text{Crude Rate of Natural Increase} = \text{C.B.R.} - \text{C.D.R.} \qquad \ldots(9\cdot89)$$

Since *CBR* (*CDR*) gives the proportion by which population increases (decreases) through births (deaths), the formula (9·62) gives the net increase (or decrease) in population through births and deaths taken together.

Another indicator of population growth based on births and deaths taken together is provided by *R. Pearle's Vital Index*, defined as follows :

$$\text{Pearle's Vital Index} = \frac{\text{Number of births in the given period } t}{\text{Number of deaths in the given period } t} = \frac{B^t}{D^t} \times 100 \qquad \ldots(9\cdot90)$$

Dividing numerator and denominator in (9·90), by the population P^t in the given period t, we get

$$\text{Pearl's Vital Index} = \frac{\text{C.B.R.}}{\text{C.D.R.}} \times 100 \qquad \ldots(9\cdot90a)$$

***Remarks* 1.** Both these measures are simple and easy to calculate. Pearle's index is regarded as a fairly reliable statistical constant reflecting the net biological status of the population as a whole. The vital index may be equal to 100, less than 100, or greater than 100. If the value of vital index is equal to 100, it indicates stagnation in the population growth. If vital index is greater than 100, then the population is regarded as having good medical care and if it is than 100, then the population is not holding its own.

2. Vital index merely gives a measure whether births exceed deaths or not. It certainly fails to give us any idea about the trend in the population growth, *i.e.*, it does not tell us anything whether population has a tendency to increase or decrease.

3. Both these measures suffer from the drawbacks of C.B.R. and C.D.R. and as such are not suitable for comparative studies.

The *Tables 9·18* and *9·18A* give the crude rate of natural increase and the vital index per annum for India.

TABLE 9·18 : CBR, CDR AND VITAL INDEX FOR INDIA

Decennium	*C.B.R.*	*C.D.R.*	*Crude Rate of Natural Increase (C.B.R. – C.D.R.)*	*Vital Index* $\left(\frac{C.B.R.}{C.D.R.} \times 100\right)$
1901—10	49·2	42·6	6·6	115·49
1991—20	48·1	47·2	0·9	101·91
1921—30	46·4	36·3	10·1	127·82
1931—40	45·2	31·2	14·0	144·87
1941—50	39·9	27·4	12·5	145·62
1951—60	40·9	22·0	18·9	185·91
1961—71	41·1	18·0	23·1	228·33

TABLE 9·18*A* : VITAL RATES BASED ON SAMPLE REGISTRATION SYSTEM BY RURAL AND URBAN

Year	*Birth rate*			*Death rate*			*Natural Growth rate*		
	R	*U*	*C*	*R*	*U*	*C*	*R*	*U*	*C*
1971	38·9	30·1	36·9	16·4	9·7	14·9	22·5	20·4	22·0
1974	35·9	28·4	34·5	15·9	9·2	14·5	20·0	19·2	20·0
1975	36·7	28·5	35·2	17·3	10·2	15·9	19·4	18·3	19·3
1976	35·8	28·4	34·4	16·3	8·5	15·0	19·5	19·9	19·4
1977	34·3	27·8	33·3	15·3	9·4	14·2	19·4	18·4	19·1
1978	34·7	27·8	33·0	16·0	9·4	14·7	18·3	18·4	18·3
1979	34·3	28·3	33·1	14·1	8·1	13·0	20·2	20·2	20·1
1980	34·6	28·1	33·3	13·7	7·9	12·6	20·9	20·2	20·7
1981	35·6	27·0	33·9	13·7	7·8	12·5	21·9	19·2	21·4
1982	35·5	27·6	33·8	13·1	7·4	11·9	22·4	20·2	21·9
1983	35·3	28·3	33·7	13·1	7·9	11·9	22·2	20·4	21·8
1984	35·3	29·4	33·9	13·8	8·6	12·6	21·5	20·8	21·3
1985	34·3	28·1	32·9	13·0	7·8	11·8	21·3	20·3	21·1
1986	34·2	27·1	32·6	12·2	7·6	11·1	22·0	19·5	21·5
1987	33·7	27·4	32·2	12·0	7·4	10·9	21·7	20·0	21·3
1988	33·1	26·3	31·5	12·0	7·7	11·0	21·1	18·6	20·5
1989	32·2	25·2	30·6	11·1	7·2	10·3	21·1	18·0	20·3
1990	31·7	24·7	30·2	10·5	6·8	9·7	21·2	17·9	20·5
1991*	30·9	24·3	29·5	10·6	7·1	9·8	20·3	17·2	19·7
1992*	30·9	23·1	29·2	10·9	7·0	10·1	20·0	16·1	19·1
1993*	30·4	23·7	28·7	10·6	5·8	9·3	19·8	17·9	19·4
1994*	30·5	23·1	28·7	10·6	5·8	9·3	19·8	17·9	19·4
1995*	30·0	22·7	28·3	9·8	6·6	9·0	20·2	16·1	19·3
1996*	29·3	21·6	27·5	9·7	6·5	9·0	19·6	15·1	18·5
1997	28·9	21·5	27·2	9·6	6·5	8·9	19·2	15·0	18·3
1998(P)$	28·0	21·0	26·5	9·7	6·6	9·0	18·3	14·4	17·4
1999	27·6	20·8	26·1	9·4	6·3	8·7	18·2	14·5	17·3

