Test 1

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Vectors & Scalars

- Scaler
    - A quantity that has magnitude only
- Vectors are quantities that have both *magnitude* and *direction*
    - Magnitude
        - The size of the vector
        - Examples: 1, 2, 3, 4
    - Direction
        - The direction the vector is pointing
        - Examples: North, South, East, West
    - Examples: Velocity, force, displacement
    - Position vector
        - A vector that points from the origin to a point in space
        - Example: home is $\langle 0, 0 \rangle$, lecture hall is $\langle -1, 3 \rangle$, and coffee shop is $\langle 2, 2 \rangle$
            - The noted as $\vec{V}$ from home to lecture hall is X **mph north**
            - $\hat{V}$ = **north**
            - $|\vec{V}|$ = X
            - To go from the lecture hall to the coffee shop, you would go X **MPH east**
            - To get the vector length, you would use the Pythagorean theorem (where vector length is the hypotenuse)
                - $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
                - $\sqrt{(2 - (-1))^2 + (2 - 3)^2}$ =
                - $\sqrt{3^2 + (-1)^2}$ =
                - $\sqrt{9 + 1}$ =
                - $\sqrt{10}$
            - To get the vector displacement, you would subtract the two vectors
                - $\langle x_2, y_2 \rangle - \langle x_1, y_1 \rangle = \langle x_2 - x_1, y_2 - y_1 \rangle$
                - $2 - (-1) = 3$ and $2 - 3 = -1$ and thus the vector displacement is $\langle 3, -1 \rangle$

Units

- Dimensionless numbers
    - Numbers that have no units
        - Examples: 1, 2, 3, 4
    - Dimensional numbers
        - Numbers that have units
            - Examples: $100W$, $10kg$, $25V$
    - Examples: The temperature in a room, the mass of an object
- Dimensional Scalers

- Dimensionless number **x** unit
  - Examples: $1m$, $2kg$, $3s$, $300,000m/s$

Thing to Measure	Unit
Length	Meters (m)
Area	Square meters (m^2)
Volume	Cubic meters (m^3)
Time	Seconds (s)
Angle	Radians (rad), 1 degree = $\pi/180$ radians
Mass	Kilograms (kg)
Speed	Meters per second (m/s)
Force	Newtons ($kg \cdot m/s^2$)
Temperature	Fahrenheit (F), Celsius (C), Kelvin (K)

Newton's Laws of Motion

Newton's First Law of Motion

- **An object subject to no external forces moves at a constant velocity.**
- Equation of predicting an object at constant velocity: $\vec{x}_f = \vec{x}_i + \vec{v} \cdot (t_f - t_i)$
  - $\vec{x}_f$ is the final position (m)
  - $\vec{x}_i$ is the initial position (m)
  - $\vec{v}$ is the velocity ($\frac{m}{s}$)
  - t_f is the final time (s)
  - t_i is the initial time (s)
- Example of finding the final position of an object at constant velocity:
  - What is $\vec{x}_f$ if $\vec{x}_i = (-4, 6, -8)$, $\vec{v} = (2, \frac{-4}{3}, 2)$, $t_f = 3$, and $t_i = 0$?
    - $\vec{x}_f = \vec{x}_i + \vec{v} \cdot (t_f - t_i)$
    - $\vec{x}_f = (-4, 6, -8) + (2, \frac{-4}{3}, 2) \cdot (3)$
    - $\vec{x}_f = (-4, 6, -8) + (6, -4, 6)$
    - $\vec{x}_f = (2, 2, -2)$

Newton's Second Law of Motion

- **The acceleration that an object experiences is equal to the net force exerted on it divided by the object's mass.**
- Equation of predicting the acceleration of an object: $\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$
  - $\vec{a}$ is the acceleration ($\frac{m}{s^2}$)
  - $\vec{F}_{\text{net}}$ is the net force ($N \cdot m$ *Newton meters*)
  - m is the mass of the object (kg)

