PORTFOLIO SELECTION FOR MANAGEMENT OF FOREIGN EXCHANGE RESERVES

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CONTENTS

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Section I

I.1 Introduction

Effective management of a country's foreign exchange reserves forms a crucial responsibility of a central bank, drawing from its role as the custodian of the reserves for the nation. Essentially, reserves management means making a choice between alternative portfolios. Portfolio management has developed over the years into a highly sophisticated art despite the fact that the basic principle of portfolio management, *viz.*, to strike a balance among three fundamental considerations of liquidity, safety and yield continues to be the same. Several developments in the international financial markets, beginning with sharper exchange rate volatility consequent upon the major currencies going into generalised floating since the mid-seventies to surges in private capital flows experienced from the late eighties, have resulted in significant increase in the size and nature

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1

of risks in the financial markets. Two important consequences, each having a bearing on reserves management, followed : first, unlike in the Bretton Woods era when the US dollar provided international liquidity and the US market the instruments for deployment of reserves, the world moved over in the eighties to a multi-currency liquidity system and several other markets grew in size and sophistication. There was the emergence of the Deutsche mark and the Japanese yen as major reserve currencies along with the US dollar. In fact, the share of the US dollar, which was about 84 per cent in 1973, fell to 59 per cent in 1996. Second, increased volatilities in exchange rate, interest rate and prices of commodities like gold and oil saw tremendous strides being made in development of risk management instruments through the so-called derivatives revolution. During the Bretton Woods era, reserves management for most of the countries meant keeping balances at the Federal Reserve and Bank of England with arrangements for investing the same in short-term bank deposits/government paper. Reserves management was, therefore, considered a passive affair with hardly any focus on risk and return. From the mid-seventies, however, the situation has undergone a noticeable change because of the deregulation of exchange rates and interest rates, leading to wider fluctuations in rates on the one hand, and redistribution of world reserves among the groups of countries, on the other. Both the Federal Reserve and Bank of England started to impress upon countries (whose reserves were hitherto being largely maintained with them) to take direct responsibility for management of their reserves. A few international security houses and banks visualised considerable business opportunities as more and more countries with large reserves could use the international currency and fixed income markets in an increasingly active fashion for managing their reserves directly. However, there was a large gap in terms of knowledge, capability and expertise among the central banks to be able to operate

in international markets. Some of these securities houses and banks provided a lead role in building up expertise in central banks on portfolio management. Ever since, there has flourished a good deal of theoretical work in the area of reserves management, enabling central banks to gain useful insights into some of the vital issues such as the purpose of holding reserves and the optimal size thereof, the desirable investment pattern, and the allocation and performance benchmarks. As a result of these developments, it is now widely recognised that identifying the desirable size of reserves as well as the designing of an appropriate management strategy are of critical importance for any central bank. Regardless of whether the authorities make any explicit mention about the objectives of reserve management, three broad patterns are noticeable. The industrialised countries, more specifically the group of five (G-5) countries, manage their reserves essentially with a view to ensuring international monetary and exchange rate stability. As a consequence, reserves in these countries evolve as an outcome of their monetary policy actions. The management of reserves is a comparatively simple affair as practically all foreign assets are held in short-maturity paper involving only one or two currencies. The exception is the UK where a predominant portion of the reserves represents a corpus of funds mobilised through medium to long-term multi-currency borrowing by the government for supporting the Government's exchange rate objectives. The Bank of England performs an agency function in managing this corpus actively.

Several countries in middle-east and south-east Asia which have accumulated substantial reserves by running persistent and large current account surpluses over years look upon the reserves as their national wealth, which could be invested in long-term assets so as to generate a steady stream of income in future. Earning from deployment of reserves is seen as a means of smoothening the inter-

temporal consumption pattern in the event of shocks. The risk profile of investments of these countries is usually large : very long-term bonds, equities as also real estate find place in their portfolios. In Singapore, investment of a part of reserves in equities is done with a strategic objective in mind *i.e.* to selectively acquire ownership stake in those overseas companies which are important from the point of view of supply of critical raw materials or intermediate products to the country. Semi-industrialised and developing countries, on the other hand, keep and manage external reserves with one or more of the following objectives :

   (i)   To provide a cushion against current and capital account shocks. The motive is guided by precautionary considerations and use of reserves in financing current account transactions is based on need.

   (ii)   To build up a portfolio of safe and liquid external assets to enhance the country's creditworthiness. This is particularly important for those countries which are prone to endemic BoP shocks, accompanied by sharp reserve losses. Here, reserves play more of a 'showcase' role implying no real spending of reserves.

   (iii)   To facilitate payment of certain categories of imports and/ or government debt service with the intention of reducing transaction costs.

The philosophy underlying the investment strategy of the reserves in almost all cases has been of low risk, both on account of exchange rate as well as interest rate movements, and easy liquidity.

In recent years, many central banks, both from the developed as well as the developing world, have preferred to be relatively open with regard to their views on the purpose and objective of holding reserves, based on country-specific macro-economic considerations.

The reserves management strategy as a first step involves adopting a scientific approach to reserves management. The second relates formally to defining the parameters that would guide investment decision and portfolio selection. Central banks have started using highly complex models and state-of-the art portfolio management techniques for seeking answers to these issues.

The main task here is to define upfront the level of risk that is acceptable and then to construct an efficient portfolio embodying that risk. A portfolio is defined as efficient if the expected return on that portfolio is the highest amongst all other portfolios having the same risk, which for the purpose of this study is defined as variance or volatility of expected returns. For a portfolio of assets, however, the aggregate risk depends not only on the variance of the returns of individual assets comprising it but also on their covariances (correlations). For analytical purposes, the risk character of a portfolio is expressed in a matrix of variances and covariances-variances of the assets it is composed of. The problem of efficient portfolio selection is to find out those combination of assets for which the risk and return are optimal. As most central banks are price takers in the international markets and hence cannot affect the expected return, the efficient portfolio is the one for which the variance-covariance matrix is the minimum for a given level of expected returns. Needless to say, the assumption here is that the historical returns on assets provide sufficient information with which future return and risk could be predicted with a reasonable degree of accuracy.

Classical mean-variance optimisation approach, however, has certain frequently mentioned shortcomings, namely;
(i) instability of the parameters, which in turn renders the optimal portfolio weights time variant,
(ii) poor out-of-sample performance of portfolios thrown up by models.

5

Michaud (1989) offered the most critical comment made so far on the mean-variance approach by arguing that mean-variance optimizers tend to operate in a manner that magnifies the errors associated with input estimates. Healy (1981), while acknowledging the shortcomings of the mean-variance technique argued that instability of the variance-covariance matrix could not be a sufficient reason for rejecting this approach.

I. 2 The Context of the Study

In the Indian situation, the case for a portfolio management study for official reserve assets arises in view of the ongoing process of liberalisation of the external sector and the sizeable build-up of reserve assets. The critical importance of holding reserves for the developing countries was evident from the recent Indian BoP problem in 1991, when reserves fell to a precariously low level equivalent of only a few weeks of imports bringing in its wake a crisis in the external payment situation. It is pertinent to note that the healthy build-up of reserves consequent to stabilisation and structural adjustment programmes enabled the country to regain its creditworthiness in the international financial markets. With the substantial regaining of strength in the balance of payments in the post-crisis period, reserve management objectives have seen reorientation. What are the issues specific to the developing countries in general and India in particular?

Movements in the levels of official reserves during any period reflect the developments in the overall balance of payments. Developing countries in general pursue a policy of supplementing domestic savings with foreign capital up to a level that is considered sustainable. A surplus in the overall balance would therefore result

only when net capital account flows exceed the actual financing gap in terms of current account deficit. Under a fixed/managed exchange rate regime such surpluses in the overall balance are generally absorbed in the form of accretion to reserve assets. Under a flexible exchange rate regime, however, such surpluses lead to upward pressures on the exchange rate. The impact of balance of payments on official reserves, therefore, largely depends on the exchange rate regime. With the shift in the exchange rate regime in India from an officially determined exchange rate regime to a market determined regime in March 1993 through a transitional phase of dual exchange rate during 1992-93 there has been a transformation in the role assigned to reserve assets. Furthermore, in response to the policy reforms instituted in the areas of international trade, investment and exchange control in general, and as a result of the policy induced shift in the composition of capital flows in particular, the balance of payment dynamics exhibit a different pattern.

The pre-reform period was characterised by restrictions on current and capital transactions and involved foreign exchange rationing at the officially determined exchange rate. As a result, transaction financing motive primarily guided the decision regarding desired levels of reserves. Capital flows were mostly policy induced and the size of net capital flows were consistent with the needs of the country in terms of financing a particular level of current account deficit as also achieving a desirable level of reserve build up. Capital flows were in the form of external assistance, commercial borrowings and NRI deposits (in the 1980s each accounted for about one third of the total net capital inflows) imparting some degree of certainty and stability to the capital account. Reserve adequacy during the pre-reform period was largely being analysed in terms of cover for several

7

months of imports since trade transactions dominated the balance of payments and a sizeable part of the imports were paid for on an official account. Reserve assets were thus assigned the primary task of insulating the economy from temporary external payment imbalances.

In the post-reform period in India, institution of measures such as allowing exchange rate to be market determined and phasing out of restrictions on most of the current and even several capital transactions should have actually contributed to a decline in the desired level of reserves. Emergence of other considerations, however, necessitated a policy of maintaining higher levels of reserves. Instead of the transaction financing motive, the precautionary motive (i.e. to provide cushion against future unanticipated shocks) for holding reserve assets thus gained precedence. The policy induced shift in the composition of capital flows also imparted certain degree of instability to the capital account as the share of normal capital flows in total declined. During the three-year period 1993-96 portfolio investments accounted for 32 to 35 per cent of the total external financing. Their share, however, fell to about 25 per cent during 1996-97. The capital account developments during the post-reform period affected the policies relating to reserve adequacy from two different counts :

(a)   exposure of the economy to higher levels of volatile external liabilities necessitated larger reserve build up, and

(b)   given the potential risk of exchange rate volatility associated with such private flows, in a market determined exchange rate regime, maintenance of extra reserves became critical.