Source : Office of the Registrar General, India

Note : The estimates exclude Bihar and West Bengal for the period 1971 to 1978

* : Excludes Jammu & Kashmir and in Mizoram the data available from 1995 onwards.

R Rural U Urban C Combined

$: Estimates at the national level exclude Nagaland (Rural) due to part-receipt of returns.

9·8·2. Gross Reproduction Rate (G.R.R.). In order to have a better idea about the rate of population growth, in addition to the age and sex composition of the population we must take into account the sex of the new born children since it is ultimately the female births who are the potential future mothers and result in an increase in the population. *The gross reproduction rate* (*G.R.R.*) is a step in this direction and is defined as the sum of age-specific fertility rates calculated *from female births for each year of reproductive period.* Symbolically, if ${}^fB_x$ is the number of female births to the women of age x during the given period in the given region, then in the usual notations, we have

$$\text{G.R.R.} = \sum_{\lambda_1}^{\lambda_2} \frac{{}^fB_x}{{}^fP_x} \times k = \sum_{\lambda_1}^{\lambda_2} {}^fi_x \qquad \ldots(9\cdot91)$$

where $$f_{i_x} = \frac{{}^fB_x}{{}^fP_x} \times k \qquad \ldots(9\cdot92)$$

is termed as the *female age-specific fertility rate* and $k = 1000$, usually. More precisely formula (9·91) gives *female gross reproduction rate*.

Gross reproduction rate is thus a modified form of total fertility rate and gives the number of females expected to be born to k newly born daughters if

(*i*) none of them is subject to risk of mortality till attaining the age λ_2, the upper limit of the reproductive period, and

(*ii*) all of them experience, thoughout the reproductive period, the current level of fertility as represented by f_{i_x}.

In other words G.R.R. exhibits the rate at which mothers would be replaced by daughters and the old generation by the new, under the above two assumptions.

Suppose now that instead of annual data, we are given the figures for different age groups of reproductive period. Let ${}_n^fB_x$ be the number of female babies born to the women, in the age group x to $x + n$, then in the usual notations, we get

$$\text{G.R.R.} = \sum_{\lambda_1}^{\lambda_2} n \left(\frac{{}_n^fB_x}{{}_n^fP_x}\right) \times k = \sum_{\lambda_1}^{\lambda_2} n\,({}_nf_{i_x}) \qquad \ldots(9\cdot93)$$

where $${}_nf_{i_x} = \frac{{}_n^fB_x}{{}_n^fP_x} \times k \qquad \ldots(9\cdot94)$$

is the age-specific fertility rate for the age group x to $x + n$ based on female births.

