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Edward H. Kaplan, Kevin Mongeon & John T. Ryan

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A Markov Model for Hockey: Manpower Differential and Win Probability Added

Edward H. Kaplan
William N. and Marie A. Beach Professor of Operations Research, Yale School of Management; Professor of Public Health, Yale School of Public Health; and Professor of Engineering, Yale School of Engineering and Applied Science (edward.kaplan@yale.edu).

Kevin Mongeon
Department of Sport Management, Brock University, 500 Glenridge Avenue, St Catharines, ON L2S 3A1 (kmongeon@brocku.ca).

John T. Ryan
Economics Department, Yale University (johntryan7@gmail.com).

Abstract—We extend the classic Poisson model of hockey based on score differential and time remaining in the game to include the effect of penalties, and derive the associated Markov win probability model given the goal/manpower differential state at any point in a hockey game. Given data from the 2008/9–2011/12 National Hockey League seasons (a total of 4,920 games) reporting second-by-second goal and manpower differentials (which results in roughly 17.7 million observations), we estimate the state dependent transition rates and win probabilities. The data reveal that even after controlling for the home edge afforded by visiting teams being penalized more frequently than home teams, the goal scoring rate for the home team is higher than for visiting teams at most equivalent manpower differential levels. We use the model to develop a new win probability added metric for evaluating individual players based on their incremental contribution to the probability of winning and illustrate its use and conservation properties.

1. INTRODUCTION

The game of hockey differs from other team sports such as baseball, basketball, football and soccer in how teams are disciplined for player or team infractions known as penalties. Typically, when a penalty is called in hockey, the offending player is removed from the ice at the next stoppage in play, which forces the penalized team to play shorthanded until the penalty expires or the opposing team scores, whichever happens first. Since goal-scoring rates are higher for teams skating on a "power play" and lower for teams engaged in "penalty killing," penalties are often crucial in determining the outcome of a hockey game. In this article, we develop a new model of the probability of winning a hockey game that explicitly accounts for the manpower differentials caused by penalties during a game. Calibrated with over 17.7 million seconds of hockey data describing every regular season hockey game in the 2008–09 through 2011–12 seasons, we show how this model can be used to create a real-time win probability scoreboard that in turn can be used to evaluate a hockey player's individual contribution to the probability of winning (win probability added).

We are by no means the first to model hockey, and our research owes much to earlier work. Mullet (1977) established the usefulness of the Poisson distribution in describing the number of goals scored during hockey games using data from the 1973–74 National Hockey League (NHL) season. Buttrey, Washburn, and Price (2011) used a Poisson model to analyze per-minute goal scoring rates across different man-advantage scenarios using data from the 2008–09 season. Using in-game Poisson models, several authors examined the consequences and optimal timing of "pulling the goalie," a unique hockey strategy whereby an extra skater is substituted for the goaltender, typically employed by the trailing team towards the end of a game. These include Morrison (1976), Morrison and Wheat (1986), Erkut (1987), Nydick and Weiss (1989), Washburn

(1991), Berry (2000), Zaman (2001), and Beaudoin and Swartz (2010), which develops a simulation program that considers various man-advantage situations and team-specific parameters. Collectively, these papers overwhelmingly suggest that teams wait too long to pull their goaltender. Other recent papers examining various aspects of hockey include Abrevaya and McCulloch (2014), Doyle and Leard (2012), Jones (2011), Marek, Šedivá and Ťoupal (2014), and Thomas (2007).

Our research builds most directly on Washburn (1991), who used the Poisson process to develop a state-space model of the probability that a team wins a hockey game given the goal differential and time left in the game. Stern (1994) develops similarly-spirited models for basketball based on Brownian motion that, while appropriate for basketball, are not well-suited for hockey where scoring is rare and the natural stochastic process is Poisson. Washburn (1991) did not consider the effect of penalties on goal scoring, but Buttrey, Washburn and Price (2011) and Beaudoin and Swartz (2010) did. In those papers, scoring rates were estimated for specific man-advantage situations (e.g. 6-on-5 or 5-on-4), whereas in our work we focus only on manpower differential (thus both 6-on-5 and 5-on-4 would be modeled as a manpower differential of +1) for purposes of creating a tractable state-space model that produces win probabilities given the goal and manpower differential at any point in a game.

The paper unfolds as follows: in the next section we briefly review the Poisson model of hockey where the state-space depends only upon goal differential at any point in the game. We then develop a new Markov model for the probability that the home team wins a hockey game given manpower differential in addition to goal differential at any point in the game. In Section 4 we discuss our dataset, which consists of the home-minus-away goal and manpower differential at every second of every regular season hockey game over four NHL seasons. We estimate the required state transition rates, present scoring rates by manpower differential and home/away status, and discuss some properties of home ice advantage that emerge from the data. Section 5 presents our numerical win probability results for the model of Section 4, and uses the model to explore an earlier analysis of Burke (2009) regarding the relative boost in win probability provided by gaining a man advantage via a penalty relative to scoring a goal. Technical material for assessing the fit of the Markov model to the data is detailed in the online Supplemental Material, where we derive explicit formulas for the variance of observed win probabilities based on the model. In Section 6, we demonstrate a real-time win probability scoreboard for keeping track of a hockey game; as opposed to changing score only when goals occur, our win probability scoreboard changes continuously over the duration of a hockey game in addition to discrete jumps that occur when goals are scored and penalties are awarded or expire. Section 7 applies this scoreboard to evaluate individual players by generalizing the +/− statistic to individual win probability added, provides a numerical example, and shows how these individual measures cumulate to team wins above average over the course of a season. Section 8 concludes with suggestions for further research and use of the model.

2. THE POISSON MODEL OF HOCKEY

We begin with a quick review of the Poisson model of hockey (see Washburn (1991) for an application to pulling the goalie). Let $\lambda(\mu)$ denote the expected number of goals scored by the home (away, or equivalently the visiting) team per unit time in a hockey game. Let $w(x,t)$ denote the conditional probability that the home team wins the hockey game, given that with t time units left in regulation, the home team leads the visitors by x goals (if x is negative, the home team trails). The Poisson model of hockey states that the home and away teams score in accord with independent Poisson processes with scoring rates λ and μ respectively. Consequently, given that the current home minus away goal differential equals x, in a sufficiently small time slice of duration Δt, three events are possible (Figure 1): (i) the home team scores, in which case the goal differential increases from x to $x+1$ (this event occurs with probability $\lambda\Delta t$); (ii) the away team scores, in which case the goal differential decreases from x to $x-1$ (this even occurs with probability $\mu\Delta t$); and (iii) no team scores in which case the goal differential remains at x (this event occurs with probability $1-(\lambda+\mu)\Delta t$).

