
# Database and its Applications

## Data Models and Mathematical Foundations

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Computer Applications

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## Set Theory: Sets, Cartesian Products, Relations

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# Introduction to Sets

- A **set** is a well-defined collection of distinct objects.
- Objects in a set are called **elements** or **members**.
- Notation:
  - $A = \{1, 2, 3\}$
  - "$2 \in A$" means 2 is an element of $A$.
  - "$5 \notin A$" means 5 is not in $A$.
- Real-world Examples:
  - Set of vowels in English = {a,e,i,o,u}.
  - Set of months in a year.

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## Types of Sets – Basic

- **Empty Set** ($\emptyset$): Contains no elements. Example: $\{x$ — x is a square root of -1 in real numbers$\}$.
- **Singleton Set**: Contains exactly one element. Example: $\{$India$\}$.
- **Finite Set**: Has a limited number of elements. Example: $\{1,2,3,4\}$.
- **Infinite Set**: Has unlimited elements. Example: $\{1,2,3,\ldots\}$.

# Types of Sets – Advanced

- **Universal Set** ($U$): Set containing all objects under consideration.
  Example: All natural numbers.
- **Subset**:

$$A \subseteq B \iff (\forall x \in A, x \in B)$$

- **Equal Sets**: $A = B$ if $A \subseteq B$ and $B \subseteq A$.
- **Power Set**:

$$P(A) = \{\text{all subsets of A}\}$$

Example: If $A = \{1, 2\}$, $P(A) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$.

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## Set Membership and Standard Sets


- Membership:
  - $a \in A$ means $a$ is an element of $A$
  - $a \notin A$ means $a$ is not an element of $A$
- Standard Sets:
  - $\mathbb{N}$ – Natural numbers $\{1, 2, 3, \ldots\}$
  - $\mathbb{Z}$ – Integers $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$
  - $\mathbb{Q}$ – Rational numbers $\frac{p}{q}, q \neq 0$
  - $\mathbb{R}$ – Real numbers
  - $\emptyset$ – Empty set
  - $U$ – Universal set
- Example: Is $5 \in \mathbb{N}$? Yes. Is $0 \in \mathbb{N}$? Depends on convention.

# Cartesian Product – Definition

- For sets $A$ and $B$, the **Cartesian Product** is:

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

- Each element is an **ordered pair**.
- Size formula: $|A \times B| = |A| \cdot |B|$.

# Cartesian Product – Example

- Example:
  - $A = \{1, 2\}, B = \{x, y, z\}$
  - $A \times B = \{(1, x), (1, y), (1, z), (2, x), (2, y), (2, z)\}$
- Note: $A \times B \neq B \times A$.

# Relations – Definition

- A **relation** $R$ from set $A$ to set $B$ is any subset of $A \times B$.
- If $A = \{1, 2\}, B = \{a, b\}$, then
  $A \times B = \{(1, a), (1, b), (2, a), (2, b)\}$.
- Example Relation:

$$R = \{(1, a), (2, b)\} \subseteq A \times B$$

# Relations – Properties

- For a relation $R$ on a set $A$:
  - Reflexive: $(a, a) \in R$ for all $a \in A$.
  - Symmetric: $(a, b) \in R \Rightarrow (b, a) \in R$.
  - Transitive: $(a, b), (b, c) \in R \Rightarrow (a, c) \in R$.
  - Antisymmetric: $(a, b), (b, a) \in R \Rightarrow a = b$.
- These lead to **equivalence relations** and **partial orders**.

# Sets and Relations in Databases

- Mapping to Databases:
  - Set $\rightarrow$ Domain (e.g., set of all student IDs).
  - Cartesian Product $\rightarrow$ Possible combinations of attributes.
  - Relation $\rightarrow$ Table with rows as tuples.
- Example: Students = $\{$Alice, Bob$\}$, Courses = $\{$DBMS, OS$\}$ Enrollment Relation $\subseteq$ Students $\times$ Courses: $\{$(Alice, DBMS), (Bob, OS)$\}$

# Problem Solving – Sets and Relations

- Q1: Let $A = \{a, b\}, B = \{1, 2, 3\}$. Find $A \times B$.
- Q2: Define a relation $R \subseteq A \times B$ such that the second element is even.
- Solution:
  - $A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$
  - $R = \{(a, 2), (b, 2)\}$


# Thank You

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