Assignment II

Q1a) Consider the following parametric curve, known as the Cornu's spiral

$$x(t) = \int_0^t \cos(\pi u^2/2)\, du \qquad y(t) = \int_0^t \sin(\pi u^2/2)\, du$$

where, the parameter t can take any real value between $-\infty$ and $+\infty$. Note that both x(t) and y(t) are odd functions of t, so that one needs to compute the integrals only for positive t.

Carry out the integrals by using the method of Gaussian quadrature for t up to 50 and plot x vs y for $t \in [-50, 50]$ to obtain the spiral.

b) Using the integrals, obtain the diffraction pattern due to a sharp edge.

Q2a) Consider the elliptic integral of the first kind:

$$F(k,\phi) = \int_0^\phi d\phi' / \sqrt{1 - k^2 \sin^2 \phi'} \qquad (k \;\&\; \phi \text{ are constants})$$

Write a function to numerically evaluate the elliptic integral for given values of $k \,\&\, \phi$, by using any of the numerical integration function.

b) Consider a simple pendulum, whose exact equation of motion is

$$\ddot{\theta} = -\,\omega_0^2 \sin\theta \qquad \omega_0^2 = g/\ell$$

Solve this equation numerically (it cannot be solved exactly) when the pendulum is released from rest at angle α with the vertical. Take $\alpha = 10^0, 20^0, ...170^0$, that is at intervals of 10 degrees and plot a graph of time period vs. Angle of release. You may take a meters pendulum $(\ell = 1\text{m})$.

c) The time period of a pendulum can be expressed in terms of the elliptic integral as follows.

$$T = 4\sqrt{\frac{\ell}{g}}\; F(k, \pi/2) \qquad \text{where, } k = \sin(\alpha/2)$$

Plot the time period vs. α using your function of part a) and compare with the similar plot of part b.

Q3) Write a function for second-order predictor corrector method for solving initial value differential equation. Apply it to solve some second-order differential equation of your choice

Q4) Consider a double pendulum, in which two identical pendulums are hung one below the other. Each pendulum consists of a mass less rod and a bob of mass m. The generalised coordinates are $\theta_1 \;\&\; \theta_2$, the angles made by the two pendulums with the vertical.

Write down the equations of motion for θ_1 & θ_2 and solve them using RK4 method for different initial conditions. Graphically display the configurations of the pendulums at several time instants for the initial conditions : $\theta_1(0) = 30^0$, $\theta_2(0) = 60^0, \dot{\theta}_1(0) = \dot{\theta}_2(0) = 0.$

Q5) Obtain the trajectory of a particle moving under the Yukawa potential

$$V(r) = -k\frac{e^{-\alpha r}}{r}$$

by numerically integrating the equations of motion. Use RK4 method.

Q6) Consider the motion of a charged particle of charge q and mass m in crossed electric and magnetic fields

$$\vec{E} = E_0\hat{x} \qquad \vec{B} = -B_0\hat{z}$$

a) Write down the equations of motion in three dimensional space and reduce them to a system of first order equation.

b) Write a program to solve the equations of motion with initial conditions.

c) If the particle is released at rest at the point x = y = z = 0, then calculate and plot the trajectory on the x-y plane. Study the motion by changing the values of E_0 & B_0.

d) By putting $E_0 = 0$, obtain the helical path of the particle around the magnetic field.

Q7) Consider a hanging chain. The differential equation for the shape of the chain is

$$\frac{d^2y}{dy^2} = a\sqrt{1+y'^2}$$

Here, x axis is the horizontal axis, y axis is the vertically up axis, and 'a' is a constant equal to μ/T , μ being mass per unit length of the chain and T being the tension at the bottom of the chain. Obviously, 'a' is not known.

The chain is hung from the two points (0,0) and (10,4). Take the length of the chain to be two times the straight-line distance between the points.

Solve the differential equation as a boundary value problem using the shooting method. In order to solve it, assign some estimated value to a. From the solution you obtained, determine the length of the chain. Keep on changing the value of 'a', until you get the given length of the chain. Plot the contour of the chain.

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