Lecture 1 (Math Review)

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Symbols & Notation

Symbol	Meaning
$=$	Equals
$\neq$	Not equal
$\propto$	Proportional to
E	Exponent value
$\ldots$	Continued series

Exponents

Exponents (or powers) are shorthand for repeated multiplication. For instance:

- $a^2 = a \times a$ ("a squared")
- $a^3 = a \times a \times a$ ("a cubed")
- $a^n = a \times a \times \cdots \times a$ ("n times")

Rules for exponents:

- $a^1 = a$
- $a^0 = 1$
- $a^{-n} = \frac{1}{a^n}$

Example powers of 10:

- $10^6 = 1,000,000$
- $10^{-3} = 0.001$

Properties:

1. $a^m \times a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$
3. $(a^m)^n = a^{m \times n}$

Scientific Notation & E Notation

Scientific notation simplifies large/small numbers by expressing them as a small number multiplied by a power of 10. For example:

- 3.56×10^3 becomes $3.56E + 3$
- 7.87×10^{-4} becomes $7.87E - 4$

Operations:

- **Multiplication:** $(2.5 \times 10^3) \times (4.3 \times 10^5) = (2.5 \times 4.3) \times 10^{3+5} = 10.75 \times 10^8 = 1.075 \times 10^9$

- **Division:** $\frac{4.3\times10^5}{2.5\times10^3} = \frac{4.3}{2.5} \times 10^{5-3} = 1.72 \times 10^2$

Addition/Subtraction:

To add or subtract in scientific notation, first convert all numbers to the same power of 10:

- $4.3 \times 10^5 - 2.5 \times 10^3 = 4.3 \times 10^5 - 0.025 \times 10^5 = (4.3 - 0.025) \times 10^5 = 4.275 \times 10^5$

Units

Units are essential for expressing physical quantities, and errors in unit specification lead to significant issues. Common SI units:

- **Length:** $1\,km = 10^3\,m$, $1\,cm = 10^{-2}\,m$
- **Mass:** $1\,g = 10^{-3}\,kg$, $1\,mg = 10^{-6}\,kg$
- **Time:** $1\,ns = 10^{-9}\,s$

Conversion example:

To convert 1 cm to nanometers: $1\,cm = 10^{-2}\,m = 10^7\,nm$

Metric System Prefixes:

- **Tera (T):** $1\,T = 10^{12}$
- **Giga (G):** $1\,G = 10^9$
- **Mega (M):** $1\,M = 10^6$
- **Kilo (k):** $1\,k = 10^3$
- **Centi (c):** $1\,c = 10^{-2}$
- **Milli (m):** $1\,m = 10^{-3}$
- **Micro (\mu):** $1\,\mu = 10^{-6}$
- **Nano (n):** $1\,n = 10^{-9}$
- **Pico (p):** $1\,p = 10^{-12}$

Numerical Precision

Precision is critical when dealing with empirical data.

- **Significant figures**: Only write digits that are meaningful and supported by the measurement's precision.
- Example: The dinosaurs were killed off **about** 6.5×10^7 years ago, not exactly 6.50×10^7 years.

To specify a number with an uncertainty:

- Example: 1.364 ± 0.003 implies the true value is between 1.361 and 1.367.

Trigonometry

For a right triangle:

- $\sin \phi = \frac{opposite}{hypotenuse}$
- $\cos \phi = \frac{adjacent}{hypotenuse}$
- $\tan \phi = \frac{opposite}{adjacent}$

Mnemonic: **SOH-CAH-TOA**

Example:

- If $a = 4\,cm$, $b = 3\,cm$, and $c = 5\,cm$, then $\sin \phi = \frac{3}{5} = 0.6$

Radians vs Degrees:

$360^\circ = 2\pi$ radians. Therefore, 1 radian = $\frac{360^\circ}{2\pi} \approx 57.296^\circ$.