In particular for the quinquennial data, $\text{G.R.R.} = 5 \sum_{\lambda_1}^{\lambda_2} {}_5f_{i_x}$ ...(9·95)

Remarks **1.** The computation of G.R.R. by the formula (9·93) requires the availability of the following data :

(*i*) the classification of the births according to the age of the mother at the time of birth and

(*ii*) the sex of the new-born babies.

Usually such data are not available. In that case, however, an approximate value of G.R.R. may be obtained under the assumption that sex-ratio at birth remains more or less constant at all the ages of the women in the reproductive period. Then we have

$$\text{Sex ratio} = \frac{\text{Number of female births}}{\text{Number of male births}} = \text{constant} \quad \Rightarrow \quad \frac{{}^fB_x}{{}^mB_x} = \text{constant} = b \text{ (say)} \;\forall\; x$$

$$\therefore \quad \frac{{}^fB_x}{{}^fB_x + {}^mB_x} = \frac{b}{b+1} = C \text{ (say)} \quad \Rightarrow \quad {}^fB_x = C({}^fB_x + {}^mB_x) = C.B_x \qquad \ldots(*)$$

where $B_x = {}^fB_x + {}^mB_x$ is the total number of births to women of age x during the given period in the given region. From (*), we get

$$\frac{{}^fB_x}{B_x} = C, \;\forall\; x \quad \Rightarrow \quad C = \frac{{}^fB_x}{B_x} = \frac{\sum_{x=\lambda_1}^{\lambda_2} {}^fB_x}{\sum_{x=\lambda_1}^{\lambda_2} B_x} = \frac{{}^fB}{B} \qquad \ldots(**)$$

where ${}^fB$ is the total number of female births and B is the total number of births.

Substituting from (**) in (*), we get $\quad {}^fB_x = \frac{{}^fB}{B} \times B_x$

Finally, substituting in (9·64), we obtain an estimate of G.R.R. as

$$\text{G.R.R.} = \frac{{}^fB}{B} \sum_{\lambda_1}^{\lambda_2} \frac{B_x}{{}^fP_x} \times k = \frac{{}^fB}{B} \times \text{T.F.R.} \qquad \text{[From (9·86)]}$$

$$= \frac{\text{Number of female births}}{\text{Total number of births}} \times \text{T.F.R.} \qquad \ldots(9\cdot96)$$

Proceeding exactly similarly the result (9·96) can be established for the data given in different age groups.

2. As a measure of fertility, G.R.R. is quite useful for comparing the fertility in different regions or in the same region at different periods of time. Gross reproduction rate may be regarded as a measure of the extent to which a sex under consideration (*i.e.*, female sex in this case) is replacing itself, unity being the criterion for exact replacement. Thus if G.R.R. is less than unity, the population would decline no matter how low the death rate may be and if G.R.R. is greater than unity then the population would increase no matter how high the death rate may be. In theory G.R.R. ranges from 0 to 5.

3. The accuracy of the G.R.R. depends upon the accuracy of the computation of f_{i_x}, the main source of error being :

(*i*) under-registration of births,

(*ii*) under-statement or inadequate statements of women's age at the time of registration, and

(*iii*) errors in enumeration or estimates of female population (${}^fP_x$) by age groups.

4. G.R.R. is computed on the hypothesis that none of the newly born female babies is subject to the risk of mortality till the end of the reproductive period of life. This is a very serious limitation of G.R.R. since all the girls born do not survive till the end of the child-bearing span. Accordingly G.R.R. leads to fallacious conclusion as it inflates the number of potential mothers. The drawback is overcome in *net reproduction rate*.

