An Analytic Model of the CAN Bus for Online Schedulability Test

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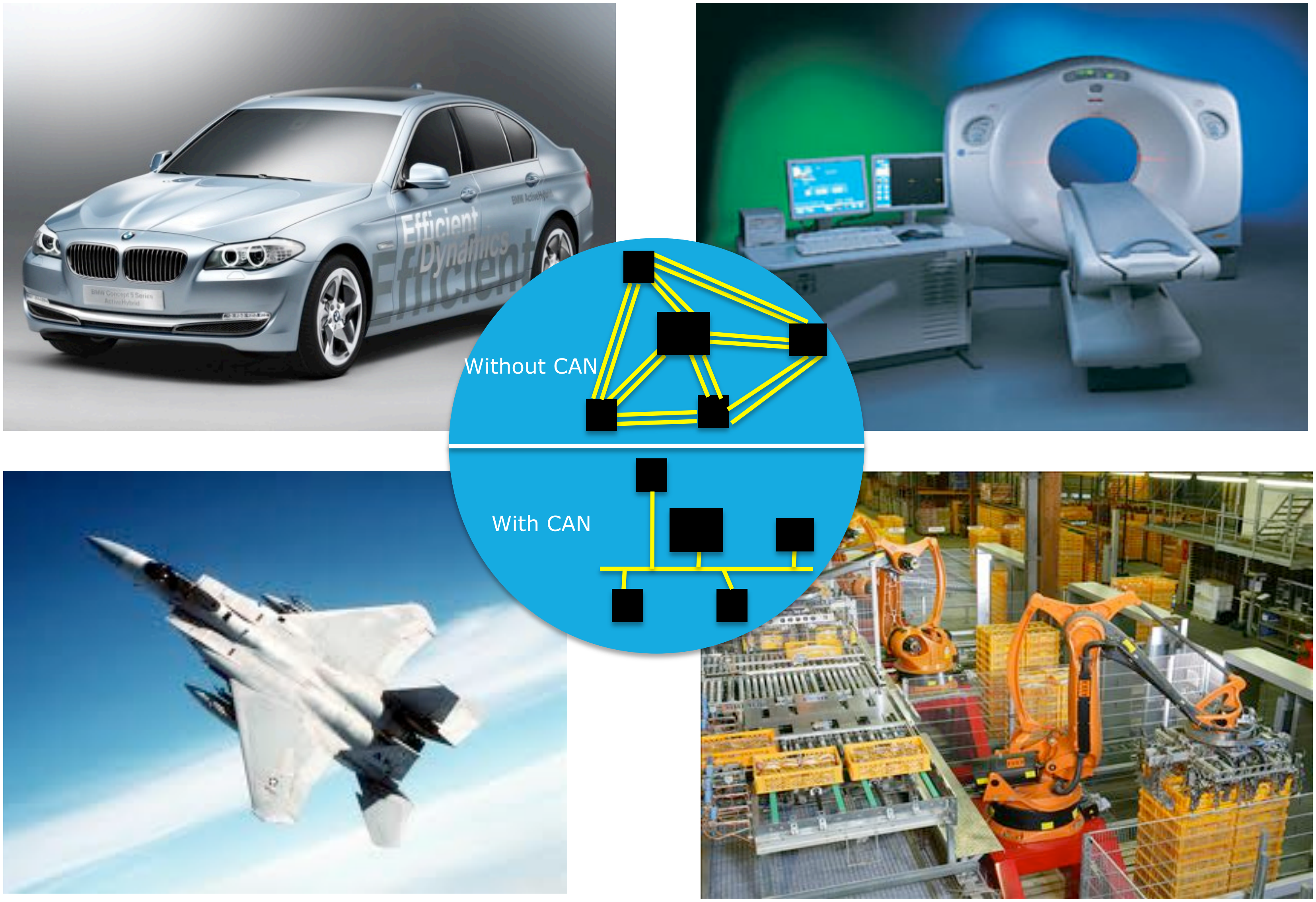
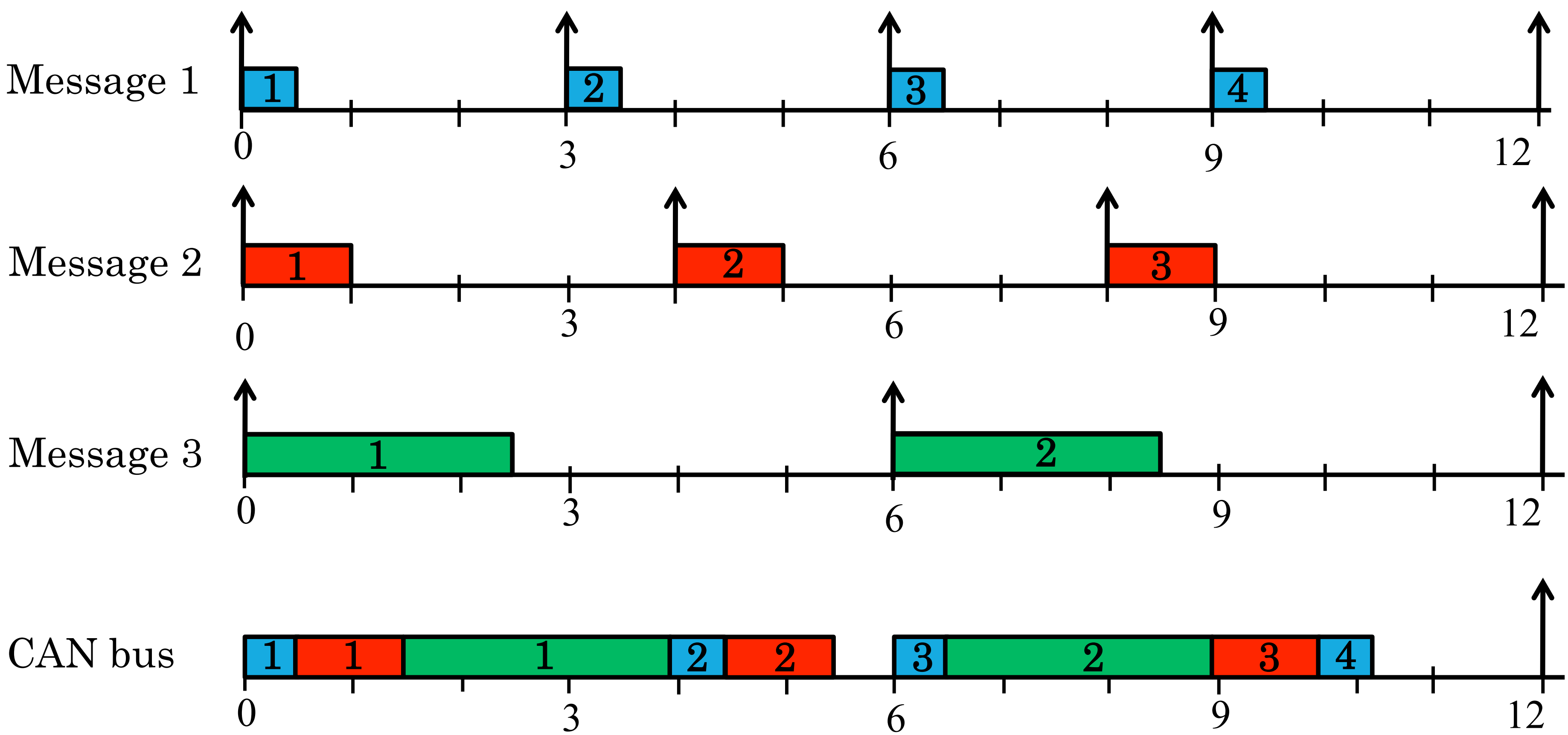
Introduction





