Definitions:

- *Undirected graph*: G=(V,E), V=vertices, E=undirected edges.
- *Directed graph*: G=(V,E), V=vertices, E=directed edges.
- Notation: |V|=n, |E|=m.
- A *simple* graph: a graph with no parallel edges or self-loops.
- *Bipartite* graph: $G=(V_1,V_2,E)$. For each edge e=(u,v), $u\in V_1$, $v\in V_2$
- *Complete* graph (*clique*) of size n. A graph in which |V|=n and $E=V\times V$ (but no self loops).
- *Complete bipartite*: A bipartite in which $E= V_1\times V_2$.
- A *path* is a sequence of vertices $v_0,v_1, \ldots,v_k$ such that $(v_i,v_{i+1}) \in E$ for all $0 \leq i < k$
- An undirected graph is *connected* if for any pair of vertices u,v, there is a path between u and v.
- A directed graph is *strongly connected* if for any pair of vertices u,v, there is a directed path from u to v.
- *Undirected Tree*: An undirected graph is a *tree* if it is connected and acyclic
- *Directed Tree:* A directed graph is a *directed tree* if it has a *root* and its underlying undirected graph is a tree.
- $r\in V$ is a *root* if every vertex $v\in V$ there is a directed path from r to v.
- *Topological sort* of a directed graph: a linear ordering of the vertices such that for any edge $(v,w)\in E$, v precedes w in the ordering.
- An Euler path is a path that contains each edge exactly once.
- An Euler cycle is an Euler path that starts and ends at the same vertex.
- Let $\Sigma = \{0, 1, ..., \sigma - 1\}$ be an alphabet. Let $L= \sigma^n$. A *de Bruijn sequence* is a (circular) sequence $a_0a_1... a_{L-1}$ over Σ such that for every word w of length n over Σ there exists a unique i such that $a_ia_{i+1} \ldots a_{i+n-1} = w$, the computation of the indices is modulo L.

Facts:

- Let G=(V,E) be an undirected graph, then $2|E|=\Sigma_v\deg(v)$.
- Any directed acyclic graph has a topological sort.
- A graph has an Euler cycle if and only if it is connected and all its vertices have even degrees
- A graph has an Euler path if and only if it is connected and exactly two of its vertices have odd degrees
- A Hamiltonian path (cycle) goes through each vertex exactly once (and returns to the starting vertex).
- The number of spanning trees over n vertices is n^{n-2}.
- In the proof of Cayley's theorem, a tree in which the degree of v is deg(v) corresponds to a word in which v appears deg(v)-1 times.
- The digraph $G_{\sigma,n}(V, E)$ built for constructing a de-bruin sequence for σ and n :
  1. V is the set of all σ^{n-1} words of length $n - 1$ over Σ.
  2. E is the set of all σ^n words of length n over Σ. The edge $b_1b_2 \ldots b_n$, starts at vertex $b_1b_2 \ldots b_{n-1}$ and ends at vertex $b_2b_3 \ldots b_n$.

<u>Algorithm for topological sort.</u> Time complexity O(n+m):

- Store each vertex's In-Degree in an array D
- Initialize queue with all "in-degree=0" vertices
- While there are vertices remaining in the queue:
    - Dequeue and output a vertex
    - Reduce In-Degree of all vertices adjacent to it by 1
    - Enqueue any of these vertices whose In-Degree became zero.
- If all vertices are output then success, otherwise there is a cycle.