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TWO-LOOP CORRECTIONS TO THE VACUUM POLARIZATIONS IN PERTURBATIVE QCD

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ABSTRACT

Closed analytic expressions for the $\mathcal{O}(\alpha\alpha_s)$ corrections to the vacuum polarizations of neutral gauge bosons induced by a massive quark and to those of charged bosons associated with a quark isodoublet of large mass splitting are derived through dispersive methods for variable external invariant mass s. Their small-s, high-s and threshold expansions are calculated. As an application, the tb contributions in $\mathcal{O}(\alpha\alpha_s)$ to the radiative parameters $\Delta\rho(0)$, Δr and the oblique correction δA_{LR} to the left-right asymmetry A_{LR} in e^+e^- collisions are studied. Handy approximation formulae for these corrections are presented which are valid for $m_t > M_W$ and take into account the next-to-leading terms in M_W/m_t.

1. INTRODUCTION

Present and imminent e^+e^- colliding beam experiments performed at LEP and SLC are capable to test the standard Glashow-Salam-Weinberg model to an as yet unmatched degree of precision. Within the electroweak sector perturbation theory can, in principle, be carried out to arbitrary order. However, the predictive power of these calculations may be considerably weakened by unknown hadronic effects. One possible way for hadronic corrections to enter electroweak processes is through oblique corrections, that is through quark loop insertions in boson lines, which may give rise to virtual gluon exchange.

It has often been stressed that, in particular, virtual heavy quark effects may delicately affect the theoretical predictions of certain "purely" electroweak variables (see *e.g.* Ref. [1]). In view of recent evidence for the lower bound on the top quark mass lying at 77 GeV [2], the characteristic ratio m_t/M_W cannot be considered small any more and it, thus, becomes necessary to take into account the QCD corrections for these effects, also. At first sight, this appears to be an enormous complication. As for the tb doublet, the situation may fortunately be alleviated in so far as m_b can now be safely neglected as against m_t. Note, however, that in order for perturbative QCD to remain applicable the constraint $m_b \gg \Lambda_{QCD}$ has still to be satisfied in the case of the quantity $\Pi_{AA}(s)/s$ at $|s| \lesssim \Lambda^2_{QCD}$ where $\Pi_{AA}(s)$ denotes the photon vacuum polarization [3].

In Ref. [4] it has been shown that in the approximation $m_b = 0$ the oblique correction $\Delta\rho(0)$ to the well-known ρ-Parameter [5] which measures the relative coupling strength of the weak neutral and charged currents receives from the tb doublet in $\mathcal{O}(\alpha\alpha_s)$ a contribution which grows quadratically with m_t. The leading m_t behaviour of both the parameter Δr which in Sirlin's electroweak renormalization scheme embodies the magnitude of the radiative corrections in the relation between α, G_F, M_W and M_Z [6] and the oblique correction δA_{LR} [7] to the asymmetry in the e^+e^- annihilation cross section with right- or left-handed polarized electron beams can be expressed in terms of $\Delta\rho(0)$. One of the major concerns of Ref. [8] was, however, to reveal that in $\mathcal{O}(\alpha\alpha_s)$ this leading term approximation for Δr and δA_{LR} fails badly for the preferred range of m_t. There, for the sake of simulating physical phenomena as realistic as possible, the tb contribution in $\mathcal{O}(\alpha\alpha_s)$ to the absorptive parts of the gauge boson vacuum polarizations were parametrized by densely spaced narrow quarkonium resonances below threshold and by the perturbative expression in the continuum choosing the scale of the running QCD coupling constant as $\mu^2 = 4\vec{p}^2$ where $\vec{p}$ denotes the relative 3-momentum of the quark system. The required real part was then evaluated numerically through dispersive methods carefully subtracting the ultraviolet divergences [9]. Such an approach is, of course, not appropriate to disclose the analytic structure of the solutions. Moreover, it is plagued by the rather technical drawback that it is too CPU-consuming to be readily installed in a Monte Carlo event generator, say. In this paper we, therefore, abandon the sophisticated treatment of the threshold region and adopt a conventional strategy. That is, we naively employ the imaginary parts as derived in perturbation theory as input for the dispersion relations throughout and take the strong coupling constant to be independent of the integration variable. It should be emphasized, however, that any deviation from the prior approach is, as a matter of principle, a higher order correction.

As usual, a high-energy cutoff Λ^2 is introduced to regulate the UV singularities connected with the still unrenormalized vacuum polarizations. As explained in Ref. [10], the explicit structure of these divergences can already be read off from the high-energy behaviour of the imaginary parts. By virtue of the renormalizability of the standard model [11], these divergent terms combine in the expressions for physical observables to yield finite remainders which, in general, are different from zero.

In the treatment of charged vector bosons it proves convenient to extract the UV divergence in such a way that the UV finite part of the self-energy function is free of mass singularities in the limit of vanishing down-type quark mass. This guarantees that for a given observable quantity the UV cancellation is unaffected; on the other hand, one has to cope with only one massive quark at a time in the actual calculation. This is particularly relevant for constructions containing the derivative of the photon vacuum polarization at zero momentum transfer, $\Pi'_{AA}(0)$, like Δr or $\delta A_{\rm LR}$ where—even for a large mass splitting within the tb doublet—m_b must not be consistently neglected.

Throughout the calculation the number of colours is taken to be $N = 3$ and all formulae already include the proper colour factor $\mathrm{tr}\, T^a T^a = (N^2-1)/2$ where T^a $(a = 1, \ldots, N^2-1)$ denote the generators of the colour gauge group SU(N). In order to translate the results to the case of virtual one-photon exchange between leptons or quarks in the vacuum bubble, one has to divide by $(N^2-1)/2$ and $(N^2-1)/(2N)$ respectively and to replace α_s by $\alpha Q_1 Q_2$ where Q_i denotes the electric charge quantum number of the fermion flavour(s).

2. $\mathcal{O}(\alpha\alpha_s)$ CORRECTIONS TO THE VACUUM POLARIZATION FUNCTIONS

The various vacuum polarization tensors representing the W, Z, A self-energy and the Z-A mixing diagrams, respectively, have a unique decomposition ($s = p^2$)

$$\Pi^{\mu\nu}(p) = \Pi(s)\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{s}\right) + \Lambda(s) p^\mu p^\nu \,. \tag{1}$$

In the following we are not concerned with the second (longitudinal) component, which, in general, is irrelevant for high energy processes. To all orders of perturbation theory the transverse vacuum polarization function $\Pi(s)$ splits up into a vector and an axial vector part. We write the QCD perturbation expansion in the conventional form

$$\Pi^{V,A} = \Pi_0^{V,A} + \frac{\alpha_s}{\pi}\Pi_1^{V,A} + \mathcal{O}\left(\left(\frac{\alpha_s}{\pi}\right)^2\right) . \tag{2}$$

Up to the first order the contributions from a quark isodoublet $(1,2)$ have the following structure

$$\begin{aligned}
\Pi_{WW}(s, m_1, m_2) &= \frac{\pi\alpha}{2s_W^2}\left(\Pi^V(s, m_1, m_2) + \Pi^A(s, m_1, m_2)\right) , \\
\Pi_{ZZ}(s, m_1, m_2) &= \frac{\pi\alpha}{4s_W^2 c_W^2}\sum_{i=1}^{2}\left(v_i^2 \Pi^V(s, m_i) + \Pi^A(s, m_i)\right) , \\
\Pi_{AA}(s, m_1, m_2) &= 4\pi\alpha \sum_{i=1}^{2} Q_i^2 \Pi^V(s, m_i) , \\
\Pi_{ZA}(s, m_1, m_2) &= \frac{\pi\alpha}{s_W c_W}\sum_{i=1}^{2} Q_i v_i \Pi^V(s, m_i) ,
\end{aligned} \tag{3}$$

where $s_W^2 = \sin^2\theta_W = 1 - c_W^2$, $v_i = 2I_i - 4s_W^2 Q_i$ and the short-hand notation $\Pi^{V,A}(s, m) = \Pi^{V,A}(s, m, m)$ has been introduced.

