Lecture 7

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Forces as a Gradient of Potential Energy

This entire section seems to not be tested

- $E = K + U_{\text{gravity}} + \ldots$
  - E is the total energy
  - K is the kinetic energy
  - U_{gravity} is the gravitational potential energy
    - $U_{\text{gravity}} = mgh$
      - m is the mass
      - g is the acceleration due to gravity ($9.81\ \frac{\text{m}}{\text{s}^2}$)
      - h is the height
        - For a ramp, $h = x \sin\theta$
          - θ is the angle of the ramp
          - x is the distance along the ramp
- $\vec{F} = -\nabla U$
  - $\vec{F}$ is the force
  - U is the potential energy
  - ∇ is the gradient operator
    - $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$
    - The gradient gives the rate of change of U in each spatial direction
  - The negative sign indicates the force points in the direction of decreasing potential energy

Half Pipe Example

- Consider a ramp shaped like a half pipe where z is the height, x is the horizontal distance, and the slope is cx
  - $z = \frac{c}{2}x^2$
    - c is a constant
  - $U_{\text{gravity}} = mgz = \frac{1}{2}mgcx^2$
  - The force is:

$$\vec{F} = -\nabla U_{\text{gravity}} = -\frac{U_{\text{gravity}}}{dx}\hat{x} = -mgcx\hat{x}$$

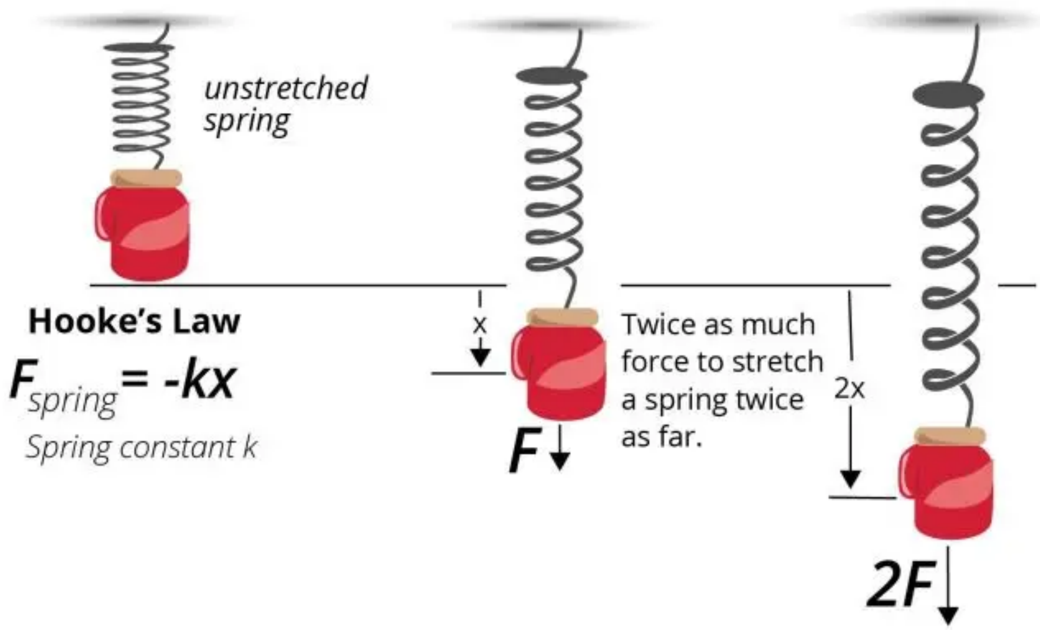
    - Here, mgc acts as the spring constant k in Hooke's Law

Elastic Potential Energy and Restoring Force

Hooke's Law

- Hook's Law states that the force exerted by a spring (or something elastic) is proportional to the displacement from equilibrium
  - $\vec{F}_{\text{restoring}} = -k\vec{x}$
    - $\vec{F}_{\text{restoring}}$ is the restoring force
    - k is the spring constant
    - $\vec{x}$ is the displacement from equilibrium
  - $U_{\text{spring}} = \frac{1}{2}kx^2$
    - U_{spring} is the spring potential energy
    - x is the displacement from equilibrium
  - $\vec{a} = -\frac{k}{m}\vec{x}$
    - $\vec{a}$ is the acceleration
    - m is the mass
    - $\vec{x}$ is the displacement from equilibrium