- Equation of predicting an object's final velocity: $\vec{v}_f = \vec{v}_i + \vec{a} \cdot (t_f - t_i)$
  - $\vec{v}_f$ is the final velocity ($\frac{m}{s}$)
  - $\vec{v}_i$ is the initial velocity ($\frac{m}{s}$)
  - $\vec{a}$ is the acceleration ($\frac{m}{s^2}$)
  - t_f is the final time (s)
  - t_i is the initial time (s)
- Equation for finding the average velocity of an object: $\vec{v}_{\text{avg}} = \frac{\vec{v}_i + \vec{v}_f}{2}$
  - $\vec{v}_{\text{avg}}$ is the average velocity ($\frac{m}{s}$)
  - $\vec{v}_i$ is the initial velocity ($\frac{m}{s}$)
  - $\vec{v}_f$ is the final velocity ($\frac{m}{s}$)
- By integrating the average velocity over the time interval, we get the equation for the final position: $\vec{x}_f = \vec{x}_i + \vec{v}_i \cdot (t_f - t_i) + \frac{1}{2}\vec{a} \cdot (t_f - t_i)^2$
  - $\vec{x}_f$ is the final position
  - $\vec{x}_i$ is the initial position
  - $\vec{v}_i$ is the initial velocity
  - $\vec{a}$ is the acceleration
  - t_f is the final time
  - t_i is the initial time

Newton's Third Law of Motion

- *For every force from an object A exterted to B, there is an equal in magnitude and opposite in direction force from B exerted to A.*
- $\vec{F}_{\text{BA}} = -\vec{F}_{\text{AB}}$
  - "The force exerted by A on B is equal in magnitude and opposite in direction to the force exerted by B on A."

Gravitational Force

Force	Symbol	Description	Direction
Gravitational	$\vec{F}_g$ or $\vec{w}$ = $\vec{\text{weight}}$	The force of attraction between two masses. $\vec{F}_g = (0, 0, -9.81\frac{m}{s^2})$	Towards the center of the Earth (downwards)
Frictional	$\vec{F}_f$	The force that opposes the motion of an object.	Opposite to the direction of motion
Normal	$\vec{F}_N$	The support force exerted by a surface perpendicular to the object.	Perpendicular to the surface (upwards)
Drag	$\vec{F}_d$	The force that opposes the motion of an object through a fluid.	Opposite to the direction of motion

- The *equivelence principle* states that the force of gravity is equivalent to the force of acceleration. This is why objects in free fall experience weightlessness.
- When dropping an object from a height, the object will accelerate downwards at a rate of $-9.81\frac{m}{s^2}$.

- Putting that into a formula gives us $\vec{x}_f = \vec{x}_i + \vec{v}_i \cdot (t_f - t_i) + \frac{1}{2}\vec{a} \cdot (t_f - t_i)^2$
  - $\vec{X}_i = (0, 0, h)$
  - $\vec{v}_i = (0, 0, 0)$
  - $\vec{a} = \vec{g} = (0, 0, -9.81\frac{m}{s^2})$
- Rearranging, $h = \frac{1}{2} \cdot -9.81 \cdot t^2$

Ramps

Ramp Anatomy

- Ramps are a simple machine that allow us to lift heavy objects with less force.

```
         *  |
        **  |
       ***  |
      ****  |
     *****  |
    ******  |
   *******  |  height
  ********  |  ^
 *********  |  |
*θ********  |  |

base -->
```

