

Building a more nuanced theory of the fractions-algebra connection

Insights from math education research

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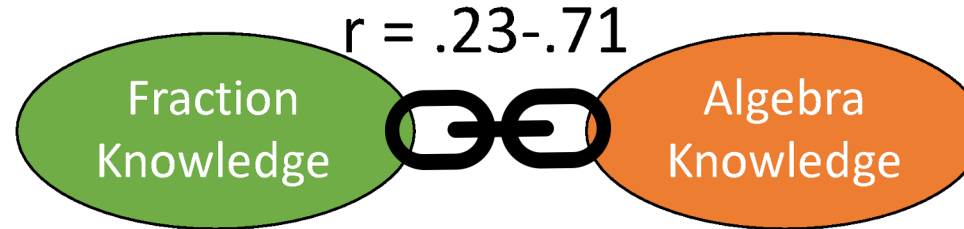


Why?

Which aspects of fractions and which aspects of algebra?

Is it a unidirectional causal relation?

Converging Evidence with Different Methods



Psychology

Often focus on quantitatively modeling relations and isolating causal impact.

Usually use brief, highly standardized measures.

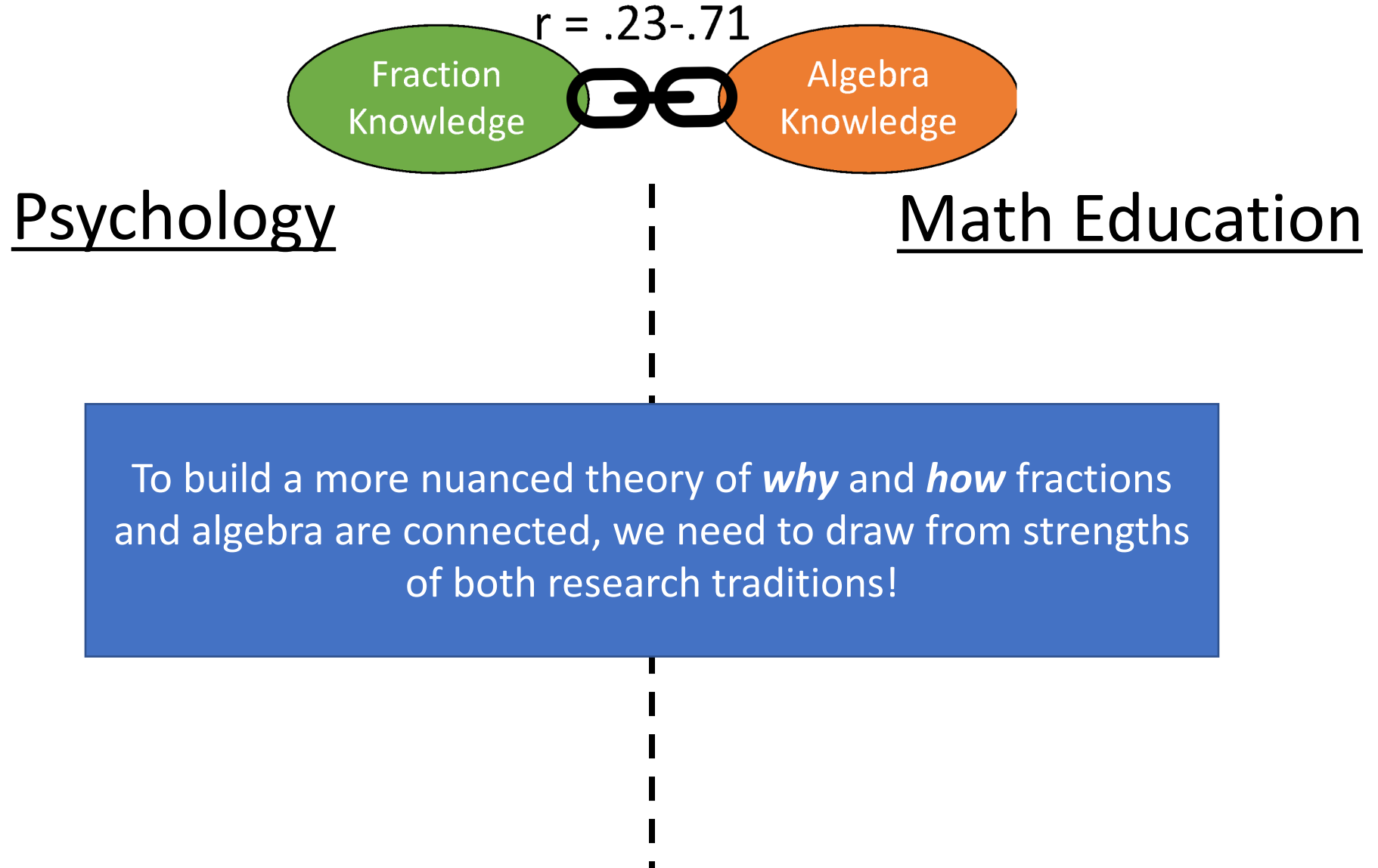
(e.g., Barbieri et al., 2021; Booth et al., 2014; Hurst & Cordes, 2018; Liang et al., 2018; Powell et al., 2019; Siegler et al., 2012)

