

Lecture 5

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Rotational Motion

Units

Translational Motion	Rotational Motion
Position: \vec{x} (m)	Angle: $\vec{\theta}$ (rad)
Velocity: \vec{v} ($\frac{m}{s}$)	Angular Velocity: $\vec{\omega}$ ($\frac{rad}{s}$)
Acceleration: \vec{a} ($\frac{m}{s^2}$)	Angular Acceleration: $\vec{\alpha}$ ($\frac{rad}{s^2}$)
Force: \vec{F} (N)	Torque: $\vec{\tau}$ (Nm)
Mass: m (kg)	Rotational Mass/Moment of Inertia: I (kg m ²)

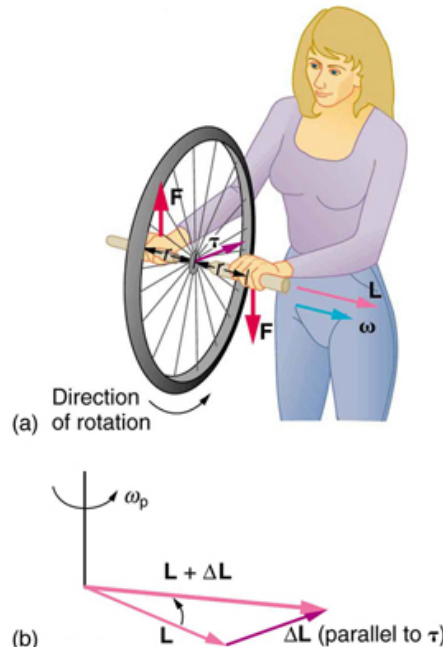
Rules

Translational Motion	Rotational Motion
No outside forces means constant \vec{v}	No outside torques means constant $\vec{\omega}$
$\vec{a} = \frac{\vec{F}_{net}}{m}$	$\vec{\alpha} = \frac{\vec{\tau}_{net}}{I}$
$\vec{F}_{BA} = -\vec{F}_{AB}$	$\vec{\tau}_{BA} = -\vec{\tau}_{AB}$

Right Hand Rule

- The right hand rule is used to determine the direction of the angular momentum vector ($\vec{\omega}$).
1. **Identify the Rotation Axis:** Determine the axis around which the object is rotating.
 2. **Curl Your Fingers:** Point the fingers of your right hand in the direction of the rotation (the direction in which the object is moving).
 3. **Thumb Direction:** Extend your thumb perpendicular to your fingers. The direction your thumb points is the direction of the angular momentum vector ($\vec{\omega}$).
- Example
 - If a wheel is rotating counterclockwise when viewed from above, you would:
 - Point your fingers in the direction of the rotation (counterclockwise).
 - Your thumb will point upwards, indicating that the angular momentum vector ($\vec{\omega}$) is directed upwards.

Visual Example



- Note $\vec{\omega} = \vec{L}$ in this example (b)

Angular Velocity

$$\vec{\omega} = \frac{\vec{\theta}_f - \vec{\theta}_i}{t_f - t_i}$$

- Example: Find the angular velocity of the second hand on a clock
 - Understanding the Rotation and Angular Velocity
 - The second hand completes one full rotation (360°) in 60 seconds. In radians, a full rotation is 2π radians.
 - Right-Hand Rule
 - For clocks, the second hand rotates clockwise. However, angular velocity and momentum are typically defined using the right-hand rule:
 - Curl the fingers of your right hand in the direction of the second hand's rotation (clockwise).

- Your thumb points in the direction of the angular velocity vector $\vec{\omega}$, which, in this case, points downward (into the clock face).
- Time for One Full Rotation
 - One complete revolution of the second hand takes 60 seconds.
- Calculating Angular Velocity
 - Angular velocity ω is defined as the change in angular displacement over time. For uniform circular motion:

