Index Number

- Most widely used **statistical devices** and there is **hardly any field** where they are not used .

- Newspapers headline the fact that **price are going up/ down** that **industrial production is rising/falling** , that **imports/Exports are increasing/decreasing**, that **crime are rising/falling** in a particular period **compared to** the **previous period** as disclosed by index number.

- They are described as '**Barometer of Economic Activity**'i.e if one wants to get an idea as to **what is happening** to an **economy**. He should look to important indices like the index number of **industrial production** , **agricultural production** , business activity, **Stock Market Index(Sensex & Nifty)** Etc.

58K 2020 33K

- Index number are **devices** for **measuring difference in the magnitude of a group of related variables.**

- An **Index number** is a **statistical measure** designed to **show changes in a variable** or a **group of related variable** with **respect to time, geographic** or other location or other characteristic such as income, profession etc. Ex Petrol Price.

- An Index number is nothing more than **a relative number**, or a 'relative' which **expresses the relationship between two figures** where one of the figure is used as a base.

- An index number is **a specialized average** designed to **measure the change in a group of related variable over a period of time.**

- Ex- When we say that the **index number of wholesale prices is 112** for january 2005 compared to january 2004, it means there is net increase in the price of wholesale commodity to the extent of 12 % during the year.

Index Number (Features)

- Index Number are **specialized average.**
- Index numbers measures the **Net Change in a group of related variables.(Ex- Sensex(30) & Nifty(50))**

BSE

NSE

- Index number measures the **Effect of change over a period of time.**

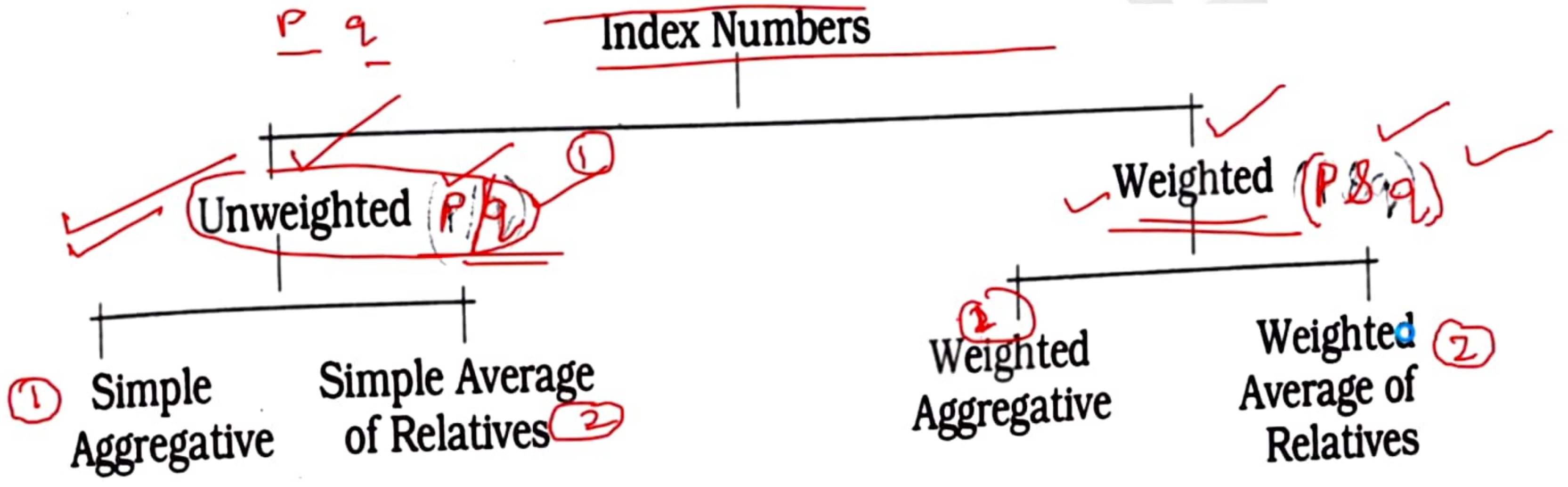
100%

0% 150 200%

Uses Of Index Number

- They help in **framing** suitable **policies**. DA
- They reveal **trends and tendencies .**
- They are important in **forecasting future economic activity.**

Methods of Constructing Index Number



- In the unweighted indices **weights are not expressly assigned** whereas in the weighted indices **weights are assigned** to the various items.