Our goal is to find explicit expressions for $\Pi_1^{V,A}(s, m)$ and $\Pi_1^{V,A}(s, m, 0)$ through an analytic evaluation of the dispersion relation. The proper subtraction prescription which is compatible with dimensional regularization and, hence, complies with the Ward identities reads [12]

$$
\begin{aligned}
\pi \Pi^{V,A}(s,m_1,m_2) = & \int\limits_{(m_1+m_2)^2}^{\Lambda^2} \frac{d\sigma}{\sigma - s - i\epsilon} \mathcal{I}m\, \Pi^{V,A}(\sigma,m_1,m_2) \\
& - \frac{1}{2}\sum_{i=1}^{2} \int\limits_{(2m_i)^2}^{\Lambda^2} \frac{d\sigma}{\sigma} \mathcal{I}m\, \Pi^{V}(\sigma,m_i),
\end{aligned} \tag{4}
$$

where the high-energy cutoff Λ^2 serves as UV regulator. The required input $\mathcal{I}m\, \Pi_1^{V,A}(s,m_1,m_2)$ may be obtained by applying Cutkosky's rule [13] to the set of two-loop graphs where a gluon traverses a quark bubble in all possible ways (see Fig. 1). The general result is found in Refs. [14], [15] and [12]. Of interest here are the special cases:

$$
\begin{aligned}
\frac{\pi}{m^2}\mathcal{I}m\, \Pi_1^V(s,m) = & \frac{16}{3}\left(r - \frac{1}{4r}\right)\left[-\mathrm{Li}_2(-\rho_-^2) + \mathrm{Li}_2(\rho_-^4) + \varphi(3\varphi - \gamma - 2\chi)\right] \\
& + \sqrt{1-\frac{1}{r}}\left[-\frac{8}{3}\left(r+\frac{1}{2}\right)(\gamma + 2\chi) + r + \frac{3}{2}\right] + 8\left(r - \frac{1}{6} - \frac{7}{48r}\right)\varphi,
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
\frac{\pi}{m^2}\mathcal{I}m\, \Pi_1^A(s,m) = & \frac{16}{3}\left(r - \frac{3}{2} + \frac{1}{2r}\right)\left[-\mathrm{Li}_2(-\rho_-^2) + \mathrm{Li}_2(\rho_-^4) + \varphi(3\varphi - \gamma - 2\chi)\right] \\
& + \sqrt{1-\frac{1}{r}}\left[-\frac{8}{3}(r-1)(\gamma + 2\chi) + r - 3 + \frac{1}{4r}\right] + 8\left(r - \frac{11}{12} + \frac{5}{48r} + \frac{1}{32r^2}\right)\varphi,
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
\frac{\pi}{m^2}\mathcal{I}m\, \Pi_1^{V,A}(s,m,0) = & \frac{1}{3}\left(x - \frac{3}{2} + \frac{1}{2x^2}\right)\left[-2\,\mathrm{Li}_2\left(\frac{1}{1-x}\right) + \beta(\alpha - \beta)\right] + \frac{1}{3}\left(x + \frac{1}{2} - \frac{1}{2x}\right)\alpha \\
& - \frac{1}{3}\left(x - \frac{3}{4} - \frac{3}{2x} + \frac{5}{4x^2}\right)\beta + \frac{1}{4}\left(x - \frac{5}{2} + \frac{2}{3x} + \frac{5}{6x^2}\right),
\end{aligned} \tag{7}
$$

where Li_2 denotes the dilogarithm [16]. For $\Pi_1^{V,A}(s,m)$ we adopt the notation of Ref. [17]

$$
\begin{aligned}
& r = \frac{s}{(2m)^2}, \\
& r_\pm = \sqrt{1-r} \pm \sqrt{-r}, & & \rho_\pm = \sqrt{r} \pm \sqrt{r-1}, \\
& f = \ln r_+ = \mathrm{Arsh}\sqrt{-r}, \quad \Phi = \arcsin\sqrt{r}, & & \varphi = \ln\rho_+ = \mathrm{Arch}\sqrt{r}, \\
& g = \ln(r_+ - r_-), & & \gamma = \ln(\rho_+ + \rho_-), \\
& h = \ln(r_+ + r_-), & & \chi = \ln(\rho_+ - \rho_-).
\end{aligned} \tag{8}
$$

For $\Pi_1^{V,A}(s,m,0)$ we use

$$
\begin{aligned}
& x = \frac{s}{m^2}, \\
& a = \ln(-x), & & \alpha = \ln x, \\
& b = \ln(1-x), & & \beta = \ln(x-1).
\end{aligned} \tag{9}
$$

Note that $\Pi_1^V(s,m,0) = \Pi_1^A(s,m,0)$ due to the γ_5 reflection property $\Pi_1^V(s,m_1,m_2) = \Pi_1^A(s,m_1,-m_2)$. In order to avoid the occurrence of trigonometric functions or the presence of the pole, it is convenient to solve the dispersion integral for negative s and to apply analytic continuation into the other kinematic zones. Incidentally, this permits two consistency checks. Firstly, for positive s below threshold the intermediately arising imaginary parts must properly cancel. Secondly, above threshold the previously inserted absorptive part must be recovered. A table of transformation rules for the analytic continuation in the variable $r = (s+i\epsilon)/(2m)^2$ can be found in Ref. [17]. After some computation we obtain

$$
\frac{\pi^2}{m^2}\Pi_1^V(s,m) = rL + V_1(r), \tag{10}
$$

$$
\frac{\pi^2}{m^2}\Pi_1^A(s,m) = \frac{3}{2}L^2 + \left(r - \frac{9}{2}\right)L + A_1(r), \tag{11}
$$

$$
\frac{\pi^2}{m^2}\Pi_1^{V,A}(s,m,0) = \frac{3}{8}L^2 + \frac{1}{4}\left(x - \frac{9}{2}\right)L + F_1(x) \tag{12}
$$

with $L = \ln(\Lambda^2/m^2)$ and

$$
\begin{aligned}
\mathcal{R}e\, V_1(r) = & 4\left(r - \frac{1}{4r}\right)\left[2\,\mathrm{Li}_3(r_-^2) - \mathrm{Li}_3(r_-^4) + \frac{8}{3}f\left(\mathrm{Li}_2(r_-^2) - \mathrm{Li}_2(r_-^4)\right) + 4f^2\left(-f + \frac{g}{3} + \frac{2}{3}h\right)\right] \\
& + \sqrt{1-\frac{1}{r}}\left\{\frac{8}{3}\left(r+\frac{1}{2}\right)\left[\mathrm{Li}_2(r_-^2) - \mathrm{Li}_2(r_-^4) + f(-3f + 2g + 4h)\right] - 2\left(r + \frac{3}{2}\right)f\right\} \\
& - 8\left(r - \frac{1}{6} - \frac{7}{48r}\right)f^2 + \frac{13}{6} + \frac{\zeta(3)}{r}, \quad r \le 0 \\
= & 4\left(r - \frac{1}{4r}\right)\left[2\,\mathrm{Cl}_3(2\Phi) - \mathrm{Cl}_3(4\Phi) + \frac{8}{3}\Phi\left(\mathrm{Cl}_2(2\Phi) - \mathrm{Cl}_2(4\Phi)\right) - \frac{4}{3}\Phi^2(\gamma + 2h)\right] \\
& + \sqrt{\frac{1}{r} - 1}\left\{\frac{8}{3}\left(r+\frac{1}{2}\right)\left[-\mathrm{Cl}_2(2\Phi) + \mathrm{Cl}_2(4\Phi) + 2\Phi(\gamma + 2h)\right] - 2\left(r + \frac{3}{2}\right)\Phi\right\} \\
& + 8\left(r - \frac{1}{6} - \frac{7}{48r}\right)\Phi^2 + \frac{13}{6} + \frac{\zeta(3)}{r}, \quad 0 \le r < 1
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
&= 4\left(r-\frac{1}{4r}\right)\left[2\,\mathrm{Li}_3(-\rho_-^2)-\mathrm{Li}_3(\rho_-^4)+\frac{8}{3}\varphi\left(\mathrm{Li}_2(-\rho_-^2)-\mathrm{Li}_2(\rho_-^4)\right)+2\left(3\zeta(2)-2\varphi^2\right)\left(\varphi-\frac{\gamma}{3}-\frac{2}{3}\chi\right)\right]\\
&\quad+\sqrt{1-\frac{1}{r}}\left\{\frac{8}{3}\left(r+\frac{1}{2}\right)\left[\mathrm{Li}_2(-\rho_-^2)-\mathrm{Li}_2(\rho_-^4)-\frac{9}{2}\zeta(2)+\varphi(-3\varphi+2\gamma+4\chi)\right]-2\left(r+\frac{3}{2}\right)\varphi\right\}\\
&\quad+4\left(r-\frac{1}{6}-\frac{7}{48r}\right)\left(3\zeta(2)-2\varphi^2\right)+\frac{13}{6}+\frac{\zeta(3)}{r},\quad r>1\,,
\end{aligned}
$$