5. The *Table 9·19* gives C.B.R., G.F.R., T.F.R. and G.R.R. for rural and urban India for 1990.

TABLE 9·19 : GROSS REPRODUCTION, GENERAL AND TOTAL FERTILITY RATES ALL INDIA, 1990

Item	*Rural*	*Urban*	*Combined*
1. Crude birth rate	31·7	24·7	30·2
2. Gross reproduction rate	2·0	1·3	1·8
3. General fertility rate	132·6	96·1	123·9
4. Total fertility	4·1	2·8	3·8

9·8·3. Net Reproduction Rate (N.R.R.). As already pointed out (*c.f.* Remark 4, § 9·8·2), the principal limitation of G.R.R. is that it completely ignores the current mortality and takes into account only the current fertility. *Net Reproduction Rate* (*N.R.R.*) is nothing but gross reproduction rate adjusted for the effects of mortality. According to Benjamin, "*N.R.R. measures the extent to which mothers produce female infants who survive to replace them. It measures the extent to which a generation of girl babies survive to reproduce themselves as they pass through the child-bearing age group.*"

Let us now take into consideration the factor of mortality of mothers also in measuring the growth of population. To formulate our ideas mathematically, to start with we construct a life table for females on the basis of age-specific death rates for females, ${}^fm_x$. The values in the L_x column of the table, denoted by ${}_n^fL_x$, give the mean size of the cohort of ${}^fl_0$ females in the age-interval x to $x + n$. In the usual notations let ${}_n^fB_x$ be the number of female births to the women in the age group x to $x + n$ at any period t, (say). Then

$$\frac{{}_n^fL_x}{{}^fl_0} \times {}_n^fB_x$$

gives the average number of female children that would be born to the cohort ${}^fl_0$ in the age group x to $x + n$. The quantity

$${}_n^f\pi_x = \frac{{}_n^fL_x}{{}^fl_0} \qquad \ldots(9\cdot97)$$

gives the life table probability of survival of a female to the age-interval x to $x+n$ and is called the *Survival rate*. This implies that out of k newly born female babies $k \times ({}_n{}^f\pi_x)$ will enter into the child bearing age-interval x to $x+n$; $k \times ({}_n{}^f\pi_{x+n})$ into the age group $x+n$ to $x+2n$ and so on.

Hence instead of multiplying $[{}_n{}^fB_x \div {}_n{}^fP_x]$ by k alone as in G.R.R., we multiply it by the factor $k\ ({}^f{}_n\pi_x)$ for each age interval x to $x+n$. Finally, a new measure of population growth, known as *(female) net reproduction rate* (N.R.R.) is given by :

$$\text{N.R.R.} = k \sum_x n \left[\frac{{}_n{}^fB_x}{{}_n{}^fP_x} \times {}_n{}^f\pi_x \right] \qquad \ldots(9{\cdot}98)$$

N.R.R. was first used by R. Blockh and was subsequently adopted by Kuczynski and many others. From practical point of view, formula (9·98) can be rewritten as :

$$\begin{aligned} \text{N.R.R.} &= k \sum_x [n\ ({}_ni_x) \times {}_n{}^f\pi_x] \\ &= k \sum_x [n \times (\text{Female Age-S.F.R.}) \times (\text{Survival Factor})] \qquad \ldots(9{\cdot}98a) \end{aligned}$$

summation being taken over all the age groups of reproductive span.

Remarks **1.** Since N.R.R. takes into account the mortality of the new born (female) babies, we get from (9·98)

$$\text{N.R.R.} \le k \sum_x n \left[\left(\frac{{}_n{}^fB_x}{{}_n{}^fP_x} \right) \right] \quad \Rightarrow \quad \text{N.R.R.} \le \text{G.R.R.} \qquad (\because\ {}_n{}^f\pi_x \le 1)$$

the sign of equality holding if and only if all the new born girls survive at least till the end of the reproductive period. Thus G.R.R. provides an upper limit to N.R.R. and hence, in theory, N.R.R. also ranges from 0 to 5 per annum.

2. It may be pointed out that out of a number of girls born to 1000 women, some die in infancy and some do not marry at all. Of the married women some become widows and it is only the balance who pass through the fertility period and thus add to the population growth. *Thus N.R.R. may be interpreted as the rate of replenishment of that population.*

3. If N.R.R. = 1, we may conclude that if the current fertility and female mortality rates prevail, in future, then a group of new born girls will exactly replace itself in the next generation. In other words, on the average, each of the females in the life table cohort will just be replaced by their daughters *i.e.*, the present female generation will exactly maintain itself. Thus in this case the population has a tendency to remain more or less constant. On the other hand if N.R.R. is greater than unity then the population has a tendency to increase while N.R.R. less than unity indicates a declining population. Accordingly, N.R.R. may be regarded as a good index of population growth.

