Computability and Complexity Theory – Exercise 1

Turing Machines

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Collaboration statement:
שיתפתי פעולה עם עצמי אך כתבתי את הפתרונות בעצמי ולא נעזרתי בפתרון כתוב כלשהו בזמן כתיבת התשובות.

Problem 1: (Elementary concepts, 25 points)

a. Write the following sets formally:

Example: The set of all strings over Σ of length less or equal to 5 may be written formally as $\{w \in \Sigma^* : |w| \leq 5\}$.

1. The set of all strings over {0,1} of even length:

   $\{w \in \Sigma^* \mid |w| \bmod 2 = 0\}$

   i. Does it contain infinite length strings? No.
   ii. What is the shortest string in the set? ε.

2. The set of all strings over {0,1} that have an even number of "0":

   $\{w \in \Sigma^* \mid \#_0(w) \bmod 2 = 0\}$

3. The set containing the empty string:

   $\{ \varepsilon \}$

4. The set of all bipartite graphs (A bipartite graph G is a graph whose vertices can be divided into two disjoint sets U and V such that every edge connects a vertex in U to a vertex in V):

   {G: G=(V,E) is a graph and $V=V_1 \cup V_2$, s.t. ∀ (w,t) and ∀ (t,w), $w \in V_1$ and $t \in V_2$}

b. Let $K = \{L \subseteq \{0,1\}^* : \forall\ w \in L\ |w| \leq 4$ and $|w|$ is odd$\}$.

1. Let $L_1, L_2 \in K$.
   i. Is $L_1 \cap L_2 \in K$? → yes.
   ii. Is $L_1L_2 \in K$? (L_1L_2 is the concatenation of L_1 and L_2) → No.
   iii. Is $\emptyset \in K$ ($\emptyset$ denotes the empty set)? → Yes.
   iv. Is $\{0,00,000\} \in K$? → No.
2. Let $L \in K$.
   i. Give a tight upper bound on $|L|$ (i.e., how large can $|L|$ be?):

      10. The set may contain strings of length 1 or 3. Assume L contains all the possible strings, hence we've got 2^1 strings of length 1 and 2^3 strings of length 3 thus $|L| = 2^1 + 2^3 = 10$
   ii. What is the concatenation of L and $\emptyset$? $\emptyset$.
   iii. What is the concatenation of L and $\{\varepsilon\}$? L.
3. What is $|K|$? 1024. Explain:

   The size of the "biggest" language is 10, hence K is the set of languages constructed from those 10 options thus K is the power set of the biggest language which yields $2^{10} = 1024$.

c. Write down the complement of the following language:
   $L = \{ P \subseteq \{0,1\}^*$: P is a legal encoding of a C program, and P terminates on all inputs that start with '0' bit$\}$

   L-COMP = $\{ P \subseteq \{0,1\}^*$: (P is not a legal encoding of a C program) $\cup$ (P does not terminate on all inputs that start with '0' bit)$\}$

Problem 2: (25 points) Let M be a Turing machine with:

$Q = \{q_0,q_1,q_{acc},q_{rej}\}$, $\Sigma = \{0,1\}$, $\Gamma = \{0,1,\_\}$ and δ remains undefined.

For each of the following pairs of configurations, C_i and C_{i+1}, determine whether it is possible under some definition of δ that C_i will yield C_{i+1}. If possible, define the specific transition that will cause M to go from C_i to C_{i+1}. If not, explain why.

a. C_i = $011q_0100$

C_{i+1} = $01q_11100$

$\delta(q_0,1)=(q_1,1,L)$

b. C_i = $011q_1001$

C_{i+1} = $0110q_001$

$\delta(q_1,0)=(q_0,0,R)$

c. C_i = $011q_001$

C_{i+1} = $01q_{acc}101$

$\delta(q_0,0)=(q_{accept},0,L)$

d. C_i = $010q_0100$

C_{i+1} = $0101q_00$

No. There's a missing '0' from the tape. Assuming *M* did delete the '0' it should have written a "space" (at least) in that position rather than nothing.

e. (0 points, not to be submitted)

C_i = $111q_0110$

C_{i+1} = $11q_00110$

f. (0 points, not to be submitted)

C_i = $011q_{acc}110$

C_{i+1} = $0101q_010$

g. (0 points, not to be submitted)