$$
\begin{aligned}
\Re e\,A_1(r) &= 4\left(r-\frac{3}{2}+\frac{1}{2r}\right)\left[2\,\mathrm{Li}_3(r_-^2)-\mathrm{Li}_3(r_-^4)+\frac{8}{3}f\left(\mathrm{Li}_2(r_-^2)-\mathrm{Li}_2(r_-^4)\right)+4f^2\left(-f+\frac{g}{3}+\frac{2}{3}h\right)\right]\\
&\quad+\sqrt{1-\frac{1}{r}}\left\{\frac{8}{3}(r-1)\left[\mathrm{Li}_2(r_-^2)-\mathrm{Li}_2(r_-^4)+f(-3f+2g+4h)\right]-2\left(r-3+\frac{1}{4r}\right)f\right\}\\
&\quad-8\left(r-\frac{11}{12}+\frac{5}{48r}+\frac{1}{32r^2}\right)f^2-3\zeta(2)+\frac{13}{6}+\left(-2\zeta(3)+\frac{1}{4}\right)\frac{1}{r},\quad r\le 0\\
&= 4\left(r-\frac{3}{2}+\frac{1}{2r}\right)\left[2\,\mathrm{Cl}_3(2\Phi)-\mathrm{Cl}_3(4\Phi)+\frac{8}{3}\Phi\left(\mathrm{Cl}_2(2\Phi)-\mathrm{Cl}_2(4\Phi)\right)-\frac{4}{3}\Phi^2(\gamma+2h)\right]\\
&\quad+\sqrt{\frac{1}{r}-1}\left\{\frac{8}{3}(r-1)\left[-\mathrm{Cl}_2(2\Phi)+\mathrm{Cl}_2(4\Phi)+2\Phi(\gamma+2h)\right]-2\left(r-3+\frac{1}{4r}\right)\Phi\right\}\\
&\quad+8\left(r-\frac{11}{12}+\frac{5}{48r}+\frac{1}{32r^2}\right)\Phi^2-3\zeta(2)+\frac{13}{6}+\left(-2\zeta(3)+\frac{1}{4}\right)\frac{1}{r},\quad 0\le r\le 1\\
&= 4\left(r-\frac{3}{2}+\frac{1}{2r}\right)\left[2\,\mathrm{Li}_3(-\rho_-^2)-\mathrm{Li}_3(\rho_-^4)+\frac{8}{3}\varphi\left(\mathrm{Li}_2(-\rho_-^2)-\mathrm{Li}_2(\rho_-^4)\right)\right.\\
&\quad\left.+2\left(3\zeta(2)-2\varphi^2\right)\left(\varphi-\frac{\gamma}{3}-\frac{2}{3}\chi\right)\right]\\
&\quad+\sqrt{1-\frac{1}{r}}\left\{\frac{8}{3}(r-1)\left[\mathrm{Li}_2(-\rho_-^2)-\mathrm{Li}_2(\rho_-^4)-\frac{9}{2}\zeta(2)+\varphi(-3\varphi+2\gamma+4\chi)\right]-2\left(r-3+\frac{1}{4r}\right)\varphi\right\}\\
&\quad+4\left(r-\frac{11}{12}+\frac{5}{48r}+\frac{1}{32r^2}\right)\left(3\zeta(2)-2\varphi^2\right)-3\zeta(2)+\frac{13}{6}+\left(-2\zeta(3)+\frac{1}{4}\right)\frac{1}{r},\quad r\ge 1\,,
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
\Re e\,F_1(x) &= \left(x-\frac{3}{2}+\frac{1}{2x^2}\right)\left[-\mathrm{Li}_3(x)-\mathrm{Li}_3\left(\frac{-x}{1-x}\right)+\frac{b}{3}\left(2\,\mathrm{Li}_2(x)-\zeta(2)\right)+\frac{b^3}{6}\right]\\
&\quad+\frac{1}{3}\left(x+\frac{1}{2}-\frac{1}{2x}\right)\mathrm{Li}_2(x)+\frac{1}{6}\left(x-\frac{3}{4}-\frac{3}{2x}+\frac{5}{4x^2}\right)b^2-\frac{1}{4}\left(x-\frac{5}{2}+\frac{2}{3x}+\frac{5}{6x^2}\right)b\\
&\quad+\zeta(3)\left(x-\frac{3}{2}\right)+\frac{\zeta(2)}{3}\left(x-\frac{7}{4}-\frac{1}{2x}\right)+\frac{13}{12}-\frac{5}{24x},\qquad x\le 1\\
&= \left(x-\frac{3}{2}+\frac{1}{2x^2}\right)\left[\mathrm{Li}_3\left(\frac{1}{1-x}\right)+\frac{2}{3}\beta\,\mathrm{Li}_2\left(\frac{1}{1-x}\right)+\left(\zeta(2)-\frac{\beta^2}{6}\right)(\alpha-\beta)\right]\\
&\quad+\frac{1}{3}\left(x+\frac{1}{2}-\frac{1}{2x}\right)\left(\mathrm{Li}_2\left(\frac{1}{1-x}\right)-\alpha\beta\right)+\frac{1}{3}\left(x-\frac{1}{8}-\frac{1}{x}+\frac{5}{8x^2}\right)\beta^2-\frac{1}{4}\left(x-\frac{5}{2}+\frac{2}{3x}+\frac{5}{6x^2}\right)\beta\\
&\quad-\frac{\zeta(3)}{2x^2}+\zeta(2)\left(\frac{1}{2}+\frac{1}{x}-\frac{5}{4x^2}\right)+\frac{13}{12}-\frac{5}{24x},\quad x\ge 1\,.
\end{aligned}
\tag{15}
$$

Here $\zeta(2)=\pi^2/6$, $\zeta(3)=1.20205690315959...$ and Li_3 and Cl_2 (Cl_3) denote the trilogarithm and the (generalized) Clausen function of second (third) order respectively [16].

All results have been checked numerically after the dispersion integral had been rendered finite by subtracting the leading high-s behaviour from the integrand. Agreement is found with Ref. [12] on $V_1'(0)$, $V_1(r)-V_1(0)-rV_1'(0)$ for $r<0$ and $r>1$ and accordingly for $A_1(r)$. The general structure of the UV divergence has been derived in Ref. [8] and reads

$$
\begin{aligned}
\pi^2\Pi_1^{V,A}(s,m_1,m_2) &= \frac{3}{8}(m_1\mp m_2)^2\,\ln^2\frac{\Lambda^2}{m_1m_2}\\
&\quad+\left[\frac{s}{4}-\frac{9}{8}(m_1\mp m_2)^2-\frac{3}{4}(m_1^2-m_2^2)\ln\frac{m_1}{m_2}\right]\ln\frac{\Lambda^2}{m_1m_2}\\
&\quad+\text{finite terms}\,.
\end{aligned}
\tag{16}
$$

For $m_1=m_2=m$ this reproduces the L-dependent terms in Eqs. (10) and (11). In order to render the limit $m_2\to 0$ possible, however, we have to split $\Pi_1^{V,A}(s,m_1,m_2)$ in a different way, viz

$$
\begin{aligned}
\pi^2\Pi_1^{V,A}(s,m_1,m_2) &= \frac{3}{8}(m_1\mp m_2)^2\,\ln^2\frac{\Lambda^2}{m_1^2}\\
&\quad+\left[\frac{s}{4}-\frac{9}{8}(m_1\mp m_2)^2\mp\frac{3}{2}m_2(m_1\mp m_2)\ln\frac{m_1}{m_2}\right]\ln\frac{\Lambda^2}{m_1^2}\\
&\quad+\text{finite terms}\,.
\end{aligned}
\tag{17}
$$

Now, setting $m_1=m$ and $m_2=0$, in fact, yields the L-dependent terms in Eq. (12). Consequently, the UV finite terms in Eq. (17) are free of mass singularities associated with m_2 and converge towards $m^2F_1(x)$. Note, however, that $\Pi_1^V(s,m_1,m_2)$ has a logarithmic Coulomb singularity [18] at threshold which is suppressed by a factor proportional to the square of the reduced mass, $m_{\rm red}=m_1m_2/(m_1+m_2)$, and, therefore, does not show up for $m_2=0$. For realistic values of m_t and m_b the substitution of $m_t^2/\pi^2\,F_1(s/m_t^2)$ for the finite piece of $\Pi_1^{V,A}(s,m_t,m_b)$ leads to an excellent approximation—except for $\Pi_1^V(s,m_t,m_b)$ in the close vicinity of threshold—as has been verified numerically.