4. It should be clearly borne in mind that the use of N.R.R. for population projections, *i.e.*, for forecasting future population changes is not desirable at all because of the following two reasons :

(*i*) It assumes that current mortality and fertility rates prevail in future, an assumption which is not true since in practice both these rates go on changing from time to time.

(*ii*) It overlooks the factor of migration. The population of a given region in any given period may be depleted more by emigration rather than by declining birth rate or it may increase as a result of fresh stock of immigrants who might be more *virile*.

Example 9·11. *The Table 9·19 gives the population of a country for the year 2001, together with the estimated number of births and deaths based on a special vital statistics enquiry conducted in the country. Calculate :*

(i) crude death rates for the total population and for males and females,

(ii) crude birth rate for the total population, *(iii) general fertility rate,*

(iv) total fertility rate, *(v) gross reproduction rate, and* *(vi) net reproduction rate.*

TABLE 9·20

Age (l.b.d)	Males		Females		Births		Survival Rates
	Population	Deaths	Population	Deaths	Males	Females	
0—4	442532	18623	434980	17308			
5—9	419042	1809	416736	1709			
10—14	393543	984	384616	1638			
15—19	308269	1233	314056	1329	3578	3343	0·914
20—24	257852	1289	269340	1481	7293	6690	0·899
25—29	230629	1776	236187	1677	6775	6361	0·844
30—34	204188	1633	203477	1465	4233	4187	0·868
35—39	182270	1588	176534	1289	2999	2685	0·852
40—44	162509	1967	145037	1233	593	725	0·834
45—49	128784	2138	122946	1352	129	128	0·819
50—54	102971	1905	96589	1188			
55—59	80717	2478	78311	1605			
60—64	58899	3099	58142	1980			
65—69	37797	2428	39099	2468			
70 and above	45099	5981	48866	7175			

TABLE 9·21 : COMPUTATION OF FERTILITY AND GROWTH RATES

	Males	Females	Total
Total Population	3055101	3024916	6080017
Deaths	48931	44897	93828
Births	25600	24119	49719

TABLE 9·21A : CALCULATIONS FOR G.F.R., T.F.R., G.R.R., N.R.R.

Age group	Population Female ${}_n{}^fP_x$	Births Male ${}_n{}^mB_x$	Births Female ${}_n{}^fB_x$	Total births ${}_nB_x = (3) + (4)$	Age S.F.R. ${}_ni_x = \frac{(5)}{(2)} \times 1000$	Female Age-S.F.R. ${}_n{}^fi_x = \frac{(4)}{(2)} \times 1{,}000$	Factor Survival ${}_n\pi_x$	${}_n{}^fi_x \times {}_n\pi_x$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9) = (7) × (8)
15—19	314056	3578	3343	6921	22·037	10·645	0·914	9·7295
20—24	269340	7293	6690	13983	51·916	24·838	0·899	22·3294
25—29	236187	6775	6361	13136	55·617	26·932	0·844	22·7306
30—34	203477	4233	4187	8420	41·381	20·577	0·868	17·808
35—39	176534	2999	2685	5684	32·198	15·210	0·852	12·9589
40—44	145037	593	725	1318	9·087	4·999	0·834	4·1692
45—49	122946	129	128	257	2·090	1·041	0·819	0·8526
Total	1467577	25600	24119	49719	214·326	104·242		90·6310

(i) C.D.R. for total population

$$= \frac{\text{Total number of deaths in the year}}{\text{Total population in the year}} \times 1{,}000 = \frac{93828}{6080017} \times 1000 = 15{\cdot}4322$$

$$\text{C.D.R. for males} = \frac{48931}{3055101} \times 1{,}000 = 16{\cdot}0162$$

$$\text{C.D.R. for females} = \frac{44897}{3024916} \times 1{,}000 = 14{\cdot}8424$$

