

A few problems to practice discrete probability

Ants. Three ants are sitting at the three corners of an equilateral triangle. Each ant randomly picks a direction and starts to move along the edge of the triangle. What is the probability that none of the ants collide?

Passports. Russian passport number can consist of any 10 digits. What is the probability that a randomly chosen Russian passport number starts with 4508 and contains 53?

Coins. I have 9 fair coins and one phony coin with heads on both sides. I picked one coin out of these 10 at random, and flipped it 3 times. It landed with heads up all the 3 times.

- What is the probability that this coin is the phony one?
- What is the probability that this coin will show heads after the next flip?

Family planning. In a country where everyone wants a boy, each family continues having babies till they have a boy. After some time, what is the proportion of boys to girls in the country? (Assuming probability of having a boy or a girl is the same).

Hitchhiker. The probability of at least one car passing a certain road intersection in a 20-minute window is 0.9. What is the probability of at least one car passing the intersection in a 5-minute window, assuming a constant probability throughout?

Sociability. In a certain social network, probability that a randomly chosen user has k subscribers is inversely proportional to $k(k + 1)$. Each user has at least one subscriber.

- Express PMF and CDF for the distribution of the number of subscribers.
- Let us call *celebrities* the users that have more subscribers than 99% of all users. How many subscribers do you need in order to become a celebrity?
- Try to find the expected number of subscribers per person in this network.

King of the hill. Consider a game of “ladder climbing”. There are 5 levels in the game, level 1 is the lowest (bottom) and level 5 is the highest (top). A player starts at the bottom. Each time, a fair coin is tossed. If it turns up heads, the player moves up one rank. If tails, the player moves down to the very bottom. Once at the top level, the player moves to the very bottom if a tail turns up, and stays at the top if head turns up. How much time (on average) does the player spend on each level?

Snail. A cube hangs on a chain attached to one corner (see the picture). A pet snail has got out of the hole in the bottom corner and is wandering from one corner to another along the edges (without treading on faces), one edge per minute. If the snail picks each next edge randomly, then how long (on average) will it wander before reaching the chain and escaping?



Credit scoring. A new client applies to a bank for a consumer loan. She has a recent application in another bank, and she has repaid a mortgage loan two years ago. We want to estimate the probability that she is a “bad” client (she won’t repay the loan). We believe that recent applications and repaid loans are independent conditionally on the client being bad/good. We know that:

- average share of bad clients is 5%;
- 80% of bad clients have recent applications in other banks, but only 30% of good clients have such applications;
- 10% of good clients have a mortgage repaid, but only 3% of bad clients have it.

So what is the probability that she is bad?