Message 1
Message 2
Message 3
CAN bus
0
3
6
9
12

Problem Formulation

- CAN bus is particularly suitable for real-time applications
  - Each message is characterized by $\{C_n^k, T_n^k, P_n^k, E_n^k\}$
  - Existing schedulability test is performed off-line, by using nominal message characteristics

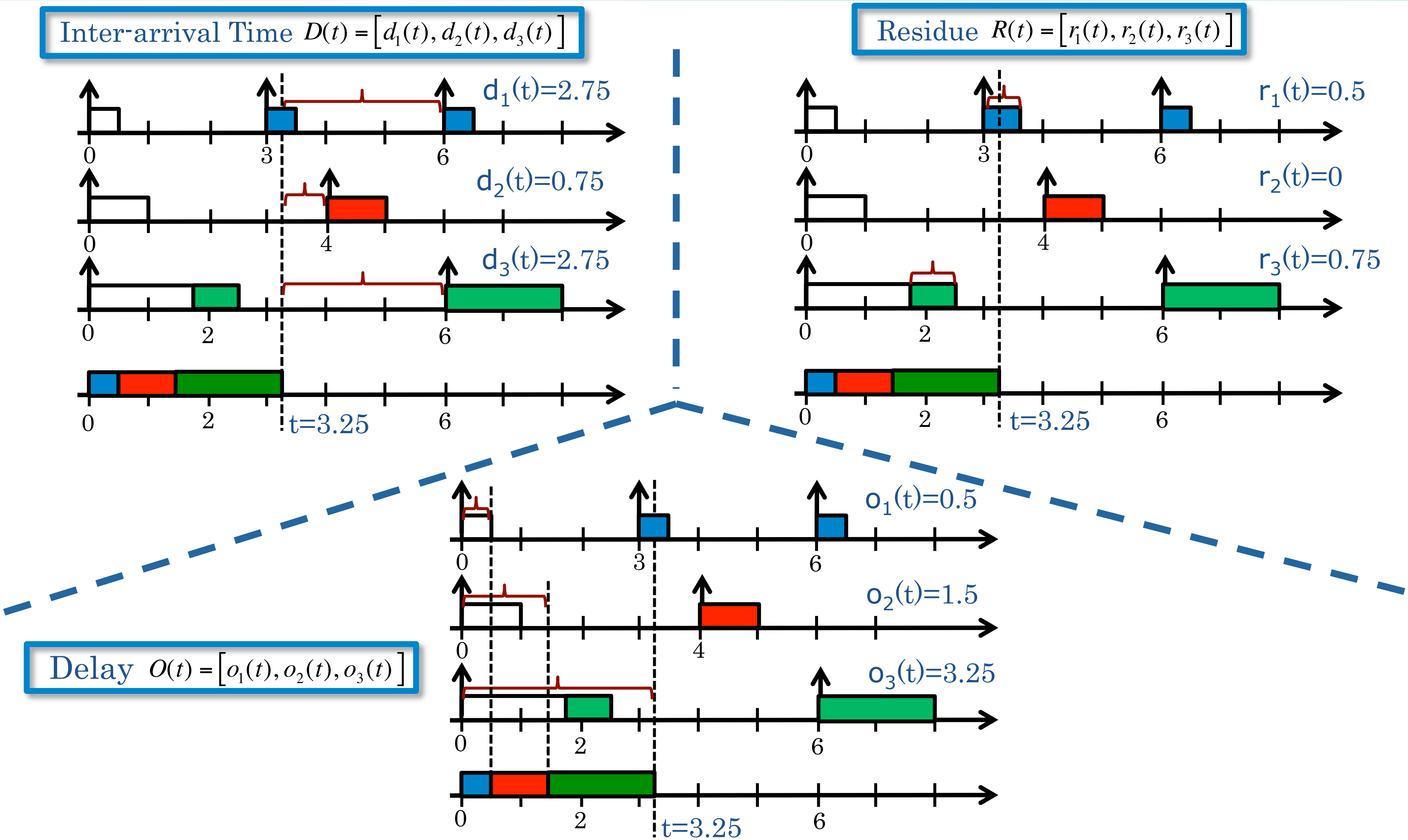
- Cyber-physical system imposes new challenges
  - Close integration of cyber and physical components
  - Operate in a dynamic environment where actual message characteristics deviate from their nominal values

