1. 用 Gauss 消元法解下列方程组.

(1)
$$\begin{cases} 2x_1 + x_2 + x_3 = 2, \\ x_1 + 3x_2 + x_3 = 5, \\ x_1 + x_2 + 5x_3 = -7, \\ 2x_1 + 3x_2 - 3x_3 = 14. \end{cases}$$

(2)
$$\begin{cases} 6x_1 + 6x_2 + 5x_3 + 18x_4 + 20x_5 = 14, \\ 10x_1 + 9x_2 + 7x_3 + 24x_4 + 30x_5 = 18, \\ 12x_1 + 12x_2 + 13x_3 + 27x_4 + 35x_5 = 32, \\ 8x_1 + 6x_2 + 6x_3 + 15x_4 + 20x_5 = 16, \\ 4x_1 + 5x_2 + 4x_3 + 15x_4 + 15x_5 = 11. \end{cases}$$

(3)
$$\begin{cases} 2x_1 + 7x_2 + 3x_3 + x_4 = 5, \\ x_1 + 3x_2 + 5x_3 - 2x_4 = 3, \\ x_1 + 5x_2 - 9x_3 + 8x_4 = 1, \\ 5x_1 + 18x_2 + 4x_3 + 5x_4 = 12. \end{cases}$$

(4)
$$\begin{cases} 2x_1 - x_2 + x_3 - x_4 = 3, \\ 4x_1 - 2x_2 - 2x_3 + 3x_4 = 2, \\ 2x_1 - x_2 + 5x_3 - 6x_4 = 1, \\ 2x_1 - x_2 - 3x_3 + 4x_4 = 5. \end{cases}$$

2．a,d 取什么值时，下面方程组有解，并求出它的解．

$$\begin{cases} 3x_1+2x_2+x_3+x_4-3x_5=a, \\ 5x_1+4x_2+3x_3+3x_4-x_5=d, \\ x_1+x_2+x_3+x_4+x_5=1, \\ x_2+2x_3+2x_4+6x_5=3. \end{cases}$$

7. 研究下列方程组的相容性并求其通解和一个特解.

(1) $\begin{cases} 3x_1+4x_2+x_3+2x_4=3, \\ 6x_1+8x_2+2x_3+5x_4=7, \\ 9x_1+12x_2+3x_2+10x_4=13; \end{cases}$

(2) $\begin{cases} 3x_1-5x_2+2x_3+4x_4=2, \\ 7x_1-4x_2+x_3+3x_4=5, \\ 5x_1+7x_2-4x_3-6x_4=3; \end{cases}$

(3) $\begin{cases} 2x_1+5x_2-8x_3=8, \\ 4x_1+3x_2-9x_3=9, \\ 2x_1+3x_2-5x_3=7, \\ x_1+8x_2-7x_3=12. \end{cases}$

8. 求方程组

$$\begin{cases} 2x_1 - x_2 + 3x_3 + 4x_4 = 5, \\ 4x_1 - 2x_2 + 5x_3 + 6x_4 = 7, \\ 6x_1 - 3x_2 + 7x_3 + 8x_4 = 9, \\ \lambda x_1 - 4x_2 + 9x_3 + 10x_4 = 11 \end{cases}$$

依赖于参数 λ 的通解.

11. 判断下列行向量组是否线性相关.

(1) (1,2,3),(4,8,12),(3,0,1),(4,5,8);

(2) (1,2,3,4,5,6),(1,0,1,0,1,0),(−1,1,1,−1,1,1),(−2,3,2,3,4,7);

(3) (1,2,3,4),(1,0,1,0),(−1,1,1,−1),(−2,3,2,3);

(4) (1,0,0,2,3,1),(0,1,0,4,6,2),(0,0,1,−2,−3,−1);

(5) (2,−3,1),(3,−1,5),(1,−4,3);

(6) (4,−5,2,6),(2,−2,1,3),(6,−3,3,9),(4,−1,5,6);

(7) (1,0,0,2,5),(0,1,0,3,4),(0,0,1,4,7),(2,3,4,11,12).

12. 对上题中每组向量,求出一个极大线性无关组.

解 (1) $\begin{bmatrix}1&4&3&4\\2&8&0&5\\3&12&1&8\end{bmatrix}\to\begin{bmatrix}1&4&3&4\\0&0&-6&-3\\0&0&-8&-4\end{bmatrix}$,

所以 α_1,α_3 为极大线性无关组;

(2) $\alpha_1,\alpha_2,\alpha_3$;

(3) $\alpha_1,\alpha_2,\alpha_3$;

(4) 自身;

(5) 自身;

(6) $\alpha_1,\alpha_2,\alpha_4$;

(7) 自身.

13. 求满足下列等式的行向量 x.

(1) $\alpha_1+2\alpha_2+3\alpha_3+4x=0$;其中

$\alpha_1=(5,-8,-1,2)$, $\alpha_2=(2,-1,4,-3)$, $\alpha_3=(-3,2,-5,4)$;

(2) $3(\alpha_1-x)+2(\alpha_2+x)=5(\alpha_3+x)$;其中

$\alpha_1=(2,5,1,3)$, $\alpha_2=(10,1,5,10)$, $\alpha_3=(4,1,-1,1)$.