Lecture 6

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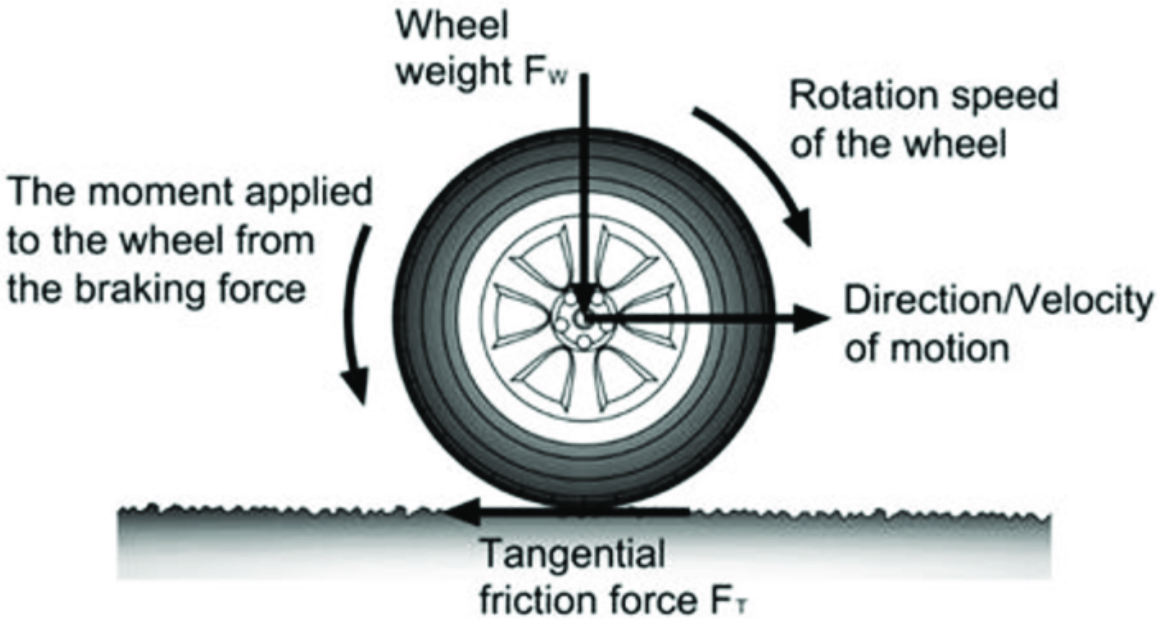
Wheels

- Wheels are a type of simple machine which are used to reduce friction and make it easier to move objects.
- $\vec{v}_{\text{ground}}$ must be 0 which means that the wheel is not slipping (because wheels *roll*) and so $\vec{v}_{\text{contact point}} = \vec{v}_{\text{ground}}$.
  - No work down by frictional forces from the ground to the wheel.
- $\vec{v}_{\text{top}} = \vec{v}_{\text{contact point}}$

Diagram of Forces on a Wheel



Angular Velocity of a Wheel

- Use right hand rule to determine the direction of the angular velocity vector.
  - $\vec{\omega} = \frac{\vec{v}_{\text{contact point}}}{r}$ where r is the radius of the wheel.

Work + Power

- Translational Motion
  - $\text{work} = W = \vec{F} \cdot \vec{\Delta x} = F_{\parallel} \Delta x$
    - W is the work done by a force.
    - $\vec{F}$ is the force applied.
    - $\vec{\Delta x}$ is the displacement of the object.
    - $F_{\parallel}$ is the component of the force parallel to the displacement.
  - $\text{power} = \vec{F} \cdot \vec{v} = \frac{W}{\Delta t}$
    - power is the rate at which work is done.
    - $\vec{F}$ is the force applied.
    - $\vec{v}$ is the velocity of the object.
    - W is the work done by a force.
    - Δt is the time interval.
- Rotational Motion
  - $\text{work} = W = \vec{\tau} \cdot \vec{\Delta\theta} = \vec{\tau} \cdot (\vec{\theta}_f - \vec{\theta}_i)$
    - W is the work done by a torque.
    - $\vec{\tau}$ is the torque applied.
    - $\vec{\Delta\theta}$ is the angular displacement of the object.
    - $\vec{\theta}_f$ is the final angle of the object.
    - $\vec{\theta}_i$ is the initial angle of the object.