These dynamics lead to the well-known birth-and-death model shown in equation (1) below which applies to regulation time of a hockey game. The model is closed by equation (2), which indicates that the home team wins with certainty if it has outscored the away team by the end of regulation time at $t = 0$ (the away team wins with certainty if it has scored more goals),

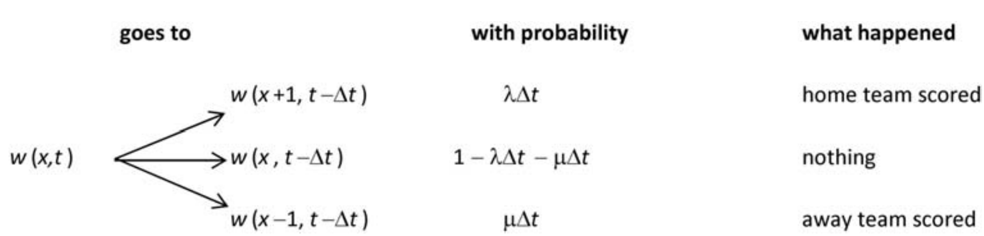

Figure 1: The Poisson Model of Hockey

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and for simplicity assigns a winning probability of 1/2 if the game is tied at the end of regulation time (later we will explicitly model overtime).

$$\frac{dw(x,t)}{dt} = \lambda w(x+1,t) + \mu w(x-1,t) - (\lambda+\mu) w(x,t) \quad (1)$$

$$w(x,0) = \begin{cases} 1 & x > 0 \\ \frac{1}{2} \quad \text{if} & x = 0 \\ 0 & x < 0 \end{cases} \quad (2)$$

As an illustrative example, consider the four NHL regular seasons spanning 2008–2012 (1,230 games per season; 4,920 games total). Over all of these games, the home team averaged 2.75 goals per game while the visiting team averaged 2.47 goals per game. Using these averages to estimate λ and μ (recall that there are 60 minutes = *3,600 seconds* in regulation time), equations (1)–(2) predict that the home team wins 54.7% of the time (that is, $w(0,3600 \text{ seconds}) = 0.547$). In fact, over the 4,920 games considered, the home team won 2,702 or 54.9% of the games.

While this is impressive considering that no win/loss data were taken into account in calibrating this model, there is more to a hockey game than solely scoring goals. As discussed in the introduction, one feature distinguishing hockey from other team sports is the imposition of penalties for various infractions that deliberately create unbalanced manpower situations. Since scoring rates are not constant across manpower advantage scenarios, an assertion with strong intuitive and empirical support, it stands to reason that the probability of winning a hockey game depends on manpower differential in addition to goal differential.

3. A MARKOV MODEL INCORPORATING MANPOWER DIFFERENTIAL

We now proceed to incorporate manpower differential into the state space and model the conditional probability that the home team wins a hockey game, given that the home team leads the visiting team by x goals while enjoying a manpower differential of y players with t time units remaining in regulation. For example, if the home team is trailing by two goals while playing with a man advantage, the (x, y) state of the game is $(-2, 1)$. Following NHL rules, we only consider manpower differentials $y = -2, -1, 0, 1,$ or 2, while for computational convenience we apply a five goal mercy rule that artificially terminates a game in favor of the home team if $x = 5$ (the visitors win if $x = -5$). The instantaneous state transition rate from a goal/manpower differential state (x,y) to a different state (x',y') is denoted by $\lambda_{xy}^{x'y'}$. These transition rates can, for example, reflect home (away) team scoring when $x' = x+1$ $(x-1)$, or an increment (decrement) in manpower differential as occurs when a penalty against the visiting team occurs (expires) in which case $y' = y+1$ $(y-1)$. Denoting the state-dependent home team win probability by $w(x,y,t)$ and measuring time in seconds (so regulation time equals 3,600 seconds), the equations of the model are:

$$\begin{aligned} \frac{dw(x,y,t)}{dt} &= \sum_{(x',y')\neq(x,y)} \lambda_{xy}^{x'y'} w(x',y',t) \\ &\quad - \left(\sum_{(x',y')\neq(x,y)} \lambda_{xy}^{x'y'} \right) w(x,y,t) \\ &\qquad x = -4,\ldots,4;\ y = -2,\ldots,2;\ 0 < t \le 3600 \end{aligned} \quad (3)$$

$$w(x,y,0) = \begin{cases} 1 & x > 0 \\ w(0,y,0) & x = 0 \quad y = -2,\ldots,2 \\ 0 & x < 0 \end{cases} \quad (4)$$

$$\begin{aligned} & w(5,y,t) = 1; w(-5,y,t) = 0; \\ &\qquad y = -2,\ldots,2;\ 0 < t \le 3600 \end{aligned} \quad (5)$$

$$\begin{aligned} \frac{dw(0,y,t)}{dt} &= \sum_{(x',y')\neq(0,y)} \lambda_{0y}^{x'y'} w(x',y',t) \\ &\quad - \left(\sum_{(x',y')\neq(0,y)} \lambda_{0y}^{x'y'} \right) w(0,y,t) \\ &\qquad y = -2,\ldots,2;\ -300 < t \le 0 \end{aligned} \quad (6)$$

$$\begin{aligned} & w(1,y,t) = 1; w(-1,y,t) = 0; \\ &\qquad y = -2,\ldots,2;\ -300 < t \le 0 \end{aligned} \quad (7)$$

$$w(0,y,-300) = \frac{1}{2};\ y = -2,\ldots,2 \quad (8)$$

Equation (3) describes the evolution of the win probability during regulation time. Equation (4) awards a certain win to the home (away) team if, at the end of regulation time, the home (away) team has scored more goals in the game. If the game is tied at the end of regulation however, equation (4) assigns a win probability that depends upon the resolution of overtime play as modeled via equation (6). Equation (5) assigns a win with certainty to the home or away team if either achieves a five goal advantage, which is taken to be insurmountable. Equation (6) mimics equation (3) in describing the win probability evolution during overtime play, except that the goal differential is restricted to zero as long as overtime ensues. Of course, if either the

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home or away team score during sudden-death overtime, the game ends with victory for the scoring team; this is enforced by equation (7). Overtime cannot last more than five minutes which is why $t > -300$ in equation (6), and if nobody scores during overtime a penalty shootout ensues; equation (8) assigns a win probability of 1/2 to both teams in such an event.