Unweighted Index Numbers(p/q)

1. Simple Aggregative Method This is the simplest method of constructing index numbers. When this method is used to construct a price index, the total of current year prices for the various commodities in question is divided by the total of base year prices and the quotient is multiplied by 100. Symbolically :

$$P_{01} = \frac{\Sigma P_1}{\Sigma P_0} \times 100.$$

ΣP_1 = total of current year prices for various commodities.
ΣP_0 = total of base year prices for various commodities.

This method of constructing the index is the simplest of all the methods. The steps required in computation are:

(i) Add the current year prices for various commodities, *i.e.*, obtain ΣP_1.
(ii) Add the base year prices for the same commodities, *i.e.*, obtain ΣP_0.
(iii) Divide ΣP_1 by ΣP_0 and multiply the quotient by 100.

Illustration 1. From the following

Commodity	Price in 2002 (Rs.)	Price in 2003 (Rs.)
A	50	70
B	40	60
C	80	90
D	110	120
E	20	20

20%

Solution. CONSTRUCTION OF PRICE INDEX

Commodity	Price in 2002 P_0	Price in 2003 P_1
A	50	70
B	40	60
C	80	90
D	110	120
E	20	20
	$\Sigma P_0 = 300$	$\Sigma P_1 = 360$

$$P_{01} = \frac{\Sigma P_1}{\Sigma P_0} \times 100 = \frac{360}{300} \times 100 = 120$$

20%

This means that as compared to 2002, in 2003 there is a net increase in the prices of commodities included in the index to the extent of 20%.

Limitations

- The **units used** in the **price or quantity** quotations can exert a **big influence** on the **value of the index**.(Kg/Pound/Per quintal).

- **No consideration** is given to the **relative importance** of the commodities.

- This index is based on the **assumption** that the various terms and their prices are quoted in the **same unit**.

2. Simple Average Of Price Relatives Method

2. Simple Average of Price Relatives Method When this method is used to construct a price index, first of all price relatives* are obtained for the various items included in the index and then average of these relatives is obtained using any one of the measures of central value, *i.e.*, arithmetic mean, median, mode, geometric mean or harmonic mean. When arithmetic mean is used for averaging the relatives, the formula for computing the index is :

$$\boxed{P_{01} = \frac{\Sigma\left(\frac{P_1}{P_0} \times 100\right)}{N}}$$

where N refers to the number of items (commodities) whose price relatives are thus averaged.

A price relative is the ratio of the price of a single commodity in a given period to its price in another period called the base period or reference period. If p_0 and p_n denote the commodity price during the base period and given period respectively, then by definition:

$$\text{price relative} = P_n / P_0$$

adding generally expressed as percentage by multiplying by 100.

If Geometric Mean

If G. M is used for averaging the price relative the formula will be–

$$P_{01} = \text{antilog}\left[\frac{\left(\Sigma \log \dfrac{P_1}{P_0} \times 100\right)}{N}\right] = \text{antilog}\ \frac{\Sigma \log P}{N}$$

Illustration 3. From the following data, construct an index for 2003 taking 2002 as base by the average of relatives method using (*a*) arithmetic mean, and (*b*) geometric mean for averaged relatives :

Commodity	Price in 2002 (Rs.)	Price in 2003 (Rs.)
A	50	70
B	40	60
C	80	90
D	110	120
E	20	20

Solution. (*a*) INDEX NUMBERS USING ARITHMETIC MEAN OF PRICE RELATIVES

Commodity	Price in 2002 (Rs.) P_0	Price in 2003 (Rs.) P_1	Price relatives $\frac{P_1}{P_0} \times 100$
A	50	70	140.0
B	40	60	150.0
C	80	90	112.5
D	110	120	109.1
E	20	20	100.0
			$\Sigma \frac{P_1}{P_0} \times 100 = 611.6$

611.6

$$\boxed{P_{01} = \frac{\Sigma \frac{P_1}{P_0} \times 100}{N}} = 122.32$$

(b) INDEX NUMBERS USING GEOMETRIC MEAN OF PRICE RELATIVES

Commodity	Price in 2002 P_0	Price in 2003 P_1	Price Relatives P $\frac{P_1}{P_0} \times 100$	Log P
A	50	70	140.0	2.1461
B	40	60	150.0	2.1761
C	80	90	112.5	2.0512
D	110	120	109.1	2.0378
E	20	20	100.0	2.0000
				$\Sigma \log P = 10.4112$

$$P_{01} = \text{Antilog}\left[\frac{\Sigma \log P}{N}\right] = \text{Antilog}\left[\frac{10.4112}{5}\right] = \text{Antilog } 2.0822 = 120.9$$

Although arithmetic mean and geometric mean have both been used, the arithmetic mean is often preferred because it is easier to compute and much better known. Some economists, notably F.Y. Edgeworth, have preferred to use the median which is not affected by extreme values. Since the argument is important only when an index is based on a very small number of commodities, it generally does not carry much weight and the median is seldom used in actual practice.

