

Closure Under “Plus”

1. For every language A over alphabet Σ , let $A^+ = \{x_1 \dots x_k \mid k \geq 1 \text{ and each } x_i \in \Sigma^*\}$. Show that the class of regular languages is closed under the “plus” operation.

- a. We know that every regular language A is closed under all regular expressions. Thus, A is closed under the “star” operation, where $A^* = \{x_0 x_1 \dots x_k \mid k \geq 0 \text{ and each } x_i \in A\}$.

We can observe that A^+ is equal to A^* with the empty string (x_0 or $\{\epsilon\}$) removed, or $A^+ = \{\epsilon\} \cup A^*$.

Because A^+ contains every string in A^* besides for the empty string, it can be written as the concatenation of {anything accepted by A } with $\{A^*\}$, or $A \circ A^*$. (Concatenating A with A^* effectively eliminates the possibility of the empty set that A^* includes.)

If A is a regular language, then A^* is also a regular language, and both are closed under concatenation. Thus, $A^+ = A \circ A^* = L$, where L is a regular language.

Thus, the class of regular languages is closed under the “plus” operation.

NFA Transformations

1. Show that a language is regular *if and only if* it can be recognized by some NFA with at most one accepting state.

- a. To prove an “if and only if,” we must prove “if” and “only if” separately.

“If”

By definition, every NFA represents a regular language; thus, *if* a language is recognized by some NFA with one accepting state, it is a regular language.

“Only If”

Every NFA can be written as an equivalent NFA with at most one accepting state. This can be proven via construction, as follows:

1. Construct an NFA N_1 with some number of accepting states.
2. Construct a new NFA, N , that **adds an epsilon transition** from each accepting state of N_1 to a single new accepting state.
3. Make this new state the only accepting state of N .

Thus, *only if* a language can be recognized by some NFA with at most one accepting state is it a regular language.