Lecture 1 (Math Review)

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Table of Contents

Symbols & Notation

Symbol	Meaning
$=$	Equals
$\neq$	Not equal
$\propto$	Proportional to
E	Exponent value
$\ldots$	Continued series

Exponents

Exponents (or powers) are shorthand for repeated multiplication. For instance:

- $a^2 = a \times a$ ("a squared")
- $a^3 = a \times a \times a$ ("a cubed")
- $a^n = a \times a \times \cdots \times a$ ("n times")

Rules for exponents:

- $a^1 = a$

- $a^0 = 1$
- $a^{-n} = \frac{1}{a^n}$

Example powers of 10:

- $10^6 = 1,000,000$
- $10^{-3} = 0.001$

Properties:

1. $a^m \times a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$
3. $(a^m)^n = a^{m \times n}$

Scientific Notation & E Notation

Scientific notation simplifies large/small numbers by expressing them as a small number multiplied by a power of 10. For example:

- 3.56×10^3 becomes $3.56E+3$
- 7.87×10^{-4} becomes $7.87E-4$

Operations:

- **Multiplication:** $(2.5 \times 10^3) \times (4.3 \times 10^5) = (2.5 \times 4.3) \times 10^{3+5} = 10.75 \times 10^8 = 1.075 \times 10^9$
- **Division:** $\frac{4.3 \times 10^5}{2.5 \times 10^3} = \frac{4.3}{2.5} \times 10^{5-3} = 1.72 \times 10^2$

Addition/Subtraction:

To add or subtract in scientific notation, first convert all numbers to the same power of 10:

- $4.3 \times 10^5 - 2.5 \times 10^3 = 4.3 \times 10^5 - 0.025 \times 10^5 = (4.3 - 0.025) \times 10^5 = 4.275 \times 10^5$

Units

Units are essential for expressing physical quantities, and errors in unit specification lead to significant issues. Common SI units:

- **Length:** $1\,km = 10^3\,m$, $1\,cm = 10^{-2}\,m$
- **Mass:** $1\,g = 10^{-3}\,kg$, $1\,mg = 10^{-6}\,kg$
- **Time:** $1\,ns = 10^{-9}\,s$

Conversion example:

To convert 1 cm to nanometers: $1\,cm = 10^{-2}\,m = 10^7\,nm$

Metric System Prefixes:

- **Tera (T):** $1\,T = 10^{12}$
- **Giga (G):** $1\,G = 10^{9}$
- **Mega (M):** $1\,M = 10^{6}$
- **Kilo (k):** $1\,k = 10^{3}$
- **Centi (c):** $1\,c = 10^{-2}$
- **Milli (m):** $1\,m = 10^{-3}$
- **Micro (\mu):** $1\,\mu = 10^{-6}$
- **Nano (n):** $1\,n = 10^{-9}$
- **Pico (p):** $1\,p = 10^{-12}$

Numerical Precision

Precision is critical when dealing with empirical data.

- **Significant figures**: Only write digits that are meaningful and supported by the measurement's precision.
- Example: The dinosaurs were killed off **about** 6.5×10^{7} years ago, not exactly 6.50×10^{7} years.

To specify a number with an uncertainty:

- Example: 1.364 ± 0.003 implies the true value is between 1.361 and 1.367.

Trigonometry

For a right triangle:

- $\sin\phi = \frac{opposite}{hypotenuse}$
- $\cos\phi = \frac{adjacent}{hypotenuse}$
- $\tan\phi = \frac{opposite}{adjacent}$

Mnemonic: **SOH-CAH-TOA**

Example:

- If $a = 4\,cm$, $b = 3\,cm$, and $c = 5\,cm$, then $\sin\phi = \frac{3}{5} = 0.6$

Radians vs Degrees:

$360^\circ = 2\pi$ radians. Therefore, 1 radian = $\frac{360^\circ}{2\pi} \approx 57.296^\circ$.

Proportional Relationships

Relationships between variables often take proportional forms:

- **Direct Proportion:** $a \propto b$
  - Example: The interest paid on a bank account is directly proportional to the balance.
- **Inverse Proportion:** $a \propto \frac{1}{b}$
  - Example: If the length of a guitar string doubles, its frequency is halved.
- **Square Proportion:** $a \propto b^2$
- **Inverse Square Proportion:** $a \propto \frac{1}{b^2}$

Proportionality can be written as an equation with a constant:

- Example: $f = \frac{C}{\ell}$, where f is frequency, ℓ is string length, and C is a constant depending on string tension and density.

Order-of-Magnitude Estimation

In physics, estimates are often good enough for practical purposes. The goal is to get the **order of magnitude** right, not exact values.

Example: Estimating the number of piano tuners in New York City.

- NYC population = 8 million.
- Estimate: 1 piano for every 100 people $\Rightarrow 80,000$ pianos.
- Each tuner tunes 500 pianos per year $\Rightarrow 160$ tuners needed.

Gravitational Law

Newton's law of universal gravitation: $F = G\frac{m_1 m_2}{d^2}$

Where:

- F is the gravitational force,
- G is the gravitational constant,
- m_1 and m_2 are the masses of the two objects,
- d is the distance between them.

Proportionality of Force:

- **Proportional to mass**: $F \propto m_1$, $F \propto m_2$.
- **Inverse-square relation to distance**: $F \propto \frac{1}{d^2}$.

Example:

- If the Earth-moon distance were halved, the gravitational force would increase by a factor of 4 due to the

inverse-square law.

Additional Exercises:

1. How many doctors are in the US?
2. How many subway cars does the NYC Transit Authority own?
3. How many humans have ever lived on Earth?

These questions invite you to apply proportional and order-of-magnitude reasoning to arrive at plausible estimates.