Here, we can find the angle θ using the relationship:

$$\tan(\theta) = \frac{\text{height}}{\text{base}}$$

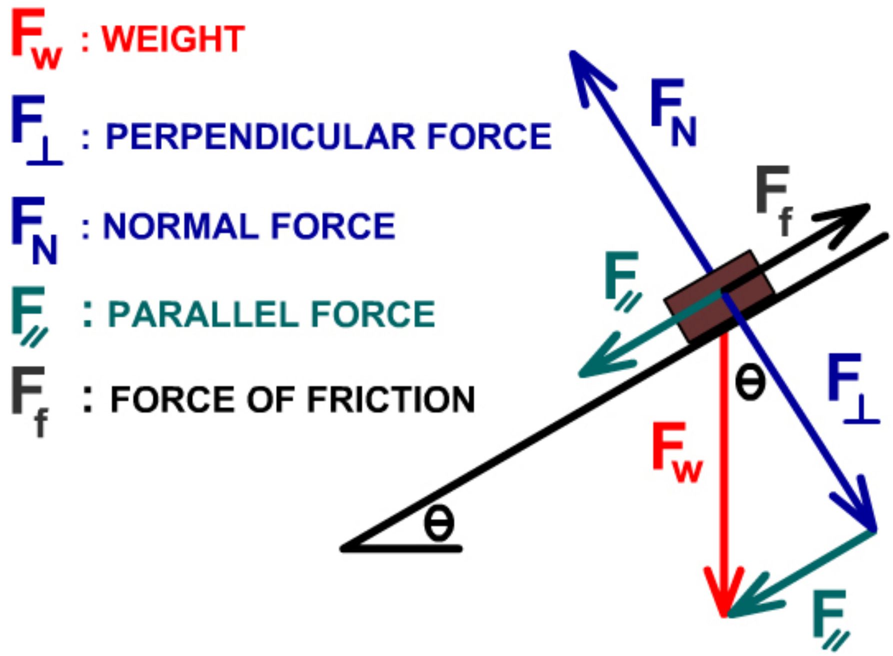
By the Pythagorean theorem, the length of the ramp (hypotenuse) can be calculated if the height and base are known:

$$\text{length} = \sqrt{\text{height}^2 + \text{base}^2}$$

Gravitational Force on an Inclined Plane

When an object is placed on an inclined plane, the gravitational force acting on it can be resolved into two components:

1. **Parallel Component ($F_{\|}$)**: This component acts parallel to the surface of the inclined plane and causes the object to slide down.
2. **Perpendicular Component ($F_{\perp}$)**: This component acts perpendicular to the surface of the inclined plane and is responsible for the normal force.



The gravitational force (F_g) acting on the object can be expressed as:

$$F_g = m \cdot g = F_w$$

where:

- m is the mass of the object
- g is the acceleration due to gravity ($9.8\mathrm{m/s}^2$)

The parallel and perpendicular components can be calculated using the angle of the incline (θ):

$$F_{\parallel} = F_g \cdot \sin(\theta) = m \cdot g \cdot \sin(\theta)$$

$$F_{\perp} = F_g \cdot \cos(\theta) = m \cdot g \cdot \cos(\theta)$$

Forces at Work on an Inclined Plane

Symbol	Name	Description	Calculation Formula
F_w	Weight	The gravitational force acting on the object.	$F_w = m \cdot g$
$F_\perp$	Perpendicular Component	The component of the gravitational force acting perpendicular to the surface of the inclined plane.	$F_\perp = F_w \cdot \cos(\theta)$
F_N	Normal Force	The force exerted by the inclined plane on the object, equal in magnitude and opposite in direction to $F_\perp$.	$F_N = -F_\perp$
$F_\parallel$	Parallel Component	The component of the gravitational force acting parallel to the surface of the inclined plane, causing the object to slide down.	$F_\parallel = F_w \cdot \sin(\theta)$

Work

Work is the transfer of energy from one object to another The *Work-Energy Principle* state the work done on an object is equal to the force applied to the object times the distance the object moves in the direction of the force

$$\text{Work} = \vec{\text{F}} \times \vec{\Delta x} = \vec{F} \cdot (\vec{x}_f - \vec{x}_i)$$