Math Education

Deep analysis of the connections between these two mathematical constructs (e.g., Empson et al., 2011)

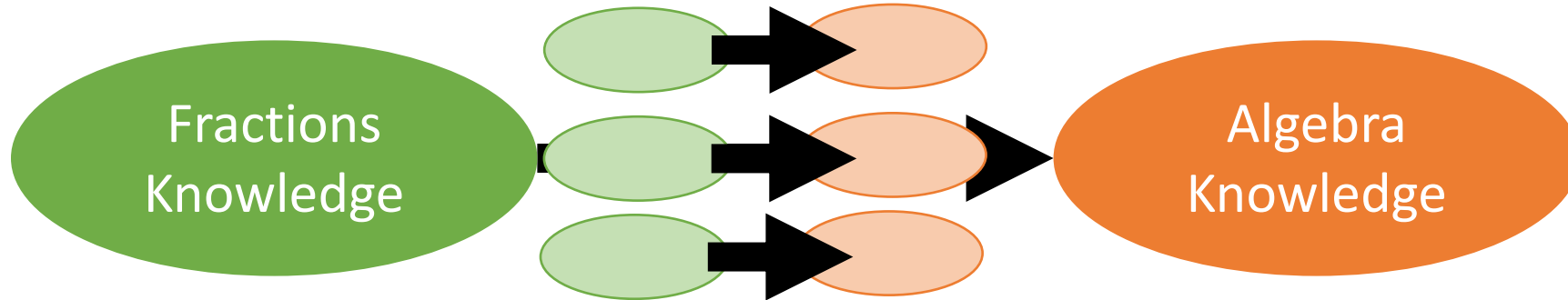
In-depth investigation of students' thinking shows that fraction *reasoning* is related to algebraic **thinking** (e.g., Hackenberg, 2013; Hackenberg & Lee, 2015; Tunc-Pekkan, 2008)

Converging Evidence with Different Methods

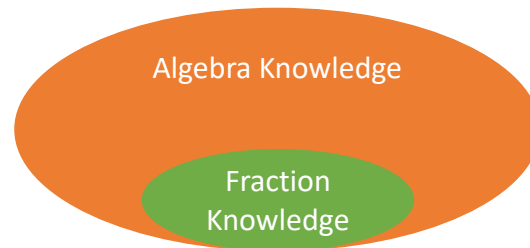


What can psychologists learn from math education research on the fractions-algebra connection?

1. More comprehensively measure *which aspects* of fractions knowledge support *which aspects* of algebra



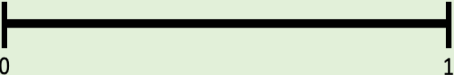
2. Consider the boundaries of fractions and algebra knowledge. Are fractions part of algebra?



Lesson 1: Which *aspects* of fractions knowledge support *which aspects* of algebra?

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Measuring fractions knowledge

$\frac{7}{8} \quad \frac{3}{5}$	$\frac{2}{3} + \frac{5}{6}$	There are 3 sandwiches to share among 5 people. How much of a sandwich does each person get?
Where is $\frac{3}{12}$? 	$\frac{1}{4} \div \frac{1}{3}$	
Magnitude	Arithmetic	Draw a picture of $\frac{1}{3}$ of $\frac{1}{7}$ of this cake. How much is that piece of the whole cake? How do you know?

Booth et al., 2014; 2021;
Siegler et al., 2012; etc.