8

Furthermore, strengthening confidence among foreign private creditors was crucial under such systems for encouraging larger inflows of capital, and a high level of reserves, on its own, serves as an indicator of strength. Reserve adequacy considerations are thus increasingly being guided by the developments in the capital account in the balance of payments without undermining the importance attached to current account developments. During the reform years, imports as percentage of GDP rose from less than 10 per cent to about 13 per cent and correspondingly the reserve adequacy requirements in terms of cover for specific months of imports also warranted an increase in the level of reserves. Periods of exchange market volatility often accentuate the leads and lags in import payments/realisation of export earnings and the resultant expansion in the mismatch between demand and supply fuels volatility in the forex market. Reserve adequacy considerations these days, therefore, also account for such leads and lags as also debt service payments. With the Reserve Bank maintaining higher levels of reserves compared to the pre-reform period and in relation to conventional reserve adequacy norms, the opportunity cost consideration also emerged as a major issue of concern. This necessitated a more active management of the reserves and as a result, along with the conventional overriding considerations of safety, liquidity, and preservation of value, profitability considerations need to be also attached some importance.

I. 3 Objectives and Scope of the Study

The main purpose of this study is to develop an implementable theoretical model for selecting portfolio weights as regards interest rate (market) exposure for a risk averse central bank and then to apply it to the Indian case to solve certain decision-making problems. This has been done within the framework of the mean-variance

optimisation approach on the assumption that the currency allocation for reserves is given. The study also attempts to address two contentious issues (as mentioned above) which are associated with the mean-variance approach *viz*, stability of the risk matrix and use of past returns to generate statistical estimates of future expected returns, which act as inputs for the model.

As a prelude to the description of the model and the results obtained we wish to briefly outline two related aspects of reserves management : *viz.*, desirable size of reserves and currency allocation. This is done in Section II. Section III presents a brief survey of the literature on portfolio selection models and the merits of different models. A justification for our choice of the mean-variance model of Markowitz (with some improvements) is also presented. Section IV describes the standard analytical approach for solving optimisation problems. We present a reformulation of the quadratic programming problem as a linear complementary problem and solve it using Lemke's complementary pivot algorithm. This section explains the approach and the history of its development. Section V presents the methodology of estimation of variance-covariance matrix and expected rates of return. There are two improvements over the approach reported in the literature. First, instead of using a historical variance-covariance matrix estimated using data for a long period (say over 20 years) which may not be reliable as the variances and covariances may not be stable over long periods, we use short term overlapping periods, over which the variances and covariances remain fairly stable. Also, instead of using just one estimated variance-covariance matrix, we treat the estimated variance-covariance matrix as a realisation from a Wishart distribution, generate over 100 such matrices from this distribution randomly, solve the problem for each such realisation and take an average of the solutions. This is similar to scenario aggregation used in multi-period

10

models. Second, the time series forecasting models have been used for estimating the expected return vector. Section VI presents the results and the conclusions. Broadly speaking, an efficient set of portfolios has been generated which can be used as a framework for defining a benchmark for investment of foreign exchange reserves in each currency segment.

Section II

Reserve Adequacy

From a theoretical standpoint, foreign exchange reserves held by a central bank are a residual, arising from a disequilibrium situation in the foreign exchange markets, in which the supply of foreign exchange exceeds the demand for it. For instance, if the inflows on the capital account are in excess of the current financing needs it is not always the fact that the exchange rate will clear the market, as there are practical problems in letting the exchange rate move freely to bring about such adjustments. In fact, if anything, the real world is far from being in such equilibrium, as different central banks exercise different degrees of control on exchange rate by intervening in the market either to deplete or build reserves to maintain a targeted level of exchange rate in their economies.

High opportunity costs of holding large reserves, and the argument that reserves to a certain extent reflect the lack of absorptive capacity or that they do not contribute directly in the form of productive investment, often affect the decision regarding reserve adequacy. However, for countries with a record of persistent current

account deficit, keeping substantial reserves as a cushion against unforeseen eventualities becomes a necessity. For one thing, a high level of reserve leads to easing of 'external constraints' to growth. For another, a comfortable reserve position facilitates the process of liberalisation and integration with the global economy. Maintaining reserves through 'borrowed' resources is even suggested for a developing country endowed with natural resources (reserves) and human capital for realising the potential growth opportunities. Ultimately, investment and productivity growth would result in conversion of available resources into tradables, leading to accretion of 'owned' reserves. In sum, despite cost implications, wider macro-economic responsibilities on the central banks often guide the reserve adequacy decisions. The degree to which the exchange rate is managed and the level of NFA to currency ratio that is preferred by the monetary authorities in their conduct of monetary policy could also affect the reserve adequacy decision.

However, regardless of the differences exhibited by central banks on what they consider as the appropriate or even the optimal level for reserves, the fact remains that during the last 10 years or so, there has been a significant surge in world reserves, accompanied by a major redistribution among different groups of countries. Since 1989, global foreign exchange reserves have more than doubled to over US$ 1.7 trillion. Reserves of semi-industrialised developing economies have risen threefold during this period, leaving them with well over half of the world stock. Reserves of industrialised countries have also increased by about 60 per cent during this period (please see Table1 as also Graphs I and II). In terms of import cover, total reserves of the world represent 30 per cent of global imports, up from 20 per cent in 1980 but of the same order as in 1972. It is

12

possible to relate the growth as well as redistribution of world reserves to increasing liberalisation and globalisation of economies. The more open an economy and the greater the variability of trade, the higher are the chances of external shocks (at least at the initial stage) and hence the requirement for higher reserves. In this context evaluating reserve position in terms of import cover does not have much of a practical relevance.

In the more recent literature [Roger (1993), Nordman (1994), Miller (1995), Pattanaik (1996), World Gold Council (1993), Report of the Committee on Capital Account Convertibility (1997)] one finds the use of multiple factors for the assessment of reserve adequacy norms such as; imports or the total currency payments, the variability of current receipts and payments, the leads and lags in receivables and payments of foreign exchange, the degree of exchange rate flexibility indicating the size of reserves that may be necessary for intervention operations, the stock of short-term debt and other highly volatile liabilities, the opportunity costs of holding reserves, the sustainable level of CAB, the stock of outstanding external liabilities, the level of international confidence and credit ratings and the ratio of foreign exchange reserves to currency in circulation and/or broad money.

As a number of considerations go into the decision regarding the 'desired' level of reserves and the fact that this may vary across countries due to their varied economy - specific needs, there can not be any standard indicator of reserve adequacy that could be applicable to all countries.

While the level of reserves could be the end-result of the macro-economic policies of the Government, active management of reserves involves a two-stage decision: first, the desirable currency allocation, and second, the investment pattern within each currency segment.

II 1. Currency Composition of Reserves

This issue of currency distribution of reserves has also been widely researched. Although central banks in general are reluctant to reveal information on actual allocation, it is now known that their decisions in this regard are fashioned by the following factors :

(i) the country's exchange rate regime and the intervention currency.

(ii) the invoicing pattern of international trade transactions in goods and services.

(iii) the currency distribution of debt stock as well as debt-service payments.

A decision on optimal currency allocation would require selecting the currencies first and then agreeing on the weights. So far as the first issue is concerned, the currencies that find a place in the allocation are those which are important from the external trade and investment point of view. As regards the weights, a decision can be arrived at following an analytical approach designed to address the objective of minimising the risk arising out of exchange rate changes. The currency risk of a portfolio is nothing but changes in the value of the portfolio with changes in exchange rates. For this purpose it would be necessary to define the value of reserves in terms of a numeraire or reporting currency. Obviously, the minimum risk configuration would be the one with overwhelming share of the numeraire currency itself. Numerous mean-variance analyses on optimal currency allocation corroborate this fact. In semi-industrialised and developing countries, reserves are invariably kept in a diversified currency portfolio with one of the major currencies (or a combination thereof) as the reporting currency. Deciding on the numeraire/reporting currency, which is nothing but a microcosm of the currency allocation, is more a question of judgement than applying any precise quantitative methodology.

14

A diversified currency portfolio provides certain flexibility in terms of import purchasing/debt servicing requirements which needs to be kept in view. However, if static composition is adhered to regardless of exchange rate changes, the intrinsic value of reserves may suffer. To illustrate this point further, if the domestic currency and the US dollar exhibit high positive correlation, then at times of weakness of the domestic currency (for instance negative shocks to terms of trade) the domestic currency would also be weak and a reserve portfolio composed pre-dominantly of the US dollar would tend to have a lower value than if it were composed of currencies that were less correlated with the domestic currency. This means that for preserving the intrinsic value of reserves, measured in any numeraire currency, the currency allocation will have to be dynamically varied alongside movements in exchange rates.

Section III

Review of Portfolio Selection Models

The portfolio selection problem faced by an investor while allocating initial wealth among 'n' risky securities available in a capital market was formally introduced in the literature on financial planning by Markowitz (1952) in his pioneering work. The approach of Markowitz to this problem consists of :

(i) estimating the future performance of securities assuming a continuous distribution with finite mean and variance for the rate of return,

(ii) analysing these estimates to obtain an efficient set of portfolios, and

(iii) selecting from the efficient set of portfolios, the one that suits the investor's performance.