$\text{Note that } \vec{F} = \vec{F}_\parallel + \vec{F}_\perp$

- You only do work if the displacement is a non-zero component of the force

Rotational Motion

Units of Motion

Translational Motion	Rotational Motion
Position: $\vec{x}$ (m)	**Angle**: $\vec{\theta}$ (rad)
Velocity: $\vec{v}$ ($\frac{m}{s}$)	**Angular Velocity**: $\vec{\omega}$ ($\frac{\text{rad}}{\text{s}}$)
Acceleration: $\vec{a}$ ($\frac{m}{s^2}$)	**Angular Acceleration**: $\vec{\alpha}$ ($\frac{\text{rad}}{s^2}$)
Force: $\vec{F}$ (N)	**Torque**: $\vec{\tau}$ (Nm)
Mass: m (kg)	**Rotational Mass/Moment of Inertia**: I (kg m^2)

Rules

Translational Motion	Rotational Motion
No outside forces means constant $\vec{v}$	No outside torques means constant $\vec{\omega}$
$\vec{a} = \frac{\vec{\text{Fnet}}}{m}$	$\vec{\alpha} = \frac{\vec{\tau}_{\text{net}}}{I}$
$\vec{F}_{\text{BA}} = -\vec{F}_{\text{AB}}$	$\vec{\tau}_{\text{BA}} = -\vec{\tau}_{\text{AB}}$

Right Hand Rule

- The right hand rule is used to determine the direction of the angular momentum vector $(\vec{\omega})$.

1. **Identify the Rotation Axis**: Determine the axis around which the object is rotating.
2. **Curl Your Fingers**: Point the fingers of your right hand in the direction of the rotation (the direction in which the object is moving).
3. **Thumb Direction**: Extend your thumb perpendicular to your fingers. The direction your thumb points is the direction of the angular momentum vector $(\vec{\omega})$.

- Example
  - If a wheel is rotating counterclockwise when viewed from above, you would:
  - Point your fingers in the direction of the rotation (counterclockwise).
  - Your thumb will point upwards, indicating that the angular momentum vector $(\vec{\omega})$ is directed upwards.
- Trick
  - Clockwise = away from you
  - Counterclockwise = towards you

Angular Velocity

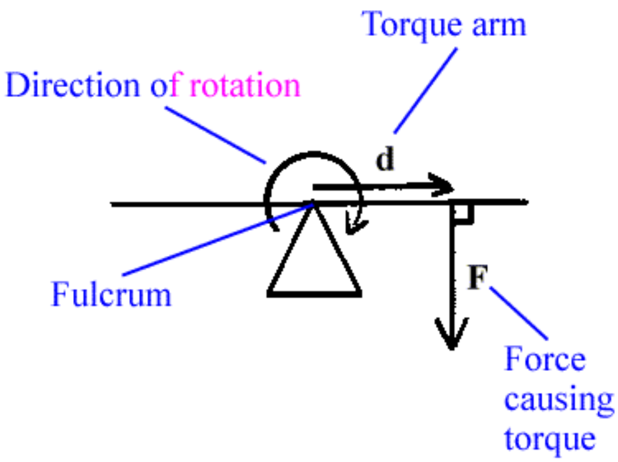
Angular velocity is the rate of change of angular displacement with respect to time.

- $\vec{\omega} = \frac{\vec{\theta}_f - \vec{\theta}_i}{t_f - t_i}$

Torque

- Torque is the rotational equivalent of force.
- $\vec{\alpha} = \frac{\vec{\tau}_{\text{net}}}{I}$
  - Units of $\vec{\tau}$ are Nm
  - Units of $\vec{\alpha}$ are $\frac{\text{rad}}{s^2}$
- I rotational mass measures $\text{mass} \times \text{distance}^2$
  - Units of I are kg m^2
- $\tau = |\vec{\tau}| = \vec{r} \times \vec{F}_\perp$
  - $\vec{r}$ is the distance from the axis of rotation
  - $\vec{F}$ is the force applied

