Assignment III

Answer Any Five Questions

Q1. A potato (same parameters as discussed in the class) at room temperature (which is to be varied between 10 degrees and 30 degrees Celsius) is put inside boiling water until the temperature at its centre reaches 70 degrees Celsius. It is then removed from inside the boiling water and allowed to cool until its temperature at the centre reaches 5 degrees above room temperature. Assume that cooling takes place only by heat diffusion outward from the surface, that is, there is no radiation cooling.

a) Find the time required for each process.

b) Plot temperature as a function of r both while heating as well as cooling, at five time instants during each process.

c) Plot the temperature at the centre as a function of time during the entire process.

Q2) Consider a string (same dimensions, mass/length) as discussed in the class. Both ends of the string are completely free to move vertically. Generate a pulse at rest at the centre and plot it at several time instants, after repeated reflections at the two ends.

Q3) Consider a string fixed at both ends. There is a heavy mass at position $3L/4$ (the problem discussed in the class). Generate a right moving pulse at the centre and study its reflections and transmissions at the heavy mass.

Q4. Consider a random walker who takes N steps. The probability that he/she takes a step to the right and left depends on the number of steps, n, that he is already away from the starting point. They are given as

$$p_R = \frac{1}{2(1-n/2N)} \qquad n \geq 0$$
$$= 1/2 \qquad n < 0$$

$$p_L = 1 - p_R$$

Obtain the distribution function for $P(m)$, the probability that he will be finally m steps away from the starting point. If his/her home is N/2 steps away, what is the probability that he/she will reach home.

Q5. Consider the PDF

$$P(x) = x e^{-x^2} \qquad 0 \leq x \leq 2$$

Generate a large number of random numbers obeying this PDF, plot their probability distribution and compare with the given PDF. The PDF should be normalized first.

Q6. Calculate the electric potential of an ellipsoid, the lower half of which carries a uniform charge density – 1 and the upper half a uniform density +1, as a function of r in different directions, and

compare that with the potentials of a dipole of the same moment as the ellipsoid. The equation of the ellipsoid is

$$\frac{x^2}{1} + \frac{y^2}{1} + \frac{z^2}{2} = 1$$

Do your calculations using Monte-Carlo integration.

Q7. Consider a disc of radius 1 unit. Its first, second, third and fourth quadrants are alternately positively and negatively charged, with density +1, -1, +1 and -1 respectively. Plot its potential distribution as a function of r in different directions from the centre. Compare the distribution with that for the potential from the quadrupole moment of the disc.

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