The Interdisciplinary Center, Herzeliya
Digital Architecture Course

Exercise 3

1. (20 pts) Implement a $2^k \rightarrow 1$ mux using **one** $2^{k-1} \rightarrow 1$ mux and $2 \rightarrow 1$ muxes as much as you like.
   Write down the recursive and the explicit equations of the cost and delay of the circuit. (Assume a $2 \rightarrow 1$ mux costs A and its delay is T).

2. (15 pts) Simplify the following functions using a Karnaugh map: (remember the order on variables!):

   a) $F = B' \cdot C + A' \cdot B' \cdot C' + A \cdot B \cdot C'$ (use a\bc)
   b) $Y = A' \cdot B' \cdot C' + A \cdot B' \cdot C' + A' \cdot B \cdot C' + A \cdot B \cdot C' + A' \cdot B \cdot C$ (use a\bc)
   c) $Z = X \cdot Y \cdot B' + X' \cdot A' + X \cdot Y \cdot A' \cdot B + X' \cdot Y \cdot A \cdot B'$ (use xy\ab)

3. (17 pts) Given the functions:
   - $F1(W,X,Y,Z) = \Sigma(0,1,4,13) + \Sigma d(5,9,12,14,15)$
   - $F2(W,X,Y,Z) = \Sigma(1,4,8,12,13) + \Sigma d(0,2,5,6,9)$

   (Bit coding presentation - example: 4 -> 0100, 5 -> 0101…
   Σ – '1' for this bit coding presentation, Σd – 'don't care' for this bit coding presentation)

   And the following definition:
   - $F3 = F1 + F2$
   - $F4 = F1 \cdot F2$

   a) Find a minimal SOP for F3, F4 (SOP- use '1')
   b) Find a minimal POS for F3, F4 (POS – use '0')

   Guidelines:
   - Draw Karnaugh maps for F1, F2
   - Build a Karnaugh map for F3, F4 based on the maps of F1, F2 and the following rules: (Why do these rules apply?)

$0 \cdot \phi = 0$	$0 + \phi = \phi$
$1 \cdot \phi = \phi$	$1 + \phi = 1$

   - Using the map, find the minimal SOP and POS of F3, F4
   - Use always WX\YZ maps!

4. (24 pts) Design and implement a modulus 6 function. The system receives a number between 0 and 15 (In a binary format using 4 bits) and should give an output of the number modulus 6 (3 bits). For example if the input is 1011, the output should be 101.

5.(24 pts) Finish the design of “7 segment” that you started in the lecture. Design parts S_2, S_5 and S_7. (use I_3 I_2 \ I_1 I_0 maps!) pay attention for don’t cares.