Merits & Limitations

- **Extreme items do not influence** the index. Equal importance is given to all the items.

- The index is **not influenced by the units in which prices are quoted** or by the absolute level of individual price . Relatives are pure numbers and are, therefore divorsed from original units.

$\frac{P_1}{P_0}$

- (–)**Difficulty** is faced with regard to the **selection of an appropriate average**. The use of the arithmetic mean is considered as questionable sometimes because it has an **upward bias**. The use of geometric mean involves difficuties of computation. Other average are almost never used while constructing index number.

Weighted Index Number

6

Weighted Aggregative Indices

Weighted Average Of Relative

- **Unweighted Index Number**.–They assign equal importance to all the items included in the index.
-
- **Weighted Index Number**– Conscious effort to **assign to each commodity a weight** in accordance with its **importance** in the total phenomenon that the index is supposed to describe.

Weighted Aggregative Indices(p & q)

Weighted Aggregative Indices These indices are of the simple aggregative type with the fundamental difference that weights are assigned to the various items included in various methods of assigning weights and consequently a large number of formulae for constructing index numbers have been devised of which some of the more important ones are:

- Laspeyres method
- Paasche method
- Dorbish and Bowley's method
- Fisher's ideal method
- Marshall-Edgeworth method, and
- Kelly's method.

$$\frac{\Sigma P_1 W}{\Sigma P_0 W}$$

1.Laspeyres Method

Laspeyres Method. The Laspeyers Price Index is a weighted aggregate price index, where the weights are determind by quantities in the base period. The formula for constructing the index is :

$$P_{01} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100$$

$$P_{01} = \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0}$$

W

Steps :

- Multiply the current year prices of various commodities with base year weights and obtain $\Sigma p_1 q_0$.
- Multiply the base year prices of various commodities with base year weights and obtain $\Sigma p_0 q_0$.
- Divide $\Sigma p_1 q_0$ by $\Sigma p_0 q_0$ and multiply the quotient by 100. This gives us the price index.

- Laspeyres index attempts to answer the question: " **What is the change in aggregate value of the base period list of goods when valued at given period price ?** q_0 $q \to q_0$

- The Laspeyres Index has an **upward bias**. When **price increase** , there is tendency to **reduce the consumption** of higher priced items. Hence by using **base year weights , to much weights will be given to those items which have increased in price the most**. Similarily when **price decline** , **consumer shift** their purchases to those items which decline the most . So **it overstate the index.**

2. Paasche Method

(ii) *Paasche's Method** The Paasche price index is a weighted aggregate price index in which the weights are determined by quantities in the given year. The formula for constructing the index is :

$$P_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$$

W

$$\frac{\Sigma P_1 q_1}{\Sigma P_0 q_1}$$

$W = q_0$

$W = q_1$

2. Paasche Method

- Disadvantage– the difficulty in computing the Passche index in practice is that **revised weights, or quantities , must be computed each year** or each period, adding to the **data collection expense** in the preparation of the index .

- For this reason , **the Paasche Index is not used frequently in practice** where the number of commodities is large.

3. Dorbish & Bowley's Method

(iii) *Dorbish and Bowley's Method.* Dorbish and Bowley have suggested simple arithmetic mean of the two indices (Laspeyres and Paasche) mentioned above so as to take into account the influence of both the periods, *i.e.*, current as well as base periods. The formula for constructing the index is :

$$P_{01} = \frac{L+P}{2} \times 100$$

where L = Laspeyres Index, P = Paasche Index

Or

$$P_{01} = \frac{\dfrac{\Sigma p_1 q_0}{\Sigma p_0 q_0} + \dfrac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}{2} \times 100$$

4. Fisher's Ideal Index

(iv) *Fisher's 'Ideal' Index.* Prof. Irving Fisher has given a number of formulae for constructing index number and of these he calls one as the 'ideal' index. The Fisher's Ideal Index is given by the formula :

$$P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100 \text{ or } P_{01} = \sqrt{L \times P}$$

- Fisher's Ideal Index is the **geometric mean of the Laspeyres & Paasche indices.**

- This Formula is known as 'Ideal' because of t following reasons:
- 1) It is based on the **geometric mean** which is theoretically considered to be the **best average** for constructing index number.
- 2) It takes into account **both current year as well as base year prices and quantities**.
- 3) It satisfies both the **time reversal test as well as factor reversal test.**