In many applications it may be sufficient to know the leading behaviour of the vacuum polarization functions in certain kinematic limits. For this reason, we list here the expansions of $V_1(r)$, $A_1(r)$ and $F_1(x)$ for $s \to \pm\infty$, $s \to 0$ and at threshold:

$$
\begin{aligned}
V_1(r) &= -2rg - 6g + \frac{1}{r}\left(-3g^2 - \frac{5}{4}g + \zeta(3) + \frac{5}{144}\right), \qquad r \ll -1 \\
&= \left(4\zeta(3) - \frac{5}{6}\right) r, \qquad |r| \ll 1 \\
&= -12\zeta(2)h - \frac{13}{2}\zeta(3) + 3\zeta(2)\left(-2\ln 2 + \frac{11}{4}\right) + \frac{13}{6} + 8\pi\sqrt{1-r} \\
&\quad + (1-r)\left[20\zeta(2)h + \frac{27}{2}\zeta(3) + \zeta(2)\left(10\ln 2 - \frac{43}{4}\right) - \frac{3}{2}\right], \qquad 0 < 1-r \ll 1 \\
&= -12\zeta(2)\chi - \frac{13}{2}\zeta(3) + 3\zeta(2)\left(-2\ln 2 + \frac{11}{4}\right) + \frac{13}{6} \\
&\quad + (r-1)\left[-20\zeta(2)\chi - \frac{27}{2}\zeta(3) + \zeta(2)\left(-10\ln 2 + \frac{43}{4}\right) + \frac{3}{2}\right] \\
&\quad + i\pi\left[6\zeta(2) - 8\sqrt{r-1} + 10\zeta(2)(r-1)\right], \qquad 0 < r-1 \ll 1 \\
&= -2r\gamma - 6\gamma + \frac{1}{r}\left(-3\gamma^2 - \frac{5}{4}\gamma + \zeta(3) + \frac{9}{2}\zeta(2) + \frac{5}{144}\right) \\
&\quad + i\pi\left[r + 3 + \frac{1}{r}\left(3\gamma + \frac{5}{8}\right)\right], \qquad r \gg 1
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
A_1(r) &= -2rg + 3\left(-2g^2 + g - \zeta(2)\right) + \frac{1}{r}\left(3g^2 + \frac{11}{4}g - 2\zeta(3) + \frac{293}{144}\right), \qquad r \ll -1 \\
&= 3\left(-2\zeta(3) - \zeta(2) + \frac{7}{4}\right) + \left(4\zeta(3) - \frac{49}{18}\right) r, \qquad |r| \ll 1 \\
&= -2\zeta(3) - \frac{3}{8}\zeta(2) + \frac{29}{12} + (1-r)\left[8\zeta(2)h + 3\zeta(3) + 2\zeta(2)(2\ln 2 - 5) - \frac{3}{2}\right], \\
&\qquad\qquad 0 < 1-r \ll 1 \\
&= -2\zeta(3) - \frac{3}{8}\zeta(2) + \frac{29}{12} + (r-1)\left[-8\zeta(2)\chi - 3\zeta(3) + 2\zeta(2)(-2\ln 2 + 5) + \frac{3}{2}\right] \\
&\quad + i\pi\, 4\zeta(2)(r-1), \qquad 0 < r-1 \ll 1 \\
&= -2r\gamma + 3\left(-2\gamma^2 + \gamma + 2\zeta(2)\right) + \frac{1}{r}\left(3\gamma^2 + \frac{11}{4}\gamma - 2\zeta(3) - \frac{9}{2}\zeta(2) + \frac{293}{144}\right) \\
&\quad + i\pi\left[r + 6\gamma - \frac{3}{2} + \frac{1}{r}\left(-3\gamma - \frac{11}{8}\right)\right], \qquad r \gg 1
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
F_1(x) &= -\frac{x}{4}a - \frac{3}{4}\left(\frac{a^2}{2} + \frac{a}{2} + \zeta(2)\right) + \frac{1}{2x}\left(\frac{3}{2}a + 1\right), \qquad x \ll -1 \\
&= -\frac{3}{2}\zeta(3) - \frac{\zeta(2)}{2} + \frac{23}{16} + x\left(\zeta(3) - \frac{\zeta(2)}{9} - \frac{25}{72}\right) + \frac{x^2}{8}\left(\zeta(2) + \frac{25}{24}\right), \qquad |x| \ll 1 \\
&= -\frac{\zeta(3)}{2} - \frac{\zeta(2)}{12} + \frac{7}{8} - (1-x)\left(\zeta(3) + \zeta(2) + \frac{13}{24}\right) \\
&\quad + (1-x)^2\left[\frac{3}{8}b^2 - \left(\zeta(2) + \frac{9}{8}\right)b - \frac{3}{2}\zeta(3) - \frac{\zeta(2)}{3} + \frac{5}{24}\right], \qquad 0 < 1-x \ll 1 \\
&= -\frac{\zeta(3)}{2} - \frac{\zeta(2)}{12} + \frac{7}{8} + (x-1)\left(\zeta(3) + \zeta(2) + \frac{13}{24}\right) \\
&\quad + (x-1)^2\left[\frac{3}{8}\beta^2 - \left(\zeta(2) + \frac{9}{8}\right)\beta - \frac{3}{2}\zeta(3) - \frac{31}{12}\zeta(2) + \frac{5}{24}\right] \\
&\quad + i\pi(x-1)^2\left(-\frac{3}{4}\beta + \zeta(2) + \frac{9}{8}\right), \qquad 0 < x-1 \ll 1 \\
&= -\frac{x}{4}\alpha + \frac{3}{2}\left(-\frac{\alpha^2}{4} - \frac{\alpha}{4} + \zeta(2)\right) + \frac{1}{2x}\left(\frac{3}{2}\alpha + 1\right) \\
&\quad + i\pi\left[\frac{x}{4} + \frac{3}{4}\left(\alpha + \frac{1}{2}\right) - \frac{3}{4x}\right], \qquad x \gg 1
\end{aligned} \tag{20}
$$

Note that the leading term for $|s| \to \infty$ is universal and leads to the well-known result for the massless case

$$
\begin{aligned}
\Pi_1^{V,A}(s,0,0) &= \frac{s}{4\pi}\ln\frac{\Lambda^2}{-s-i\epsilon} \\
&= \Pi_0^{V,A}(s,0,0),
\end{aligned} \tag{21}
$$

which is identical with the corresponding free field theory result.

3. APPLICATIONS

We now discuss the implications of the above calculation on three genuinely electroweak quantities: the ρ-parameter, Δr and the left-right asymmetry. The oblique corrections to these observables are given by [8]

$$\Delta\rho(0) = \frac{\Pi_{ZZ}(0)}{M_Z^2} - \frac{\Pi_{WW}(0)}{M_W^2}, \tag{22}$$

$$\Delta r = \frac{c_W^2}{s_W^2}\mathcal{R}e\left(\frac{\Pi_{WW}(M_W^2)}{M_W^2} - \frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2}\right) + \Pi'_{AA}(0) + \frac{1}{M_W^2}\left(\Pi_{WW}(0) - \mathcal{R}e\,\Pi_{WW}(M_W^2)\right), \tag{23}$$

$$\delta A_{\rm LR} = -\frac{64 s_W^4 c_W^2}{(v_e^2+1)^2}\left[\frac{1}{M_Z^2}\mathcal{R}e\left(\frac{c_W^2 - s_W^2}{s_W c_W}\Pi_{ZA}(M_Z^2) - \Pi_{ZZ}(M_Z^2)\right) + \Pi'_{AA}(0) + \frac{\Pi_{WW}(0)}{M_W^2}\right]. \tag{24}$$

In the approximation $m_b = 0$ we find for the tb contribution to $\Delta\rho(0)$ in $\mathcal{O}(\alpha\alpha_s)$

$$\begin{aligned}(\Delta\rho(0))_1^{tb} &= -\frac{1}{4}\left(\zeta(2) + \frac{1}{2}\right)\frac{\alpha}{\pi}\frac{m_t^2}{s_W^2 M_W^2} \\ &= -\frac{4}{3}\left(\zeta(2) + \frac{1}{2}\right)(\Delta\rho(0))_0^{tb}\end{aligned} \tag{25}$$

in accordance with Ref. [4]. As mentioned above, setting $m_b = 0$ causes perturbation theory to break down for parameters involving $\Pi'_{AA}(0)$. For Δr and $\delta A_{\rm LR}$ we, therefore, impose the condition $\Lambda_{QCD} \ll m_b \ll \min(m_t,\ M_Z/2)$ and obtain

$$\begin{aligned}(\Delta r)_1^{tb} = \frac{\alpha}{\pi}\Bigg\{&-\frac{1}{8 s_W^4}\left[\frac{1}{2r}\left(v_t^2 V_1(r) + A_1(r)\right) - (v_b^2+1)\gamma\right] \\ &+ \frac{1}{s_W^2 x}\left[\left(\frac{1}{s_W^2} - 2\right)F_1(x) - \frac{3}{2}\zeta(3) - \frac{\zeta(2)}{2} + \frac{23}{16}\right] \\ &+ \left(2\zeta(3) - \frac{5}{12}\right)(1+Y^2) + Q_b^2 \ln\frac{m_t^2}{m_b^2}\Bigg\},\end{aligned} \tag{26}$$

$$\begin{aligned}(\delta A_{\rm LR})_1^{tb} = \frac{\alpha}{\pi}\frac{64 s_W^2}{(v_e^2+1)^2}\Bigg\{&\frac{1}{r}\left[\left(\frac{1}{16} - \frac{1}{2}I_t Q_t + s_W^2 c_W^2 Q_t^2\right)V_1(r) + \frac{1}{16}A_1(r)\right] \\ &- \left(\frac{1}{4} - I_b Q_b + 2 s_W^2 c_W^2 Q_b^2\right)\gamma + \frac{c_W^2}{2x}\left(3\zeta(3) + \zeta(2) - \frac{23}{8}\right) \\ &- s_W^2 c_W^2\left[\left(2\zeta(3) - \frac{5}{12}\right)(1+Y^2) + Q_b^2\ln\frac{m_t^2}{m_b^2}\right]\Bigg\},\end{aligned} \tag{27}$$

where $r = M_Z^2/(2m_t)^2$, $x = M_W^2/m_t^2$ and $Y = 1/3$ denotes the weak hypercharge of the tb

doublet.

