ASSIGNMENT 4
Probability and Statistics

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Question 1:

The probability distribution of X, the number of imperfections per 10 meters of a synthetic fabric in continuous rolls of uniform width, is given

x	0	1	2	3	4
$p(x)$	0.41	0.37	0.16	0.05	0.01

Find the average number of imperfections per 10 meters of this fabric. (Try functions sum( ), weighted.mean( ), c(a %*% b) to find expected value/ mean

OUTPUT:

```
> x <- c(0, 1, 2, 3, 4)
> px <- c(0.41, 0.37, 0.16, 0.05, 0.01)
> expval <- sum(x * px)
> #OR
> expval <- weighted.mean(x, px)
> #OR
> expval <- c(x %*% px)
> cat("avg value: ", expval)
avg value:  0.88
```

Question 2:

The time T, in days, required for the completion of a contracted project is a random variable with probability density function f(t) = 0.1 e(-0.1t) for t > 0 and 0 otherwise. Find the expected value of T. Use function integrate( ) to find the expected value of continuous random variable T.

OUTPUT:

```
> f <- function(t){
+   t*0.1*exp(-0.1*t)
+ }
>
> expval <- integrate(f, lower = 0, upper = Inf)
> print(expval$value)
[1] 10
```

Question 3:

A bookstore purchases three copies of a book at $6.00 each and sells them for $12.00 each. Unsold copies are returned for $2.00 each. Let X = {number of copies sold} and Y = {net revenue}. If the probability mass function of X is

x	0	1	2	3
$p(x)$	0.1	0.2	0.2	0.5

Find the expected value of Y.

OUTPUT:

```
> x <- c(0, 1, 2, 3)
> px <- c(0.1, 0.2, 0.2, 0.5)
> y <- 12*x - 18 + (3-x)*2
>
> expval <- sum(y* px)
> print(expval)
[1] 9
```

Question 4:

Find the first and second moments about the origin of the random variable X with probability density function f(x) = 0.5e -lxl , 1 < x < 10 and 0 otherwise. Further use the results to find Mean and Variance. (kth moment = E(Xk), Mean = first moment and Variance = second moment – Mean2 .

OUTPUT:

```
> f1 <- function(x){
+   x * 0.5 * exp(-abs(x))
+ }
>
> m1 <- integrate(f1, lower = 1, upper = 10)
> cat("M1: ", m1$value)
M1:  0.3676297>
> f2 <- function(x){
+   (x^2) * 0.5 * exp(-abs(x))
+ }
>
> m2 <- integrate(f2, lower = 1, upper = 10)
> cat("M2: ", m2$value)
M2:  0.9169292> cat("mean:", m1$value)
mean: 0.3676297> cat("variance:", m2$value - (m1$value ^ 2))
variance: 0.7817776
```

Question 5:

Let X be a geometric random variable with probability

$$f(x) = \frac{3}{4}\left(\frac{1}{4}\right)^{x-1}, x = 1,2,3, \ldots$$

Write a function to find the probability distribution of the random variable Y = X2 and find probability of Y for X = 3. Further, use it to find the expected value and variance of Y for X = 1,2,3,4,5.

OUTPUT:

```
> x <- c(1, 2, 3, 4, 5)
> y <- x^2
> fy <- function(y){
+     (3/4) * ((1/4) ^ (sqrt(y)-1))
+ }
> py <- fy(y)
> m1 <- sum(y * py)
> m2 <- sum(y*y*py)
> var <- m2 - (m1^2)
> print(m1)
[1] 2.182617
> print(var)
[1] 7.614112
```