15

Following Markowitz (op. cit) there have been a number of papers dealing with this problem, especially with regard to the second stage, namely, computation of efficient sets of portfolios. It may be noted that the second of the three step procedure outlined by Markowitz involves the solution of a number of quadratic programming problems. To understand this clearly, one needs to note that if the rate of return on a security is modeled as a random variable, there would be two parameters that could be used to measure its performance; namely, the expected rate of return and the variance of return. With 'n' securities in the portfolio, we may model the vector of returns on these securities as a multivariate random variable with a variance-covariance matrix. Variance on the return of a security can be thought of as a measure of the risk faced by an investor. If a portfolio of securities is defined as a vector of the proportions of the initial wealth to be invested in 'n' securities, one can compute its expected return and variance using the given mean vector and the variance-covariance matrix. As one is looking for a portfolio that minimises risk for a given level of expected return one is led to the following quadratic programming problem :

Minimise $(\frac{1}{2}) x^t V x - \Theta \sum x_i \mu_i$

Subject to $\sum_{i=1}^{n} x_i \leq 1; \ x_i \geq 0; \ 1 \leq i \leq n$ .... (1)

Where 'x_i' denotes the proportion to be invested in security 'i', μ_i denotes the expected return on security 'i' and 'V' denotes the variance-covariance matrix of the returns on the 'n' securities. In the above formulation 'Θ' denotes a parameter representing the risk aversion of the investor, the higher the value of Θ the less being the aversion of the investor to risk. For example, if $\Theta = 1$ then risk and return are assigned equal weights by the investor.

The computational complexity of the portfolio selection problem, therefore, is mainly due to the requirement of having to solve a large

16

number of quadratic programming problems with 'n' variables each in order to evaluate the efficient portfolios. The development of the literature in this area is also centred on the question of solving efficiently convex quadratic programming problems. Sharpe (1962) developed what he calls a critical line method to solve the quadratic programming problems and claims that as many as 2000 securities can be analysed at an extremely low computing cost. Later on, following the work of Lemke (1965) and Cottle & Dantzig (1968) an efficient pivoting method has been developed for maximising a concave quadratic function, subject to linear inequalities. We shall discuss this method in some detail in the next section as this has been used in this study. It is to be noted here that the model proposed by Markowitz is a single-period static model. In the literature, certain other single-period static models for portfolio selection have been proposed. These models introduce a concave utility function for money and formulate the problem as one of maximising utility at the end of the planning horizon. The utility function has to be built incorporating a risk tolerance parameter and investor's preferences. In addition to the computational issue of obtaining an optimal portfolio in these models, there is also the problem of determining a suitable utility function. [For a comparison of utility functions used in these models see K.G. Kallberg and W.T. Ziemba (1983)]. However, the effect of risk aversion on the optimal portfolios obtained by maximising the utility function at the end of the planning horizon is not clear and is the subject of many papers [Li and Ziemba (1989)]. Although such an approach has many interesting theoretical properties, its use in practice is limited. We have also not found it suitable for the purpose of our study, as there are no accepted guidelines for developing a utility function of money for a central bank. The more widely used approach is based on the Markowitz model and we use this in our study as in this approach both expected return and risk could be explicitly introduced as measurable quantities.

17

As mentioned earlier, the mean-variance approach has been criticised in the literature. Among the major criticisms is a feature that it is a single-period static model that does not allow for review and changes in the investment decisions to be made in shorter intervals of the fixed period of the planning horizon. An investor willing to take risks in a capital market will prefer a model that allows review of and changes in the investment decisions more often than just once in the beginning of a one year period of planning horizon. Since the late 1980s dynamic models for financial planning have been considered in the literature. Dynamic problems of portfolio management under uncertainty can be modeled as multistage stochastic programs thereby capturing the dynamic aspects of the problem as well as allowing for a number of realisations of the uncertain quantities. The models that have been suggested use a concave utility function incorporating a risk tolerance parameter and investor's preferences for maximisation. In addition, a number of other constraints also could be introduced. However, models of multi-stage stochastic programs exhibit a multiplicative growth in size with the number of decision stages and the number of realisations admitted in each stage. This has restricted the practical applications of such models. It is hoped that in the near future it would be possible to devise suitable applications of such models. It is hoped that in the near future it would be possible to devise suitable application software for tackling the enormous computational requirements of a multi-variable dynamic programming model. To get an idea of the growth in the size of the problem let us consider the following situation : We use a four stage model and with only five different securities. Suppose we admit only 5 different values for the rate of return on each security at each stage. This will give us a problem with 5^4 different scenarios, a very large number of variables indeed.

. The multi-period stocha   ve been recently proposed for selecting portfolios of fixed income securities include the models proposed by In reference Zenios (1991) and Hiller & Eckstein (1991). The multi-period generalised network models incorporating uncertainty in asset returns involving a non-linear objective function in the stochastic program can be solved by the progressive hedging algorithm of Rockafeller and Wets (1991). [Also, general model proposed by Mulvey and Vladimirou (1992)]. But as noted earlier, application of these ideas in practice, with a sufficient number of scenarios being allowed, belongs to the future when one has much better computing power than at present. In this context see the remark in Mulvey (1994).

We are concerned with the problem of portfolio selection for an investor such as a central bank which is not allowed, by law or by custom, to take too much risk. Our problem is therefore restricted and we need to consider only the fixed income asset categories of low duration for the investment of the foreign exchange reserve. Thus only five asset categories have been identified in each of the four major foreign currencies considered in the model. Given the assumption of low risk, it would be sufficient for our purpose if we develop a static model for a one year planning horizon. This restriction on the choice of our model is also partly due to the computational difficulty mentioned earlier.

Section IV

On the Algorithm used for Optimisation

As mentioned earlier, the quadratic programming problem encountered in the static mean-variance model of Markowitz can be

solved efficiently by applying Lemke's complementary pivoting method. In this section, we give a brief outline of this technique.

Lemke's (1965) complementary pivoting algorithm was an extension of the algorithm proposed by Lemke and Howson (1964) for proving constructively the existence of a Nash equilibrium point of the bimatrix game problem. For a description of this issue, consider two players, say, Player I and Player II each with a finite number of actions. Suppose Player I has 'm' actions and Player II 'n' actions. Suppose Player I plays his action 'i', and Player II plays his action 'j'. Then the cost incurred by Player 1 is a_{ij} and the cost incurred by Player II is b_{ij}. This is called a nonzero-sum game as the loss of Player I does not necessarily yield an equal gain to Player II or vice versa. (In other words, the matrix A+B is not the 0 – matrix.) At the outset, it is not clear how such games can be optimally played by the two players when we assume that there is no co-operation between them. John Nash (1951) defined the notion of a pair of equilibrium strategies, under non-co-operation, as follows : First, we extend the notion of action to that of a strategy by defining a strategy for Player I as a probability vector over the set of finite actions available to Player I. Thus if $q = (q_1, q_2 ....q_m)$ is a probability vector, i.e., $q_I \geq 0$, for all i and $\Sigma q_i = 1$, then q_i specifies the probability that Player I will choose his action i. Similarly the notion of a strategy of Player II is defined as a probability vector over the set of actions available to Player II. Let S_1 and S_2 be the set of strategies of the two players respectively. Note that the unit column vectors of order m, e^i_m with 1 as its i-th co-ordinate and 0 as other co-ordinates are in S_1. Similarly, the vectors e^i_n are in S_2. Suppose x and y are the strategies chosen by Players I and II, respectively. It is easy to note that the expected cost to Player I works out to be $x^t Ay$ and that of Player II to be $x^t By$ where A and B

are the given m x n cost matrices. A pair of strategies x*, y* is called
a Nash equilibrium pair if :

$$x^{*t}Ay^* \leq x^tAy^* \text{ for all } x \in S_1 \qquad \ldots (2.1)$$

$$x^{*t}By^* \leq x^{*t}By^* \text{ for all } y \in S_2 \qquad \ldots (2.2)$$

If (x^*,y^*) is an equilibrium pair, equation (2.1) states that as long as
Player II plays y^* there is no motivation for Player I to play any
strategy other than x^* as this is his best response against y^*. Similarly
equation (2.2) essentially means that Player II would be punished for
deviating from y^* when Player I plays x^*. In this framework, the pair
is an equilibrium pair of strategies and the corresponding pair of
expected payoffs $(x^{*t}Ay^*, x^{*t}By^*)$ is called a Nash equilibrium point in
the cost space. There may be more than one equilibrium point for a
game and it is possible that some equilibria are better in the sense of
lowering costs to each of the players than some other equilibria. Nash
suggested that in a non-co-operative set-up the players tend to settle
for a pair of equilibrium strategies and he proved using the notion of
fixed points for continuous functions, the existence of a pair of
equilibrium strategies for the bi-matrix game. His proof, however,
gave no clue to computing such a pair of strategies. The first step
towards a computational approach to this problem consists in noting
that the problem can be reformulated as follows :

Note that as equation (2.1) holds for all $x \in S_1$, it holds when
    $x = e^i_m$, $1 \leq i \leq m$.
This gives the equations :

$$(Ay^*)_i \geq x^{*t} Ay^*, 1 \leq i \leq m \qquad \ldots (2.3)$$

Similarly, one has $(x^{*t}B)_j \geq x^{*t}By^*, 1 \leq j \leq n \qquad \ldots (2.4)$

Also, note that $x^*_i = 0$ for i such that $(Ay^*)_i > x^{*t} Ay^*$

Similarly, $(x^{*t}B)_j > x^{*t}By^* \Rightarrow y^*_j = 0$. These are called the complementarity conditions. We may assume without loss of generality that the cost matrices A and B are strictly positive. In this case $x^{*t}Ay^*$ and $x^{*t}By^*$ are strictly positive, too. Hence we can divide the equation (2.3) by $x^{*t}Ay^*$ and the equation (2.4) by $x^{*t}By^*$ to obtain :

$$Ay^{**} \geq e; \quad B^t x^{**} \geq e; \quad (Ay^{**}-e)^t x^{**} = 0; \quad (B^t x^{**}-e)^t y^{**} = 0 \qquad \text{.... (2.5)}$$

Where $y^{**} = y^* / (x^{*t} Ay^*)$ and $x^{**} = x^* / (x^{*t} By^*)$

Thus the problem of computing a pair of equilibrium strategies for the players reduce to finding a solution for the following problem :

Find vectors w and z, such that
$$w-Mz = q; \quad w \geq 0; \quad z \geq 0; \quad w^t z = 0 \qquad \text{.... (2.6)}$$

where $w^t = [u^t, v^t]$, $\quad z^t = [x^t, y^t]$, $\quad q^t = [-e^t, -e^t]$

$$M = \begin{bmatrix} O & A \\ B^t & O \end{bmatrix}$$

The problem defined in equation (2.6) is known as a linear complementarity problem. Lemke and Howson [4] presented a pivotal algorithm to solve a linear complementarity problem when the matrix M and the vector q are of the form defined above, corresponding to a given bimatrix problem. The algorithm is unique in the sense that for its convergence it does not depend on the monotonic nature of any objective function over the iterations, unlike many of the optimisation algorithms known at that time.