Depending on the required precision, it may suffice to insert the following interpolating formulae which properly take into account the threshold behaviour and the high-mass limit ($y = \sqrt{1-r}$)

$$V_1(r) = -4\zeta(2)\left(3 - 5y^2\right)h + v_1 + 8\pi y + v_2 y^2 + v_3 y^3 + v_4 y^4, \quad 0 \le r < 1$$

$$\begin{aligned}
v_1 &= -\tfrac{13}{2}\zeta(3) + 3\zeta(2)\left(-2\ln 2 + \tfrac{11}{4}\right) + \tfrac{13}{6} &&\approx 1.0829 \\
v_2 &= \tfrac{27}{2}\zeta(3) + \zeta(2)\left(10\ln 2 - \tfrac{43}{4}\right) - \tfrac{3}{2} &&\approx 8.4465 \\
v_3 &= 7\zeta(3) + \zeta(2)\left(12\ln 2 - \tfrac{7}{2}\right) - 24\pi - \tfrac{22}{3} &&\approx -66.3923 \\
v_4 &= -14\zeta(3) + 6\zeta(2)\left(-4\ln 2 + 1\right) + 16\pi + \tfrac{20}{3} &&\approx 22.6086
\end{aligned} \tag{28}$$

$$A_1(r) = 8\zeta(2) y^2 h + a_1 + a_2 y^2 + a_3 y^3 + a_4 y^4, \quad 0 \le r \le 1$$

$$\begin{aligned}
a_1 &= -2\zeta(3) - \tfrac{3}{8}\zeta(2) + \tfrac{29}{12} &&\approx -0.6043 \\
a_2 &= 3\zeta(3) + 2\zeta(2)(2\ln 2 - 5) - \tfrac{3}{2} &&\approx -9.7824 \\
a_3 &= -14\zeta(3) + \zeta(2)\left(-24\ln 2 + \tfrac{35}{2}\right) + \tfrac{80}{9} &&\approx -6.5179 \\
a_4 &= 7\zeta(3) + 3\zeta(2)\left(4\ln 2 - \tfrac{27}{8}\right) - \tfrac{41}{9} &&\approx 0.8861
\end{aligned} \tag{29}$$

$$F_1(x) = (1-x)^2 b\left(\tfrac{3}{8}b - \zeta(2) - \tfrac{9}{8}\right) + f_1 + f_2 x + f_3 x^2 + f_4 x^3 + f_5 x^4 + f_6 x^5, \quad 0 \le x \le 1$$

$$\begin{aligned}
f_1 &= \tfrac{1}{2}\left(-3\zeta(3) - \zeta(2) + \tfrac{23}{8}\right) &&\approx -1.1881 \\
f_2 &= \zeta(3) - \tfrac{10}{9}\zeta(2) - \tfrac{53}{36} &&\approx -2.0979 \\
f_3 &= \tfrac{1}{8}\left(13\zeta(2) + \tfrac{277}{24}\right) &&\approx 4.1157 \\
f_4 &= \tfrac{1}{2}\left(-3\zeta(3) + \tfrac{117}{36}\zeta(2) - \tfrac{197}{32}\right) &&\approx -2.2082 \\
f_5 &= 3\zeta(3) - \tfrac{187}{72}\zeta(2) + \tfrac{2513}{576} &&\approx 3.6968 \\
f_6 &= \tfrac{1}{2}\left(-3\zeta(3) + \tfrac{7}{4}\zeta(2) - \tfrac{349}{96}\right) &&\approx -2.1815
\end{aligned} \tag{30}$$

The functions $V_1(r)$, $A_1(r)$ and $F_1(x)$ and their approximations are depicted in Fig. 2. For $m_t \ge M_W$ one has already $r \le 1/(4c_W^2) \approx 0.3$, which suggests to approximate $V_1(r)$ and

$A_1(r)$ by their small-r expansions as given in Eqs. (18) and (19). Inserting also the quantum numbers for the tb doublet, one ends up with

$$(\Delta r)_1^{tb} = -\frac{c_W^2}{s_W^2}(\Delta\rho(0))_1^{tb} + \frac{\alpha}{\pi}\left[\frac{1}{2s_W^2}\left(-\frac{1}{4s_W^2}+\frac{1}{3}\right)\ln u_t - \frac{1}{9}\ln u_b + \frac{c_1}{s_W^4} + \frac{c_2}{s_W^2} + c_3 \right.$$
$$\left. + \frac{1}{s_W^2}\left(\frac{1}{s_W^2}-2\right)\left.\frac{F_1(x)-F_1^\infty(x)}{x}\right|_{x=c_W^2/u_t}\right], \tag{31}$$

$$(\delta A_{\mathrm{LR}})_1^{tb} = \frac{64 s_W^4 c_W^2}{(v_e^2+1)^2}\left\{(\Delta\rho(0))_1^{tb} + \frac{\alpha}{\pi}\left[\frac{1}{s_W^2 c_W^2}\left(\frac{1}{24}\ln u_t + c_4\right) + \frac{1}{9}\ln u_b - c_3\right]\right\}, \tag{32}$$

where $u_q = m_q^2/M_Z^2$ $(q=t,b)$, $F_1^\infty(x) = f_1 + (f_2 + \zeta(2) + 9/8)x$ and

$$\begin{aligned} c_1 &= \tfrac{\zeta(3)}{2} - \tfrac{\zeta(2)}{9} - \tfrac{1}{8} && \approx\ 0.2933\,, \\ c_2 &= \tfrac{1}{3}\left(-2\zeta(3) + \tfrac{2}{3}\zeta(2) + \tfrac{5}{4}\right) && \approx -0.0192\,, \\ c_3 &= \tfrac{1}{9}\left(4\zeta(3) - \tfrac{5}{6}\right) && \approx\ 0.4417\,, \\ c_4 &= -\tfrac{1}{6}\left(\zeta(3) + \tfrac{1}{2}\right) && \approx -0.2837\,. \end{aligned}$$

Apart from the well-known leading behaviour proportional to $\Delta\rho(0)$, these equations reveal the subleading terms for $m_t \gg M_W$ also. The term in Δr proportional to $(F_1(x) - F_1^\infty(x))/x$ is relevant for values of m_t just above M_W and dies out for $m_t \gg M_W$. Recalling Sirlin's definition $c_W = M_W/M_Z$ [6], Eq. (31) gives $(\Delta r)_1^{tb}$ as a function of M_W, M_Z, m_t and m_b. It may, thus, be used in combination with the full $\mathcal{O}(\alpha)$ expression for Δr and the $\mathcal{O}(\alpha\alpha_s)$ contribution from the light quarks to find M_W as the self-consistent solution of Eq. (10) in Ref. [8]. That reference also explains in detail how light quarks are properly incorporated.

Fig. 3 displays our analytic result for $(\Delta r)_1^{tb}$ together with its approximation by Eq. (31) with and without the last term therein for $M_Z = 91.1$ GeV, $s_W^2 = 0.23$ and $m_b = 4.9$ GeV. For $\alpha_s(\mu^2)$ we employ the representation in the modified minimal-subtraction ($\overline{\mathrm{MS}}$) scheme as of Eq. (6) in Ref. [19] with $\Lambda^{(5)}_{\overline{MS}} = 240$ MeV [20] and $\mu = m_t + m_b$ [21]. Note that the spike at $m_t = M_W - m_b$ which is expected to arise due to the logarithmic Coulomb singularity is suppressed in our approach, as we set $m_b = 0$ in the finite part of the W self-energy function (see Chap. 2). Similarly, Fig. 4 contrasts $(\delta A_{\mathrm{LR}})_1^{tb}$ with its asymptotic expansion Eq. (32). Here no spike is omitted at $m_t = M_W - m_b$, as T_b threshold effects are absent anyway.

Fig. 5 shows the complete $\mathcal{O}(\alpha)$ prediction for Δr in the framework of the minimal standard model (three generations, one Higgs) computed on the basis of the formulae in Refs. [22] and [23] for $M_Z = 91.1$ GeV, $M_H = 100$ GeV (1 TeV) and variable m_t. For $M_H = 100$ GeV, in addition, the effect of the $\mathcal{O}(\alpha\alpha_s)$ corrections is visualized as they result from both our analytic calculation (dashed line) and the comprehensive numerical approach of Ref. [8] (solid line). The difference between these two curves may be regarded as the hadronic uncertainty and is small for $m_t < 150$ GeV. For higher values of m_t, however, the solid line is supposed to represent the more realistic estimate and will, therefore, be referred to in the subsequent discussion. For m_t between 100 and 250 GeV the inclusion of the $\mathcal{O}(\alpha\alpha_s)$ contributions amounts to a shift of M_W between -36 and -85 MeV, which has to be compared with the envisaged experimental accuracy of $\delta M_W \approx 100$ MeV at LEP 200 [24]. When M_W and, thus, Δr have been measured these corrections will raise the range for m_t allowed by the uncertainty of the Higgs mass by about 10 GeV for heavy top. This turns out to be relatively important, since the Higgs contribution slowly varies with M_H as a consequence of the well-known screening theorem [25]. As is evident from Fig. 5, the $\mathcal{O}(\alpha\alpha_s)$ corrections are of the same order of magnitude as the uncertainty induced by the as yet unknown Higgs mass. Once top mesons or toponium have been observed, high precision measurements of Δr could be sensitive to virtual effects associated with the Higgs boson. QCD corrections would then considerably lower the estimate for M_H.