Factor reversal test is satisfied when

$$P_{01} \times Q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

Time reversal test is satisied when

$$P_{01} \times P_{10} = 1$$

12:46 53%

5. Marshall-Edgeworth Method

$$P_{01} = \frac{\Sigma (q_0 + q_1) p_1}{\Sigma (q_0 + q_1) p_0} \times 100$$

$$\frac{\Sigma P_1 (q_0 + q_1)}{\Sigma P_0 (q_0 + q_1)}$$

6. Kelly's Method

$$P_{01} = \frac{\sum p_1 q}{\sum p_0 q} \times 100; \text{ where } q = \frac{q_0 + q_1}{2}$$

Weighted Aggregative Indices

(i) Laspeyre's Index

$$P_{01} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100$$

$W = q_0$ $\quad \frac{\Sigma P_1 W}{\Sigma P_0 W}$

(ii) Paasche's Index

$$P_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100$$

$W = q_1$

4

(iii) Bowley's Index

$$P_{01} = \frac{L + P}{2}$$

A M

(iv) Fisher's Ideal Index

$$P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100$$

G.M

(v) Marshall-Edgeworth's Index

$$P_{01} = \frac{\Sigma p_1 (q_0 + q_1)}{\Sigma p_0 (q_0 + q_1)} \times 100$$

$W = (q_0 + q_1)$

(vi) Kelly's Index

$$P_{01} = \frac{\Sigma p_1 Q}{\Sigma p_0 Q} \times 100$$

$W = Q$ $\quad Q = \frac{q_1 + q_0}{2}$

Illustration 5. Construct index numbers of price from the following data by applying :

1. Laspeyres method,
2. Paasche method,
3. Bowley's method,
4. Fisher's Ideal method, and
5. Marshall-Edgeworth method.

Commodity	2002 Price (p_0)	2002 Quantity (q_0)	2003 Price (p_1)	2003 Quantity (q_1)
A	2	8	4	6
B	5	10	6	5
C	4	14	5	10
D	2	19	2	13

Solution.

CALCULATION OF VARIOUS INDICES

Commodity	2002 Price p_0	2002 Qty q_0	2003 Price p_1	2003 Qty. q_1	$p_1 q_0$	$p_0 q_0$	$p_1 q_1$	$p_0 q_1$
A	2	8	4	6	32	16	24	12
B	5	10	6	5	60	50	30	25
C	4	14	5	10	70	56	50	40
D	2	19	2	13	38	38	26	26
					$\Sigma p_1 q_0 = 200$	$\Sigma p_0 q_0 = 160$	$\Sigma p_1 q_1 = 130$	$\Sigma p_0 q_1 = 103$

1. *Laspeyres Method* : $P_{01} = \dfrac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100$; where $\Sigma p_1 q_0 = 200$, $\Sigma p_0 q_0 = 160$

$$P_{01} = \frac{200}{160} \times 100 = 125.$$

2. *Paasche's Method* : $P_{01} = \dfrac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100$; where $\Sigma p_1 q_1 = 130$, $\Sigma p_0 q_1 = 103$

$$P_{01} = \frac{130}{103} \times 100 = 126{\cdot}21$$

Accounting Masterclass

3. ***Bowley's Method*** :

$$P_{01} = \frac{\dfrac{\Sigma p_1q_0}{\Sigma p_0q_0} + \dfrac{\Sigma p_1q_1}{\Sigma p_0q_1}}{2} \times 100 = \frac{\dfrac{200}{160} + \dfrac{130}{103}}{2} \times 100$$

$$= \frac{1.25 + 1.262}{2} \times 100 = \frac{2.512}{2} \times 100 = 125.6.$$

or

$$P_{01} = \frac{L + P}{2} = \frac{125 + 126.2}{2} = 125.6$$

4. ***Fisher's Ideal Method*** :

$$P_{01} = \sqrt{\frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times \frac{\Sigma p_1q_1}{\Sigma p_0q_1}} \times 100 = \sqrt{\frac{200}{160} \times \frac{130}{102}} \times 100$$

$$= \sqrt{1.578} \times 100 = 1.256 \times 100 = 126.6$$

5. ***Marshall-Edgeworth Method*** :

$$P_{01} = \frac{\Sigma (q_0 + q_1)p_1}{\Sigma (q_0 + q_1)p_0} \times 100 = \frac{\Sigma p_1q_0 + \Sigma p_1q_1}{\Sigma p_0q_0 + \Sigma p_0q_1}$$

$$= \frac{200 + 130}{160 + 103} \times 100 = \frac{330}{263} \times 100 = 125.47$$

2. Weighted Average Of Relative

- In the **weighted aggregative method** discussed above **price relatives were not computed**. However like Unweighted relative method it is also possible to compute weighted average of relative .
- For purpose of averaging we may **use either the arithmetic mean or the geometric mean**.

$$P_{01} = \frac{\sum PV}{\sum V}, \text{ where } P = \text{Price Relative}$$

$$\frac{p_1}{p_0} \times 100$$

*V = Value Weights, *i.e.*, p_0q_0

A price relative is the ratio of the price of a single commodity in a given period to its price in another period called the base period or reference period. If p_0 and p_n denote the commodity price during the base period and given period respectively, then by definition:

$$\text{price relative} = P_n / P_0$$

adding generally expressed as percentage by multiplying by 100.