(ii) C.B.R. for the total population $= \frac{\text{Total births}}{\text{Total Population}} \times 1000 = \frac{49719}{6080017} \times 1{,}000 = 8{\cdot}1774$

$$= \frac{\text{Total births} \times 1{,}000}{\text{(Total female population in the reproductive span)}}$$

(iii) $$\text{G.F.R.} = \frac{\sum_x {}_nB_x}{\sum_x {}_n^fP_x} \times 1{,}000$$

$$= \frac{49719}{1467577} \times 1{,}000 = 33{\cdot}8788$$

(iv) $\text{T.F.R} = 5 \sum_x {}_ni_x = 5 \times 214{\cdot}326 = 1071{\cdot}63$

(v) $\text{G.R.R.} = 5 \sum_x {}_ni_x = 5 \times 104{\cdot}242 = 521{\cdot}210$

(vi) $\text{N.R.R.} = 5 \sum ({}_n^fi_x \times {}_n\pi_x) = 5 \times 90{\cdot}6310 = 453{\cdot}155.$

Example 9·12. *Compute (i) G.F.R., (ii) S.F.R., (iii) T.F.R. and (iv) the gross reproduction rate, from the data given below :*

Age group of child bearing females	15—19	20—24	25—29	30—34	35—39	40—44	45—49
Number of women ('000)	16·0	16·4	15·8	15·2	14·8	15·0	14·5
Total Births	260	2244	1894	1320	916	280	145

Assume that the proportion of female births is 46·2 per cent.

Solution.

TABLE 9·22 : COMPUTATION OF G.F.R., S.F.R., T.F.R., AND G.R.R.

Age (year)	*Number of women ('000)* ${}_n^fP_x$	*Number of births* ${}_nB_x$	*Age-SFR* ${}_ni_x = \frac{{}_nB_x}{{}_n^fP_x} \times 1{,}000$
15—19	16·0	260	16·25
20—24	16·4	2244	136·83
25—29	15·8	1894	119·87
30—34	15·2	1320	86·84
35—39	14·8	916	61·89
40—44	15·0	280	18·67
45—49	14·5	145	10·00
Total	107·7	7059	450·35

(i) $$\text{G.F.R.} = \frac{\sum_x {}_nB_x}{\sum_x {}_n^fP_x} \times 1000$$

$$= \frac{7059}{107700} \times 1000$$

$= 65{\cdot}543$ per thousand

(ii) Age-S.F.R.'s are given in the last column of *Table 9·22*.

(iii) $\text{T.F.R.} = 5 \times \sum_x {}_ni_x$

$= 5 \times 450{\cdot}35 = 2251{\cdot}75$ per thousand.

(iv) In this case we cannot compute G.R.R. by formula (9·91), since we are not given the groupwise break up of female births. However, since it is given that the proportion of female births is 46·2%, we can obtain the value of G.R.R. by approximate formula (9·69), *i.e.*,

$$\text{G.R.R.} = \frac{\text{Number of female births}}{\text{Total births}} \times \text{T.F.R.} = \frac{46{\cdot}2}{100} \times 2251{\cdot}75 = 1040{\cdot}31 \text{ per thousand.}$$

Example 9·13. *Calculate the general fertility rate, total fertility rate and the gross reproduction rate from the following data, assuming that for every 100 girls 106 boys are born.*

Age of women	15—19	20—24	25—29	30—34	35—39	40—44	45—49
Number of women	212,619	198,732	162,800	145,362	128,109	106,211	86,753
Age-SFR (per 1000)	98·0	169·6	158·2	139·7	98·6	42·8	16·9

Solution. In the usual notations, we have age-specific fertility rate given by :

$${}_ni_x = \frac{{}_nB_x}{{}_n^fP_x} \times 1{,}000 \quad \Rightarrow \quad {}_nB_x = \frac{{}_ni_x \times {}_n^fP_x}{1000}$$

TABLE 9·23 : CALCULATIONS FOR G.R.R. AND T.F.R.