Diagram of Torque



- Note $\vec{F} = \vec{F}_{\perp}$

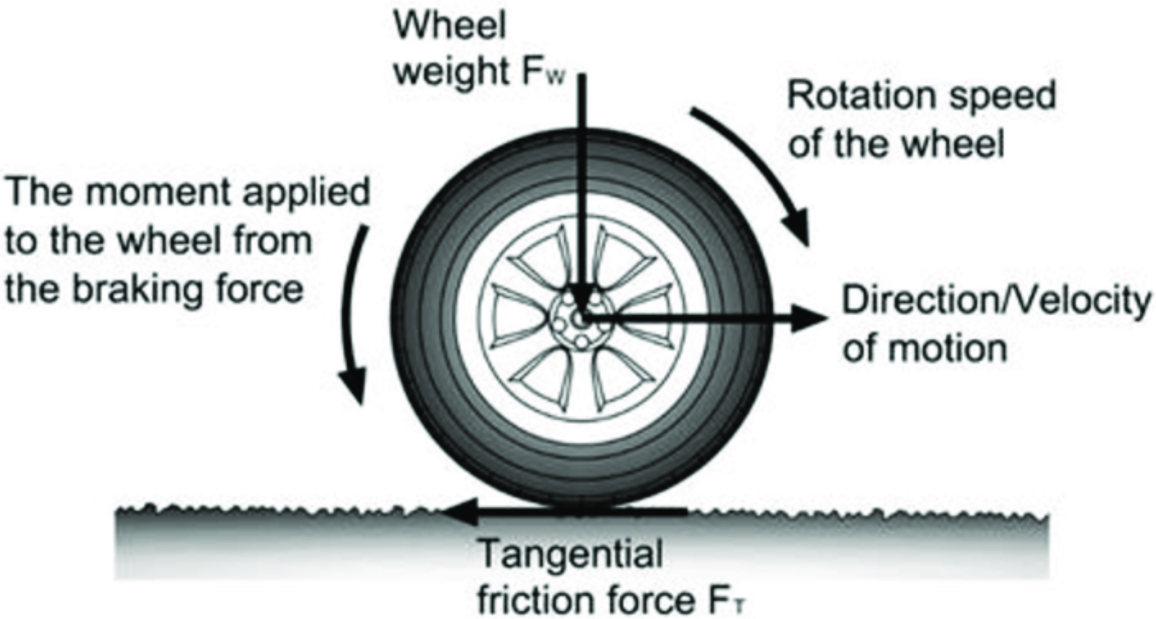
Friction

- Friction is a force that opposes motion.
  - Static friction prevents motion
  - Sliding friction slows motion
- $\vec{F}_{\text{sliding friction}} = \mu_{\text{sliding friction}} \cdot \vec{F}_{\text{support}}$
  - μ is the coefficient of friction
  - $\vec{F}_{\text{support}}$ is the support force
- $\hat{F}_{\text{sliding friction}} = -\hat{v}$ if $\vec{v} > 0$
  - $\hat{v}$ is the velocity of the object
- $F_{\text{static friction}} = F_{\text{push}}$ if $F_{\text{push}} < \mu_{\text{static friction}} \cdot F_{\text{support}}$
  - F_{push} is the force applied to the object
  - $\mu_{\text{static friction}}$ is the coefficient of static friction
  - F_{support} is the support force

Wheels

- Wheels are a type of simple machine which are used to reduce friction and make it easier to move objects.
- $\vec{v}_{\text{ground}}$ must be 0 which means that the wheel is not slipping (because wheels *roll*) and so $\vec{v}_{\text{contact point}} = \vec{v}_{\text{ground}}$.
  - No work down by frictional forces from the ground to the wheel.
- $\vec{v}_{\text{top}} = \vec{v}_{\text{contact point}}$

Diagram of Forces on a Wheel



Angular Velocity of a Wheel

- Use right hand rule to determine the direction of the angular velocity vector.
  - $\vec{\omega} = \frac{\vec{v}_{\text{contact point}}}{r}$ where r is the radius of the wheel.