4. SUMMARY

We have calculated the tb contribution in $\mathcal{O}(\alpha\alpha_s)$ to the vacuum polarization functions of the vector bosons in the standard model using dispersive methods. The treatment of the finite part of the W self-energy is greatly simplified in the limit of the bottom quark being massless. We have applied these results to the radiative parameter Δr and the oblique correction δA_{LR} for the left-right asymmetry and determined the subleading terms in m_t^2/M_W^2. As for the full Δr, it turns out that the $\mathcal{O}(\alpha\alpha_s)$ corrections are comparable to the variation caused by the actual lack of knowledge of the precise value of the Higgs mass.

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FIGURE CAPTIONS

1) Feynman diagrams representing the first order QCD corrections to the vacuum polarizations of the vector bosons.

2) Exact result and approximation for a) $V_1(r)$, b) $A_1(r)$ and c) $F_1(x)$ for $0 < r, x < 1$.

3) Perturbative result for the tb contribution to Δr in $\mathcal{O}(\alpha\alpha_s)$ together with its asymptotic high-m_t expansion and the fit for $m_t \geq M_W$.

4) Perturbative result for the tb contribution to $\delta A_{\rm LR}$ in $\mathcal{O}(\alpha\alpha_s)$ together with its asymptotic high-m_t expansion.

5) Full standard model prediction for Δr. The effect of first order QCD corrections is compared with the variation with the Higgs mass.