Illustration 8. From the following data compute price index by supplying weighted average of price method using :

(*a*) arithmetic mean, and
(*b*) geometric mean.

Commodity	p_0 *(Rs.)*	q_0	p_1 *(Rs.)*
Sugar	3.0	20 kg.	4.0
Flour	1.5	40 kg.	1.6
Milk	1.0	10 lt.	1.5

Solution.

(*a*) INDEX NUMBER USING
WEIGHTED ARITHMETIC MEAN OF PRICE RELATIVES

Commodity	p_0	q_0	p_1	p_0q_0 V	$\frac{p_1}{p_0} \times 100$ P	PV
Sugar	Rs. 3.0	20 kg.	Rs. 4.0	60	$\frac{4}{3} \times 100$	8,000
Flour	Rs. 1.5	40 kg.	Rs. 1.6	60	$\frac{1.6}{1.5} \times 100$	6,400
Milk	Re. 1.0	10 lt.	Rs. 1.5	10	$\frac{1.5}{1.0} \times 100$	1,500
				$\Sigma V = 130$		$\Sigma PV = 15{,}900$

$$p_{01} = \frac{\Sigma PV}{\Sigma V} = \frac{15{,}900}{130} = 122.31$$

This means that there has been a 22.3 per cent increase in prices over the base level.

(b) INDEX NUMBER USING GEOMETRIC MEAN OF PRICE RELATIVES

Commodity	p_0	q_0	p_1	V	p	Log p	V Log p
Sugar	Rs. 3.0	**20 kg.**	Rs. 4.0	60	133.3	2.1249	127.494
Flour	Rs. 1.5	40 kg.	Rs. 1.6	60	106.7	2.0282	121.692
Milk	Re. 1.0	10 lt.	Rs. 1.5	10	150.0	2.1761	21.761
				$\Sigma V = 130$			$\Sigma V \log p = 270.947$

$$p_{01} = \text{Antilog}\left[\frac{\Sigma V \log p}{\Sigma V}\right] = \text{Antilog}\left[\frac{270.947}{130}\right] = \text{Antilog } 2.084 = 120.9$$

Imp Q–Which Weighted Aggregative Method gives the same result as by the Weighted Arithmetic Mean of Price Relative Method ?

The result obtained by applying the Laspeyres method would come out to be the same as obtained by weighted arithmetic mean of price relatives method as shown below :

PRICE INDEX BY LASPEYRES METHOD

Commodity	p_0	q_0	p_1	$p_1 q_0$	$p_0 q_0$
Sugar	Rs. 3.0	20 kg.	Rs. 4.0	80	60
Flour	Rs. 1.5	40 kg.	Rs. 1.6	64	60
Milk	Re. 1.0	10 lt.	Rs. 1.5	15	10
				$\Sigma p_1 q_0 = 159$	$\Sigma p_0 q_0 = 130$

$$p_{01} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100 = \frac{159}{130} \times 100 = 122.3$$

The answer is the same as that obtained by weighted arithmetic mean of price relative method. This is because the weighted average of price relative method can be transferred to simple aggregative method (given by Laspeyers) as follows :

$$\frac{\Sigma \frac{p_1}{\cancel{p_0}} \times \cancel{p_0} q_0}{\Sigma p_0 q_0} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}$$

$\frac{\Sigma RV}{\Sigma V}$

Value Index Number

pq

- The **value** of a single commodity is the **product of its price and quantity** . Thus a Value Index V is the **sum of the value of a given year divided by the sum of the value of the base year.**

$$V = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} \times 100,\ V = \text{Value index}$$

where $\Sigma p_1 q_1$ = Total value of all commodities in the given period, and
$\Sigma p_0 q_0$ = Total value of all commodities in the base period.

Since in most cases the value figures are given the formula can be stated more simply

$$V = \frac{\Sigma V_1}{\Sigma V_0}$$

$$\frac{\Sigma V_1}{\Sigma V_0}$$