The invention of this algorithm in computational optimisation is at least as significant as the invention of the simplex method for

22

the linear programming problem. Apart from providing a constructive proof for the existence of an equilibrium pair of strategies and corresponding costs for a bi-matrix game problem, a modification of this algorithm has provided a computational procedure for the solution of the composite primal and dual linear programming problem, convex quadratic programming problem and the linear fractional programming problem. Further, it has been found useful in the numerical solution of many engineering problems and in variational inequalities. It has also significantly influenced the development of algorithms for the computation of an equilibrium price vector in the general equilibrium models of economics. For references on the impact of this algorithm in various areas requiring a computational approach see Cottle, Pang and Stone (1994). More recent applications include the use of this algorithm in some special cases of zero-sum and nonzero-sum stochastic game problems [See Mohan *et al* (1994)].

In our optimisation model of the portfolio selection problem we shall be using the algorithm of concave quadratic maximisation problems that arise in the model. Earlier work in this area includes the paper of Pang, Kaneko and Hallman (1979) who use a parametric linear complementarity problem and a modification of Lemke's algorithm. However, they address the problem of a general investor who deals with a very large number of asset categories of all kinds and, therefore, have to solve a significantly large size problem, although the model is static. The modifications used by them may not be of any advantage for the problem we consider in this work. We use a simple and straightforward application of Lemke's algorithm using our own FORTRAN code. The linear complementarity problems solved in the optimizer have the following form :

$$w - Mz = q; \quad w \geq 0; \quad z \geq 0; \quad w^t z = 0 \qquad \dots (2.7)$$

$$w^t = [u^t, v^t], \quad z^t = [x^t, y^t], \quad q^t = [-\Theta\mu^t, -1] \text{ and}$$

$$M = \begin{bmatrix} V & e \\ e^t & o \end{bmatrix}$$

This solves the quadratic maximisation problem

Minimise $(1/2)x^t V x - \Theta\mu^t x$

Subject to $e^t x \leq 1, \quad x \geq 0;$

The Karush Kuhn Tucker conditions of the above problem leads to the linear complementarity problem defined in (2.7).

As already mentioned, another criticism regarding the Markowitz approach is that it relies on an estimation of the average returns and the variance-covariance matrix using the past data assuming that these parameters remain stable over time, at least over the period for which the data collected is used for estimation. It is clear that such an assumption is not realistic, especially when for the sake of accuracy of estimation it is required to use data spanning a sufficiently long period in the past. We do test for the stability of the variance-covariance matrix. In addition we look upon the variance-covariance matrix V computed from the past as the expectation of a matrix valued random variable with a Wishart distribution whose expectation is V. We then generate scenarios of various possible variance-covariance matrices, solve the problem for each of these and aggregate the solutions over the scenarios which in this case reduces to taking an average of the solutions obtained for the scenarios. The details of this procedure are presented in the next section.

Section V

Estimation of Return Vector and Variance-Covariance Matrix

The empirical exercise for selection of an optimum portfolio of fixed income assets for deployment of foreign exchange reserves, in the backdrop of the optimising framework earlier discussed, involves mainly two steps: (i) forecast of expected return vector, and (ii) working out of the risk matrix for the set of assets considered for this purpose.

The two-step optimisation process has been undertaken for a multi-currency multi-market portfolio comprising the US dollar, the German mark, the Japanese yen and the Pound sterling[1], with five fixed income asset classes in each currency, namely :

    (i)   Money market instruments of maturity up to and inclusive of 1 year,

    (ii) Government bonds of maturity between 1 to 3 years,

    (iii) Government bonds of maturity between 3 to 5 years,

    (iv) Government bonds of maturity between 5 to 7 years,

    (v) Government bonds of maturity between 7 to 10 years.

The optimisation exercise has been undertaken for seeking solutions to two problems :

First, to estimate optimal asset-class weights for investments within each currency segment. Obviously, the assumption here is that the currency composition is exogenously determined and is given.

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1. Euro cannot be considered at this stage since it was introduced only from January 01, 1999 and there are not many observations for purposes of analysis.

25

Second, to estimate optimal currency as well asset class weights (within each currency segment) for each currency segment. However, since for the second exercise it would be necessary to treat return and risk of assets across different currencies in a unified manner, it would be essential to express these in terms of a single currency, which is also the numeraire, reporting or base currency. We adopt the US dollar for this purpose. Historical monthly return figures for the five asset classes in the four currency segments, both in terms of local currency as also in US dollar terms for the last 11 years have been taken from the publications of Salomon Brothers. It may be mentioned that certain large international security houses and banks publish periodic data regarding the performance of various equity and fixed income markets world-wide. For fixed income instruments, since the rate of return is different for instruments (having the same credit) with different maturities, the yield curve is divided into suitable (finite) maturity buckets and the aggregate return for instruments in each bucket is calculated on the basis of a valuation index. The rate of return in respect of each standard bucket in local currency terms could then be converted into a US dollar rate of return by applying the following formula :

$$(1 + r) * [E1 / E2] - 1 = R, \text{ where}$$

'r' is the return in local currency;

'R' is the return in numeraire currency;

'E1' is the beginning-of-the-period exchange rate of the numeraire currency against the local currency;

'E2' is the end-of-the-period exchange rate of the numeraire currency against the local currency.

For example, if the return in a particular asset class in the Japanese market is, say 2 per cent and the beginning- and end-of-

the-period US$/Japanese yen exchange rates are 110 and 112 respectively, the return in terms of US dollar for the same period would be :

(1 + 2) * (110/112) - 1 = 1.95

It is obvious that the return in numeraire currency terms would combine the effects of interest rate movement in local currency and the movement of the exchange rate of the local currency *vis-à-vis* the numeraire currency. Thus, when we take the historical return data in terms of the US dollar for estimating the optimal currency and asset weights for a multi-currency multi-market portfolio both the exchange rate and interest rate effects are captured.

A look at the return data used in the present exercise would reveal that returns are extremely volatile from year to year even in respect of own currency return. The volatility (i.e., standard deviation) increases manifold when returns in US dollar terms are considered.

It is a general practice to implement the mean-variance optimisation technique by calculating a historical variance-covariance matrix based on a fairly long return series relating to chosen set of securities. However, serious doubts have been raised in various studies about the stability of such a historical variance-covariance matrix. Elton, Gruber and Urich (1978) have compared the ability of various methods to forecast the correlation structure between stock-market securities and their conclusion is that "the historical correlation matrix was the poorest of all techniques". In fact, we carried out certain statistical tests to examine the stability of the observed variance-covariance between selected securities. Under the assumption that the return vectors follow a multivariate normal distribution – a rather strong assumption indeed – a likelihood ratio

27

based test for testing the equality of two or more covariance matrices may be undertaken (For details please see Appendix I). Bifurcating the 11 year period under study into two non-overlapping periods of equal duration, it is observed that equality of two underlying variance-covariance matrices is rejected for returns data of various currencies. However, the equality of variance-covariance matrices can not be rejected when regimes are overlapping and of shorter duration, say, two to three years. On close scrutiny of the data it was found that a variance-covariance matrix based on last two years returns data would give a fair approximation of the risk structure that is likely to obtain in the coming one year, which is the forecast period for our exercise. Accordingly, the optimisation procedure was implemented with variance-covariance matrix based on the two years data ending March 1996 for each currency. Since the observed variance-covariance matrix is based on a realisation of the underlying data generating process, the "true" variance-covariance matrix may differ from it by an unknown degree of error. It would, therefore, be appropriate to work out the optimum portfolio of securities for different realisations of the underlying "true" risk matrix since it is expected to be much more robust in that it would not be too sensitive to small perturbations to the unknown "true" risk-matrix. Accordingly, for the current exercise, it is assumed, without any loss of generality, that the "true" risk matrix follows a Wishart distribution, based on an underlying multivariate normal distribution with zero mean vector. A random sample of 100 such variance-covariance matrices was drawn from this Wishart distribution and optimum portfolios were worked out for each such randomly drawn variance-covariance matrix. The final optimum portfolio, for a given risk-tolerance parameter is based on average of these 100 portfolios.

The expected return vector forms one of the inputs to the optimisation procedure. In the current exercise, the time horizon for

28

decision period is taken to be one year. Accordingly, forecasts have to be generated for 12 lead periods based on the monthly returns data. High volatility makes prediction of yields in fixed income markets a highly hazardous job. Although all the monthly return series except the money market returns are found to be stationary in their original levels, the series mean is often a very poor forecast of short-term movement in yields because of large variance of the innovation process. This also makes any time series forecast of future yields very fragile. Furthermore, time series forecast, particularly the univariate forecasting of returns of individual asset classes, suffers from a very important shortcoming, which is that an investor in a fixed income market is interested in future shape of the yield curve. She wants to know whether it is possible to identify in advance when the reward for duration extension is likely to be very high or very low. Economic theories postulate that the current shape of the yield curve provides valuable information about the time-varying risk premium and, hence, about the expected future return on assets. For example, the popular of them all viz. the expectations theory of term-structure of interest rates states that all fixed income instruments of the same credit risk must give the same rate of return over any specific time horizon. Thus, the next period's expected return for any bond is simply the one-period spot rate. This conclusion undergoes modification if it is recognised that bonds may be mispriced. Then the expected return will be a function of one-period spot rate as well as of the mispricing. If an alternative term-structure theory is considered to be a better describer of the observed term-structure then the estimation of the expected return may have to be modified accordingly. Several researches have attempted to identify variables that can enhance the quality of forecasts regarding the short-term excess bond return. In the current exercise, we proceed on the assumption that the expected returns vector given by the pure time-series forecast, univariate and perhaps multivariate, provides an efficient and unbiased estimate of the future returns for different asset classes.