- An online schedulability test over $[t_a, t_b]$ checks if all message instances are able to meet their deadlines within $[t_a, t_b]$

Analytic Timing Model

1. State Vector
2. State Evolution

State Vector Z(t)=[D(t) R(t) O(t)]



Hybrid Automaton

- A hybrid automaton $H = \{Z, Q, F, \mathrm{Dom}, \mathrm{Guard}, \mathrm{Reset}\}$ for the evolution of $Z(t)$
  - — $Q = \{q_0, \cdots, q_3\}$: modes of the CAN bus. Each mode is listed below

Mode	q_0	q_i, where $1 \le i \le 3$
Meaning	No message is being transmitted	Message i is being transmitted

  - — $F(q_i, Z)$: the evolution dynamics of $Z(t)$ in any mode q_i

Flow Map	$\dot{D}(t)$	$\dot{R}(t)$	$\dot{O}(t)$
$F(q_2, Z)$	$\dot{D}(t) = [-1, -1, -1]$	$\dot{R}(t) = [0, -1, 0]$	$\dot{O}(t) = [\mathrm{sgn}(r_1(t)), \mathrm{sgn}(r_2(t)), \mathrm{sgn}(r_3(t))]$

  - — $\mathrm{DOM}(q_i)$: a set of $Z(t)$ that evolve continuously in any mode q_i

Domain	q_0	q_i, where $1 \le i \le 3$
$\mathrm{Dom}(q_i)$	$\left\{Z(t) \middle\| \min_{1\le i\le 3}\{D(t)\} > 0, \mathrm{Card}(G(t)) = 0\right\}$	$\left\{Z(t) \middle\| \min_{1\le i\le 3}\{D(t)\} > 0, 0 < r_i \le C_i^k\right\}$

