Lecture 5

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9/17/2024

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Rotational Motion

Units

Translational Motion	Rotational Motion
Position: $\vec{x}$ (m)	**Angle**: $\vec{\theta}$ (rad)
Velocity: $\vec{v}$ ($\frac{m}{s}$)	**Angular Velocity**: $\vec{\omega}$ ($\frac{\text{rad}}{\text{s}}$)
Acceleration: $\vec{a}$ ($\frac{m}{s^2}$)	**Angular Acceleration**: $\vec{\alpha}$ ($\frac{\text{rad}}{s^2}$)
Force: $\vec{F}$ (N)	**Torque**: $\vec{\tau}$ (Nm)
Mass: m (kg)	**Rotational Mass/Moment of Inertia**: I (kg m^2)

Rules

Translational Motion	Rotational Motion
No outside forces means constant $\vec{v}$	No outside torques means constant $\vec{\omega}$
$\vec{a} = \frac{\vec{\text{Fnet}}}{m}$	$\vec{\alpha} = \frac{\vec{\tau}_{\text{net}}}{I}$
$\vec{F}_{\text{BA}} = -\vec{F}_{\text{AB}}$	$\vec{\tau}_{\text{BA}} = -\vec{\tau}_{\text{AB}}$

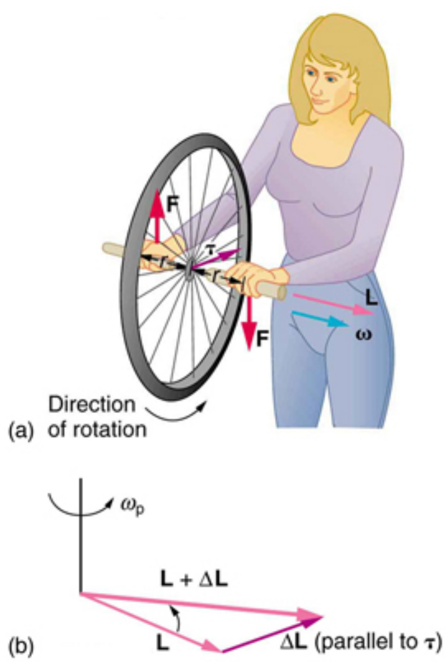
Right Hand Rule

- The right hand rule is used to determine the direction of the angular momentum vector $(\vec{\omega})$.

1. **Identify the Rotation Axis**: Determine the axis around which the object is rotating.
2. **Curl Your Fingers**: Point the fingers of your right hand in the direction of the rotation (the direction in which the object is moving).
3. **Thumb Direction**: Extend your thumb perpendicular to your fingers. The direction your thumb points is the direction of the angular momentum vector $(\vec{\omega})$.

- Example
  - If a wheel is rotating counterclockwise when viewed from above, you would:
  - Point your fingers in the direction of the rotation (counterclockwise).
  - Your thumb will point upwards, indicating that the angular momentum vector $(\vec{\omega})$ is directed upwards.

Visual Example of Right Hand Rule



- Note $\vec{\omega} = \vec{L}$ in this example

Trick for Right Hand Rule

- Clockwise = away from you
- Counterclockwise = towards you

Angular Velocity

$\vec{\omega} = \frac{\vec{\theta}_f - \vec{\theta}_i}{t_f - t_i}$

Example: Find the angular velocity of the second hand on a clock

- Understanding the Rotation and Angular Velocity
  - The second hand completes one full rotation (360°) in 60 seconds. In radians, a full rotation is 2π radians.
- Right-Hand Rule
  - For clocks, the second hand rotates clockwise. However, angular velocity and momentum are typically defined using the right-hand rule:
    - Curl the fingers of your right hand in the direction of the second hand's rotation (clockwise).
    - Your thumb points in the direction of the angular velocity vector $\vec{\omega}$, which, in this case, points downward (into the clock face).
- Time for One Full Rotation
  - One complete revolution of the second hand takes 60 seconds.
- Calculating Angular Velocity
  - Angular velocity ω is defined as the change in angular displacement over time. For uniform circular motion:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