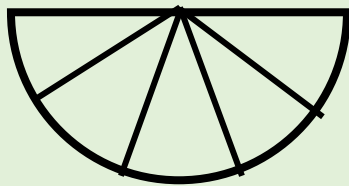
Hurst & Cordes, 2018;
Liang et al., 2019; etc.

Hackenberg, 2013;
Norton & Hackenberg, 2010;
Thompson & Saldanha, 2003; etc.

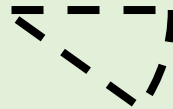
Measuring fraction thinking: Example 1

Disembedding & other fractions “operations” support reasoning about & symbolizing algebraic relations (Hackenberg, 2013)

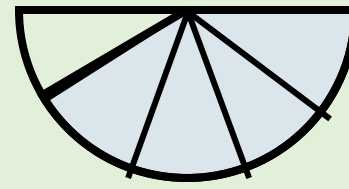
*Your piece of pie is $\frac{4}{5}$ as big as the piece shown below.
Draw your piece of pie.*



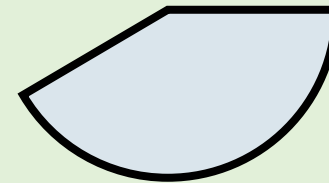
Partition



Disembed



Iterate

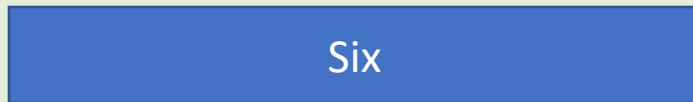


Disembed

Measuring fraction thinking: Example 2

Coordinating 3 levels of multiplicative units supports algebraic equation writing (Hackenberg & Lee, 2015)

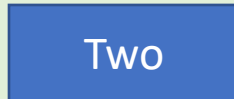
(3 levels)



Six

Six contains three 2's, which each contain two 1's

(2 levels)



Two

Each Two contains two 1's

(1 level)



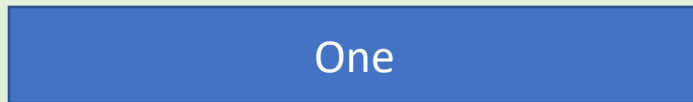
1

Simplest unit

Measuring fraction thinking: Example 2

Coordinating 3 levels of multiplicative units supports algebraic equation writing (Hackenberg & Lee, 2015)

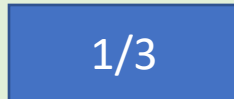
(3 levels)



One

One contains three $\frac{1}{3}$'s, which each contain two $\frac{1}{6}$'s

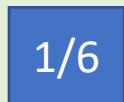
(2 levels)



$\frac{1}{3}$

Each $\frac{1}{3}$ contains two $\frac{1}{6}$

(1 level)