Hybrid Automaton

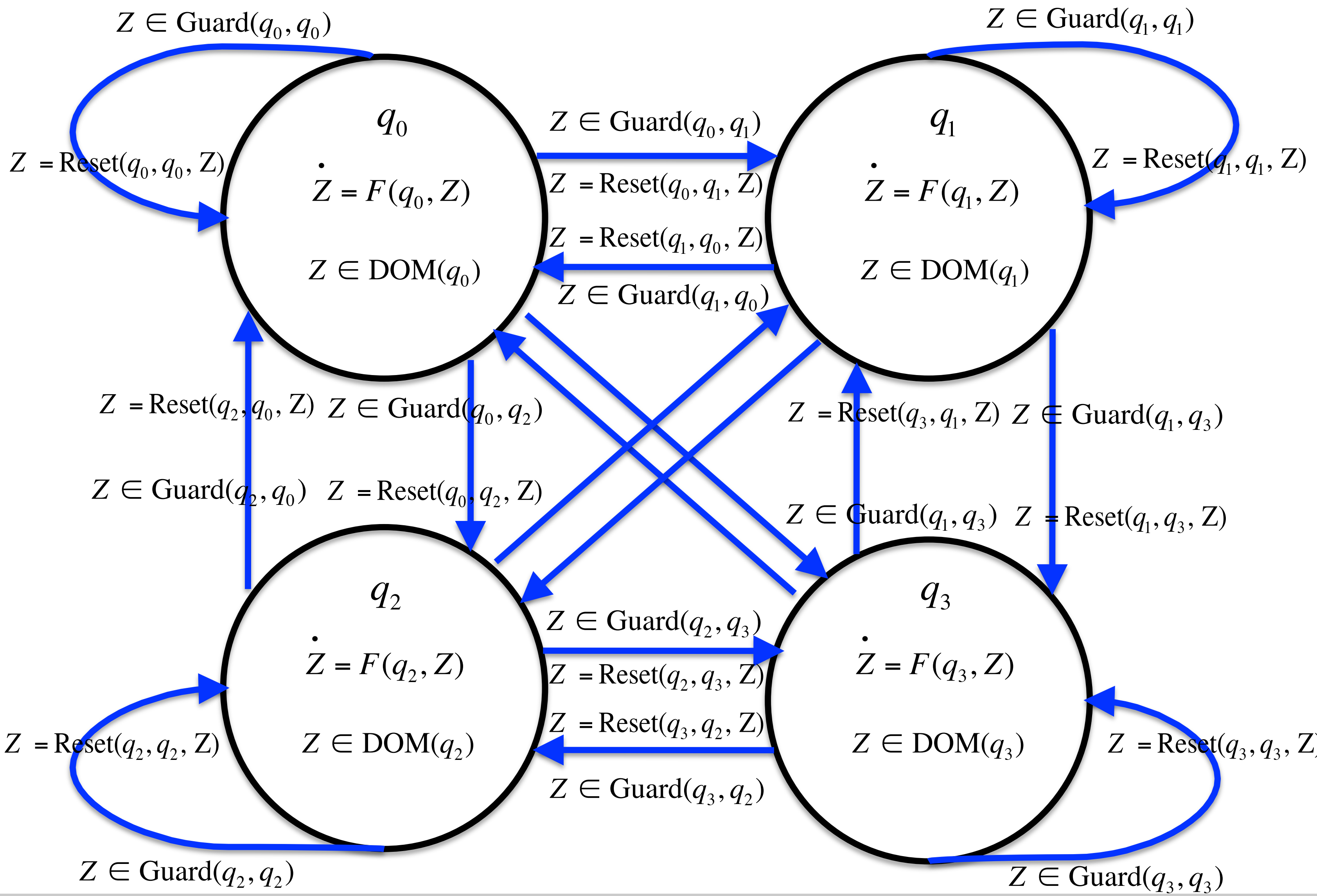
- A hybrid automaton $H = \{Z, Q, F, \text{Dom}, \text{Guard}, \text{Reset}\}$ for the evolution of $Z(t)$
  - $\text{Guard}(q_i, q_j)$: a set of $Z(t)$ that trigger $q_i \Rightarrow q_j$

Jump Conditions	$i = j$	$i = 0,\ 1 \le j \le 3$	$1 \le i \le 3,\ j = 0$	$1 \le i, j \le 3,\ i \ne j$
$\text{Guard}(q_i, q_j)$	$\left\{\min\limits_{1 \le n \le 3}\{D(t)\} = 0\right\}$	$\left\{\text{Card}(G(t)) > 0,\ j = \arg\min\limits_{n \in G(t)} P_n^k\right\}$	$\{r_i(t) = 0, \text{Card}(G(t)) = 0\}$	$\left\{r_i(t) = 0,\ j = \arg\min\limits_{n \in G(t)} P_n^k\right\}$

  - $\text{Reset}(q_i, q_j, Z)$: values to which $Z(t)$ is reset during $q_i \Rightarrow q_j$

Reset	$D(t)$	$R(t)$	$O(t)$
$\text{Reset}(q_i, q_j, Z) = [D(t)\ R(t)\ O(t)]$	$D(t) = \begin{bmatrix} d_1(t^-) + T_1^k \,\text{sgn}(d_1(t^-)) \\ d_2(t^-) + T_2^k \,\text{sgn}(d_2(t^-)) \\ d_3(t^-) + T_3^k \,\text{sgn}(d_3(t^-)) \end{bmatrix}$	$R(t) = \begin{bmatrix} r_1(t^-) + C_1^k \,\text{sgn}(d_1(t^-)) \\ r_2(t^-) + C_2^k \,\text{sgn}(d_2(t^-)) \\ r_3(t^-) + C_3^k \,\text{sgn}(d_3(t^-)) \end{bmatrix}$	$O(t) = \begin{bmatrix} o_1(t^-)\text{sgn}(d_1(t^-)) \\ o_2(t^-)\text{sgn}(d_2(t^-)) \\ o_3(t^-)\text{sgn}(d_3(t^-)) \end{bmatrix}$

