

1. Mean squared error

$$\frac{\partial J}{\partial W_{jk}} = \frac{\partial J}{\partial y_k} \frac{\partial y_k}{\partial in_k} \frac{\partial in_j}{\partial W_{jk}}$$

where

$$in_k = \sum_j W_{jk} x_j + b_k$$

and y_k is the sigmoid function of in_j

$$\begin{aligned} \frac{\partial J}{\partial y_k} &= -(t_k - y_k) \\ \frac{\partial y_k}{\partial in_j} &= y_k(1 - y_k) \\ \frac{\partial in_j}{\partial W_{jk}} &= \frac{\partial}{\partial W_{jk}} \sum_j W_{jk} x_j + b_k = x_j \\ \frac{\partial J}{\partial W_{jk}} &= -(t_k - y_k) y_k (1 - y_k) x_j \end{aligned}$$

This partial derivative for the bias works the same way, except

$$\begin{aligned} \frac{\partial J}{\partial y_k} &= -(t_k - y_k) \\ \frac{\partial y_k}{\partial in_j} &= y_k(1 - y_k) \\ \frac{\partial in_j}{\partial b_k} &= 1 \\ \frac{\partial J}{\partial b_k} &= -(t_k - y_k) y_k (1 - y_k) \end{aligned}$$

Thus, the update for the weights and biases become:

$$\begin{aligned} W_{jk} &\leftarrow W_{jk} - \eta \frac{\partial J}{\partial W_{jk}} \\ b_k &\leftarrow b_k - \eta \frac{\partial J}{\partial b_k} \end{aligned}$$

2. Cross-entropy error

The only term that changes here is $\frac{\partial J}{\partial y_k}$ which becomes:

$$\frac{1 - t_k}{1 - y_k} - \frac{t_k}{y_k}$$

so the entire gradient becomes

$$\frac{\partial J}{\partial y_k} = (\frac{1-t_k}{1-y_k} - \frac{t_k}{y_k})y_k(1-y_k)x_j$$