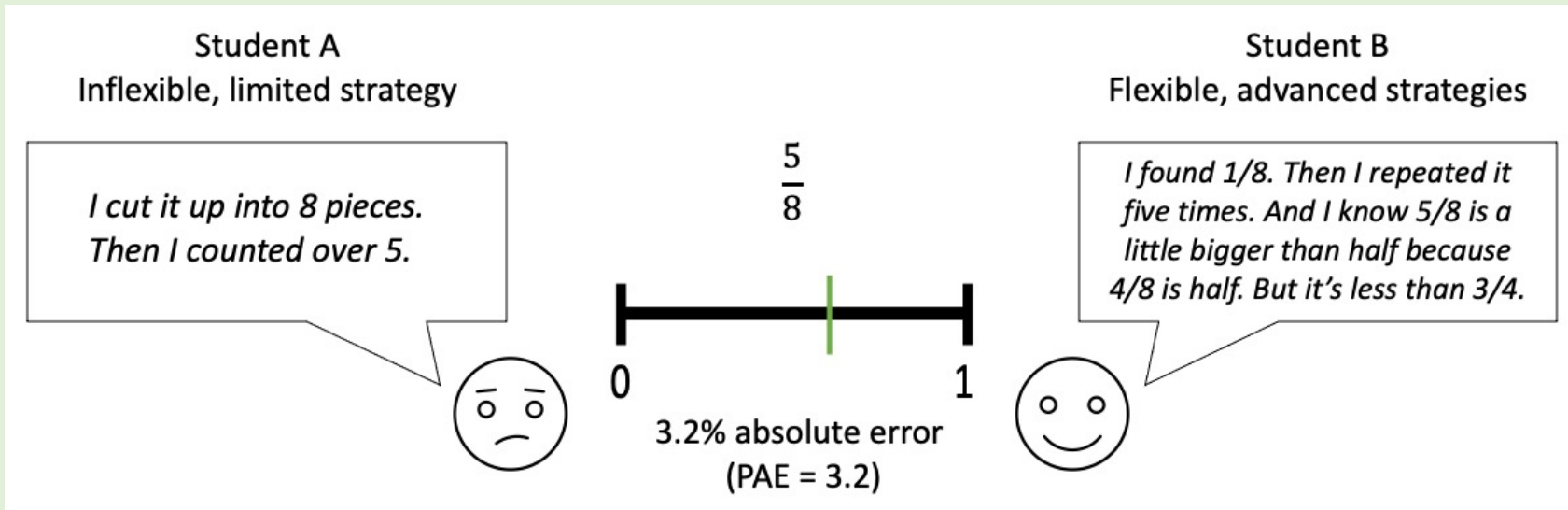


$\frac{1}{6}$

Simplest unit

Why is it important to attend to student thinking?

Dominant measures from psychology might gloss over important individual differences.



Math education research suggests that these differences might have a big impact on students' algebraic thinking.

Psych might also be missing important areas of algebra!

Many psych assessments of algebra start by thinking about Algebra I classes, textbooks, or standardized tests.

Sample Tasks

Solve: $4x + 5 = 8$

Is $4x - 3$ equivalent to $3 - 4x$?

Simplify: $2(x + y) - (2x - y)$

$a = 3$ and $b = -1$

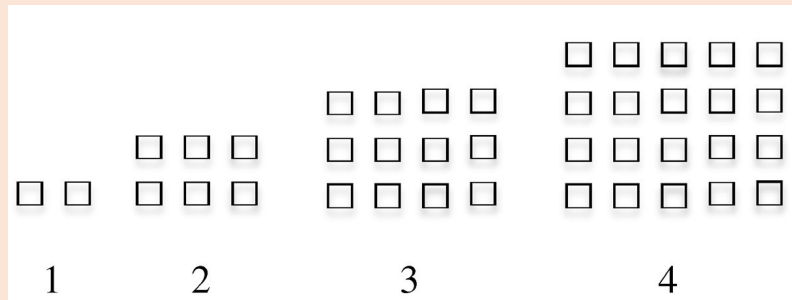
What is the value of $2a + 3(2 - b)$

E.g., Booth et al., 2014; Booth et al., 2021; DeWolf et al., 2015; Hurst & Cordes, 2018; etc.

Measuring algebra knowledge

A focus on algebra *performance* might fail to capture differences in algebraic *thinking*.

Below are the first four stages in a growing pattern.



- A. Describe stage 10 in a way that makes sense to you (e.g., draw).
- B. Find a formula that gives the total number of squares S at any stage n . Explain why your formula is true.
- C. Is 9,900 a rectangular number? Explain your answer.

Rivera & Becker, 2016

Measuring algebra knowledge

A focus on algebra *performance* might fail to capture differences in algebraic *thinking*.

A friend gives you some money. Can you tell which is larger?

