



AEROSPACE PALACE ACADEMY, NIGERIA
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LESSON 29: DIVISIBILITY RULES

A useful skill in arithmetic is knowing when a large number can be divided evenly by a smaller number. The “divisibility rules” are taught in school, but the reasons they work are not usually taught. Most of the reasons are not difficult to understand.

GRADES K-2

A wonderful way to prepare your students for multiplication and division is to sort a set of objects into equal subsets. Given a group of 20 blocks, how many ways can they discover to split them into smaller groups? Two sets of 10, four sets of 5, etc. Try different starting amounts and see what the students observe about their subsets. Do any of them notice that an even number can be sorted into an even number of subsets, but an odd number cannot?

Another useful exercise is to have students practice their skip-counting. Counting by 2s, 5s, or 10s is a small step in the direction of learning to multiply. An easy place to start is with counting feet or fingers. If each student has 2 feet, then how many feet are in the whole class? If $5 + 5 = 10$ fingers per student, then how many fingers in the whole class? From there you can count chair legs, table legs, and other everyday objects.

Useful read-alouds: *What Comes in 2s, 3s, and 4s?* by Suzanne Aker, *What’s a Pair?* *What’s a Dozen?* by Steve Swinburne, *One Hundred Hungry Ants* by Elinor J. Pinczes

GRADES 3-5

The divisibility rules are these:

- ☐ If a number is even, meaning that its last digit can be divided by two, then the entire number can be divided by two.
- ☐ If the sum of a number’s digits can be divided by three, then the entire number can be divided by three.
- ☐ If the last two digits of a number can be divided by four, then the entire number can be divided by four.

GRADES 3-5 (CONTINUED)

- ✓ If the last digit of a number is zero or five, then the entire number can be divided by five.
- If a number can be divided by both two and three, then it can be divided by six.
- If the last three digits of a number can be divided by eight, then the entire number can be divided by eight.
- If the sum of a number's digits can be divided by nine, then the entire number can be divided by nine.
- If a number ends with a zero, it can be divided by ten.

The divisibility rules for two and five are easily explained. Any number can be split into its ones digit and the rest of the number. The value of the number is the value of its ones digit, plus ten times the value of the rest of the number. Ten is divisible by two; thus the value of the entire number without the ones digit is always divisible by two. If the ones digit can be divided by two, then the entire number can be divided by two.

Dividing by five works the same way. The value of rest of the number without the ones digit is always divisible by five because it is ten times something. If the ones digit is also divisible by five, then the entire number can be divided by five.

GRADES 6-8

The divisibility rule for four works because 100 is a multiple of four. Any number can be divided into its last two digits and the rest of the number. The value of the number is the value of the last two digits plus one hundred times the value of the rest of the number. One hundred is divisible by four; if the number formed from the last two digits is also divisible by four, then the entire number is divisible by four.

Dividing by eight is similar, but requires three digits. This is because 1000 is divisible by eight.

If you add nine to a number, either you change the ones digit from zero to nine or else you carry. If you carry, you subtract one from the ones digit and add one to the tens digit.

GRADES 6-8 (CONTINUED)

Without carrying, you add nine to the sum of the digits; if you carry, then the sum of the digits stays the same. If the tens digit is a nine, then it turns into a zero and you add one to the hundreds digit; the net result is that you subtract nine from the sum of the digits. Starting with the number zero, the sum of the digits is zero, which is divisible by nine. Each time you add nine, the sum of the digits continues to be divisible by nine.

The divisibility rule for three is similar to the one for nine. When you add three to a number, either you increase the ones digit by three or else you carry. In the first case, you increase the sum of the digits by three. In the second case, you reduce the ones digit by seven and increase the tens digit by one, reducing the sum of the digits by six. If you need to carry to the hundreds place, the tens-to-hundreds carrying reduces the sum of the digits by another nine. Six and nine are multiples of three, so the rule works.

There is another divisibility rule, for eleven. Starting with the leftmost digit, alternately add and subtract the digits of the number. If your running total is too less than the next digit so that you cannot subtract, add eleven to it. Similarly, if your running total is larger than eleven, you can subtract eleven from it. For example, 280258 will give you $2 (+ 11) - 8 + 0 - 2 + 5 - 8 = 0$. If the result is divisible by eleven, then the entire number is divisible by eleven.

GRADES 9-12

Divisibility rules also work in other numbering systems. For example, there are the following rules in octal (base 8):

- ✓ If the last digit of a number in octal can be divided by two, the entire number can be divided by two. Note that 10_8 is a multiple of two.
- ✓ The add-and-subtract rule for divisibility by eleven described in the previous grades works for three in octal. Note that 11_8 is a multiple of three.
- ✓ If the last digit of a number in octal can be divided by four, the entire number can be divided by four. Note that 10_8 is a multiple of four.
- ✓ If a number can be divided by both two and three, then it can be divided by six.

GRADES 9-12 (CONTINUED)

- ✓ If the sum of the digits of a number in octal can be divided by seven, the entire number can be divided by seven. Note that $7 = 10_8 - 1$.
- ✓ If the last digit of a number in octal is a zero, the entire number can be divided by 10_8 , which is eight.
- ✓ The add-and-subtract rule for divisibility by eleven described in the previous grades works for 11_8 , which is nine.

Another commonly-used numbering system is hexadecimal, or base 16. Here are the divisibility rules:

- ✓ If the last digit of a number in hexadecimal can be divided by two, the entire number can be divided by two. Note that 10_{16} is a multiple of two.
- ✓ If the sum of the digits of a number in hexadecimal can be divided by three, the entire number can be divided by three. Note that three is a factor of fifteen, which is $10_{16} - 1$.
- ✓ If the last digit of a number in hexadecimal can be divided by four, the entire number can be divided by four. Note that 10_{16} is a multiple of four.
- ✓ If the sum of the digits of a number in hexadecimal can be divided by five, the entire number can be divided by five. Note that five is a factor of fifteen, which is $10_{16} - 1$.
- ✓ If a number can be divided by both two and three, then it can be divided by six.
- ✓ If the last digit of a number in hexadecimal can be divided by eight, the entire number can be divided by eight. Note that 10_{16} is a multiple of eight.
- ✓ If a number can be divided by both two and five, then it can be divided by ten, which is A_{16} .
- ✓ If a number can be divided by both three and four, then it can be divided by twelve, which is C_{16} . Note that three and four have no common factors; if a number is divisible by two and six, we cannot conclude that it is divisible by twelve.

GRADES 9-12 (CONTINUED)

- ✓ If the last digit of a number in hexadecimal is a zero, the entire number can be divided by 10_{16} , which is sixteen.
- ✓ The add-and-subtract rule for divisibility by eleven described in the previous grades works for 11_{16} , which is seventeen.

Sixty Years Ago in the Space Race:

December 24, 1956: "Aviation Week" magazine reported that [William O'Sullivan of the American National Advisory Committee on Aeronautics \(NACA\) proposed using an inflatable sphere as a satellite to measure the density of air at extremely high altitudes.](#) The United States later put the spherical "Echo" satellites into orbit.