29

For each of the asset classes, three types of time series forecasts were generated, viz., univariate ARIMA forecast, ARMAX forecast and a state-space model based forecast. For ARMAX forecast, one particular asset class in a given currency group was selected as an additional input to the ARIMA model for other securities in the same currency group, the selection of the input asset class being done on the basis of cross-correlation structure between the input series and the forecasted series. The underlying assumption is that the particular asset class would provide some early signal for the future movement of returns in respect of other asset classes. Alternatively, it may be said that the changes in the returns of the particular asset class act as a leading indicator for others and that there is positive feedback. In all currency segments, the 1 to 3 year asset class was selected for this purpose on the basis of various criteria, like, cross-correlation, final prediction error etc. The state-space models have been selected on the basis of similar automatic model selection procedure.

The optimum portfolio for each currency has been generated following Lemke's algorithm of solving a quadratic programming problem. As mentioned in Section III, the optimal portfolio depends on a given risk-tolerance parameter (Θ), which determines the trade off between risk and return for a given portfolio selection strategy. Such optimal portfolios have been worked out for various values of the risk-tolerance parameter.

It has been observed that above a certain range of Θ, the gain in return is much less as compared to the increase in risk, thereby rendering such values of the parameter practically unacceptable.

As we have already noted, the volatility of the return series becomes manifold when we consider return of different markets in terms of the US dollar, indicating the high volatility of the foreign exchange market. As a result, the forecast of expected return vector becomes extremely difficult. Moreover, as the number of securities becomes much larger for the exercise (20 securities all taken together), a multivariate forecasting procedure, like the state-space model based forecast breaks down. It is also not possible to develop an ARMAX model as the structure of correlation between various assets does not provide any unambiguous clue to the choice of a small set of explanatory input variables. We have, therefore, used only the univariate ARIMA procedure to work out the expected return vector. The risk-matrix has been worked out in the same manner as in the case of estimating optimal asset allocation for different currency segments. The result of this exercise is given in Table II and Graphs III, IV, V and VI.

The broad features of the optimum portfolios for each of the four currencies considered in this exercise are summarized in the following table.

Currency	Low Risk		Moderate Risk		High Risk	
	Return	Risk	Return	Risk	Return	Risk
Dollar	4.8	1.4	5.6	9.1	6.6	31.1
DM	3.1	3.6	5.8	29.5	7.0	46.1
Pound Sterling	6.2	0.8	7.3	9.0	8.5	29.5
Yen	1.2	6.7	2.9	26.9	4.7	80.8

Note: - The return figures are averages of annualized rates of return on the selected portfolio for three forecasts of return vector. The risk is an average of standard deviation expressed as percentage to portfolio rate of return. The value of risk tolerance parameter (θ) for the above three categories of risks are low risk 0.10 moderate risk 0.50 and high risk 1.0.

The results are on the expected line. The risk return characteristics of the optimum portfolio clearly brings out the positive association between the riskiness of portfolio as measured by the standard deviation of the portfolio returns and the expected return of the portfolio. As the optimum portfolio is contingent upon the given value of risk tolerance parameter in the objective function, composition of optimum portfolio has been worked out for different values of these parameters. It is observed that beyond a certain level of the risk tolerance parameter, gains in returns are negligible while the risk continues to be increased.

The expected return for the least risky portfolio for the dollar securities ranges between 4.5 to 5 per cent per annum depending on the choice of the forecasting model of expected return vector. The average risk associated with this portfolio works out to a range between 1 to 2 per cent of the expected rate of return. The resulting portfolio is overwhelmingly dominated by the securities of shortest maturity class (below one year) while a small proportion of around 2 per cent of total investible amount is shared by securities of a next maturity band (1 to 3 yr.) The composition of the portfolio changes uniformly in favour of securities in the maturity band of one to three years as we increase the value of risk tolerance parameter. What is interesting to note is that at no reasonable level of risk tolerance parameter securities with any higher maturity period gets selected in the optimum portfolio. The risk of optimum portfolio for moderate level of risk tolerance parameter is observed to be quite high at around 9 per cent of rate of return, and this increases to around 31 per cent for high value of θ.

For sterling denominated securities the expected return in terms of own currency is much higher at 6 to 6.4 per cent for the lowest level of risk tolerance parameter. The expected risk of sterling

denominated portfolio is found to be among the lowest among all currencies for the comparable level of risk tolerance parameter. The composition of portfolio in terms of securities of various maturity bands looks almost similar to the one obtained in respect of dollar securities.

For the securities denominated in Deutsche Mark the annualized rate of return for the least risky portfolio is found to be much lower at 2.2 to 3 per cent. For a moderate level of risk parameter the expected return of the portfolio increases to around 5.8 per cent and for higher level risk the return goes up to around 7 per cent. The riskiness of the portfolio for the lowest value of the tolerance risk parameter in respect of these securities is relatively much higher (3 to 5 per cent of the expected return) as compared to dollar and sterling denominated securities. As regards composition of the portfolio, the share of the lowest maturity period securities is somewhat lower as compared to dollar and sterling denominated securities. For a moderate level of risk tolerance parameter the portfolio appears to be overwhelmingly dominated by the securities of next asset class (i.e. maturity period ranging between 1 to 2 years). In respect of Yen securities the expected return is found to be the lowest at between 1.1 to 1.5 per cent per annum even for the least risky portfolio. The expected risk is also much higher for this portfolio with lowest value of risk tolerance parameters. The rate of expected return increases to around 2.8 per cent for moderate level of risk tolerance parameter and rises to around 4.6 to 4.8 for high level of risk tolerance parameter. The composition of the portfolio is again biased in favour of shortest maturity period but with a much higher share accruing to securities falling in the next maturity band i.e. within 3 to 5 years.

When all the securities are considered in terms of the returns denominated in numeraire currency i.e. the US dollar, the resulting

portfolio that emerges out of the current exercise appears to be quite interesting. It is observed that the optimum portfolio for the lowest level of risk tolerance parameter provides that a certain amount of the portfolio to be kept in cash; in fact around 17 per cent of the portfolio is recommended to be kept in cash as per the current exercise. The securities which get selected are obviously dominated by U.S.$ securities, that too, overwhelmingly in favour of securities in the shortest maturity band. Apart from dollar dominated securities the only other currency that gets selected in the optimum portfolio relates to those of Sterling securities. The share of the Sterling securities is around 20 per cent for the moderate level of risk tolerance parameter and its share rises to around 30 per cent for a pretty high risk of risk tolerance parameter.

Section VI

Conclusions

In the present study certain cardinal issues relating to the management of foreign exchange reserves by a risk averse central bank have been addressed. On the question of the optimal size for the reserves of a country, the study provided a general discussion, articulating the main issues. The view that emerges here is that a normative approach, based either on considerations of ideal and safe import cover or on a more rigorous cost-benefit analysis does not fully encompass all the relevant issues.

On management of foreign currency reserves, the study concludes that a two-stage decision process would be involved : first, with regard to currency composition and second, with regard to asset allocation within each currency segment.

34

In respect of currency composition, the study corroborates the result of many similar attempts made elsewhere *viz.* once the numeraire or reporting currency for the reserves (which could be a single currency, like the US dollar or a composite currency index to which the domestic currency is pegged or a synthetic currency unit such as the SDR) is identified, the minimum risk currency composition is the one closely resembling the numeraire itself. The study estimated optimal currency composition weights for a four-currency portfolio with the US dollar as the numeraire currency. The minimum risk configuration turned out to be the one with an overwhelming share of the US dollar and only a very small presence of the Pound Sterling.

The implication of this conclusion is that if macro-economic considerations indicate the need for maintaining a diversified portfolio in terms of currencies, which is equivalent to saying that if natural exposures in all/some of major currencies are justified, *a priori*, on macro-economic grounds (and there are good reasons for doing so), then estimating the optimal currency composition through a mean-variance optimisation exercise may not be of much help for the central banks. Put differently, the numeraire currency itself represents the currency composition at the lowest risk level.

Balance sheets of central banks are drawn in their respective domestic currencies. So, if the domestic currency is pegged to a single currency that is also used as the numeraire currency for all accounting purpose (like the US dollar, or to an index) there would be no valuation gain/loss for the foreign exchange reserves. In all other cases, i.e., where there is a divergence between the currency composition of reserve assets and the numeraire currency used for accounting of transactions, valuation changes in terms of the numeraire would result. Valuation changes arising on account of fluctuation of the domestic exchange rate are however different.

35

Insofar as valuation changes (in terms of domestic currency) are concerned they have both balance sheet and fiscal implications. This brings forth a very crucial question : how to tackle the adverse balance sheet impacts of a policy involving currency composition of reserves in the short-term? For central banks, which treat the currency revaluation gain/loss as revenue items, a fall in the value of reserves in domestic currency terms would lead to a fall in the current income, other things remaining the same. Even where the revaluation gain/ loss is treated as a balance sheet item – being routed through a 'provision/reserve' account – any sharp fall thereof would warrant more retention of profit for rebuilding the internal reserve – a step which would affect disposable surplus. The following framework for addressing currency valuation risks is suggested :

(i)   Valuation shocks arising out of movements in the exchange rate of the domestic currency against the currency composition is, in reality, an exchange rate policy-related issue and may, therefore, be kept outside the focus of portfolio optimisation analyses. The only safeguard against adverse implication of an appreciation of the domestic currency on the value of reserves is a sufficient cushion in the balance sheet of the central banks. For tackling the impact of *inter se* movements of the exchange rates of the currencies comprising the reserves, it would be necessary to vary the composition thereof dynamically for reducing valuation loss.

