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Design and Analysis of Algorithms
Homework: KT 1.1, 1.2, 1.3

1.1. False.

Proof by Counterexample:
In a group consisting of {x, y} and {i, j}, in which their preferences are:

x: {i, j}   i: {y, x}
y: {j, i}   j: {x, y}

No pair of ({x, y}, {i, j}) exists such that both sides are left completely satisfied (each gets their #1 choice).

To show this exhaustively, here is every matching:
(x, i)—i's 2nd choice; **(x, j)**—x's 2nd choice; **(y, i)**—y's 2nd choice; **(y, j)**—j's 2nd choice.

1.2. True.

Because w is first on m's preference list, we *know* that m will try to match with w.

Next, according to Gale-Shapley, one of three outcomes will occur:
1. If w is not yet matched, m-w will match. In this outcome, they end up matching.
2. Else, if m prefers w to m's current partner, m-w will match. In this outcome, too, they end up matching.
3. Else, m rejects w. This outcome will **not** happen, because m is first on w's preference list.

Thus, in every stable matching, m will always match with w.

1.3. There isn't always a stable pair.

Proof by Counterexample:

Suppose A and B each have 2 shows (n=2). Let's call A's shows a and b, and B's shows c and d. Assume A's shows are rated 1 & 3 respectively, and B's shows are rated 2 & 4 respectively.

A's scheduling options are a followed by b, or b followed by a.
Similarly, B's schedule can be either c followed by d, or d followed by c.

Thus, there are a total of four scheduling scenarios. Here are all possible schedules:
1. {a,b}, {c,d}: A wins none. A will thus want to flip their shows (see outcome #3) to win one.
2. {a,b}, {d,c}: Same as outcome #3. B will flip to arrive back at outcome #1.
3. {b,a}, {c,d}: Both win one. B will thus want to flip their shows (see outcome #4) to win two.
4. {b,a}, {d,c}: Same as outcome #1. A wins none, and will flip to arrive at outcome #2.

Thus, no stable set of schedules exist for this set of tv shows and ratings.