In the numerical implementation of the model, equations (3) and (6) were converted to discrete-time difference equations with one second time steps to match the resolution of the data. As these equations are first-order, numerical implementation is a simple task; indeed all of the computations reported in this paper were performed in Excel.

The model above does simplify the treatment of penalties to ensure model tractability. Unlike the duration of a typical penalty that lasts for either two minutes or the time until a power play goal is scored, whichever comes first, penalties in the model are all exponentially distributed in duration, but with onset and expiration rates per game that match the actual rates with which penalties occur and end in the data. Explicitly modeling penalties as the minimum of time to a goal or the penalized time would require keeping track of elapsed penalty time, which would destroy the discrete state-space of the model (thus creating severe data sparsity), and complicate calculations considerably. Happily, as shown in the online Supplemental Material, the model formulated above does fit the observed data, which we now describe.

4. DATA, PARAMETER ESTIMATION, AND HOME ICE ADVANTAGE

4.1. The Data

Our initial data were drawn from the play-by-play reports that are publicly available at nhl.com. In addition to the identity of the home and away teams, these reports record and time-stamp many different events that take place during games, including goals scored and penalties. For each goal and penalty, the reports contain associated information relating to the identity of the team and the number of team-goals scored prior to the occurrence of the event. In the case of penalty events, the reports contain associated information relating to the penalty duration (e.g., 2 minutes). Based on the rules of the game that determine the expiration of penalty events from either elapsed time or opposing team goals scored, an algorithm was developed to identify the score and manpower differential states for each game-second. Our final data set consists of over 17.7 million season-game-second-level specific score and manpower differential states, relative to the home team, during games played from the 2007–08 through 2011–12 seasons (17,712,000 seconds = 60 seconds/minute × 60 minutes/game × 1230 games/season × 4 seasons).

4.2. Parameter Estimation

Estimating the state-to-state transition rates from the data described above is straightforward in principle. For any two states $s = (x,y)$ and $s' = (x',y') \neq s$, define $n(s,s')$ as the number of transitions observed in the data from state s to state s', and let $\tau(s)$ denote the total time spent in state s over the entire data set. For example, if $s = (0,0)$, then aggregating over all 4,920 hockey games, $\tau(s)$ is the total time spent by teams playing at even strength while the score is tied. With these definitions, the maximum likelihood estimate for the transition rate $\lambda_s^{s'}$ is equal to

$$\lambda_s^{s'} = \frac{n(s,s')}{\tau(s)}. \tag{9}$$

However, with nine goal differential ($x = -4,\ldots,4$) and five manpower differential ($y = -2,\ldots,2$) possibilities (see equation (3)), there are 45 possible states and thus 45×44 = 1,980 conceivable state-to-state transition rates. Most such transitions are impossible. For example, the goal differential cannot change by more than one in absolute value in any single state transition. Similarly, manpower differential cannot change by more than two in a single transition. To further simplify, we decided to use the much smaller set of parameters that results from removing the dependence of state transition rates upon initial goal differential. An implication of this assumption is that while goal scoring rates depend upon manpower differential, they do not depend upon goal differential. Though this is an assumption, note that it generalizes the Poisson model of Section 2 where goal scoring rates are completely independent of both goal and manpower differential. In addition, we only allowed transitions that would change goal differential by at most one, manpower differential by at most two, and eliminated other state transitions we deemed unlikely a priori based on our knowledge of hockey (e.g. from playing two men down to simultaneously scoring while attaining even strength).

The estimated transition rates per game appear in Table 1. Cells with a dashed line represent either self-loops (both goal and manpower differential remain unchanged), or transitions we deemed infeasible as discussed above. Of the 17.7 million total transitions in the data, only 53 fell outside of the feasible

TABLE 1.
State transition rates per 3600 seconds of hockey.

Change in Goal Differential	x' - x	1	1	1	1	1	0	0	0	0	0	-1	-1	-1	-1	-1
Final Manpower Differential	y'	2	1	0	-1	-2	2	1	0	-1	-2	2	1	0	-1	-2
Initial	2	0.00	11.92	0.37	---	---	---	35.24	3.29	---	---	0.51	---	---	---	---
Manpower	1	0.00	0.16	5.99	---	---	2.45	---	28.77	0.08	---	---	0.86	0.03	---	---
Differential	0	---	0.00	2.49	0.00	---	0.07	4.22	---	3.92	0.06	---	0.00	2.25	0.00	---
(y)	-1	---	---	0.01	0.97	---	---	0.09	28.98	---	2.17	---	---	5.59	0.12	0.00
	-2	---	---	---	---	0.65	---	---	3.41	33.65	---	---	---	0.13	12.40	0.00

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TABLE 2.
Goal scoring rates per 3600 seconds of hockey for different manpower differentials.

Manpower State	Home Goals/Hour	Away Goals/Hour	State Probability
Home Up 2	12.28	0.51	0.006
Home Up 1	6.15	0.89	0.099
Even Strength	2.49	2.26	0.797
Home Down 1	0.98	5.71	0.093
Home Down 2	0.65	12.53	0.005
Overall	**2.75**	**2.47**	

transition set. We reassigned such transitions to the closest feasible category, and then applied equation (9) to the adjusted data. After eliminating transition dependence upon initial goal differential, had we either allowed all transitions to be feasible, or ignored those deemed infeasible, we would have still obtained the numerical results shown in Table 1 given so few transitions appear infeasible relative to the total number of transitions observed.

4.3. Manpower Differential, Scoring Rates, and Home Ice Advantage

Note that since a transition with $x'-x=1$ implies that the home team scored a goal, summing along the rows across the first five columns of Table 1 produces the home team scoring rates for each manpower differential; similarly summing along the rows across the last five columns (corresponding to $x'-x=-1$) yields the manpower differential dependent scoring rates for the visiting team. By construction (equation (9)), these scoring rates are the same as the empirically observed number of goals scored per unit time across the different manpower states over all games in our data. The resulting scoring rates are displayed in Table 2, which makes clear how important manpower differential is in hockey. For both the home and away teams, the scoring rates increase monotonically with manpower advantage. There is about a 20-fold increase in the goal scoring rates as teams move from playing two men short to having a two man advantage. Note also from Table 2 that about 80% of all hockey is played at even strength, 19% is played with a one-man advantage, and 1% is played with a two man advantage. These rates are broadly consistent with those reported in Table 1 of Buttrey, Washburn and Price (2011) based on one NHL season that conditions on the actual number of skaters on ice (and not just their difference), though Buttrey, Washburn and Price do not distinguish scoring rates for home versus away; see also Beaudoin and Swartz (2010).