(ii)   In applying the Markowitz mean-variance optimisation technique in estimating an optimal asset mix, it was found that the variance-covariance matrix representing the risk profile of the portfolios was not stable over two non-overlapping sub-periods comprising the period, 1985-1996. However, the variance-covariance matrix for a two/three-year overlapping period indicated reasonable degree of stability over time. Hence, for the purpose of making

forecasts of risks for a one year time horizon, variance-covariance matrix calculated on the basis of the previous two years return data were used. For generating forecasts about returns of different asset classes in the one year horizon period, various time-series forecasting models *viz.* ARIMA, ARMAX and state space were considered and statistically univariate ARIMA forecasts were found to yield superior results. The lowest risk configuration of asset classes in all currencies showed a concentration at the short-end of the yield curves.

(iii) The optimiser provided different asset allocations for different levels of risk in all the four currency groups. The optimal risk/return co-ordinates could be used to identify the efficiency frontiers. For setting up benchmarks for investments in different currency segments a particular point on the frontier needs to be chosen which represents ·the acceptable level of risk and return for the central bank. If the central bank operates at any one point on the frontier for the implicit tolerable level of risk the probability of return being minimised would be quite high. For active management of reserve assets, tolerable risk bands on both sides of the benchmark point could be fixed by the authorities. While identifying the band a reasonable balance needs to be maintained between the considerations of liquidity and safety of a country's reserves and the need to optimise return.

The endeavour of a researcher would remain incomplete unless the unresolved questions and an agenda for future studies are explicitly stated. To us, the following two areas appear to be of critical importance for any central bank.

(i) Identification of an appropriate variance-covariance matrix for the time period for which an optimal portfolio is to be estimated remains the most important pre-requisite for a successful optimisation process. The

37

present exercise proposes one robust method for arriving at the risk matrix on the basis of return for example, data of recent vintage. There are other ways of doing this, namely, a multivariate ARCH or GARCH model could be fitted to the observed return data for projecting the future risk matrix. It is necessary to make a comparative study of the usefulness of various methodologies that may be applied to the problem.

(ii) The limitations of a static mean-variance optimiser have been referred to in this study. Multi-period stochastic programming models have been proposed in the literature for capturing the dynamic aspects of a portfolio optimisation problem. Operationalisation of such an approach at the policy level promises substantial gains but would require enormous data and computational capability.

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40

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Table-I

GLOBAL FOREIGN EXCHANGE RESERVES AND IMPORTS

Year	Non-gold Foreign Exchange Reserve (in U. S . dollar million) World	Devel- oped Countries	Devel- oping Countries	Asia	Imports (U.S.$ mn) World	Reserves Reserves as % of Imports (2) / (6)	Cover Number of weeks of imports
1	2	3	4	5	6	7	8
1971	94517	72735	21781	5919	345000	0.273961	14.3
1972	120408	88701	31708	7898	405800	0.296719	15.5
1973	140951	96449	44503	10357	560000	0.251698	13.1
1974	176315	97757	78556	12602	822500	0.214365	11.2
1975	185788	99231	86555	12226	867300	0.214214	11.2
1976	216824	108715	108109	18542	972400	0.222979	11.6
1977	277526	145828	131698	26066	1113400	0.249260	13.0
1978	319822	188660	131162	29443	1288000	0.248309	13.0
1979	359457	204059	155401	34537	1625700	0.221109	11.5
1980	409758	237272	172485	37882	2000500	0.204828	10.8
1981	383751	217088	166663	42635	1983900	0.193433	10.2
1982	361738	204691	157047	47562	1854300	0.195081	10.3
1983	379353	215876	163478	57159	1794000	0.211457	11.2
1984	399012	218607	177464	66040	1912400	0.208645	11.0
1985	444677	252017	192660	72794	1936400	0.229641	12.0
1986	512063	307394	204669	97625	2115400	0.242065	12.6
1987	720074	460114	259960	139238	2474800	0.290962	15.2
1988	730333	475120	255213	148555	2823600	0.258653	13.5
1989	776667	502306	274362	157609	3042700	0.255256	13.3
1990	907286	589129	318156	180691	3466500	0.261730	13.6
1991	960967	573103	387864	222716	3608000	0.266343	13.9
1992	953825	545526	408301	224011	3861000	0.247041	13.0
1993	1051988	567907	484083	259946	3774900	0.278680	14.5
1994	1203492	633332	570160	333129	4291300	0.280449	14.6
1995	1412096	724956	687141	375199	5120800	0.275757	14.3
1996	1566048	789219	776829	425821	5366000	0.291846	15.2
1997	1678093	779485	898608	503410	5549000	0.302414	15.7

Source : Various issues of International Financial Statistics , IMF.

Table II

Optimum Portfolio Selected Out of 20 International Bonds, When All Returns Are in Numeraire Currency US($)[1]

| Theta[2]	1 Year US	3 Year US	5 Year US	7 Year US	10 Year US	1 Year Yen	3 Year Yen	5 Year Yen	7 Year Yen	10Year Yen
0.1	0.8086	0.0387	0	0	0	0	0	0	0	0
0.2	0.7528	0.137	0	0	0	0	0	0	0	0
0.3	0.6168	0.2298	0	0	0	0	0	0	0	0
0.4	0.4943	0.3253	0	0	0	0	0	0	0	0
0.5	0.358	0.4172	0	0	0	0	0	0	0	0
0.6	0.2283	0.5031	0	0	0	0	0	0	0	0
0.7	0.1211	0.5683	0	0	0	0	0	0	0	0
0.8	0.0521	0.5974	0	0	0.0001	0	0	0	0	0
0.9	0.0165	0.5936	0	0	0.001	0	0	0	0	0
1	0.0065	0.5619	0.001	0	0.0037	0	0	0	0	0
1.5	0	0.3147	0.0165	0	0.0444	0	0	0	0	0
2	0	0.0929	0.0283	0	0.0725	0	0	0	0	0
2.5	0	0.0134	0.0104	0	0.0433	0	0	0	0	0
3	0	0.0004	0	0	0.0137	0	0	0	0	0
3.5	0	0	0	0	0.0019	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
4.5	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
5.5	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0

Note :1. Figures represent the share of the bond in the optimum portfolio
   2. Theta : Risk Tolerance Parameter

44

Table II (Contd.....)

Optimum Portfolio Selected Out of 20 International Bonds, When All Returns Are in Numeraire Currency US($)[1]

Theta[2]	1 Year DM	3 Year DM	5 Year DM	7 Year DM	10 Year DM	1 Year UK	3 Year UK	5 Year UK	7 Year UK	10Year UK
0.1	0	0	0	0	0	0.0427	0.0002	0.0004	0	0
0.2	0	0	0	0	0	0.0855	0.0013	0.0033	0.0001	0
0.3	0	0	0	0	0	0.1226	0.0026	0.0079	0.0004	0
0.4	0	0	0	0	0	0.1611	0.004	0.0145	0.0008	0
0.5	0	0	0	0	0	0.1973	0.0053	0.0209	0.0012	0
0.6	0	0	0	0	0	0.2323	0.0063	0.028	0.002	0
0.7	0	0	0	0	0	0.262	0.0075	0.0375	0.0035	0
0.8	0	0	0	0	0	0.2814	0.0088	0.0545	0.0057	0
0.9	0	0	0	0	0	0.2912	0.0092	0.0801	0.0084	0
1	0	0	0	0	0	0.2951	0.0089	0.1124	0.0106	0
1.5	0	0	0	0	0	0.3163	0.0089	0.2794	0.0198	0
2	0	0	0	0	0	0.316	0.0069	0.4562	0.0273	0
2.5	0	0	0	0	0	0.2445	0.0048	0.6546	0.029	0
3	0	0	0	0	0	0.1668	.0.004	0.7829	0.0321	0
3.5	0	0	0	0	0	0.1087	0.0026	0.8375	0.0494	0
4	0	0	0	0	0	0.0688	0.0011	0.8495	0.0806	0
4.5	0	0	0	0	0	0.0436	0	0.8254	0.131	0
5	0	0	0	0	0	0.0265	0	0.7715	0.202	0
5.5	0	0	0	0	0	0.0152	0	0.6905	0.2943	0
6	0	0	0	0	0	0.0086	0	0.6008	0.3905	0

Note :1. Figures represent the share of the bond in the optimum portfolio
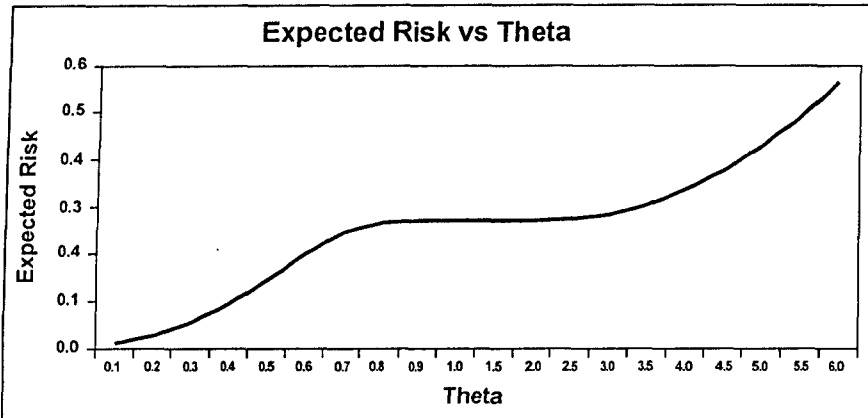2. Theta : Risk Tolerance Parameter