Hybrid Automaton



Hybrid Automaton

- The hybrid automaton $H=\{Z, Q, F, \mathrm{Dom}, \mathrm{Guard}, \mathrm{Reset}\}$ is non-blocking and deterministic

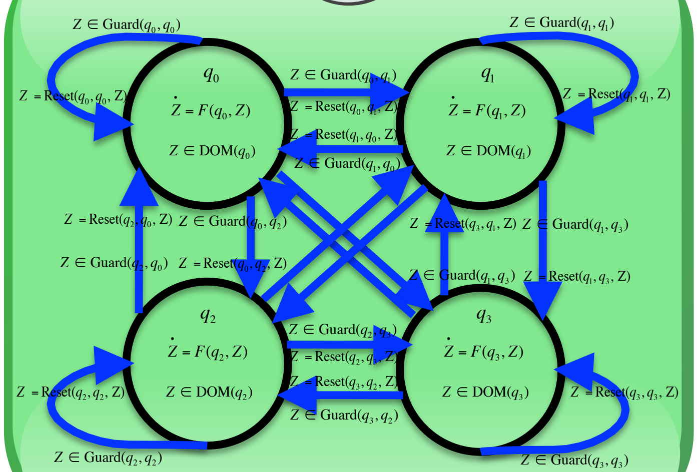
- At any time t_a, given the initial condition $Z(t_a)$ and $Q(t_a)$, and message characteristics $\{C_n^k, T_n^k, P_n^k, E_n^k\}_{n=1}^{N}$ within any time interval $[t_a, t_b]$
  - $\left[Z(t),\ Q(t)\right]=H\left(\left[Z(t_a), Q(t_a)\right],\ \{C_n^k, T_n^k, P_n^k,\ E_n^k\}_{n=1}^{N}\right)$ for $t\in\left[t_a, t_b\right]$
  - The trajectory $\left[Z(t),\ Q(t)\right]$ for $t\in\left[t_a,\ t_b\right]$ is unique

Online Schedulability Test

- Consider a set of messages $\Gamma = \{\tau_1, \cdots, \tau_N\}$ being simultaneously transmitted on the CAN bus within any time interval $[t_a, t_b]$
- $\Gamma = \{\tau_1, \cdots, \tau_N\}$ is schedulable within any time interval $[t_a, t_b]$ if and only if each message $\tau_n \in \Gamma$ is schedulable within $[t_a, t_b]$
- A message $\tau_n \in \Gamma$ is schedulable within $[t_a, t_b]$ if and only if for all $t \in [t_a, t_b]$ such that $d_n(t) = 0$, we have $r_n(t) = 0$

Online Schedulability Test

1



$[Z(t), Q(t)]$

for $t \in [t_a, t_b]$

2

for $n = 1$ **to** N **do**
  while $t < t_b$ **do**
    if $d_n(t) == 0$ **and** $r_n(t) == 0$ **then**
      $\mathrm{Sch}_n = [\mathrm{Sch}_n \;\; 0]$;
    else if $d_n(t) == 0$ **and** $r_n(t) > 0$ **then**
      $\mathrm{Sch}_n = [\mathrm{Sch}_n \;\; 1]$;