Work + Power

- Translational Motion
  - $\text{work} = W = \vec{F} \cdot \vec{\Delta x} = F_{\|}\Delta x$
    - W is the work done by a force.
    - $\vec{F}$ is the force applied.
    - $\vec{\Delta x}$ is the displacement of the object.
    - $F_{\|}$ is the component of the force parallel to the displacement.
  - $\text{power} = \vec{F} \cdot \vec{v} = \frac{W}{\Delta t}$
    - power is the rate at which work is done.
    - $\vec{F}$ is the force applied.
    - $\vec{v}$ is the velocity of the object.
    - W is the work done by a force.
    - Δt is the time interval.
- Rotational Motion
  - $\text{work} = W = \vec{\tau} \cdot \vec{\Delta\theta} = \vec{\tau} \cdot (\vec{\theta}_f - \vec{\theta}_i)$
    - W is the work done by a torque.
    - $\vec{\tau}$ is the torque applied.
    - $\vec{\Delta\theta}$ is the angular displacement of the object.
    - $\vec{\theta}_f$ is the final angle of the object.
    - $\vec{\theta}_i$ is the initial angle of the object.

  - $\text{power} = \vec{\tau} \cdot \vec{\omega} = \frac{W}{\Delta t}$
    - power is the rate at which work is done.
    - $\vec{\tau}$ is the torque applied.
    - $\vec{\omega}$ is the angular velocity of the object.
    - W is the work done by a torque.
    - Δt is the time interval.

Kinetic Energy

- Kinetic energy is the energy of motion.
  - This means that the total energy of a system is constant.

Formulas for Kinetic Energy

- $K = \frac{1}{2}mv^2$ for translational motion.
- $K = \frac{1}{2}I\omega^2$ for rotational motion.
- Total energy is *conserved*
  - $u = mgh$
    - u is the potential energy of an object.
    - m is the mass of the object.
    - g is the acceleration due to gravity ($-9.8\frac{m}{s^2}$).
    - h is the height of the object.
  - $\text{total energy} = K + u + heat + ...$

Momentum and Impulse

Momentum

- Momentum is the product of mass and velocity.
  - Refers to the quantity of motion an object has.
- Translational Motion
  - $\vec{p} = m\vec{v}$
    - $\vec{p}$ is the momentum of an object ($kg\frac{m}{s}$).
    - m is the mass of the object.
    - $\vec{v}$ is the velocity of the object.
  - If no forces are applied, momentum is conserved.
- Rotational Motion
  - $\vec{L} = I\vec{\omega}$
    - $\vec{L}$ is the angular momentum of an object ($kg\frac{m^2}{s}$).
    - I is the moment of inertia of the object.
    - $\vec{\omega}$ is the angular velocity of the object
  - No $\vec{\tau}$ means $\vec{L}$ is constant.