45

Graph - I : Levels of non-gold Reserves

Legend: World — Developed — Developing — Asia
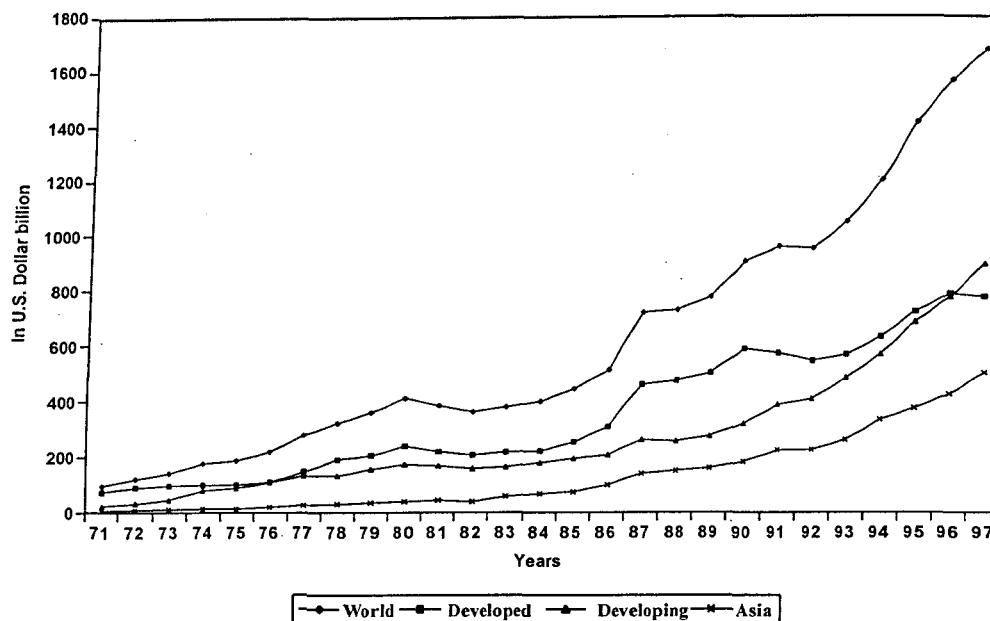
Graph - II : Reserve Cover for imports (World)

Years

46
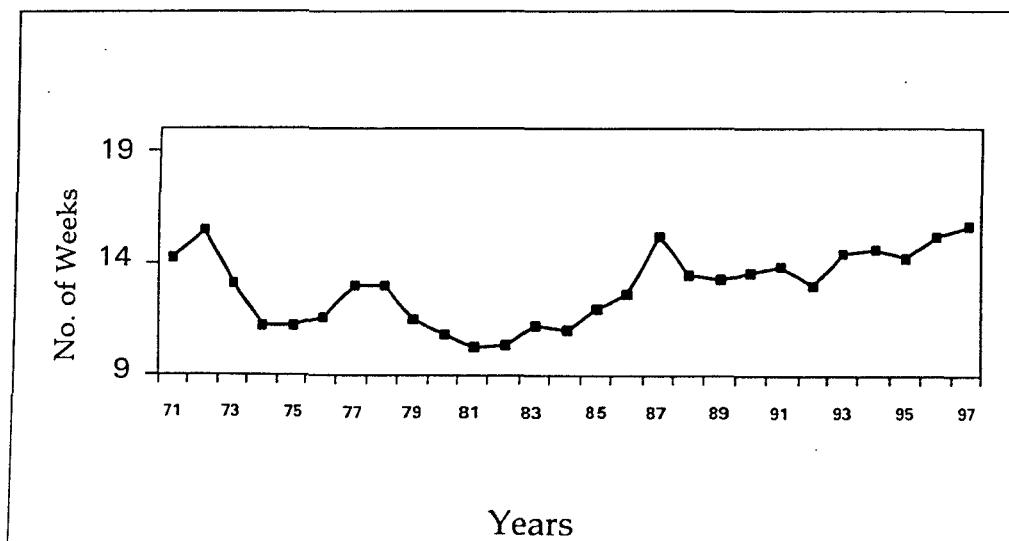
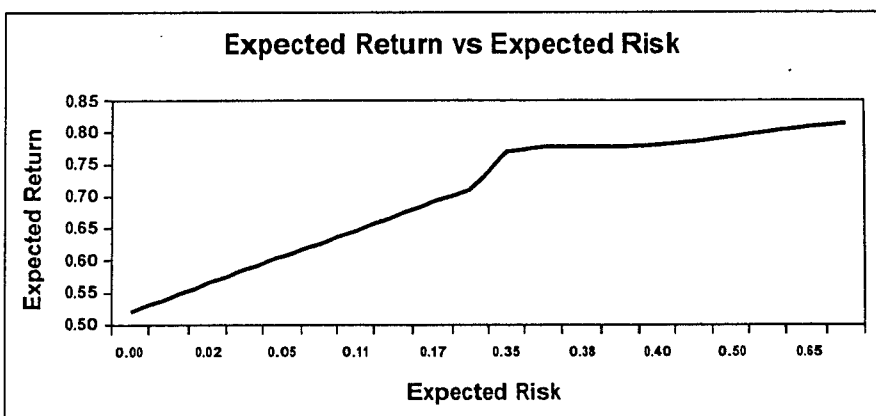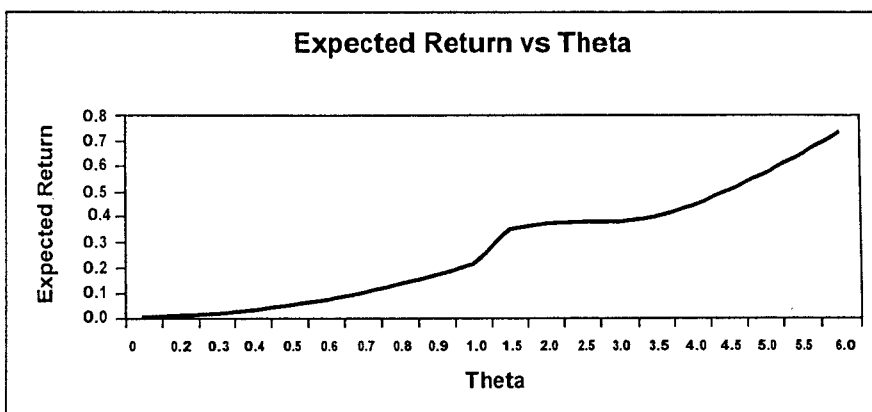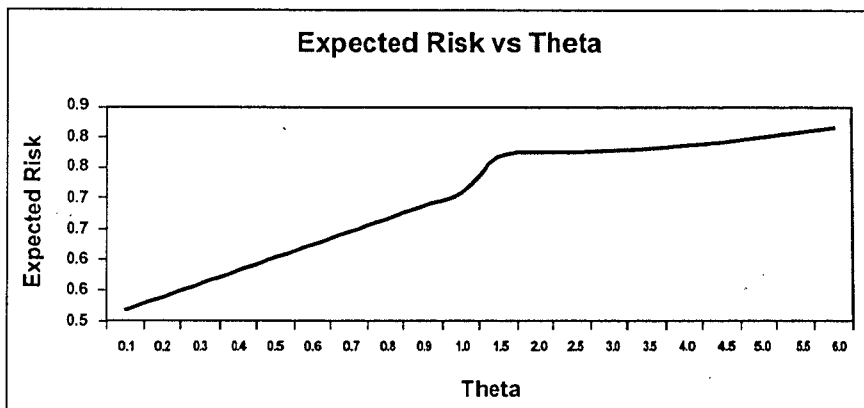## Graph III
Relationship between Theta, Expected Risk and Expected Return: US Dollar
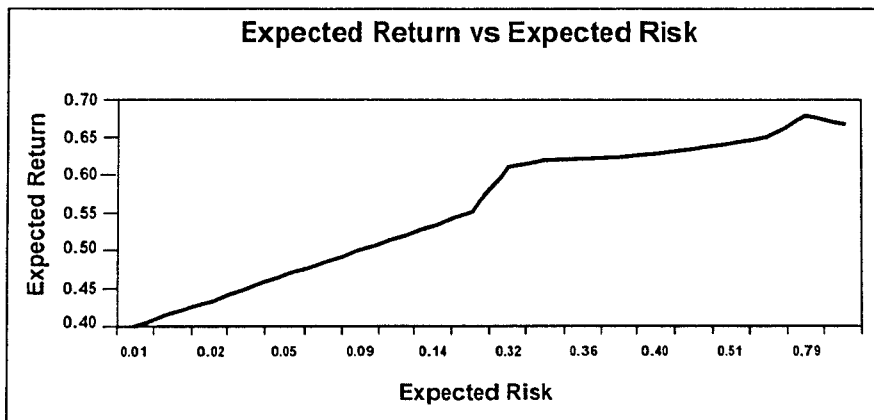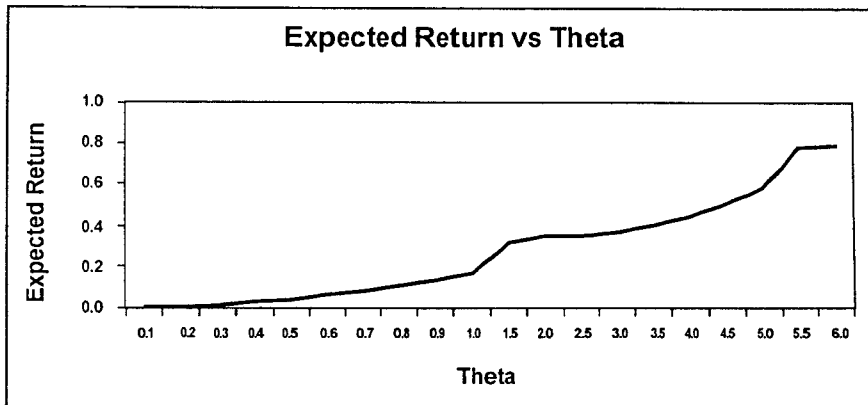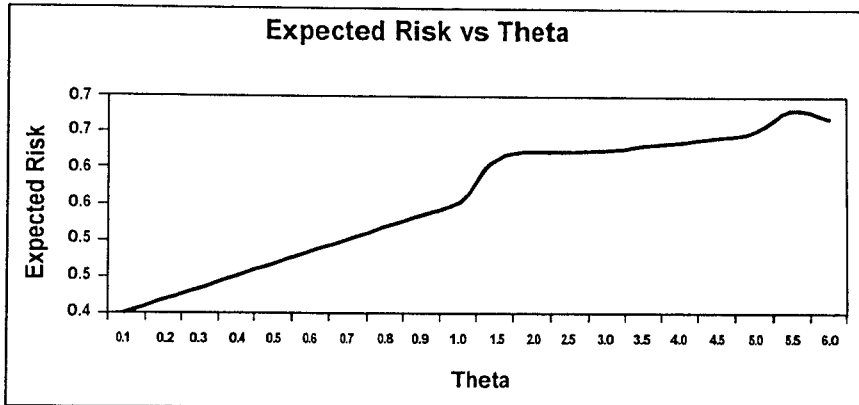
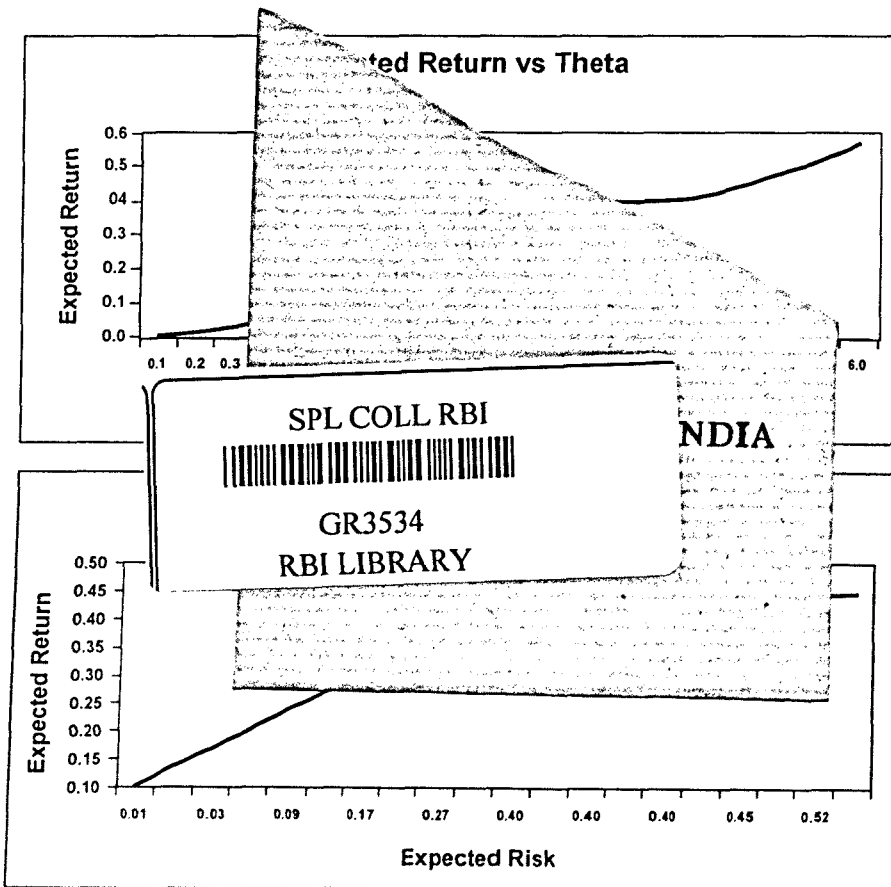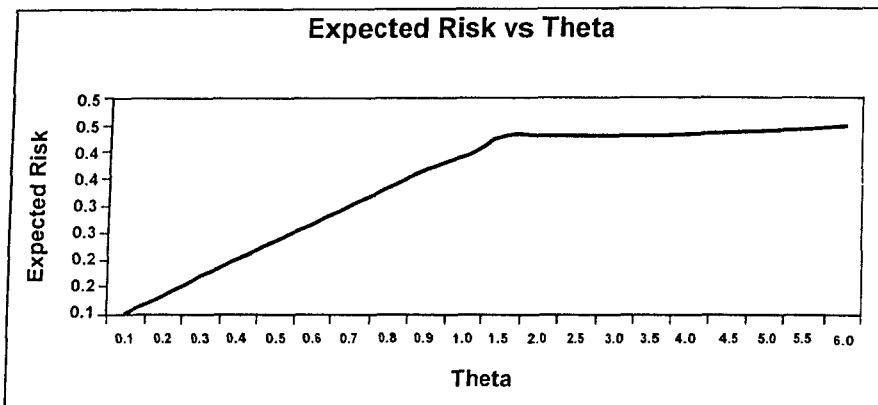## Graph - IV :
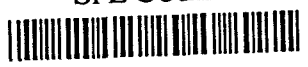## Relationship between Theta, Expected Risk and Expected Return: Deutsche Mark