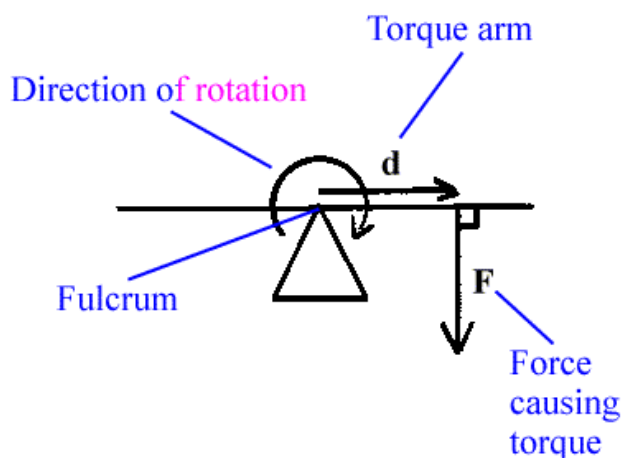
$$\omega = \frac{\Delta\theta}{\Delta t}$$

- Here, $\Delta\theta = 2\pi$ radians (full circle) and $\Delta t = 60$ s.
- Angular Velocity (in radians per second)

$$\omega = \frac{2\pi \text{ rad}}{60 \text{ s}} = \frac{\pi}{30} \text{ rad/s}$$

Torque

- Torque is the rotational equivalent of force.
- $\vec{\alpha} = \frac{\vec{\tau}_{\text{net}}}{I}$
 - Units of $\vec{\tau}$ are Nm
 - Units of $\vec{\alpha}$ are $\frac{\text{rad}}{\text{s}^2}$
- I rotational mass measures mass \times distance²
 - Units of I are kg m²
- $\tau = |\vec{\tau}| = \vec{r} \times \vec{F}_{\perp}$
 - \vec{r} is the distance from the axis of rotation



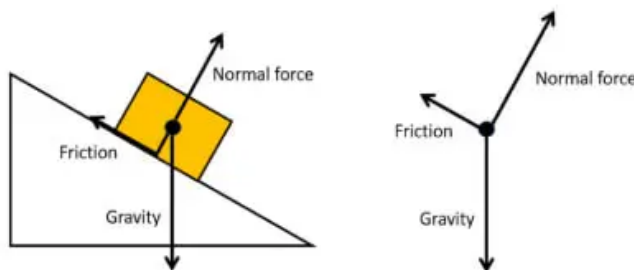
- \vec{F} is the force applied

Friction

- Friction is a force that opposes motion.
 - Static friction prevents motion
 - Sliding friction slows motion

- $\vec{F}_{\text{sliding friction}} = \mu_{\text{sliding friction}} \cdot \vec{F}_{\text{support}}$
 - μ is the coefficient of friction
 - \vec{F}_{support} is the support force
- $\hat{F}_{\text{sliding friction}} = -\hat{v}$ if $\vec{v} > 0$
 - \hat{v} is the velocity of the object
- $F_{\text{static friction}} = F_{\text{push}}$ if $F_{\text{push}} < \mu_{\text{static friction}} \cdot F_{\text{support}}$
 - F_{push} is the force applied to the object
 - $\mu_{\text{static friction}}$ is the coefficient of static friction
 - F_{support} is the support force

Visual Example



- Note $\vec{F}_{\text{support}} = \vec{F}_{\text{normal}}$

PolI EV Answers

$$D\left(\frac{2\pi}{\text{day}} = 0.000727 \frac{\text{radians}}{\text{s}}\right)$$

- What is the angular speed of Earth around its axis?

To solve for the angular speed of the Earth around its axis, we can use the given information:

The Earth completes one full rotation (2π radians) in one day (24 hours).

First, convert the time period from days to seconds:

$$1 \text{ day} = 24 \text{ hours}$$

$$24 \text{ hours} = 24 \times 60 \text{ minutes} = 1440 \text{ minutes}$$

$$1440 \text{ minutes} = 1440 \times 60 \text{ seconds} = 86400 \text{ seconds}$$

Now, the angular speed (ω) is given by:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Where:

- $\Delta\theta = 2\pi$ radians
- $\Delta t = 86400$ seconds

So,

$$\omega = \frac{2\pi \text{ radians}}{86400 \text{ seconds}}$$

Simplify the expression:

$$\omega = \frac{2\pi}{86400} \approx 0.0000727 \text{ radians/second}$$

Thus, the angular speed of the Earth around its axis is approximately 0.0000727 radians/second.