Stability of a laser cavity with non-parabolic phase transformation elements

Igor A. Litvin*

CSIR National Laser Centre, PO Box 395, Pretoria 0001, South Africa
*ILitvin@csir.co.za

Abstract: In this paper we present a general approach to determine the stability of a laser cavity which can include non-conventional phase transformation elements. We consider two pertinent examples of the detailed investigation of the stability of a laser cavity firstly with a lens with spherical aberration and thereafter a lens axicon doublet to illustrate the implementation of the given approach. In the particular case of the intra–cavity elements having parabolic surfaces, the approach comes to the well–known stability condition for conventional laser resonators namely $0 \le (1 - z / R_1)(1 - z / R_2) \le 1$.

OCIS codes: (140.3410) Laser resonators; (140.3300) Laser beam shaping.

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1. Introduction

The recent development of the new types of optical phase transformation devices lead to new implementation of these devices in the laser cavity to control the spatial intensity and phase distribution of laser modes [1, 2], maximization of mode–pump matching for increasing pump efficiency [3] and thermal lensing compensation [4]. Such devices include aspheric optical elements [5], high quality diffractive optical elements [6], kinoforms [7], Zernike plates [8], deformable mirrors [4] and spatial light modulators [9] and so on [10, 11]. One of the most important parameters in constructing such laser systems is the stability of a particular laser cavity [12]. In the case of conventional phase transformation elements such as mirrors and

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lenses with spherical surfaces there exists a simple relationship between the laser cavity geometry and the stability of a particular cavity, however, in implementing non-conventional phase transformation elements, intra-cavity, we cannot use such a relation. Consequently the intra-cavity implementation of any non-conventional phase transformation elements or taking into account the thermal lensing which in general has a non-parabolic phase transformation [13], leads to a solution of the complicated Fox–Li eigenvalue problem [1,14] in order to determine the real stability of such a cavity.

In this paper we have presented a simple formula to investigate the stability of a laser cavity that includes non-conventional phase transformation elements. We have presented several important examples of investigations of the stability of laser cavities which include such elements. We have shown in general that the stability of a laser cavity is radially dependent and this radial dependence can be a useful tool in the discrimination of undesirable modes. We have concluded that the intra–cavity phase transformation which includes first and second order aberrations (the corresponding phase transformation elements are the lens and axicon) are responsible for the stability of the central on axis part of a laser cavity and consequently responsible for the stability of the fundamental mode. The method outlined in this paper may be modified to any type of cavity for which the intra–cavity propagation of rays can be described with ABCD matrix multiplication.

2. Stability of a laser cavity concept

A simple relation for the cavity stability with arbitrary shapes of the intra–cavity elements namely non-conventional parabolic surfaces (see Fig. 1(a)) can be derived based on the solution of the Fresnel integral with use of the stationary phase approximation. We can follow a conventional route in finding the stability of a laser cavity where we start from a general stability condition $|A + D|\leq 2$ for ABCD matrix representation [15] of ray propagation in a laser cavity. The given condition is a general condition which does not take into account the quadratic phase transformation on the laser mirrors and is applicable to any resonator. In order to find the coefficients A and D we implement a well-known condition for a stationary phase approximation and apply this to the Fresnel diffraction integral. This leads to a relationship of the radial coordinate of a ray at some initial plane (r_0) and its radial coordinate upon intersecting some interest plane (r_i) (see Fig. 1(b)) separated by a distance z namely $\partial_{r0} f\left(r_0\right)+\left(r_0-r_i\right)/z=0$ where $k_0 f(r_0)$ is the phase distribution of an electromagnetic field at the initial plane and $k_0 = 2\pi/\lambda$.