48
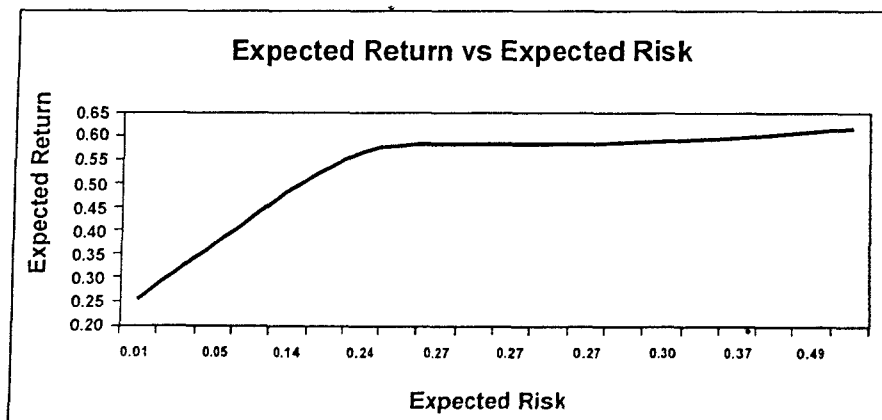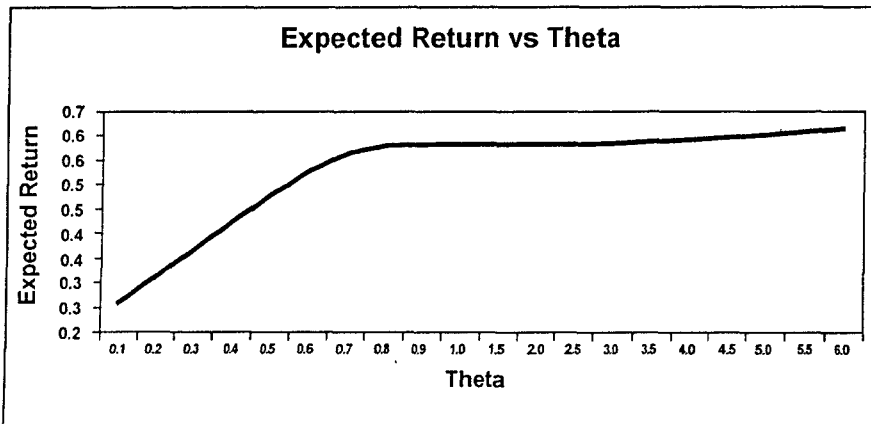
Graph-V
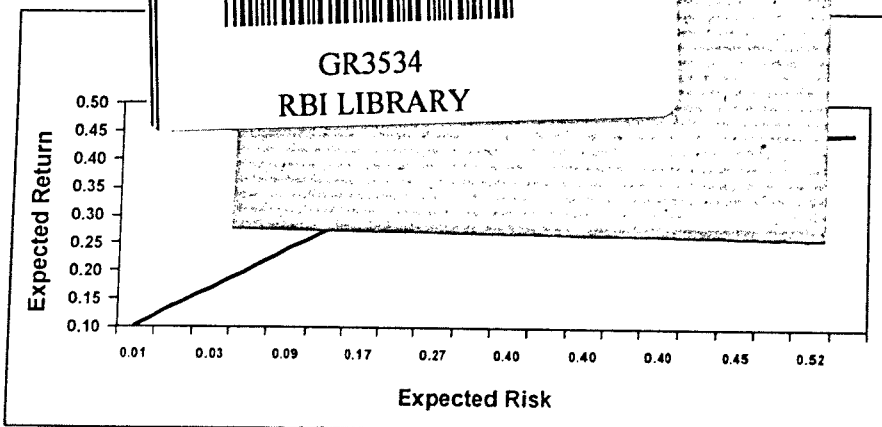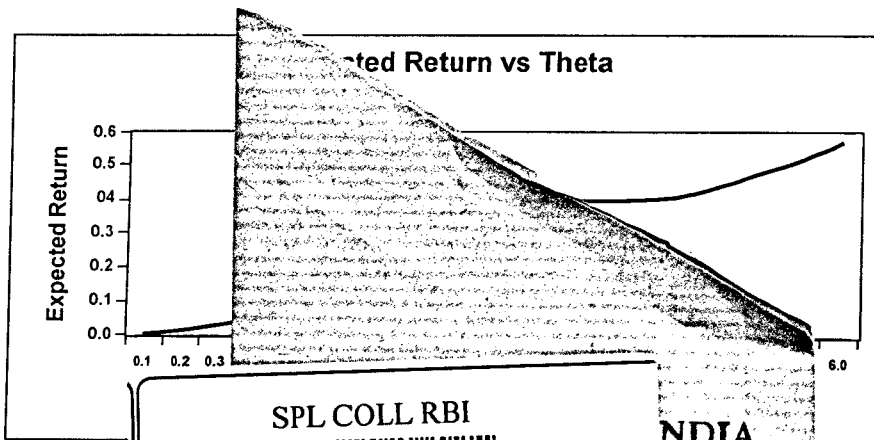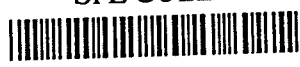## Relationship between Theta, Expected Risk and Expected Return: Pound Sterling

Expected Risk vs Theta

Expected Return vs Theta

Expected Return vs Expected Risk

49

Graph VI
Relationship between Theta, Expected Risk and Expected Return :Japanese Yen

Expected Risk vs Theta

...ed Return vs Theta

50

Graph VI
Relationship between Theta, Expected Risk and Expected Return :Japanese Yen

Expected Risk vs Theta

~ted Return vs Theta

50