That money plus six dollars



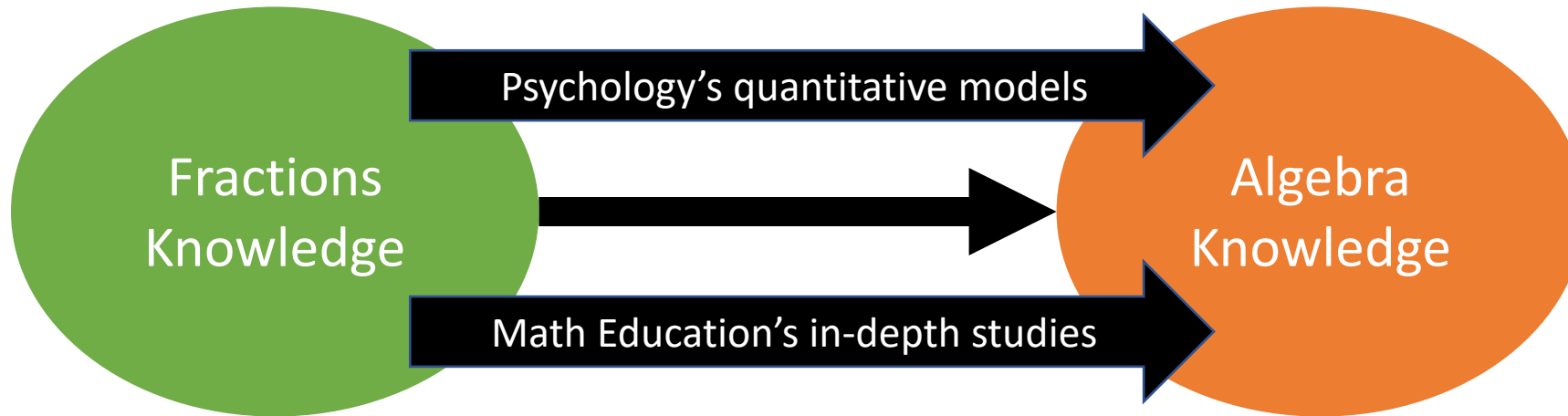
Three times that money



Knuth et al., 2005; Stephens et al., 2021

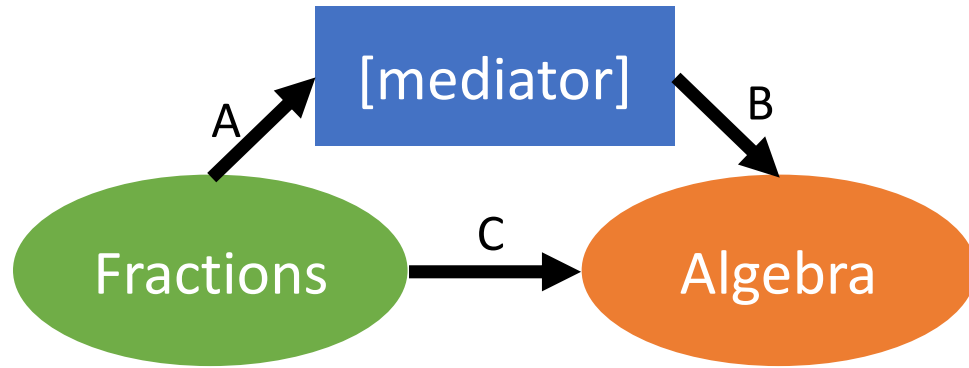
Fractions are likely important for algebra in multiple ways.

Are Psychology and Math Education capturing the same relation?
Or are they capturing unique paths/mechanisms?



To find out, we should consider combining both types of measures with the same kids!

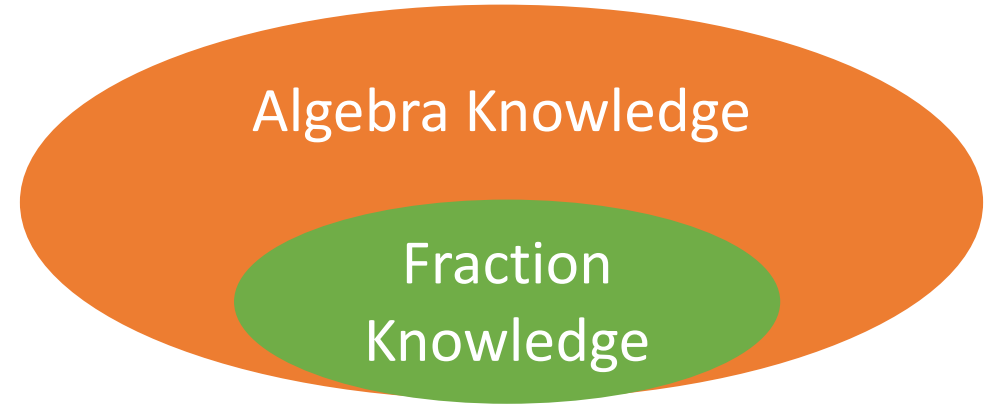
Where are the *boundaries* between these categories of mathematical knowledge?



Causal: Fractions-to-Algebra

Longitudinal evidence:

- Booth et al. (today)
- Gesuelli et al. (today)
- Liang et al. (2018)
- Siegler et al. (2012)



Constitutive: Algebra includes fractions

As Jay & Alibali suggest, fractions help students practice with **generalizing mathematical relationships**.

Nabors (2003), Empson et al. (2011) make similar arguments.

Thank you!



Percival Matthews



Ana Stephens



Martha Alibali



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Learning & Development Lab



Anderson
Norton



Martin Simon



Jon Star



Eric Knuth

