Number System and Conversion

Contents

1. Introduction

1. **Decimal Number** Base = 2, means it has 2 digits: 0, 1
2. **Binary Number** Base = 10, means it has 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
3. **Hexadecimal Number** Base = 16, means it has 16 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

- Binary Numbers is what used by Computers to represent data, not easy to read and write.
- Decimal Numbers is what we are comfortable with but computers do not represent data in Decimal
- Hexadecimal Numbers is the middle ground we can easily convert to and from Binary, easier to read and write then Binary

Relation Between Decimal Hexadecimal and Binary

Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Hex	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Decimal	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111

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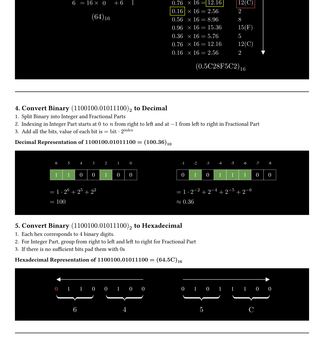
2. Convert Decimal 100.36 to Binary

1. Split Decimal into Integer (100) and Fractional (0.36) parts
2. Repeatedly Divide 100 by 2 until its 0 and read the remainder from bottom to top
3. Multiply 0.36 by 2 and extract the Integer Part from the Product
4. Repeat it with Fractional Part of Product until its 0 or upto few steps and read extracted integer part from top to bottom

Binary Representation of 100.36 = $(1100100.01011100)_2$



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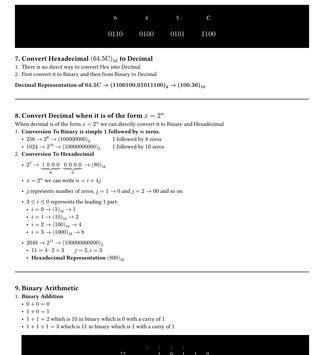
3. Convert Decimal 100.36 to Hexadecimal

1. Same as **Decimal To Binary** just use Hexadecimal's Base = 16

Hexadecimal Representation of 100.36 = $(64.5C28F5C2)_{16}$



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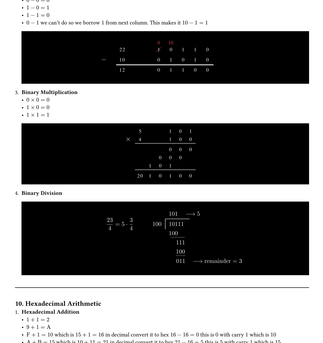
4. Convert Binary $(1100100.01011100)_2$ to Decimal

1. Split Binary into Integer and Fractional Parts
2. Indexing in Integer Part starts at 0 to n from right to left and at −1 from left to right in Fractional Part
3. Add all the bits, value of each bit is = bit · 2^{index}

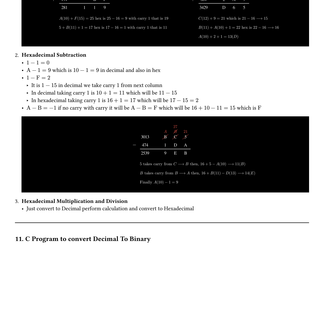
Decimal Representation of 1100100.01011100 = $(100.36)_{10}$



5. Convert Binary $(1100100.01011100)_2$ to Hexadecimal

1. Each hex corresponds to 4 binary digits.
2. For Integer Part, group from right to left and left to right for Fractional Part
3. If there is no sufficient bits pad them with 0s

Hexadecimal Representation of 1100100.01011100 = $(64.5C)_{16}$



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6. Convert Hexadecimal $(64.5C)_{16}$ to Binary

1. Expand each hex digit to its corresponding 4 bit binary

Binary Representation of 64.5C = $(1100100.01011100)_2$

7. Convert Hexadecimal $(64.5C)_{16}$ to Decimal

1. There is no direct way to convert Hex into Decimal
2. First convert it to Binary and then from Binary to Decimal

Decimal Representation of 64.5C → $(1100100.01011100)_2 \rightarrow (100.36)_{10}$