Based on the above relation for the stationary phase condition we are able now to determine the ABCD matrix for the single lossless phase transformation element. The coefficients of such matrix will be: A=1, B=0, C= $\partial_{r0} f\left(r_0\right)/r_0$ and D=1 where r_0 – the radial coordinate of the ray at the plane of the phase transformation element. Now we can find the ABCD matrix of propagation in the cavity and extract the required coefficients A and D. We arrive at the following formula for the stability of a laser cavity with intra–cavity elements of the arbitrary shapes:

$$0 \leq \frac{1}{4}\left(2+\mathrm{C}_1\left(r\right)\mathrm{z}\right)\left(2+\mathrm{C}_2\left(r\right)\mathrm{z}\right) \leq 1, \tag{1}$$

where $\mathrm{C}_1\left(r\right) \equiv \partial_r f_1\left(r\right)/r$ and $\mathrm{C}_2\left(r\right) \equiv \partial_r f_2\left(r\right)/r$.

The functions $f_1(r)$ and $f_2(r)$ present the phase transformation of the phase of the incident beam on the first and second mirrors respectively (see Fig. 1(a)). $C_1(r)$ and $C_2(r)$ are now the new coefficients for the stability of the laser cavity ($f\left(r\right)=-r^2/R$ in the case of a mirror having a radius of curvature R).

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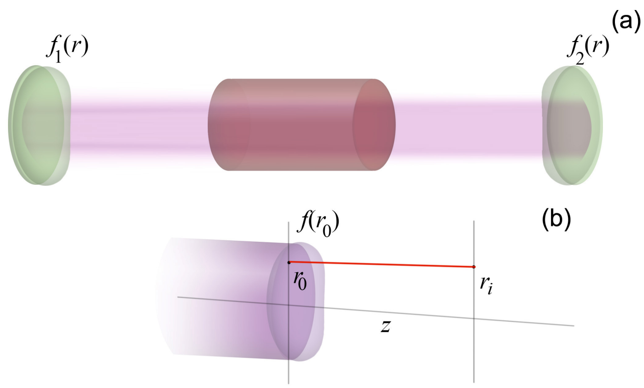

Fig. 1. (a) A schematic representation of the laser cavity with non-conventional mirrors and (b) the stationary phase approximation.

Equation (1) represents the general stability condition for non–conventional cavities consisting of two circular symmetric mirrors with a surface of an arbitrary shape. We can see from Eq. (1) that the stability formula is in general radially dependant. In the case of the intra–cavity elements having particular surfaces, namely, parabolic surfaces, Eq. (1) comes to a well–known non–radially dependent stability condition for conventional laser resonators: $0 \le (1-z\ /R_1)(1-z\ /R_2) \le 1$ [12].

2.1. Spherical aberration

As an example, we implement the obtained stability condition (see Eq. (1)) to investigate a laser cavity that consists of a conventional mirror of focal length *f* with an induced spherical aberration and plane second mirror. The spherical aberration can be a result of the mirror itself or a thermal effect of the laser crystal positioned close to the second mirror [13]. The resulting stability coefficients for such a cavity will be:

$$\mathrm{C}_1(r) = 0,\ \mathrm{C}_2(r) = -4\beta r^2 - \frac{1}{f}, \tag{2}$$

where β is the spherical aberration coefficient.

Let us depict the behavior of the obtained cavity stability dependent on the radial coordinate and the aberration coefficients *f* (defocus) and β (spherical) (see Fig. 2 (a)).