Proportional Relationships

Relationships between variables often take proportional forms:

- **Direct Proportion:** $a \propto b$
  - Example: The interest paid on a bank account is directly proportional to the balance.
- **Inverse Proportion:** $a \propto \frac{1}{b}$
  - Example: If the length of a guitar string doubles, its frequency is halved.
- **Square Proportion:** $a \propto b^2$
- **Inverse Square Proportion:** $a \propto \frac{1}{b^2}$

Proportionality can be written as an equation with a constant:

- Example: $f = \frac{C}{\ell}$, where f is frequency, ℓ is string length, and C is a constant depending on string tension and density.

Order-of-Magnitude Estimation

In physics, estimates are often good enough for practical purposes. The goal is to get the **order of magnitude** right, not exact values.

Example: Estimating the number of piano tuners in New York City.

- NYC population = 8 million.

- Estimate: 1 piano for every 100 people $\Rightarrow 80,000$ pianos.
- Each tuner tunes 500 pianos per year $\Rightarrow 160$ tuners needed.

Gravitational Law

Newton's law of universal gravitation: $F = G\frac{m_1 m_2}{d^2}$

Where:

- F is the gravitational force,
- G is the gravitational constant,
- m_1 and m_2 are the masses of the two objects,
- d is the distance between them.

Proportionality of Force:

- **Proportional to mass**: $F \propto m_1$, $F \propto m_2$.
- **Inverse-square relation to distance**: $F \propto \frac{1}{d^2}$.

Example:

- If the Earth-moon distance were halved, the gravitational force would increase by a factor of 4 due to the inverse-square law.

Vector Math

Vectors are quantities that have both magnitude and direction. They are often represented as arrows in a coordinate system.

Vector Notation

- A vector can be written as **v** or $\vec{v}$.
- In component form: $\vec{v} = \langle v_x, v_y, v_z \rangle$.

Vector Addition

- To add two vectors, add their corresponding components:

$$\vec{u} + \vec{v} = \langle u_x + v_x, u_y + v_y, u_z + v_z \rangle$$

Scalar Multiplication

- To multiply a vector by a scalar, multiply each component by the scalar:

$$c \cdot \vec{v} = \langle c \cdot v_x, c \cdot v_y, c \cdot v_z \rangle$$

Dot Product

- The dot product of two vectors is a scalar:

$$\vec{u} \cdot \vec{v} = u_x v_x + u_y v_y + u_z v_z$$

Cross Product

- The cross product of two vectors is another vector:

$$\vec{u} \times \vec{v} = \langle u_y v_z - u_z v_y, u_z v_x - u_x v_z, u_x v_y - u_y v_x \rangle$$

Magnitude of a Vector

- The magnitude (or length) of a vector $\vec{v}$ is:

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

Unit Vector

- A unit vector has a magnitude of 1 and points in the same direction as the original vector:

$$\hat{v} = \frac{\vec{v}}{|\vec{v}|}$$

Vector Example

- Given $\vec{a} = \langle 2, 3, 4 \rangle$ and $\vec{b} = \langle 1, 0, -1 \rangle$:
  - $\vec{a} + \vec{b} = \langle 3, 3, 3 \rangle$
  - $2 \cdot \vec{a} = \langle 4, 6, 8 \rangle$
  - $\vec{a} \cdot \vec{b} = 2 \cdot 1 + 3 \cdot 0 + 4 \cdot (-1) = -2$
  - $\vec{a} \times \vec{b} = \langle 3 \cdot (-1) - 4 \cdot 0, 4 \cdot 1 - 2 \cdot (-1), 2 \cdot 0 - 3 \cdot 1 \rangle = \langle -3, 6, -3 \rangle$
  - $|\vec{a}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$
  - $\hat{a} = \frac{\vec{a}}{\sqrt{29}} = \langle \frac{2}{\sqrt{29}}, \frac{3}{\sqrt{29}}, \frac{4}{\sqrt{29}} \rangle$