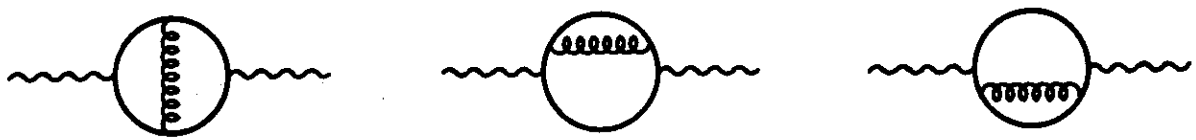


Fig. 1

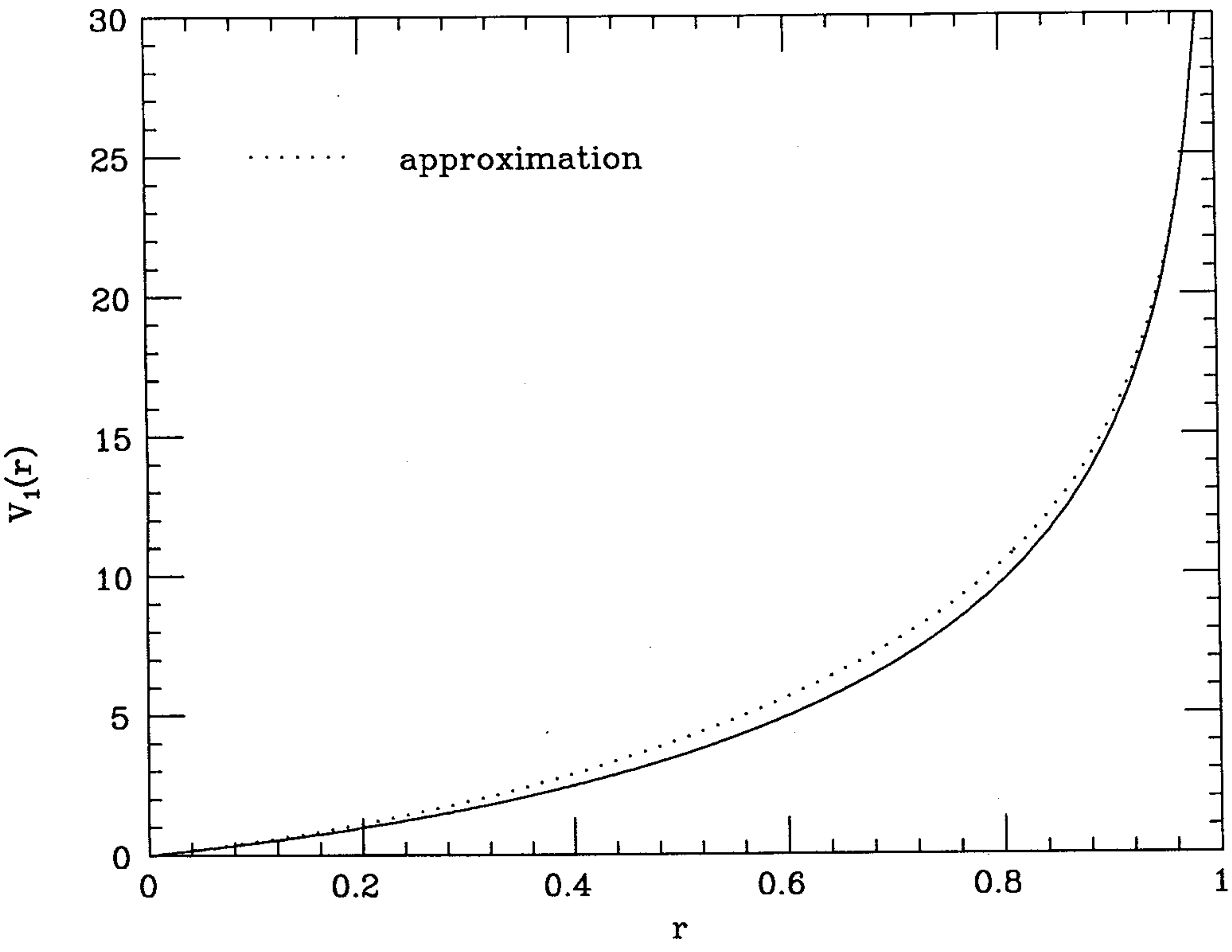


Fig. 2a

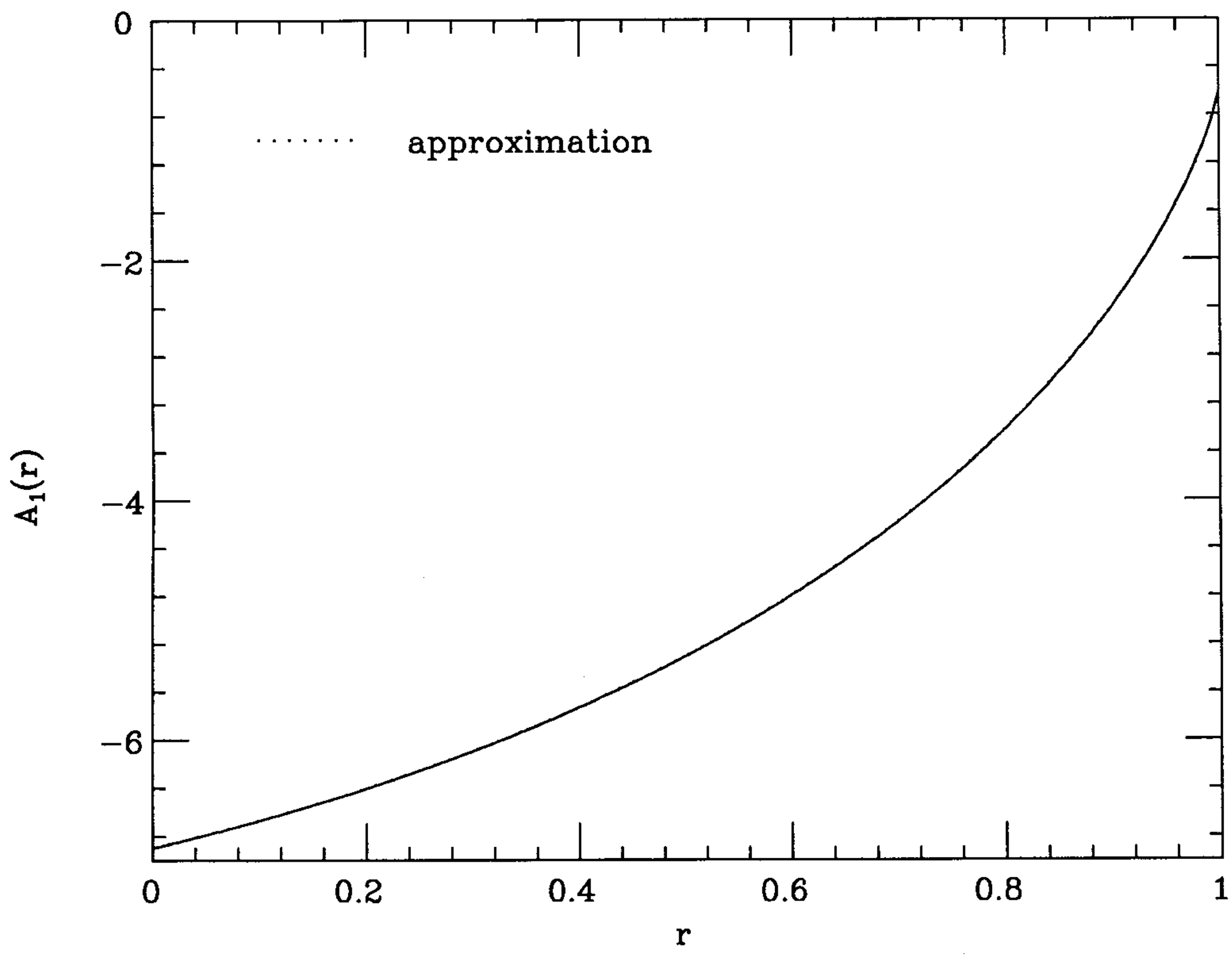


Fig. 2b

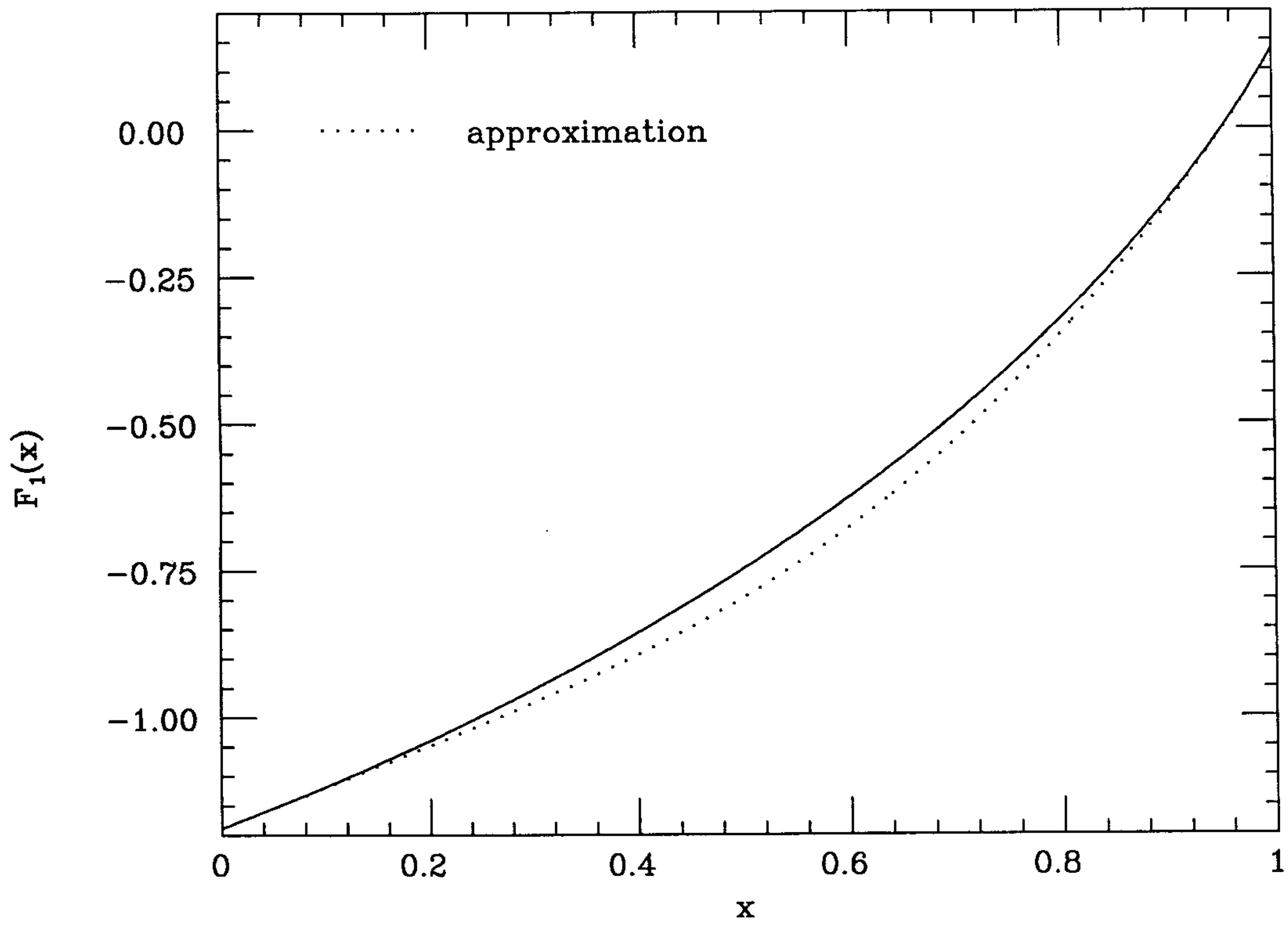


Fig. 2c

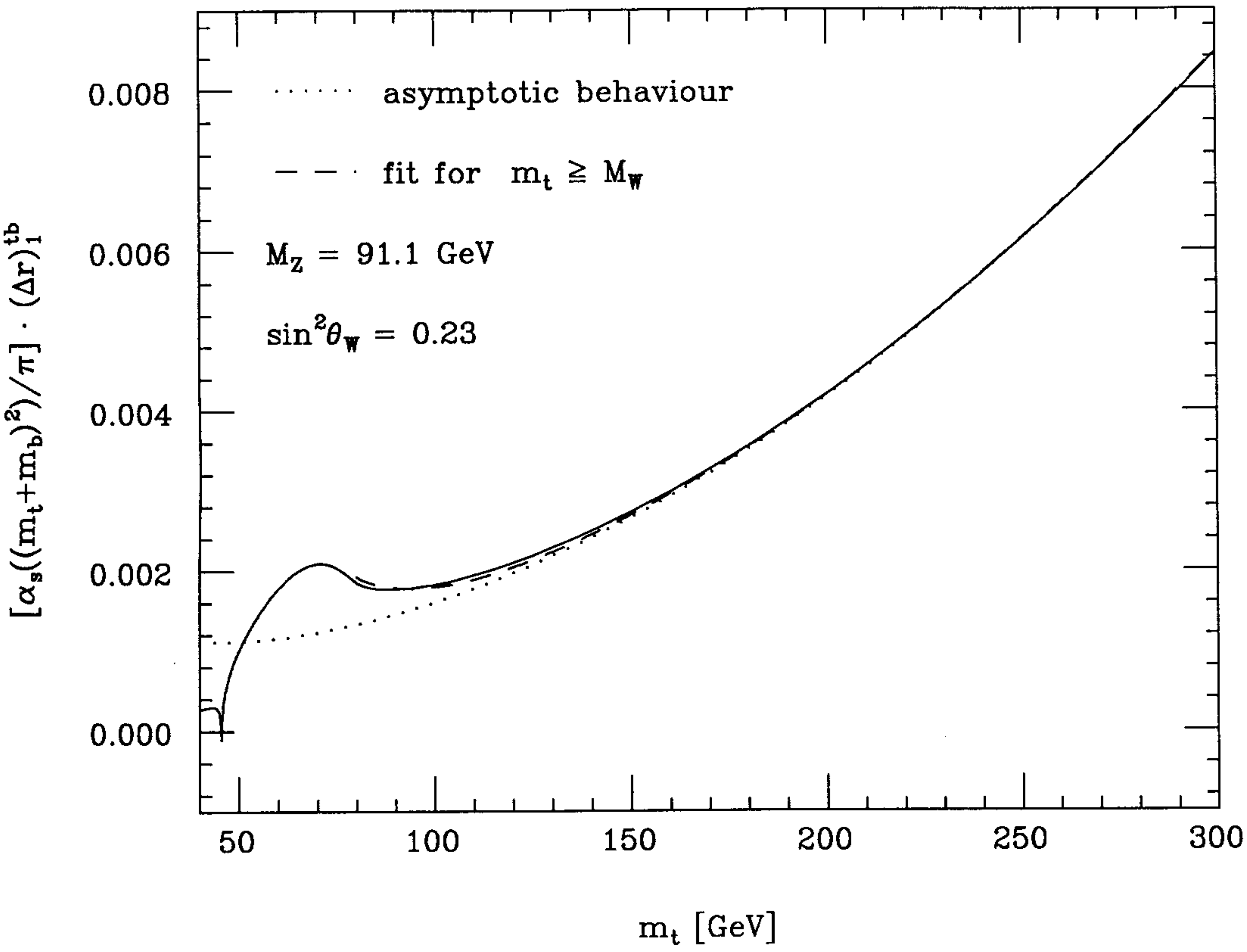

asymptotic behaviour
fit for $m_t \geqq M_W$
$M_Z = 91.1$ GeV
$\sin^2\theta_W = 0.23$
0.008
0.006
0.004
0.002
0.000
$[\alpha_s((m_t+m_b)^2)/\pi] \cdot (\Delta r)_1^{tb}$
50
100
150
200
250
300
m_t [GeV]