There is an additional observation of interest regarding home ice advantage that follows directly from these rates. That home teams enjoy an edge in baseball, basketball, and football is well known (Moskowitz and Werthem, 2011), and hockey is no exception; as discussed earlier, in our data the home team averages 2.75 goals per game while the away team averages 2.47, a highly significant difference ($z = 8.6$ for difference of Poisson rates). Moskowitz and Werthem (2011) have argued that the main source of the home edge stems from referee bias in favor of the home team, perhaps in response to pressure from exuberant home team fans. In hockey, this would translate into referees calling more penalties against the visiting team than the home team, and indeed this does occur: over the four years in our data, home teams were penalized on average 4.64 times per game while the visiting team averaged 4.99 penalties per game, also a highly significant result ($z = -7.9$; see also Abrevaya and McCulloch (2014) and Beaudoin and Swartz (2010)). However, Table 2 also shows that the home team outscores the visiting team at four out of five manpower differential levels (the exception being a two man advantage). These data easily reject the hypothesis that scoring rates are equal once one controls for manpower differential ($\chi^2 = 54.4$ at 5 degrees of freedom). Given these results, it seems that the home scoring edge cannot be completely explained by referees penalizing visiting teams more often.

5. IN-GAME WIN PROBABILITIES INCORPORATING MANPOWER DIFFERENTIAL

Using the estimated transition rates reported in Table 1, we numerically solved equations (3)-(8) to obtain $w(x,y,t)$, the home team win probability given the home-minus-away goal (x) and manpower (y) differentials with t seconds remaining in the game. Win probabilities for goal differentials between −2 and +2 at all manpower differential levels are displayed in Figure 2 (goodness-of-fit details appear in the Online Supplement). The curves in this graph neatly divide into five different groups of five curves each, one group for each goal differential and five curves within each group corresponding to the five different manpower differentials. Focusing on the middle group first, at the start of a game, the score is tied, teams are at even strength, and the home team win probability equals 55%, consistent with the observed data. By the end of regulation time (3,600 seconds), the home win probability ranges from 35% if the home team enters overtime playing two men down to 66% if the home team enjoys a two man advantage; this is the widest margin due to penalties that can occur at any time during a game for any fixed goal differential. Starting overtime at equal strength (meaning four skaters per team as per NHL rules as opposed to the usual five skaters), the home team win probability equals 51%. If there is no scoring during five minutes of overtime play, then the win probability equals exactly 50% via equation (8); this explains why the win probabilities converge to 1/2 for all manpower differentials in a tie game.

The other four groups of curves correspond to games where the home team has a two goal lead (highest set of five curves), a one goal lead (next highest set of five curves), a one goal deficit (second lowest set of five curves), and a two goal deficit (lowest set of five curves). All of these win probability curves converge to either one or zero depending upon whether the home team is leading or trailing when regulation time expires.

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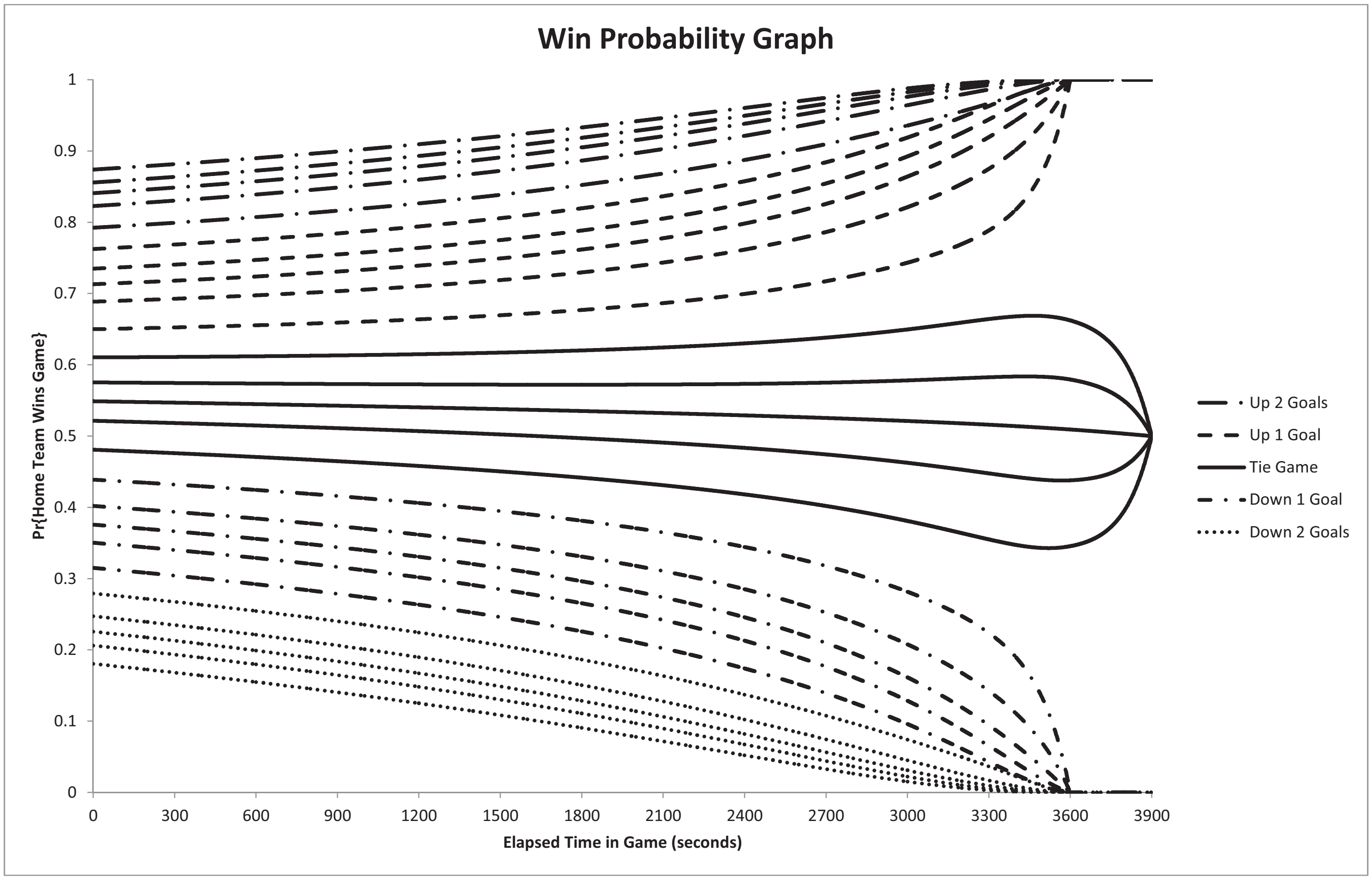

Figure 2: In-game NHL goal and manpower differential-dependent home team win probabilities based on transition rates of Table 1.