Example Diagram of a Spring



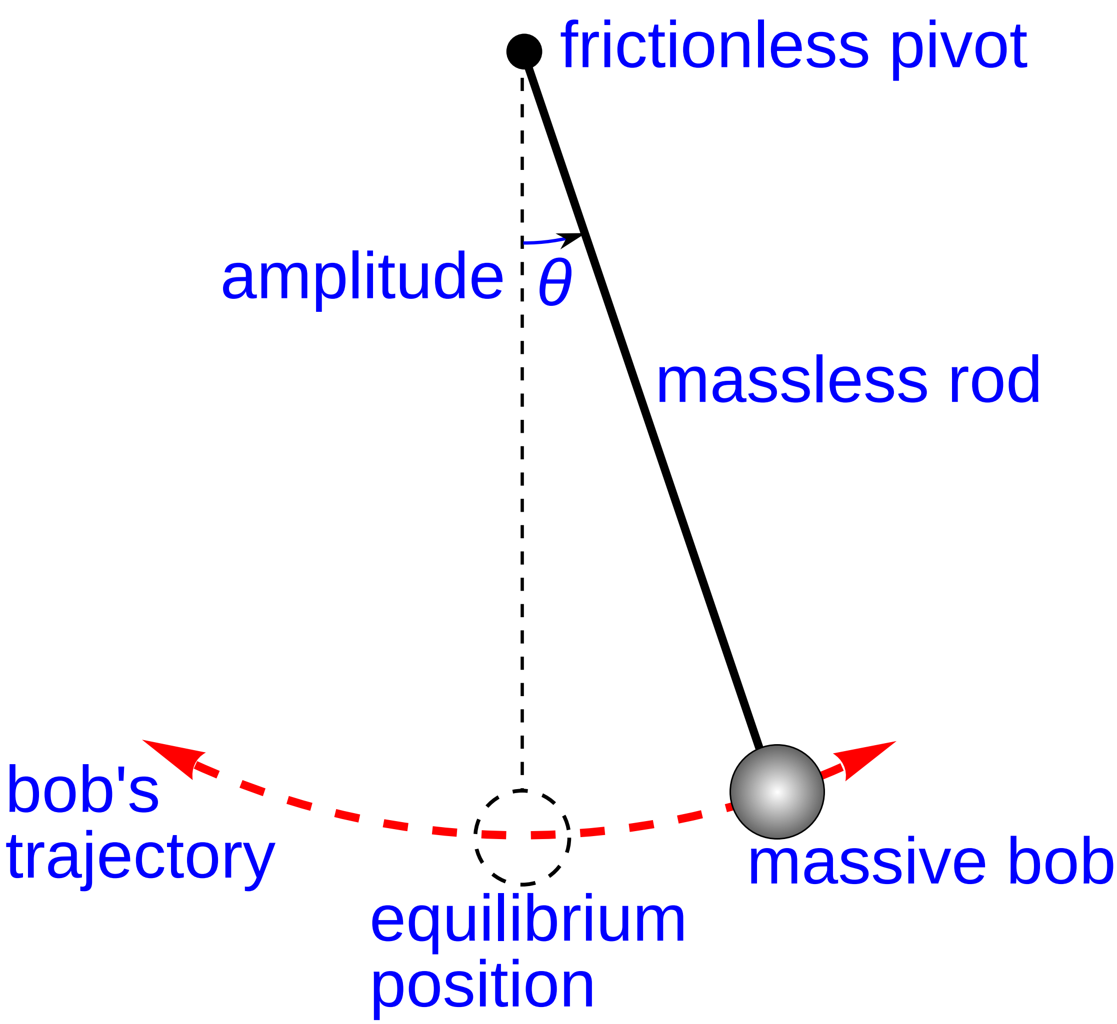
Harmonic Oscillators

- Harmonic oscillators are systems that oscillate about an equilibrium point
  - Examples include springs, pendulums, and electric circuits
  - The period of oscillation does not depend on the amplitude of the oscillation
- $T = 2\pi\sqrt{\frac{m}{k}}$
  - T is the period of the oscillation (time for one complete cycle)
  - m is the mass
  - k is the spring constant

Pendulum and Clocks

- A pendulum is a mass on a string that oscillates back and forth
  - The period of a pendulum is given by $T = 2\pi\sqrt{\frac{L}{g}}$
    - T is the period
    - L is the length of the pendulum
    - g is the acceleration due to gravity
  - The period of a pendulum is independent of the mass of the pendulum
  - $U_{\text{gravity}} = \frac{mg}{2L}x^2$
    - U_{gravity} is the gravitational potential energy
    - m is the mass of the pendulum
    - x is the displacement from equilibrium
  - $k_{\text{pendulum}} = \frac{mg}{L}$
    - k_{pendulum} is the spring constant of the pendulum
- Clocks use pendulums to keep time
  - The period of a pendulum is constant, so the clock will keep time accurately

Pendulum Diagram



- $\text{massless rod} = L$

PollEV Answers

- *D* **All of the above are correct** *we will all get full credit for this question*
    - What are the units of the spring constant "k" in Hooke's law? [J = Joule, kg = kilogram, m = meter, s = second, N = Newton.]

1. **Hooke's Law**:
    - Hooke's law is given by $F = kx$, where F is the force applied to the spring, k is the spring constant, and x is the displacement of the spring.

2. **Units of Force (F)**:
    - The unit of force F is the Newton (N).

3. **Units of Displacement (x)**:

- The unit of displacement x is the meter (m).

4. **Units of Spring Constant (k)**:
   - Rearranging Hooke's law to solve for k: $k = \frac{F}{x}$.
   - Therefore, the units of k are $\frac{\mathrm{N}}{\mathrm{m}}$.

5. **Dimensional Analysis**:
   - Newton (N) can be expressed in terms of base units: $1\ \mathrm{N} = 1\ \mathrm{kg} \cdot \mathrm{m/s}^2$.
   - Substituting this into the units of k: $k = \frac{\mathrm{kg} \cdot \mathrm{m/s}^2}{\mathrm{m}} = \mathrm{kg/s}^2$.

6. **Energy Perspective**:
   - The potential energy stored in a spring is given by $U = \frac{1}{2}kx^2$.
   - The unit of energy U is the Joule (J).
   - Rearranging to solve for k: $k = \frac{2U}{x^2}$.
   - Therefore, k can also be expressed in terms of energy per unit displacement squared: $\frac{\mathrm{J}}{\mathrm{m}^2}$.

Given these analyses, the spring constant k can be expressed in multiple units:

- $\frac{\mathrm{N}}{\mathrm{m}}$
- $\mathrm{kg/s}^2$
- $\frac{\mathrm{J}}{\mathrm{m}^2}$

Thus, the answer "all of the above" is correct because the spring constant k can be represented in terms of Joules (J), kilograms (kg), meters (m), seconds (s), and Newtons (N).