Fig. 3

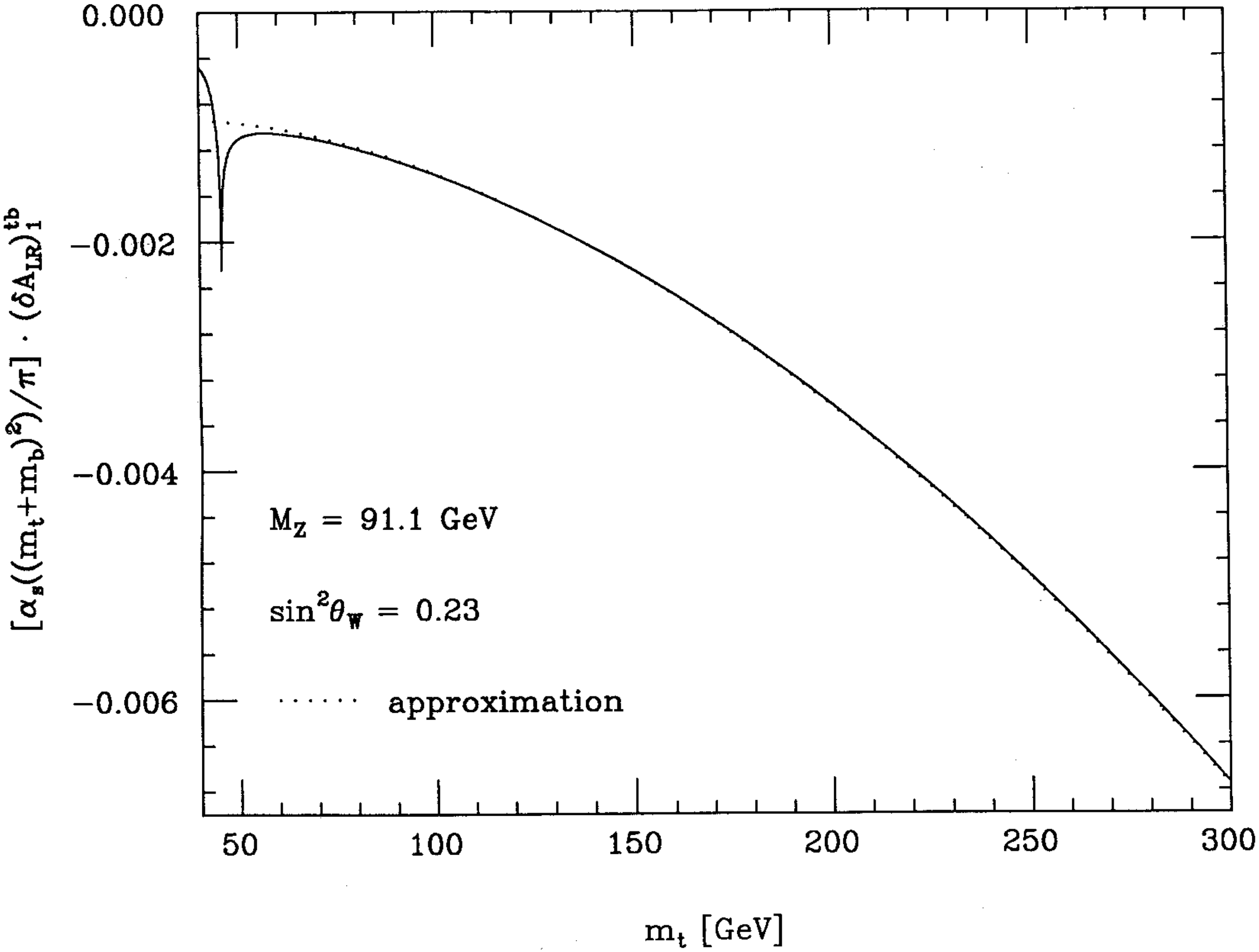
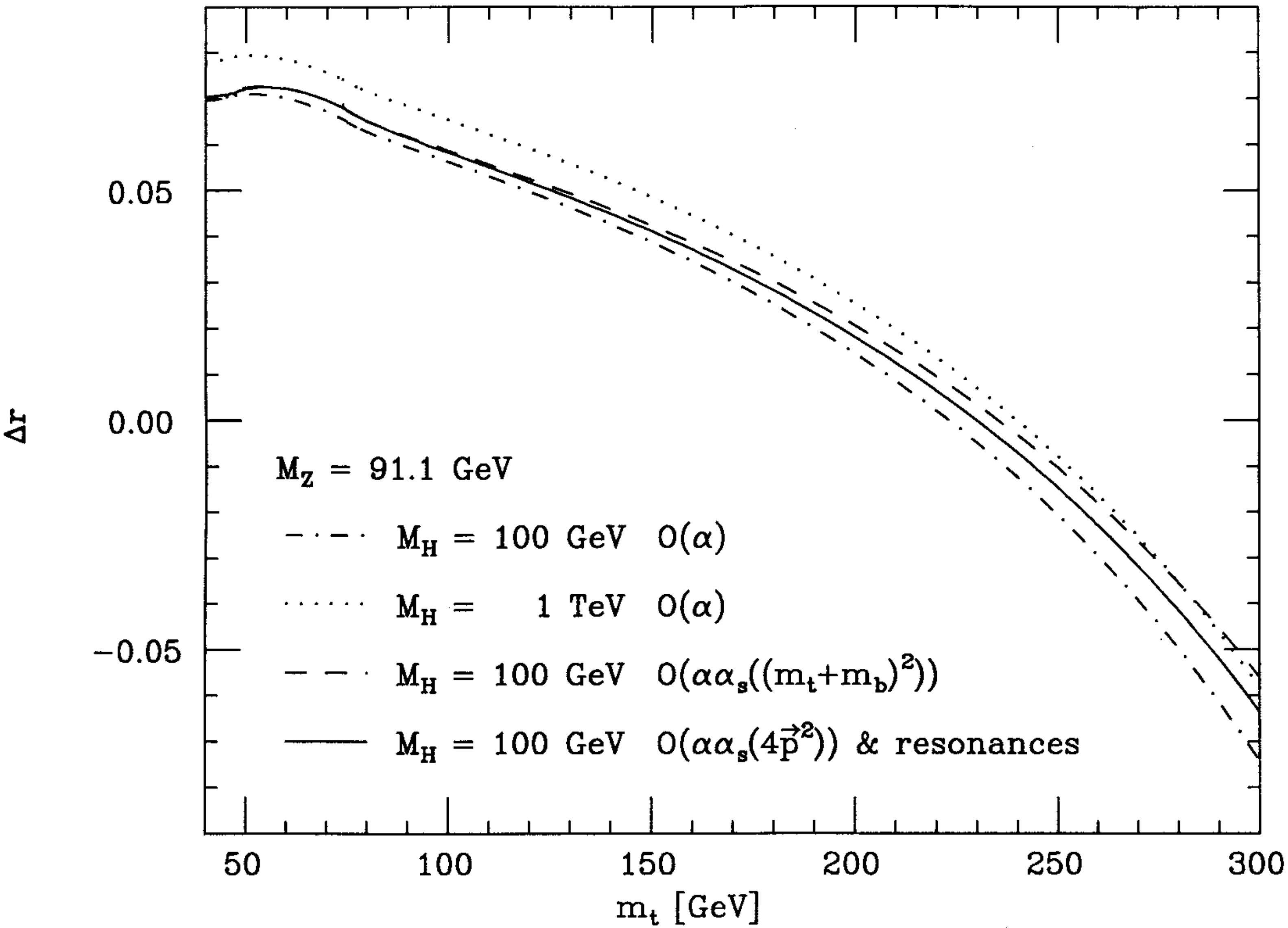

0.000
−0.002
−0.004
−0.006
$[\alpha_s((m_t+m_b)^2)/\pi] \cdot (\delta A_{LR})_1^{tb}$
M_Z = 91.1 GeV
$\sin^2\theta_W$ = 0.23
approximation
50
100
150
200
250
300
m_t [GeV]

Fig. 4



0.05
0.00
−0.05
Δr
50
100
150
200
250
300
m_t [GeV]
M_Z = 91.1 GeV
M_H = 100 GeV O(α)
M_H = 1 TeV O(α)
M_H = 100 GeV O(αα_s((m_t+m_b)^2))
M_H = 100 GeV O(αα_s(4p⃗^2)) & resonances

Fig. 5