

$$4) P_i(x) = a_i(x-x_i)^2 + (x-x_i)b_i + c_i$$

a) Conditions

$$1) P_i(x_i) = y_i \quad i=1, 2, \dots, N-1$$

$$2) P_i(x_{i+1}) = y_{i+1} \quad i=1, 2, \dots, N-1$$

$$3) P_i'(x_{i+1}) = P_{i+1}'(x_{i+1}) \quad i=1, 2, \dots, N-2$$

$$\text{Also } x_{i+1} - x_i = h$$

From 1

$$y_i = c_i$$

From 2

$$\begin{aligned} y_{i+1} &= a_i h^2 + b_i h + c_i \\ &= a_i h^2 + b_i h + y_i \end{aligned}$$

$$\Rightarrow a_i = (y_{i+1} - y_i)/h^2 - b_i/h \quad (1)$$

From 3

$$2a_i h + b_i = b_{i+1}$$

Using (1)

$$2(y_{i+1} - y_i)/h - 2b_i + b_i = b_{i+1}$$

$$\Rightarrow b_{i+1} + b_i = 2(y_{i+1} - y_i)/h \quad i=1, 2, \dots, N-2$$

$$b) \text{ If } P_i''(x_1) = 0 \Rightarrow a_1 = 0$$

Using (1)

$$a_1 = (y_2 - y_1)/h^2 - b_1/h = 0$$

$$\Rightarrow (y_2 - y_1)/h = b_1$$