Exercise 2 in Algorithms

The Interdisciplinary Center, Herzliya

Due: 19.11.14

Problem 1 (20 pts.):

Let G=(V,E) be a connected undirected graph.

a. Prove that the number of vertices with an odd degree in G is even.

b. Prove that it is possible to add at most $\lfloor n/2 \rfloor$ edges to E such that the resulting graph has an Euler cycle.

Problem 2 (16 pts.):

Find an Euler path in the following undirected graph, represented by a matrix. Whenever you have multiple ways to advance, prefer vertices with lower labels. Note: the matrix entries give the number of edges between the respective nodes. For example, there are two parallel edges connecting a andb. An empty entry corresponds to 0.

Describe the stages performed in the path construction.

	a	b	c	d	e	f	g
a		2	1			1	
b	2		1		1		
c	1	1		2			
d			2			1	1
e		1				1	
f	1			1	1		1
g				1		1	

Problem 3 (24 pts.):

In each of the following questions, select the correct answer andjustify.

3.1 Let $G = (V_1, V_2, E)$ be an undirected bipartite graph with an Euler cycle.

a. $|V_1| = |V_2|$
b. $|V_1|$ and $|V_2|$ are even.
c. $\sum_{u \in V_1} \deg(u) = \sum_{v \in V_2} \deg(v)$

3.2 Given a complete graph over 183 vertices (K_{183}), two edges with a common endpoint are removed from the graph. In the resulting graph:

a. There is an Euler cycle.
b. There is an Euler path which is not a cycle.
c. There is no Euler path and no Euler cycle.

3.3 Let G be a graph in which every Hamiltonian cycle is also an Euler cycle.

a. G must be a clique

b. G must be a cycle

c. G is not necessarily a clique or a cycle.

Problem 4 (20 pts.):

Let G be an undirected connected graph, such that all its vertices have evendegrees. A vertex *s* is called an Euler-center if every application of the algorithmfor finding an Euler cycle in G, starting at *s*, produces an Euler cycle in a singleiteration, no matter what is the order in which edges are selected. In other words,there is no need to merge different parts of the cycle into others. Prove that *s* is anEuler-center if and only if *s* appears in every simple cycle in G.

Problem 5 (20 pts.):

Consider the De-Bruijn digraph for the alphabet $\{0,1\}$and $n = 7$. Assume we remove from this graph the two vertices (000000) and (100000) together with all the edges adjacent to these vertices. Denote the resulting graph by G'.

Determine for each of the following claims if it is true or false. Justify your answer.

1. G' has a directed Euler path.

2. The underlying graph of G' has an Euler path.