Breaking. A stick of length 4cm is broken in the random position (let it be distributed uniformly). Find the CDF, PDF, and the mean of the length of the longest piece.

Strontium Let T be an exponentially distributed random variable with parameter λ.
We call half-life the time h such that $P(T > h) = \frac{1}{2}$. Express h in terms of λ.
Strontium 90 is a dangerous radioactive isotope of strontium, which is released after a nuclear explosion. An atom of strontium 90 remains radioactive for a random time which follows an exponential distribution, after which it decays. Its half-life is approximatively 28 years.

- Find the value of λ.
- Find the probability that a given atom of strontium 90 has not decayed after 50 years.
- Calculate the number of years to wait so that 99$\%$ of the strontium 90 produced during a nuclear reaction disappears.

Portfolio. An investor puts $2,000 into a deposit account with a fixed rate of return of 10% per year. A second sum of $1,000 is invested in a fund with an expected rate of return of 16% and a standard deviation of 8% per year.

- Find the expected value of the total amount of money this investor will have after a year.
- Find the standard deviation of the total amount after a year.
- If the fund return is distributed normally, what is the probability of a loss?

Plane. The Aeroflot flight to Saint Petersburg on an Airbus has 380 seats. Suppose that any customer who booked this flight will cancel with probability $1 - p = 0.1$, independently of other customers. Suppose that the company sells a fixed number N of tickets, with $N \geq 380$. Let X be the number of customers present the day of the flight.

- What is the distribution of X?
- Using the central limit theorem, approximate the probability $P(X \geq n)$: plug the mean and variance from the original distribution into normal.
- What is the largest number of bookings the company can accept if it does not want to refuse customers with probability more than 0.05.
- Suppose the company chooses N equal to the value calculated in the previous question. What is the probability that at least 35 seats are left unoccupied?

Waste. A company manufactures metal poles. Suppose the length of a pole is a random variable X, with mean μ_X and probability density function $f_X(x)$. Poles are cut to obtain an exact length L. If the initial length of the pole is less than L, the entire pole is lost.

If it is greater than L, the pole will be cut down to L, and the section left over is lost. We are interested in the random variable Y, defined as the length of each piece lost.

- Sketch the graph of the function g that maps the pole length x to the lost length y, and so derive $\mu_Y = \mathbb{E}(Y)$ as a function of $f_X(x)$ and μ_X.
- Suppose that X follows a normal distribution with mean μ_X and variance σ_X^2. Show that there exists a value μ^* of μ_X that minimizes μ_Y.
- Let $L = 2m$ and $\sigma_X = 0.02m$. What is the value of μ_X that minimizes the amount of lost material?