Since teams playing with a man advantage score on approximately 20% of power plays while teams skating shorthanded score on only 2% of such occasions, Burke (2009) reasoned that gaining a man advantage should increase the probability of winning by a little less than 20% of the difference between the win probability with the current goal differential and the win probability at the next highest goal differential based on the Poisson model ignoring manpower differential (see equations (1)-(2)). We can check Burke's intuition using Figure 2 by calculating $((w(x,y+1,t)-w(x,y,t))/(w(x+1,y,t)-w(x,y,t))$, which is the fraction of the win probability jump from scoring a goal at a fixed manpower level that is covered by the win probability jump from gaining a man advantage without scoring a goal, and comparing the result to 20%.

Figure 3 reports one such comparison, where we consider the impact of increasing manpower differential by one from even strength in games where the game is tied or the home team leads or trails by one goal. This figure supports Burke's observation over the first half of regulation play, in that for the first 30 minutes of hockey, the impact of gaining a man advantage accounts for between 15% (when the home team is trailing by a goal) and 19% (when the home team is up a goal) of the incremental win probability a goal would bring. However, the impact of penalties relative to goals changes dramatically later in the game. For example, with 5 minutes left and the home team leading by one goal, the impact of gaining a man advantage on win probability increases to 35% of the value of scoring a goal; this shows that gaining a power play with a one goal lead at the end of a game is worth much more than achieving a man advantage with the same goal differential near the beginning or middle of the game.

6. REAL-TIME WIN PROBABILITY SCOREBOARD

The win probability model reported in Figure 2 can be used to create an in-game win probability scoreboard based on the goal and manpower differential at any point in time during a game. To illustrate, Table 3 reports the official NHL game summary for the October 9, 2010 game between the visiting Dallas Stars and the home team New York Islanders. This summary contains all of the information necessary to deduce the goal and

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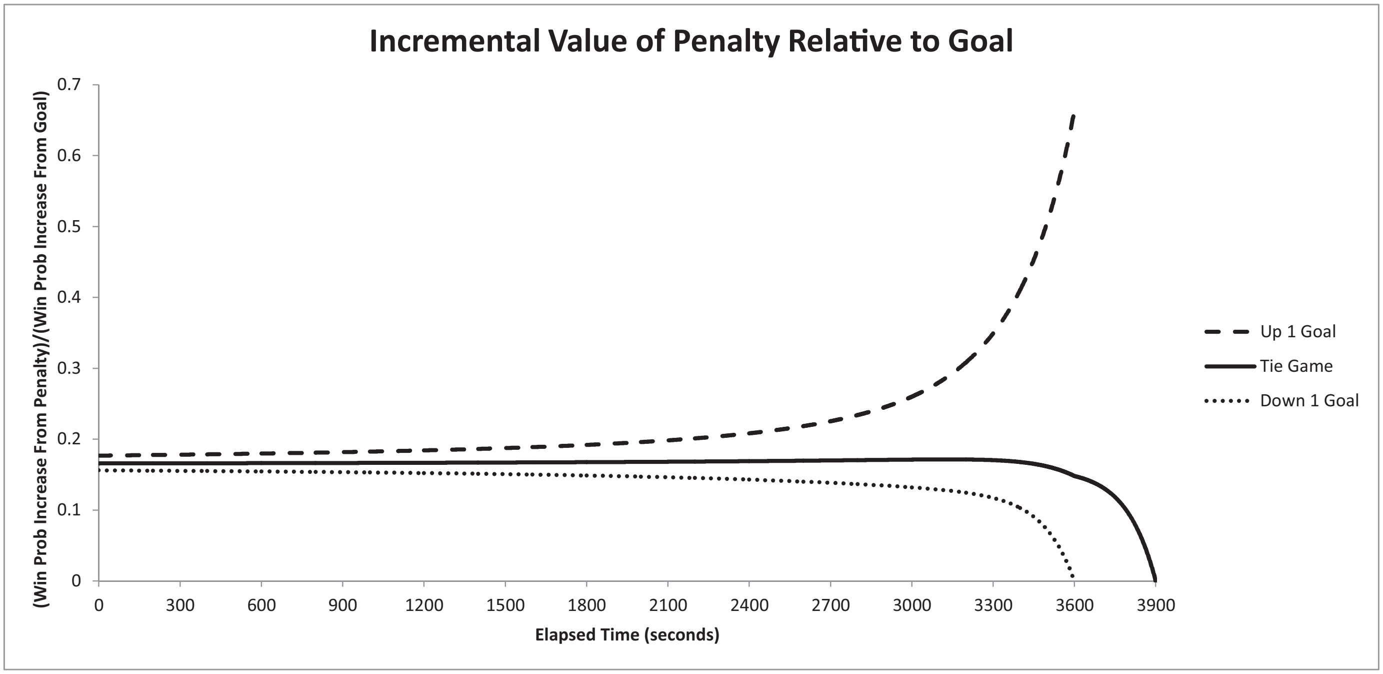

Figure 3: Percentage increase in win probability from winning a penalty versus scoring a goal.

manpower differentials at any point in the hockey game providing one understands the rules of hockey. For example, the New York penalty that occurred at 8:49 of the first period gave Dallas a man advantage (so $y = -1$ since New York is the home team), while the power play Dallas goal scored at the 9:52 of the first period ended the penalty and returned the game to even strength ($y = 0$) but gave Dallas a one goal lead ($x = -1$).

Figure 4 shows the result of mapping all of the game state information inferred from Table 3 onto the win probability graph of Figure 2; we refer to the result as the real-time win probability scoreboard. The mapping simply selects the appropriate win probability curve corresponding to the state of the game at all points in time. Specifically, the underlying state-specific curves from Figure 2 are shown as dotted gray lines in Figure 4, while the solid curve highlights the win probability associated with the actual goal/manpower differential game state at all points during the game. Note how the Dallas goal scored at 9:52 of the first period (592 seconds into the game) manifests as a vertical drop from the curve corresponding to the home team playing down one man in a tie game to the curve corresponding to even strength while trailing by a goal. This corresponds to a drop in the win probability from 0.52 to 0.36, thus the combined effect of surrendering a goal and regaining a skater dropped New York's win probability by 16 percentage points. While this example was constructed after the fact from Table 3, there is absolutely no reason why graphs like Figure 4 could not be constructed in real time using the publicly available live data feed from nhl.com.