Age group	*Number of women* ${}_n^fP_x$	*Age S.F.R. (per 1000)* ${}_ni_x$	*Number of births* ${}_nB_x = \frac{(2) \times (3)}{1000}$
15—19	212,619	98·0	20836 ≃ 20837
20—24	198,732	169·6	33704·95 ≃ 33705
25—29	162,800	158·2	25754·96 ≃ 25755
30—34	145,362	139·7	20307·07 ≃ 20307
35—39	128,109	98·6	12631·55 ≃ 12631
40—44	106,211	42·8	4545·83 ≃ 4546
45—49	86,753	16·9	1466·13 ≃ 1466
Total	10,40,586	723·8	1,19,247

(i) $$\text{G.F.R.} = \frac{\sum_x {}_nB_x}{\sum_x {}_n^fP_x} \times 1000$$

$$= \frac{1{,}19{,}247}{10{,}40{,}586} \times 1{,}000$$

$= 114{\cdot}596$

$\text{T.F.R.} = 5 \times \sum_x {}_ni_x$

$= 5 \times 723{\cdot}8 = 3619$

Since it is given that for every 100 girls, 106 boys are born, an approximation to G.R.R. is given by :

$$\text{G.R.R.} = \frac{100}{100 + 106} \times \text{T.F.R.}$$

$$= \frac{100}{206} \times 3619 = 1756{\cdot}7961.$$

Example 9·14. *From the data given in Table 9·24, calculate the gross re-production rate and the net reproduction rate.*

TABLE 9·24

Age-group	*Number of children born to 1,000 women passing through the age-group*	*Mortality rate (per 1000)*
16—20	150	120
21—25	1500	180
26—30	2000	150
31—35	800	200
36—40	500	220
41—45	200	230
46—50	100	250

Sex ratio being males : females : : 52 : 48.

Solution. TABLE 9·24(A) : CALCULATIONS FOR G.R.R. AND N.R.R.

Age-group	*Number of children born to 1000 women passing through the age-group*	*Number of female children* ${}_nfB_x$ *= (48%) of the (2)*	*Survival rate :* ${}_n\pi_x = 1 -$ *[Mortality rate per woman]*	*Number of female children survived* $= {}_nfB_x \times {}_n\pi_x$
(1)	(2)	(3) = 0·48 × (2)	(4)	(5) = (3) × (4)
16—20	150	150 × 0·48 = 72	1 − 0·12 = 0·88	63·36
21—25	1,500	1500 × 0·48 = 720	1 − 0·18 = 0·82	590·40
26—30	2,000	2000 × 0·48 = 960	1 − 0·15 = 0·85	816·00
31—35	800	800 × 0·48 = 384	1 − 0·20 = 0·80	307·20
36—40	500	500 × 0·48 = 240	1 − 0·22 = 0·78	187·20
41—45	200	200 × 0·48 = 96	1 − 0·23 = 0·77	73·92
46—50	100	100 × 0·48 = 48	1 − 0·25 = 0·75	36·00
Total	5,250	2520		2074·08

∴ Gross Reproduction Rate per woman

$$= \frac{\text{Total number of female children born}}{1000} = \frac{2520}{1000} = 2{\cdot}52$$

Net Reproduction Rate per woman

= [Total number of female children born and survived to 1000 women] ÷ 1000

$$= \frac{\sum {}_nfB_x \times {}_n\pi_x}{1000} = \frac{2074{\cdot}08}{1000} = 2{\cdot}07.$$

9·9. GRADUATION OF MORTALITY RATES

The computation of the age-SDR's m_x for any population from the causes data or sample registration data is subject to a number of irregularities. In order to use these rates for further mathematical work, specially in the construction of life tables, we should smooth out these irregularities. hence, we need to obtain some explicit expressions for m_x in terms of x. In practice, instead we try to obtain an explicit expression for μ, the force of mortality at age x, [as defined in (9·52)] which is related to m_x by the relation :

$$m_x \simeq \mu_{x+\frac{1}{2}} \qquad \text{[For proof, see (9·53), (9·5·4)]}$$

9·9·1. Makham's Graduation Formula. A number of attempts have been made to develop a suitable formula for μ_x, from time to time but the most successful of them seems to be one due to British actuary Makeham.