Impulse

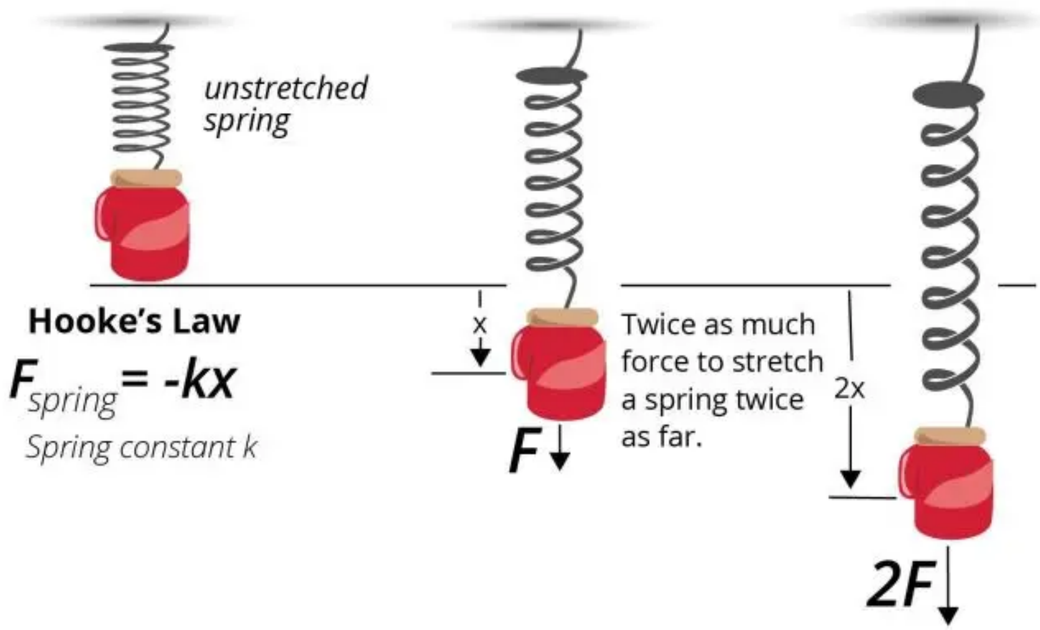
- Impulse is the change in momentum.
  - Quantifies the overall effect of a force on an object across a time interval.
- Translational Motion
  - $\Delta\vec{p} = \vec{p_f} - \vec{p_i} = \vec{F} \cdot \Delta t$
    - $\Delta\vec{p}$ is the change in momentum.
    - $\vec{F}$ is the force applied.
    - Δt is the time interval.
- Rotational Motion
  - $\Delta\vec{L} = \vec{L}_f - \vec{L}_i = \vec{\tau} \cdot \Delta t$
    - $\Delta\vec{L}$ is the change in angular momentum.
    - $\vec{\tau}$ is the torque applied.
    - Δt is the time interval.

Elastic Potential Energy and Restoring Force

Hooke's Law

- Hook's Law states that the force exerted by a spring (or something elastic) is proportional to the displacement from equilibrium
  - $\vec{F}_{\text{restoring}} = -k\vec{x}$
    - $\vec{F}_{\text{restoring}}$ is the restoring force
    - k is the spring constant
    - $\vec{x}$ is the displacement from equilibrium
  - $U_{\text{spring}} = \frac{1}{2}kx^2$
    - U_{spring} is the spring potential energy
    - x is the displacement from equilibrium
  - $\vec{a} = -\frac{k}{m}\vec{x}$
    - $\vec{a}$ is the acceleration
    - m is the mass
    - $\vec{x}$ is the displacement from equilibrium