It is important to point out that Figure 4 does not correspond to the actual probabilities that New York defeats Dallas, for the underlying model from Figure 2 was calibrated for all games over four seasons. Thus, the real-time win probability does not attempt to consider the skill levels of the two teams involved. However, while the win probabilities are themselves based on all games, the state transitions that give rise to the jumps in Figure 4 are the actual game transitions that result from the play of the two teams in the game. This is how the model creates an alternative scoreboard. Rather than keep track of a game solely on the basis of goals scored, the model keeps score in units of home team win probability based on Figure 2. The score thus changes both continuously with time and with jumps corresponding to changes in the goal and manpower differential state of the game.

7. WIN PROBABILITY ADDED

A common problem faced by managers and analysts of team sports is how to assess the marginal contribution of an individual player to team performance, and hockey is no exception. The NHL's adopted approach to this problem is via the "+/−" statistic that, whenever an even strength, shorthanded or empty net goal is scored, awards "+1" to all players on the ice for the

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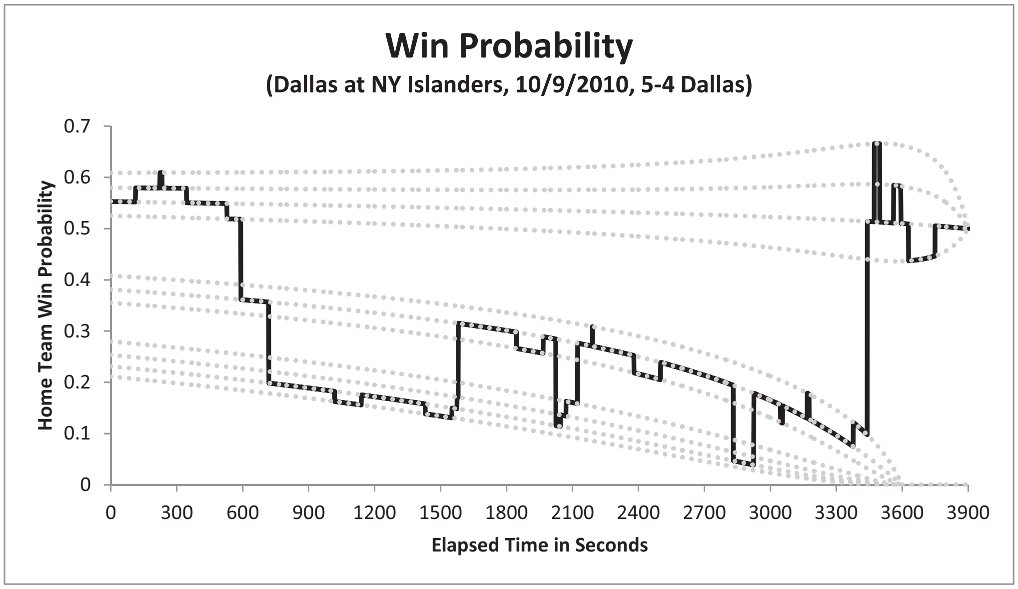

Figure 4: The win probability scoreboard for Dallas at NY Islanders based on the events in Table 3.

scoring team excluding the goalie, while penalizing the skaters on the ice for the scored-upon team by "−1" (again excluding the goalie). These +/− points are aggregated across games and seasons, and used as information to discern a player's net contribution to winning.

Various authors have attempted to estimate a hockey player's marginal contribution to net goals scored using statistical models. MacDonald (2010) and Schuckers (2011) applied regression methods to shift-level data (that is, data collected while the same collection of players is on ice) to produce "adjusted +/− statistics." Gramacy, Jensen and Taddy (2013) used logistic models to estimate the amount of "credit" individual players should receive when goals are scored. Regardless of approach, previous research has found it difficult to obtain accurate estimates of a player's marginal contribution to net goals for non-elite players.

Given that goals are relatively rare in hockey (recall that both home and away teams average fewer than three goals per game), while the overarching objective of a team is to win, we seek to score players at all moments they are on the ice during a game in terms of their contribution to the win probability scoreboard. Summing such incremental win probability contributions over time can be thought of as assessing the statistical number of games each player wins for the team. Such "win probability added" statistics have existed for some time in baseball (Tango, Lichtman and Dolphin, 2007; Winston, 2009), and are routinely updated in real-time at fangraphs.com, for example. The win probability scoreboard provides a basis for meeting this objective in hockey, as we will now show.

Excluding the goalie, let

$$\xi_i^H(t) = \begin{cases} 1 & \text{home team player } i \text{ on ice at time } t \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

and

$$n_H(t) = \sum_i \xi_i^H(t)$$
$$= \text{number home team skaters on ice at time } t. \tag{11}$$

Further, define $\omega(t)$ as the home team win probability scoreboard at elapsed time t, an example of which is the solid curve in Figure 4. With these definitions, the *win probability added* (*WPA*) for the i^{th} player on the home team equals

$$WPA_i^H = \int_0^\tau \frac{\xi_i^H(t)}{n_H(t)}\, d\omega(t), \tag{12}$$

while the analogous measure for the j^{th} player on the away team equals

$$WPA_j^A = -\int_0^\tau \frac{\xi_j^A(t)}{n_A(t)}\, d\omega(t) \tag{13}$$

where $\xi_j^A(t) = 1$ if visiting player j is on the ice at time t and otherwise equals zero, $n_A(t)$ is the number of away team skaters on the ice at time t, and τ now refers to the duration of the hockey game (including overtime if applicable). We do not attempt

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TABLE 3.
NHL game summary from http://www.nhl.com/scores/htmlreports/20102011/GS020015.HTM

VISITOR | **HOME**

Game Summary

5 — DALLAS STARS, Game 2 Away Game 2

Saturday, October 9, 2010
Attendance 13,351 at Nassau Coliseum
Start 7:16 EDT; End 9:55 EDT
Game 0015
Final