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8. Convert Decimal when it is of the form $x = 2^n$

When decimal is of the form $x = 2^n$ we can directly convert it to Binary and Hexadecimal

1. **Conversion To Binary is simple 1 followed by n zeroes.**
   - $256 = 2^8 \rightarrow (100000000)_2$ 1 followed by 8 zeros
   - $1024 = 2^{10} \rightarrow (10000000000)_2$ 1 followed by 10 zeros
2. **Conversion To Hexadecimal**
   - $2^7 \rightarrow \underbrace{1\,0\,0\,0}_{8}\ \underbrace{0\,0\,0\,0}_{0} \rightarrow (80)_{16}$
   - $x = 2^n$ we can write $n = i + 4j$
   - j represents number of zeros, $j = 1 \rightarrow 0$ and $j = 2 \rightarrow 00$ and so on
   - $3 \leq i \leq 0$ represents the leading 1 part:
     - $i = 0 \rightarrow (1)_2 \rightarrow 2$
     - $i = 1 \rightarrow (10)_2 \rightarrow 2$
     - $i = 2 \rightarrow (100)_2 \rightarrow 4$
     - $i = 3 \rightarrow (1000)_2 \rightarrow 8$
   - $2048 = 2^{11} \rightarrow (10000000000)_2$
     - $11 = 4 \cdot 2 + 3 \quad j = 2, i = 3$
     - **Hexadecimal Representation** $(800)_{16}$

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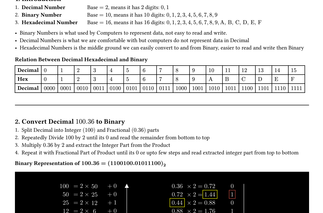
9. Binary Arithmetic

1. **Binary Addition**
   - $0 + 0 = 0$
   - $1 + 0 = 1$
   - $1 + 1 = 2$ which is 10 in binary which is 0 with a carry of 1
   - $1 + 1 + 1 = 3$ which is 11 in binary which is 1 with a carry of 1

2. **Binary Subtraction**
   - $0 - 0 = 0$
   - $1 - 0 = 1$
   - $1 - 1 = 0$
   - $0 - 1$ we can't do so we borrow 1 from next column. This makes it $10 - 1 = 1$

3. **Binary Multiplication**
   - $0 \times 0 = 0$
   - $1 \times 0 = 0$
   - $1 \times 1 = 1$



4. **Binary Division**

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10. Hexadecimal Arithmetic

1. **Hexadecimal Addition**
   - $1 + 1 = 2$
   - $9 + 1 = A$
   - $F + 1 = 10$ which is $15 + 1 = 16$ in decimal convert it to hex $16 - 16 = 0$ this is 0 with carry 1 which is 10
   - $A + B = 15$ which is $10 + 11 = 21$ in decimal convert it to hex $21 - 16 = 5$ this is 5 with carry 1 which is 15
   - $F + F = 1E$ which is $15 + 15 = 30$ in decimal convert it to hex $30 - 16 = 14$ this is 14(E) with carry 1 which is 1E

2. **Hexadecimal Subtraction**
   - $1 - 1 = 0$
   - $A - 1 = 9$ which is $10 - 1 = 9$ in decimal and also in hex
   - $1 - F = 2$
     - It is $1 - 15$ in decimal we take carry 1 from next column
       - In decimal taking carry 1 is $10 + 1 = 11$ which will be $11 - 15$
       - In hexadecimal taking carry 1 is $16 + 1 = 17$ which will be $17 - 15 = 2$
   - $A - B = -1$ if no carry with carry it will be $A - B = F$ which will be $16 + 10 - 11 = 15$ which is F

3. **Hexadecimal Multiplication and Division**
   - Just convert to Decimal perform calculation and convert to Hexadecimal

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11. C Program to convert Decimal To Binary