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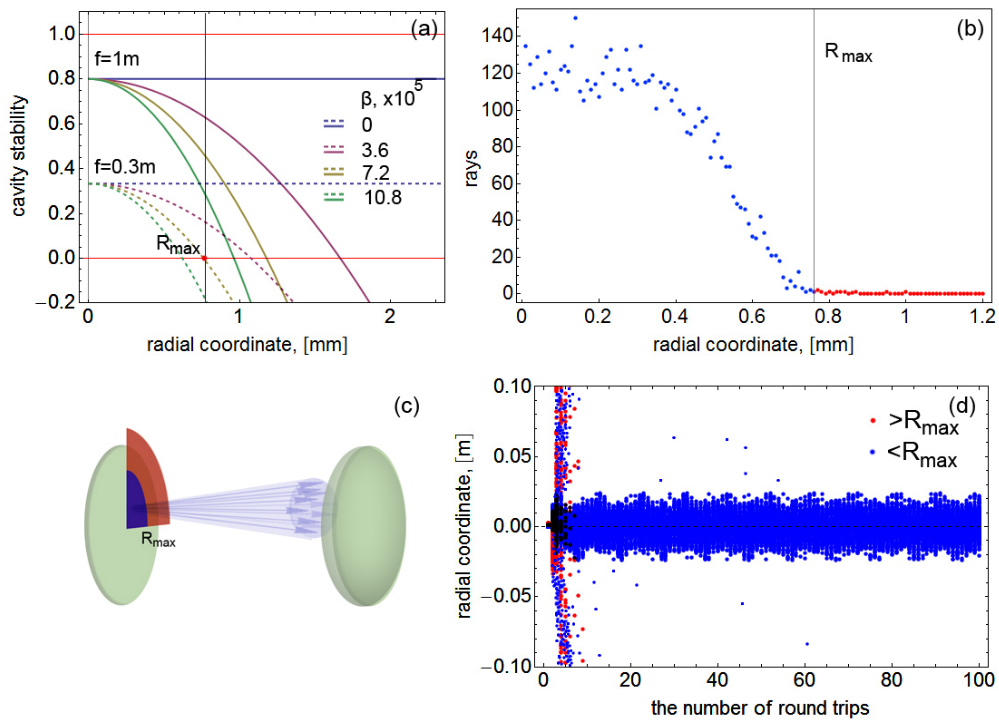

Fig. 2. (a) The dependence of the cavity stability on a spherical aberration value (β) for a constant focal length intra–cavity lens f = 1m (solid), f = 0.3m (dashed) and constant cavity length z = 0.4 m.(b, c, d) The Monte-Carlo method of propagation of rays in the particular laser cavity namely (b) the dependence of initial coordinate of rays and the number of rays which did not leave the particular laser cavity and (d) the radial coordinate dependence on the number of intra–cavity passes for the randomly generated rays in the stability region (red points) (see R_{max} point of Fig. 2(a)) and outside the stability region (blue points) for similar parameters of laser cavity. Following parameters of the laser cavity were used namely the intra–cavity lens with a focal length of f = 1m and a spherical aberration parameter $\beta = 7.2\ 10^5$ on one side of the cavity and the plane mirror on another side of the cavity. For the generation Fig. 2(b) and 2(d) 400 rays were generated at every radial position with step of 0.01 mm and with a random initial angle (see Fig. 2(c)), 200 passes were completed to generate Fig. 2(b).

We can see that the stability of the obtained cavity is radially dependant and increasing the induced spherical aberration moves the stability boundary (R_{max} of Fig. 2(a)) close to the central region of the cavity. As a result we identify the dramatically decreasing eigenvalues (the round trip losses) of the higher order modes as presented in Table 1. The given behavior of the dependence of the cavity stability on the radial coordinate can be implemented as one of the techniques of higher order mode discrimination.

Table 1. The dependence of the eigenvalues of the first three radial modes on the spherical aberration coefficient for a constant focal length of the intra–cavity lens f = 1m (solid curves of Fig. 2(a)) with a cavity length z = 0.4 m. The eigenvalues were calculated by Fox–Li method [1, 14].

β, x10^5	TEM_{00}	TEM_{10}	TEM_{20}
0	0.9999	0.9991	0.9984
3.6	0.9986	0.9700	0.9539
7.2	0.9683	0.9289	0.9613
10.8	0.9739	0.9175	**0.5427**

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In order to understand the meaning of the radially dependant stability regions (see Fig. 2(a)) we can implement a well–known Monte-Carlo method to the rays in the cavity. We will send the rays with random radial coordinates and random initial angles to the cavity (see Fig. 2(c)) to monitor the propagation behavior of every ray passing through an ABCD matrix of the cavity. The given behavior is depicted in Fig. 2(b) where we can see from this figure that the rays which are initially within the boundary of the laser cavity stability ((R_{max} of Fig. 2(a))) have the stable oscillations in the cavity (independent of the initial angle) and conversely, the rays which are initially positioned outside the region of R_{max} are unstable and will disperse out of this particular laser cavity.