C_i = $101q_1101$

C_{i+1} = $1010q_301$

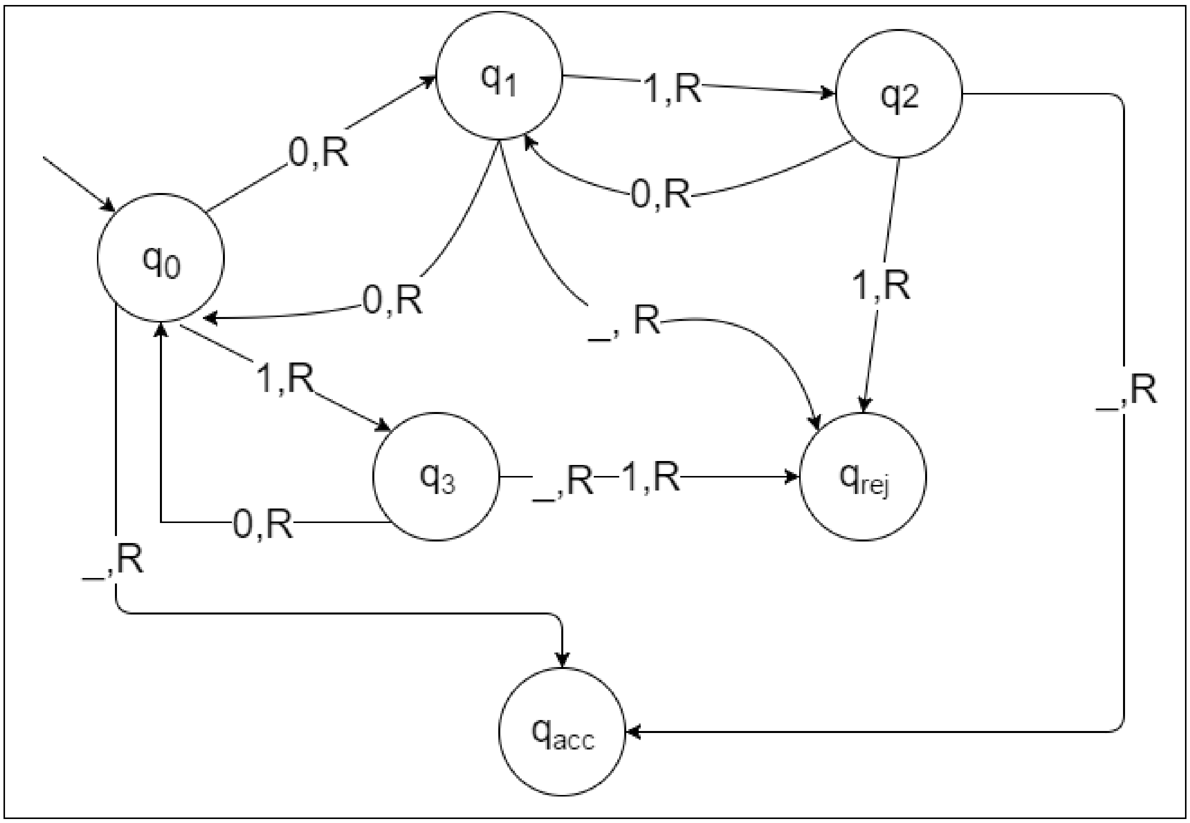
h. (0 points, not to be submitted)

C_i = $010q_1101$

C_{i+1} = $0100q_001$

Problem 3: (25 points)

Let L = $\{w \in \{0,1\}^* \; : |w|$ is even and w does not contain '11' as a subsequence$\}$.
Draw a state diagram of a Turing machine M that decides the language L (reminder: a Turing machine decides a language L if M accepts every w∈L and rejects every w∉L).



Explain in words each component of the construction.

q_{acc} and q_{rej} are self-describing and doesn't have any logic following them.

q_0 is the starting state and marks even length *legal* strings (or the empty string). q_1 will mark strings with odd length that start by '0', q_2 indicates strings of even length that end with '1', q_3 indicates odd strings starting with '1'. Each state that accessed directly by reading '1' will cause redirection towards q_{rej} upon reading another '1'.

For each of the input strings below give the configurations sequence of M on the corresponding inputs:

a. 1001

$q_0 1001 \Rightarrow 1q_3 001 \Rightarrow 10q_0 01 \Rightarrow 100q_1 1 \Rightarrow 1001q_2\_ \Rightarrow 1001\_q_{acc}$

b. 0110

$q_0 0110 \Rightarrow 0q_1 110 \Rightarrow 01q_2 10 \Rightarrow 011q_{rej} 0$

c. 100

$q_0 100 \Rightarrow 1q_3 00 \Rightarrow 10q_0 0 \Rightarrow 100q_1 \Rightarrow 100\_q_{rej}$

Problem 4: (25 points)

Consider the following Turing machine:

$$Q = \{q_0,q_1,q_2,q_3,q_{acc}\} \;,\; \Sigma = \{0,1\} \;,\; \Gamma = \{0,1,\#,\_\} \quad \text{and}$$

$$\delta : Q\times\Gamma \rightarrow Q\times\Gamma\times\{L,R\} \text{ is defined as follows:}$$

$\delta(q_0,0) = (q_1,\#,R)$ $\quad$ $\delta(q_1,0) = (q_1,0,R)$ $\quad$ $\delta(q_2,0) = (q_1,1,R)$

$\delta(q_0,1) = (q_2,\#,R)$ $\quad$ $\delta(q_1,1) = (q_2,0,R)$ $\quad$ $\delta(q_2,1) = (q_2,1,R)$

$\delta(q_1,\_) = (q_3,0,L)$ $\quad$ $\delta(q_3,0) = (q_3,0,L)$

$\delta(q_2,\_) = (q_3,1,L)$ $\quad$ $\delta(q_3,1) = (q_3,1,L)$ $\quad$ $\delta(q_3,\#) = (q_{acc},0,R)$

a. What does this TM output on input 11100101011?
   0 (tape snapshot: 011100101011)

b. Describe (in words) the function computed by this Turing machine. Your description should be clear and unambiguous.

The machine adds a 0 to the given string (so visually it looks like a shit-right, but it's not the function shift-right)

Problem 5: (0 points, not to be submitted)

Provide a detailed description (including Q, Σ, Γ and δ) of a Turing machine that, given an input x over {0,1}, outputs 0y, where y is identical to x except the last character, which is negated (for example, on input 01110 the machine should output 001111). You may assume that the input contains at least one symbol.

Problem 6: (0 points, not to be submitted)

Let L⊆{0,1}* be some language. Define

Max(L) = {w∈L : there is no x∈{0,1}* of size > 0 such that wx∈L }.

1. Define Max(L) for each of the following languages:
   a. $L_1 = \{0^n 1^n 0^i \mid n \geq 0, i = 0\}$
   b. $L_2 = \{0^n 1^n 0^i \mid n \geq 0, i \geq 0\}$
   c. $L_3 = \{0^n 1^n 0^i \mid n \leq 2, i \leq 2\}$