$[Z(t_a), Q(t_a)] \quad \{C_n^k, T_n^k, P_n^k, E_n^k\}_{n=1}^{N}$

$$\begin{Bmatrix} \mathrm{Sch}_1 \\ \vdots \\ \mathrm{Sch}_N \end{Bmatrix}$$

Simulation

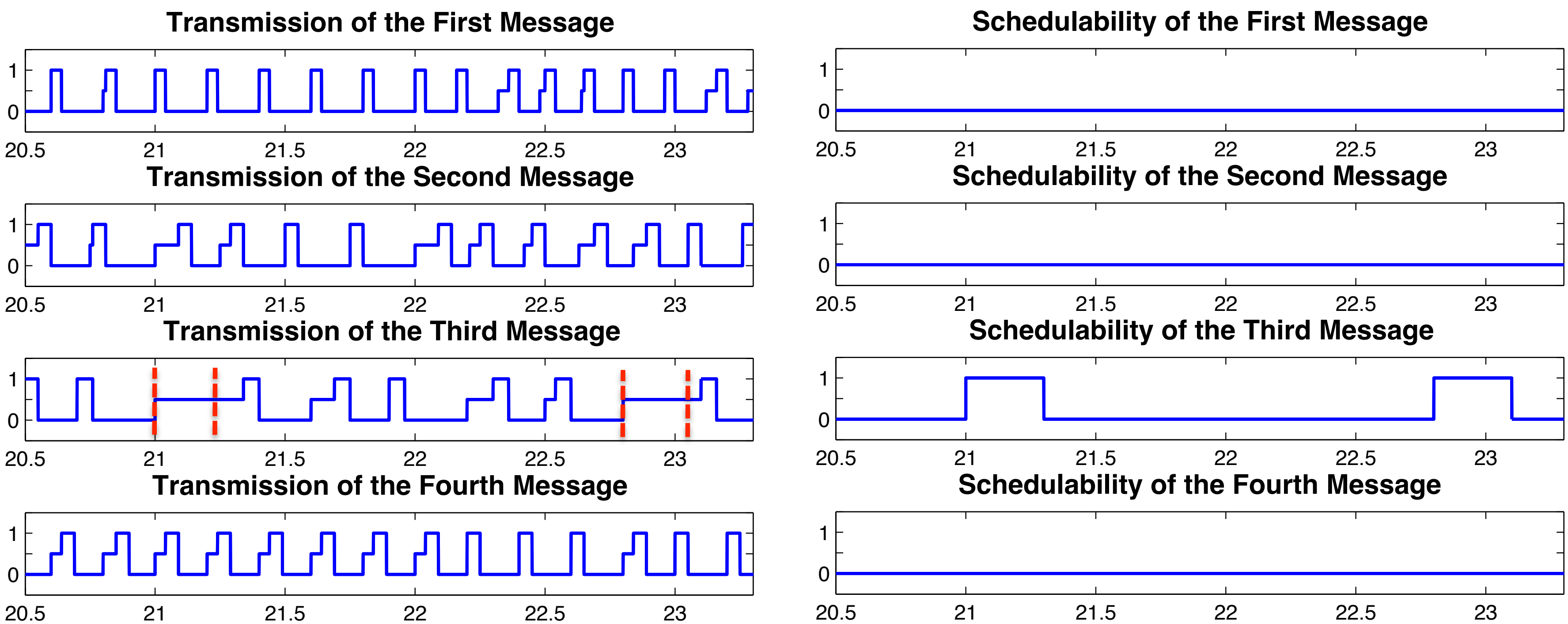
- A set of messages with the following nominal characteristics

Description	C_n^k	T_n^k	E_n^k	P_n^k
Message 1	40 ms	200 ms	100 ms	100
Message 2	50 ms	250 ms	175 ms	221
Message 3	60 ms	300 ms	200 ms	1000

- Within the time interval [20.5, 23.5]s, the communication requirements changes as follows

Message 4	$C_4^k = 50$ ms	$T_4^k = 200$ ms	$E_4^k = 200$ ms	$P_n^k = 150$
Period Adjust	T_1^k : 200 ms $\rightarrow$ 160 ms		T_2^k : 250 ms $\rightarrow$ 210 ms	

Simulation



- 0: Transmission finishes
- 0.5: Transmission is blocked
- 1: Transmission is continuing

- 0: Message is schedulable
- 1: Message is un-schedulable

Thank You !