2.2. Intra–cavity axicon

We can see from Eq. (1) and Fig. 2(a) and 3(a) that the stability of the central part of the cavity (the area of the fundamental mode TEM_{00}) depends strongly on the first and second order aberrations which include defocus (lens) and an axicon (the general view of a phase transformation equation of the axicon in cylindrical coordinates is $-k_0(n-1)\gamma r$ where γ is the axicon base angle). This is primarily due to the value of the central part of the higher order aberrations which are negligible in this area (see Fig. 3(a)). Consequently the central part of an unstable cavity in terms of the conventional stability condition $0 \le (1-z/R_1)(1-z/R_2) \le 1$ can be moved to a stable region through the use of an axicon (see Fig. 3(a)).

In Fig. 3(a) we have represented the behavior of a laser cavity with lens–axicon doublet positioned close to one of output couplers (see Fig. 3(b)) where the focal length of the lens is fixed at f = –0.9 m and second mirror is plane. The resulting stability coefficients for such a cavity will be:

$$C_1(r) = 0,\ C_2(r) = -(n-1)\frac{\gamma}{r} - \frac{1}{f}, \tag{3}$$

where n is the refractive index of axicon and γ is its base angle.

We can see from Fig. 3(a) that increasing the axicon angle moves the stability of the cavity from an unstable position to a stable one. As a result the eigenvalue (round trip losses) of the fundamental mode increases (see Fig. 3(a) and Table 2).

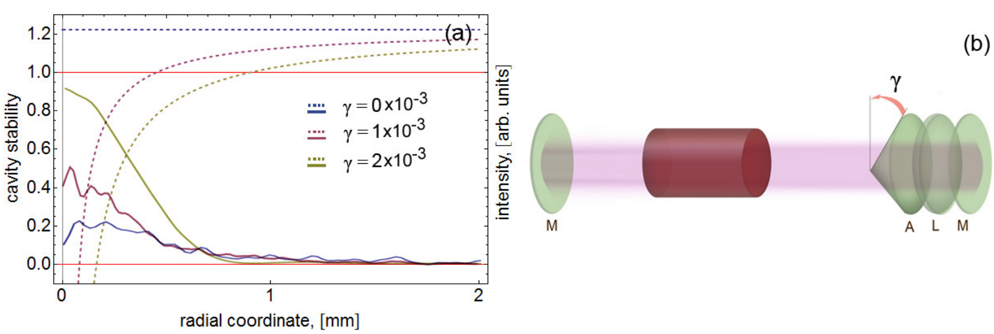

Fig. 3. (a) The behavior of the laser cavity stability with a lens–axicon doublet and the corresponding intensity of the TEM_{00} mode for the following fixed parameters of the laser cavity, f = –0.9 m, z = 0.4 m, n = 1.5. Where the dashed lines present the stability graph and the solid lines present the corresponding intensity distribution of the TEM_{00} mode. (b) A schematic representation of the laser cavity with a lens-axicon doublet where M, are the laser mirrors (output couplers), A, is an axicon and L, is a lens.

Table 2. The behavior of the eigenvalue of the fundamental TEM_{00} mode of the laser cavity with a lens–axicon doublet that depends on the axicon cone angle.

axicon cone angle, x10^{-3} [rad]	eigenvalue of TEM_{00} mode
0	**0.57**
1	**0.78**
2	**0.97**

3. Conclusion

In this work we have shown one of the ways of determining the stability of a laser cavity which can include non-conventional phase transformation elements. The given approach may be modified to any type of cavity for which the intra–cavity propagation of rays can be described with ABCD matrix multiplication. We have investigated two pertinent examples where we investigate in detail the stability of a laser cavity firstly with a lens with spherical aberration and thereafter a lens axicon doublet to illustrate the implementation of the given approach. We have shown that the stability of a laser cavity is radially dependent in general and the given behavior can be implemented as one of the techniques of higher order mode discrimination. We have shown that a lens and an axicon are primary phase transformation elements that are responsible for the stability of the central on axis part of a laser cavity and consequently responsible for the stability of the fundamental mode.

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