Makeham assumes that deaths occur to two causes: (*i*) Accidents and (*ii*) Diseases.

He further assumes that :

(*a*) The effect of accidents is constant throughout the life span because although, the younger people are more active, dynamic and adventurous than the older people, thus being more prone to accidents, they have greater recovery rate than the older people.

(*b*) The capacity of the human body to resist diseases decreases as age increases. In other words, the force of mortality would vary inversely as some function of age x, say, $g(x)$, which represents the force of resistance to disease.

Consequently, he takes : $$\mu_x = A + \frac{B}{g(x)}, \qquad \text{...(9·99)}$$

where $A > 0$, $B > 0$ and $g(x)$ is a decreasing function of x.

Makeham further assumes that in a short span, a person loses a constant proportion ($r > 0$) of such force of resistance to disease as he/she still has. In other words, Makeham takes :

Instant rate of decrease in $g(x)$ = constant

$$\Rightarrow \quad -\frac{1}{g(x)}\cdot\frac{d}{dx}(g(x)) = r \quad \text{or} \quad \frac{1}{g(x)}\cdot\frac{d}{dx}[g(x)] = -r,\ r > 0$$

Integrating *w.r.t.* x, we get

$$\log g(x) = -rx + c_1, \text{ where } c_1 \text{ is the constant of integration.}$$

$$\Rightarrow \quad g(x) = e^{c_1 - rx} \quad \text{or} \quad g(x) = c_2\, e^{-rx},\ (r > 0)$$

where $c_2 = e^{c_1}$ is arbitrary constant. ...(9·100)

Substituting the value of $g(x)$ in (9·99), we get

$$\mu_x = A + \frac{B}{c_2\ e^{-rx}} = A + Dc^x \qquad \text{...(9·100a)}$$

where A and $D = B/c_2$ are arbitrary constants and $c = e^r > 0$.

$$\Rightarrow \quad -\frac{1}{l_x}\cdot\frac{d}{dx}(l_x) = A + Dc^x \qquad \text{[From (9·52)]}$$

Integrating *w.r.t.* x, we get

$$\log(l_x) = -\left[Ax + \frac{Dc^x}{\log_e c} + E\right] \Rightarrow l_x = e^{-E}.(e^{-A})^x.[e^{-D/\log_e c)}]^{c^x} \quad i.e., \quad l_x = k.\, s^x\, p^{c^x} \qquad \text{...(9·101)}$$

where $k = e^{-E}$, $s = e^{-A}$, c and $p = e^{-D/\log c}$ ...(9·101*a*) are arbitrary constants, termed as the four parameters of the function l_x.

The formula (9·101), known as *Makeham's formula*, is used to graduate the l_x figures in a life table, from which the force of mortality μ_x can be computed.

Remark. Makeham's mortality law (1860) is given by [*c.f.* (9·100*a*)].

$$\mu_x = A + D\,c^x \qquad \text{...(9·101b)}$$

In Makeham's models (9·101*b*), the parameters A, D and c, usually lie in the following ranges.

$$0{\cdot}001 < A < 0{\cdot}003 \quad ; \quad 10^{-6} < D < 10^{-3} \quad ; \quad 1{\cdot}08 < c < 1{\cdot}12 \qquad \text{...(9·101c)}$$

9·9·2. Gompertz Makeham, Formula For Mortality Prior to Makeham, Gompertz had given the idea of mortality by considering only the force of resistance to diseases exactly in the same manner as Makeham did but he completely ignored the factor of accidents. This leads to :

$$A = 0 \text{ in (9·101)} \quad \Rightarrow \quad s = e^{-A} = 1$$

Hence, taking $s = 1$ in (9·101), the Gompertz graduation formula for l_x becomes :

$$l_x = k\,p^{c^x}, \qquad \text{...(9·102)}$$

which is a three parameter (k, p and c) function of x.

Remark. **Taking $A = 0$ in (9·100*a*), Gompertz mortality law (1825) is given by :**

$$\mu_x = Dc^x \qquad \text{...(9·102a)}$$