Example Diagram of a Spring



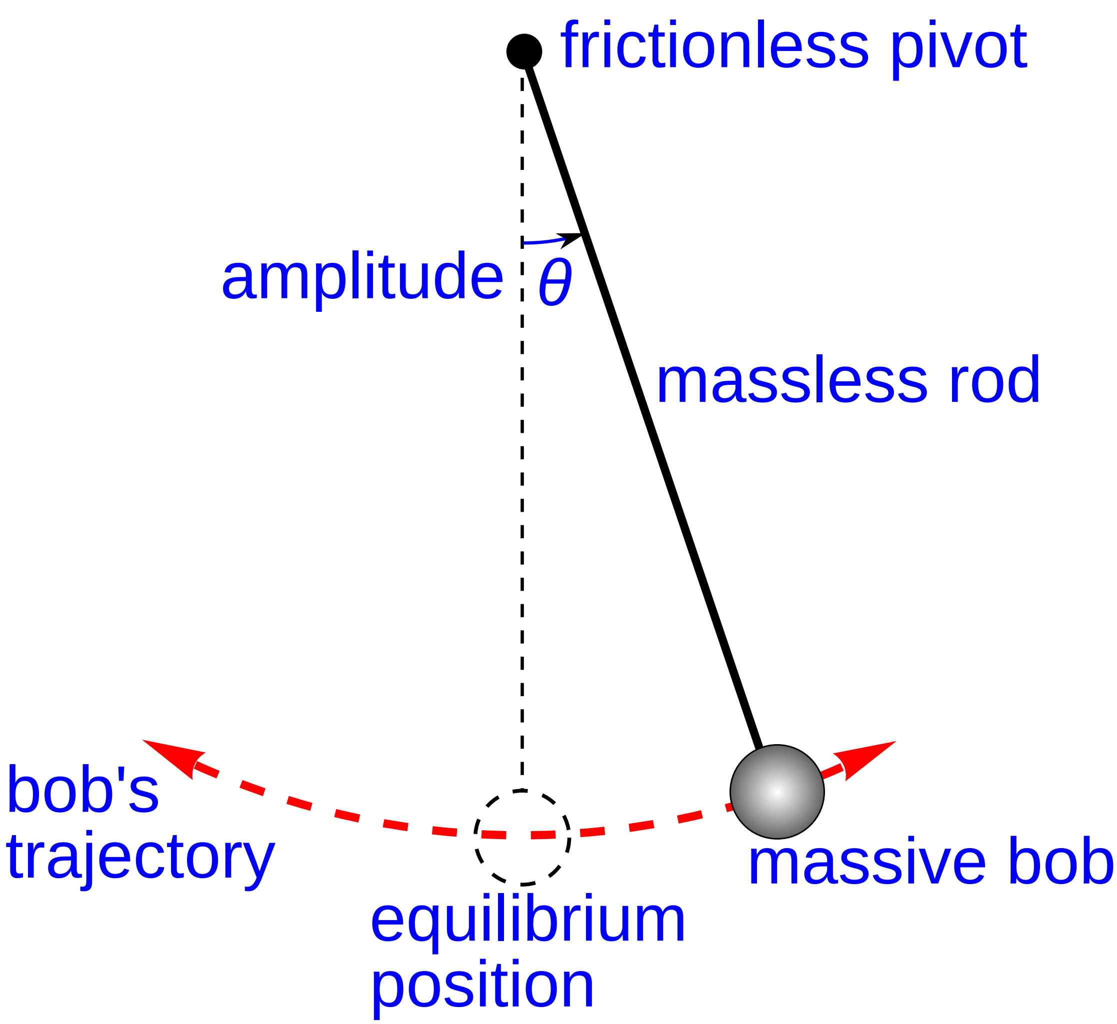
Harmonic Oscillators

- Harmonic oscillators are systems that oscillate about an equilibrium point
  - Examples include springs, pendulums, and electric circuits
  - The period of oscillation does not depend on the amplitude of the oscillation
- $T = 2\pi\sqrt{\frac{m}{k}}$
  - T is the period of the oscillation (time for one complete cycle)
  - m is the mass
  - k is the spring constant

Pendulum and Clocks

- A pendulum is a mass on a string that oscillates back and forth
  - The period of a pendulum is given by $T = 2\pi\sqrt{\frac{L}{g}}$
    - T is the period
    - L is the length of the pendulum
    - g is the acceleration due to gravity
  - The period of a pendulum is independent of the mass of the pendulum
  - $U_{\text{gravity}} = \frac{mg}{2L}x^2$
    - U_{gravity} is the gravitational potential energy
    - m is the mass of the pendulum
    - x is the displacement from equilibrium
  - $k_{\text{pendulum}} = \frac{mg}{L}$
    - k_{pendulum} is the spring constant of the pendulum
- Clocks use pendulums to keep time
  - The period of a pendulum is constant, so the clock will keep time accurately

Pendulum Diagram



- $\text{massless rod} = L$

Notes

- Please refer to the equation sheet for what equations you will have for the exam
- I have left out *Forces as a Gradient of Potential Energy* as it seems like it will not be tested