- $\text{power} = \vec{\tau} \cdot \vec{\omega} = \frac{W}{\Delta t}$
  - power is the rate at which work is done.
  - $\vec{\tau}$ is the torque applied.
  - $\vec{\omega}$ is the angular velocity of the object.
  - W is the work done by a torque.
  - Δt is the time interval.

Kinetic Energy

- Kinetic energy is the energy of motion.
  - This means that the total energy of a system is constant.

Formulas for Kinetic Energy

- $K = \frac{1}{2}mv^2$ for translational motion.
- $K = \frac{1}{2}I\omega^2$ for rotational motion.
- Total energy is *conserved*
  - $u = mgh$
    - u is the potential energy of an object.
    - m is the mass of the object.
    - g is the acceleration due to gravity ($-9.8\frac{m}{s^2}$).
    - h is the height of the object.
  - $\text{total energy} = K + u + heat + ...$

Momentum and Impulse

Momentum

- Momentum is the product of mass and velocity.
  - Refers to the quantity of motion an object has.
- Translational Motion
  - $\vec{p} = m\vec{v}$
    - $\vec{p}$ is the momentum of an object ($kg\frac{m}{s}$).
    - m is the mass of the object.
    - $\vec{v}$ is the velocity of the object.
  - If no forces are applied, momentum is conserved.
- Rotational Motion
  - $\vec{L} = I\vec{\omega}$
    - $\vec{L}$ is the angular momentum of an object ($kg\frac{m^2}{s}$).
    - I is the moment of inertia of the object.
    - $\vec{\omega}$ is the angular velocity of the object
  - No $\vec{\tau}$ means $\vec{L}$ is constant.

Impulse

- Impulse is the change in momentum.
  - Quantifies the overall effect of a force on an object across a time interval.
- Translational Motion
  - $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = \vec{F} \cdot \Delta t$
    - $\Delta\vec{p}$ is the change in momentum.
    - $\vec{F}$ is the force applied.
    - Δt is the time interval.
- Rotational Motion
  - $\Delta\vec{L} = \vec{L}_f - \vec{L}_i = \vec{\tau} \cdot \Delta t$
    - $\Delta\vec{L}$ is the change in angular momentum.
    - $\vec{\tau}$ is the torque applied.
    - Δt is the time interval.

PollEV Answers

- *A* (+v, 0, -v)
  - What is the velocity vector of right middle point of a wheel's rim?
- *C* ($50\frac{m}{s}$)
  - A roller coaster car drops from rest at a height of $150m$ along a complicated track. Ignoring friction, what is the velocity of the car $25m$ above the ground?
    - To determine the speed of the roller coaster car at 25 meters above the ground, we can use the principle of **conservation of mechanical energy**. This principle states that in the absence of non-conservative forces (like friction), the total mechanical energy (potential + kinetic) of an object remains constant.

Given:

- **Initial Height (h_i)**: 150 meters
- **Final Height (h_f)**: 25 meters
- **Initial Speed (v_i)**: 0 m/s (since it starts from rest)
- **Acceleration due to Gravity (g)**: 9.8 m/s^2

Steps:

1. **Calculate the Change in Height (Δh)**:

$$\Delta h = h_i - h_f = 150\,\mathrm{m} - 25\,\mathrm{m} = 125\,\mathrm{m}$$

2. **Apply Conservation of Energy**:

$$\text{Potential Energy Initial} + \text{Kinetic Energy Initial} = \text{Potential Energy Final} + \text{Kinetic Energy Fina}$$

$$mgh_i + \frac{1}{2}mv_i^2 = mgh_f + \frac{1}{2}mv^2$$

Since the car starts from rest ($v_i = 0$), the equation simplifies to:

$$mgh_i = mgh_f + \frac{1}{2}mv^2$$

The mass (m) cancels out:

$$gh_i = gh_f + \frac{1}{2}v^2$$

3. **Solve for Final Speed (v)**:

$$gh_i - gh_f = \frac{1}{2}v^2$$

$$v^2 = 2g(h_i - h_f) = 2 \times 9.8\,\text{m/s}^2 \times 125\,\text{m} = 2450\,\text{m}^2/\text{s}^2$$

$$v = \sqrt{2450\,\text{m}^2/\text{s}^2} \approx 49.5\,\text{m/s}$$

Final Answer: The roller coaster car would be moving at approximately **49.5 meters per second** when it is 25 meters above the ground.