4 — NEW YORK ISLANDERS, Game 1 Home Game 1

SCORING SUMMARY

G	Per	Time	Str	Team	Goal Scorer	Assist	Assist	DAL on Ice	NYI on Ice
1	1	9:52	PP	DAL	14 J.BENN(1)	91 B.RICHARDS(2)	29 S.OTT(1)	6, 10, 14, 29, 32, 91	4, 24, 26, 39, 51
2	1	11:59	EV	DAL	3 S.ROBIDAS(1)	91 B.RICHARDS(3)	6 T.DALEY(1)	3, 6, 18, 21, 32, 91	4, 24, 26, 39, 91
3	2	6:21	PP	NYI	20 J.WISNIEWSKI(1)	93 D.WEIGHT(1)	15 P.PARENTEAU(1)	3, 14, 29, 32, 37	12, 15, 20, 26, 39, 93
4	2	13:45	EV	DAL	10 B.MORROW(2)	63 M.RIBEIRO(2)	2 N.GROSSMAN(1)	2, 3, 10, 16, 32, 63	10, 15, 20, ,26, 28, 39
5	2	15:24	PP	NYI	93 D.WEIGHT(1)	15 P.PARENTEAU(2)	20 J.WISNIEWSKI(1)	3, 32, 37, 63	12, 15, 20, 26, 39, 93
6	3	7:12	EV	DAL	10 B.MORROW(3)	63 M.RIBEIRO(3)		5, 10, 29, 32, 37, 63	16, 27, 39, 47, 57, 93
7	3	8:45	EV	NYI	57 B.COMEAU(1)	24 R.MARTINEK(1)	12 J.BAILEY(1)	6, 13, 28, 29, 32, 91	4, 12, 24, 28, 39, 57
8	3	17:21	PP	NYI	26 M.MOULSON(1)	51 F.NIELSEN(1)	93 D.WEIGHT(2)	2, 10, 28, 32, 63	7, 20, 26, 39, 51, 93
9	SO			DAL	63 M.RIBEIRO				

PENALTY SUMMARY

DALLAS STARS

#	Per	Time	Player	PIM	Penalty
1	1	1:53	29 S.OTT	2	Hooking
2	1	3:44	18 J.NEAL	2	Tripping
3	2	6:15	17 T.PETERSEN	2	Hooking
4	2	14:06	2 N.GROSSMAN	2	Interference
5	2	14:31	29 S.OTT	2	Tripping
6	3	10:56	91 B.RICHARDS	2	Holding
7	3	16:17	3 S.ROBIDAS	2	Delaying Game-Puck over glass
8	3	17:53	3 S.ROBIDAS	2	Delaying Game-Puck over glass
TOT (PN-PIM) 8-16					
Power Plays (Goals-Opp./PPTime) 1-7/11:12					

NEW YORK ISLANDERS

#	Per	Time	Player	PIM	Penalty
1	1	8:49	57 B.COMEAU	2	Boarding
2	1	16:59	4 M.EATON	2	Interference
3	2	3:51	TEAM	2	Too many men/ice - bench
4	2	10:46	93 D.WEIGHT	2	Hi-sticking
5	2	19:40	20 J.WISNIEWSKI	2	Slashing
6	3	10:47	16 J.SIM	2	Roughing
7	OT	0:29	57 B.COMEAU	2	Cross checking
TOT (PN-PIM) 7-14					
Power Plays (Goals-Opp./PPTime) 3-8/09:35					

to apply *WPA* to shootouts that occur when overtime fails to determine game winner (the +/− statistic also ignores shootouts).

The logic behind equations (12-13) is straightforward: $d\omega(t)$ is the instantaneous change in the home team win probability at time t in the game (and $-d\omega(t)$ is the instantaneous change in win probability for the away team). Presuming that all skating players are equally responsible for their team's change in win probability (a similar assumption applies to standard +/− statistics, namely, that all skating players share responsibility for goals scored or allowed), the ratio $\xi_i^H(t)/n_H(t)$ $(\xi_j^A(t)/n_A(t))$ captures home player i's (away player j's) share of this change. Cumulating these win probability changes over all time for each player in the game yields equations (12-13). Note that unlike the +/− statistics, win probability added includes skaters on power plays in addition to even strength or short-handed situations, and as the number of skaters on ice changes for either team, the "shared responsibility" for changes in win probability is divided accordingly. We implemented equations (12) and (13) by setting $d\omega(t) = \omega(t)-\omega(t-1)$ (time is measured in seconds) and replacing the integrals with sums.

To illustrate, consider again the Dallas Stars at New York Islanders game reported in Table 3 and Figure 4. Table 4 reports the +/− results for this game. Josh Bailey was the only Islander with a positive +/− statistic, while Doug Weight and James Wisniewski both have +/− statistics of −1. For the Stars, Brenden Morrow, Mike Ribeiro and Stephane Robidas each have +/− statistics equal to +2. Of note is that Morrow, Wisnewski and Weight were selected as the "three stars" of this particular

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TABLE 4.
Plus/minus statistics for Dallas at New York Islanders from http://scores.espn.go.com/nhl/boxscore?gameId=301009012.

New York Islanders				Dallas Stars			
Player	G	A	+/−	Player	G	A	+/−
J. Bailey LW	0	1	1	K. Barch RW	0	0	−1
B. Comeau LW	1	0	0	Jamie Benn LW	1	0	0
M. Eaton D	0	0	0	A. Burish RW	0	0	1
T. Gillies LW	0	0	0	T. Daley D	0	1	0
T. Hunter RW	0	0	0	L. Eriksson LW	0	0	1
M. Jurcina D	0	0	−1	M. Fistric D	0	0	−1
Z. Konopka C	0	0	0	N. Grossmann D	0	1	1
A. MacDonald D	0	0	−1	B. Morrow LW	2	0	2
R. Martinek D	0	1	0	J. Neal LW	0	0	1
M. Mottau D	0	0	−1	M. Niskanen D	0	0	1
M. Moulson LW	1	0	−2	S. Ott C	0	1	0
N. Niederreiter RW	0	0	0	T. Petersen C	0	0	0
F. Nielsen C	0	1	0	M. Ribeiro C	0	2	2
P. Parenteau RW	0	2	−1	B. Richards C	0	2	0
J. Sim RW	0	0	−1	S. Robidas D	1	0	2
J. Tavares C	0	0	−1	K. Skrastins D	0	0	1
D. Weight C	1	2	−1	B. Sutherby C	0	0	0
J. Wisniewski D	1	1	−1	T. Wandell C	0	0	0

game (http://www.nhl.com/scores/htmlreports/20102011/GS020015.HTM), which in the case of Wisnewski and Weight is perhaps surprising given their negative +/– statistics.

