Computability and Complexity Theory – Exercise 1

Turing Machines

ID:____________________

Due date: *Tuesday, March 15th, please follow the posted submission instructions.*

Please write your answers in the designated spaces.

Also don't forget to write down a *collaboration statement* (either “ שיתפתי פעולה עם ... אך כתבתי את הפתרונות בעצמי ולא נעזרתי בפתרון כתוב כלשהו בזמן כתיבת התשובות” or “ נעזרתי בחומר מ... אך כתבתי את הפתרונות בעצמי ולא נעזרתי בפתרון כתוב כלשהו בזמן כתיבת התשובות”).

Collaboration statement:
____
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Problem 1: (Elementary concepts, 25 points)

a. Write the following sets formally:

   Example: The set of all strings over Σ of length less or equal to 5 may be written formally as $\{w \in \Sigma^* : |w| \leq 5\}$.

   1. The set of all strings over {0,1} of even length:

      ____

      i. Does it contain infinite length strings? _______.
      ii. What is the shortest string in the set? _______.

   2. The set of all strings over {0,1} that have an even number of "0":

      ____

3. The set containing the empty string:

   ____

4. The set of all bipartite graphs (A bipartite graph G is a graph whose vertices can be divided into two disjoint sets U and V such that every edge connects a vertex in U to a vertex in V):

   {G: G=(V,E) is a graph and (____________________

   ____)

b. Let K = {L⊆{0,1}*: ∀ w∈L |w|≤ 4 and |w| is odd}.

1. Let L_1,L_2∈K.

   i. Is $L_1 \cap L_2 \in$K? ______

   ii. Is $L_1L_2\in$K? (L_1L_2 is the concatenation of L_1 and L_2) ______

   iii. Is ∅∈K (∅ denotes the empty set)? ______

   iv. Is {0,00,000}∈K? ______

2. Let L∈K.

   i. Give a tight upper bound on |L| (i.e., how large can |L| be?):

      _______________. Explain: _____________________________

      ____

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   ii. What is the concatenation of L and ∅? _____.

   iii. What is the concatenation of L and {ε}? _____.

3. What is |K|? ____________. Explain:

   ____

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c. Write down the complement of the following language:
$L = \{ P \subseteq \{0,1\}^*$: P is a legal encoding of a C program, and P terminates on all inputs that start with '0' bit$\}$

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Problem 2: (25 points) Let M be a Turing machine with:

$Q = \{q_0, q_1, q_{acc}, q_{rej}\}$, $\Sigma = \{0,1\}$, $\Gamma = \{0,1,\_\}$ and δ remains undefined.

For each of the following pairs of configurations, C_i and C_{i+1}, determine whether it is possible under some definition of δ that C_i will yield C_{i+1}. If possible, define the specific transition that will cause M to go from C_i to C_{i+1}. If not, explain why.

a. C_i = $011q_0100$
C_{i+1} = $01q_11100$

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b. C_i = $011q_1001$
C_{i+1} = $0110q_001$

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c. C_i = $011q_001$
C_{i+1} = $01q_{acc}101$

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d. C_i = $010q_0100$
C_{i+1} = $0101q_00$

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e. (0 points, not to be submitted)

C_i = $111q_0110$
C_{i+1} = $11q_00110$

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f. (0 points, not to be submitted)

C_i = 011q_{acc}110

C_{i+1} = 0101$q_0$10

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g. (0 points, not to be submitted)

C_i = 101$q_1$101

C_{i+1} = 1010$q_3$01

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h. (0 points, not to be submitted)

C_i = 010$q_1$101

C_{i+1} = 0100$q_0$01

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Problem 3: (25 points)

Let L = $\{w \in \{0,1\}^* : |w|$ is even and w does not contain '11' as a subsequence$\}$.
Draw a state diagram of a Turing machine M that decides the language L (reminder: a Turing machine decides a language L if M accepts every w$\in$ L and rejects every w$\notin$L).

Explain in words each component of the construction.

For each of the input strings below give the configurations sequence of M on the corresponding inputs:

a. 1001

b. 0110

c. 100

Problem 4: (25 points)

Consider the following Turing machine:

$$Q = \{q_0,q_1,q_2,q_3,q_{acc}\} \ , \ \Sigma = \{0,1\} \ , \ \Gamma = \{0,1,\#,\_\} \quad \text{and}$$

$$\delta : Q\times\Gamma \rightarrow Q\times\Gamma\times\{L,R\} \text{ is defined as follows:}$$

$\delta(q_0,0) = (q_1,\#,R)$	$\delta(q_1,0) = (q_1,0,R)$	$\delta(q_2,0) = (q_1,1,R)$
$\delta(q_0,1) = (q_2,\#,R)$	$\delta(q_1,1) = (q_2,0,R)$	$\delta(q_2,1) = (q_2,1,R)$
$\delta(q_1,\_) = (q_3,0,L)$	$\delta(q_3,0) = (q_3,0,L)$	
$\delta(q_2,\_) = (q_3,1,L)$	$\delta(q_3,1) = (q_3,1,L)$	$\delta(q_3,\#) = (q_{acc},0,R)$

a. What does this TM output on input 11100101011?

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b. Describe (in words) the function computed by this Turing machine. Your description should be clear and unambiguous.

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Problem 5: (0 points, not to be submitted)

Provide a detailed description (including Q, Σ, Γ and δ) of a Turing machine that, given an input x over {0,1}, outputs 0y, where y is identical to x except the last character, which is negated (for example, on input 01110 the machine should output 001111). You may assume that the input contains at least one symbol.

Problem 6: (0 points, not to be submitted)

Let L⊆{0,1}* be some language. Define

Max(L) = {w∈ L : there is no x∈ {0,1}* of size > 0 such that wx∈ L }.

1. Define Max(L) for each of the following languages:
   a. $L_1 = \{0^n1^n0^i \mid n \geq 0, i = 0\}$
   b. $L_2 = \{0^n1^n0^i \mid n \geq 0, i \geq 0\}$
   c. $L_3 = \{0^n1^n0^i \mid n \leq 2, i \leq 2\}$