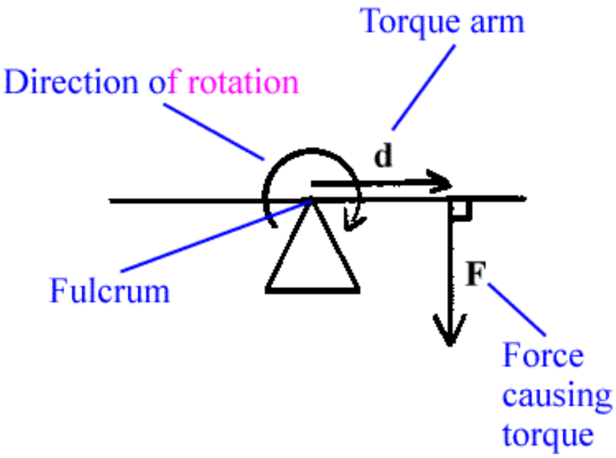
    - Here, $\Delta\theta = 2\pi$ radians (full circle) and $\Delta t = 60\ \text{s}$.
- Angular Velocity (in radians per second)

$$\omega = \frac{2\pi\ \text{rad}}{60\ \text{s}} = \frac{\pi}{30}\ \text{rad/s}$$

Torque

- Torque is the rotational equivalent of force.
- $\vec{\alpha} = \frac{\vec{\tau}_{\text{net}}}{I}$
  - Units of $\vec{\tau}$ are Nm
  - Units of $\vec{\alpha}$ are $\frac{\text{rad}}{s^2}$
- I rotational mass measures $\text{mass} \times \text{distance}^2$
  - Units of I are kg m^2
- $\tau = |\vec{\tau}| = \vec{r} \times \vec{F}_{\perp}$
  - $\vec{r}$ is the distance from the axis of rotation
  - $\vec{F}$ is the force applied

Diagram of Torque

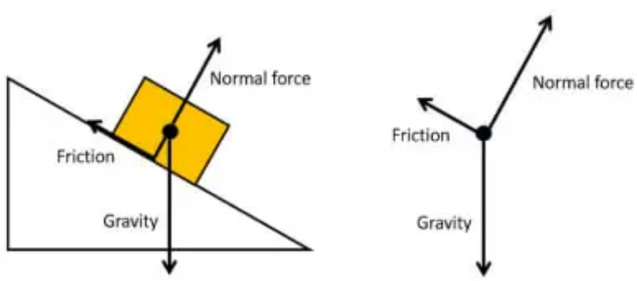


- Note $\vec{F} = \vec{F}_{\perp}$

Friction

- Friction is a force that opposes motion.
  - Static friction prevents motion
  - Sliding friction slows motion
- $\vec{F}_{\text{sliding friction}} = \mu_{\text{sliding friction}} \cdot \vec{F}_{\text{support}}$
  - μ is the coefficient of friction
  - $\vec{F}_{\text{support}}$ is the support force
- $\hat{F}_{\text{sliding friction}} = -\hat{v}$ if $\vec{v} > 0$
  - $\hat{v}$ is the velocity of the object
- $F_{\text{static friction}} = F_{\text{push}}$ if $F_{\text{push}} < \mu_{\text{static friction}} \cdot F_{\text{support}}$
  - F_{push} is the force applied to the object
  - $\mu_{\text{static friction}}$ is the coefficient of static friction
  - F_{support} is the support force

Visual Example of Friction



- Note $\vec{F}_{\text{support}} = \vec{F}_{\text{normal}}$

PollEV Answers

D ($\frac{2\pi}{\text{day}} = 0.000727 \frac{\text{radians}}{s}$)

- What is the angular speed of Earth around its axis?

To solve for the angular speed of the Earth around its axis, we can use the given information:

The Earth completes one full rotation (2π radians) in one day (24 hours).

First, convert the time period from days to seconds:

$$1 \text{ day} = 24 \text{ hours}$$

$$24 \text{ hours} = 24 \times 60 \text{ minutes} = 1440 \text{ minutes}$$

$$1440 \text{ minutes} = 1440 \times 60 \text{ seconds} = 86400 \text{ seconds}$$

Now, the angular speed (ω) is given by:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Where:

- $\Delta\theta = 2\pi \text{ radians}$
- $\Delta t = 86400 \text{ seconds}$

So,

$$\omega = \frac{2\pi \text{ radians}}{86400 \text{ seconds}}$$

Simplify the expression:

$$\omega = \frac{2\pi}{86400} \approx 0.0000727 \text{ radians/second}$$

Thus, the angular speed of the Earth around its axis is approximately $0.0000727 \text{ radians/second}$.