Table 5 reports our win probability added measures for each player in this game as calculated via equations (12-13) with $\omega(t)$ following Figure 4. Comparing to the +/– statistics, note that while Josh Bailey of the Islanders has the highest *WPA* for the Islanders (just as he had the highest +/– statistic), Doug Weight and James Wisnewski have the third and fourth highest *WPA* measures for the Islanders. This is a reassuring result, in that these two players were both recognized as stars of this game, and their positive *WPA* measures reinforce that view. Similarly, Brenden Morrow of the Stars, another three star selection, also recorded positive *WPA*. Recalling that Morrow was one of three Dallas skaters who received +/– statistics of +2, it is interesting that the other two (Mike Ribeiro and Stephane Robidas) recorded *negative WPA* measures by our calculations. This example illustrates that *WPA* and +/– are not capturing the same thing, and though admittedly anecdotal, it is reassuring that the selection of the three stars of the game is more consistent with *WPA* than with +/–.

There is another property of *WPA* evident in Table 5. Note that summing the *WPA* measures for the Islanders yields –0.05 while doing the same for Dallas gives +0.05. This is not a coincidence. Inspection of Figure 2 shows that at the start of a game (with both goal and manpower differentials of zero), the home team has a 55% probability of winning. At the end of overtime in a tie game, the home team has a 50% probability of winning. Thus, the change in home team win probability from the start of a game to the end of unresolved overtime is given by 0.50 –0.55 = –0.05 (and by symmetry, the change in away team win probability equals +0.05). If the home team wins a hockey game (whether in regulation or overtime), the win probability jumps to 1, and thus the sum of *WPA* for the home team must equal 1 –0.55 = 0.45; conversely if the home team loses, the sum of individual player *WPA* will equal –0.55. Summarizing, at the end of a game, the sum of player *WPA* for the home team equals

$$WPA^H = \sum_i WPA_i^H = \begin{cases} 0.45 & \text{home team wins} \\ -0.05 & \text{OT tie} \\ -0.55 & \text{home team loses} \end{cases} \tag{14}$$

TABLE 5.
Individual win probability added for the skaters in Dallas at NY Islanders.

Islanders	WPA	Mins	Stars	WPA	Mins
Josh Bailey	0.1097	22.15	Trevor Daley	0.0700	25.78
Trent Hunter	0.0660	17.35	Adam Burish	0.0530	12.35
Doug Weight	0.0549	20.33	Steve Ott	0.0381	16.47
James Wisniewski	0.0353	24.17	Brad Richards	0.0348	24.58
Nino Niederreiter	0.0305	11.45	Brenden Morrow	0.0292	19.97
Frans Nielsen	0.0303	18.25	Toby Petersen	0.0272	11.38
Andrew MacDonald	-0.0007	23.58	James Neal	0.0271	20.42
Zenon Konopka	-0.0031	11.38	Matt Niskanen	0.0229	17.75
Trevor Gillies	-0.0049	1.95	Karlis Skrastins	0.0168	19.60
Matt Moulson	-0.0100	21.52	Loui Eriksson	0.0091	23.58
Jon Sim	-0.0113	12.52	Jamie Benn	-0.0097	13.23
Milan Jurcina	-0.0119	18.18	Brian Sutherby	-0.0129	6.55
Pa Parenteau	-0.0373	17.08	Stephane Robidas	-0.0141	26.05
John Tavares	-0.0467	5.30	Tom Wandell	-0.0147	10.25
Radek Martinek	-0.0470	20.60	Krystofer Barch	-0.0236	6.00
Mark Eaton	-0.0575	18.82	Mike Ribeiro	-0.0256	22.20
Blake Comeau	-0.0609	20.90	Nicklas Grossman	-0.0696	19.33
Mike Mottau	-0.0884	21.10	Mark Fistric	-0.1051	12.13

while for the visiting team, we have

$$WPA^{A} = \sum_{j} WPA_{j}^{A} = \begin{cases} 0.55 & \text{away team wins} \\ 0.05 & \text{OT Tie} \\ -0.45 & \text{away team loses} \end{cases} \cdot \quad (15)$$

For a given team, over the course of an entire regular season, let $\#HW$, $\#HT$, $\#HL$, $\#AW$, $\#AT$, and $\#AL$ reflect the number of games the team wins at home, ties after overtime at home, losses at home, wins away, ties after overtime away, and losses away respectively. Noting that there are 41 home and 41 away games for each team in an 82 game NHL regular season, we can write

$$\#HL = 41 - \#HW - \#HT \quad (16)$$

and

$$\#AL = 41 - \#AW - \#AT. \quad (17)$$

Summing all player WPAs over all games yields team WPA for the entire season, and making use of equations (14–17), we see that

$$\begin{aligned} \text{Team } WPA &= 0.45 \times \#HW - 0.05 \times \#HT - 0.55 \times \#HL \\ &\quad + 0.55 \times \#AW + 0.05 \times \#AT - 0.45 \times \#AL \\ &= \#HW + \#AW - 41 + \frac{\#HT + \#AT}{2} \quad (18) \\ &= \text{Wins} - 41 + \frac{\text{Ties}}{2} \end{aligned}$$

which is wins over average adjusted for ties. An alternative expression for team WPA is given by

$$\text{Team } WPA = 0.5 \times (\text{Wins} - \text{Losses}) \quad (19)$$

which follows from equation (18) by noting that Ties = 82 – Wins – Losses. Equations (18) and (19) complete the connection from individual player WPAs for each team in each game to end-of-season records. Also, summing individual player WPAs across an entire season records each player's contribution to Team WPA.

8. CONCLUSIONS

We have developed a new Markov model for NHL hockey incorporating manpower differential in addition to goal differential into the state space. The model was calibrated based on more than 17.7 million second-by-second state observations during regulation time of every regular-season NHL game played between the fall of 2008 and spring of 2012. We showed

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that the model can provide an in-game win probability scoreboard that in turn can be used to evaluate the contributions of individual hockey players to the probability that their team wins the game. We also showed that win probability added cumulates across players and games to equal the number of wins over average adjusted for ties for any given team.

Future research opportunities include deploying our approach to estimating win probability added to all players (excluding goalies) in all games across a given season to see how the resulting player evaluation measures compare to those already computed. As with adjusted +/−, it might also prove important to adjust *WPA* to account for lineup correlation and variable opponent skill, for as formulated, our WPA metric has the same limitations as most other player-level metrics in accounting for the quality of linemates and opposing players. Our method currently does not apply to goaltenders, but surely goalie contributions to win probability matter and should be accounted for in some way. There are also some timing issues that perhaps deserve additional attention. For example, consider a penalty that is successfully killed by two different shifts. In the present approach, only the shift on ice when the penalty expires receives the jump in win probability that accompanies the favorable transition in manpower differential, even though the first unit surely deserves some credit for killing the penalty. Nonetheless, we believe that our model does suggest new ways to think about the relative gains and losses associated with goals and penalties while offering a direct method for valuing the incremental contributions of individual players to the probability their team wins hockey games.

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