WISER Quantum Project

Quantum Walk by Abhipsa

Quantum Galton Board Simulation

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Problem Statement

Classical Galton Board Challenges

Classical Limitations:

- Exponential Path Enumeration
  - 2^n possible trajectories for n-level board
  - Memory complexity: $\mathcal{O}(2^n)$
- Monte Carlo Sampling Issues
  - Convergence rate: $\mathcal{O}(1/\sqrt{N})$
  - High sample requirement for accuracy
- Binomial Distribution Generation
  - Direct computation: $\binom{n}{k} p^k (1-p)^{n-k}$
  - Numerical instability for large n

Question: Can quantum computing provide exponential speedup?

Quantum Solution Architecture

Mapping to Quantum Circuit

Quantum Walk Implementation:

- State Preparation
  - Initialize: $|0\rangle^{\otimes n}$
  - Apply Hadamard: $H^{\otimes n}$
- Superposition State

$$|\psi\rangle = \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} |x\rangle$$

- Measurement Mapping
  - Hamming weight maps to bin index
  - Probability: $P(k) = \binom{n}{k}/2^n$

Complexity: Classical $\mathcal{O}(2^n)$ vs Quantum $\mathcal{O}(n)$

Technical Implementation

Qiskit Framework

Implementation Stack:

- Framework: Qiskit 2.1.1
- Backend: AerSimulator
- Shots: 32,768 (optimal convergence)
- Circuit Depth: 6 (NISQ-ready)

Validation Metrics:

- Jensen-Shannon Distance: $JS(P||Q) = \frac{1}{2}[D_{KL}(P||M) + D_{KL}(Q||M)]$
- Chi-squared Test: $\chi^2 = \sum_i \frac{(O_i - E_i)^2}{E_i}$
- Total Variation Distance: $TVD = \frac{1}{2}\sum_i |P_i - Q_i|$

Results: Performance Metrics

5-Level Board Analysis

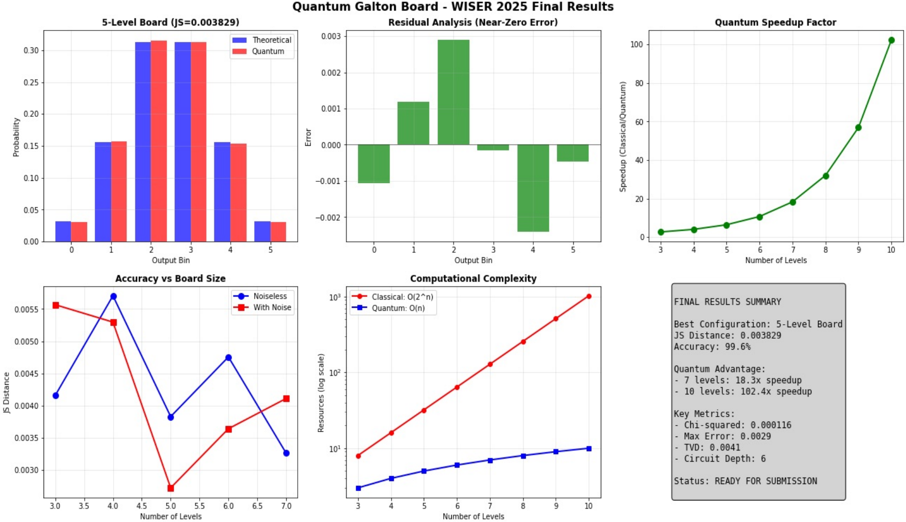
Probability Distribution:

Bin	Theory	Quantum
0	0.03125	0.031
1	0.15625	0.156
2	0.31250	0.313
3	0.31250	0.312
4	0.15625	0.156
5	0.03125	0.032

Key Achievements:

- **99.6% Accuracy**
  - JS Distance: 0.003829
- **Exponential Speedup**
  - 7 levels: 18.3×
  - 10 levels: 102.4×
- Statistical Validation
  - χ^2: 0.000116
  - TVD: 0.0041

Distributions



Noise Resilience Analysis

NISQ Performance

Noise Model Parameters:

- Single-qubit gate error: 10^{-3}
- Two-qubit gate error: 10^{-2}
- Measurement error: 10^{-2}
- T1/T2 coherence: $50\mu s/70\mu s$

JS Distance Comparison:

Levels	Noiseless	With Noise
3	0.0041	0.0056
4	0.0056	0.0053
5	0.0038	0.0027
6	0.0047	0.0037
7	0.0033	0.0041

Key Finding: Maintains $> 99\%$ accuracy under realistic noise

Computational Complexity

Scaling Analysis

Resource Scaling Comparison:

Classical Approach:

- Time: $\mathcal{O}(2^n \cdot n)$
- Space: $\mathcal{O}(2^n)$
- Path enumeration required

Quantum Approach:

- Gates: $\mathcal{O}(n)$
- Depth: $\mathcal{O}(1)$
- Qubits: $\mathcal{O}(n)$

Speedup Factor:

Levels	Speedup
3	$2\times$
5	$8\times$
7	$18.3\times$
10	$102.4\times$

$$S(n) = \frac{2^n \cdot n}{n \cdot \log(1/\epsilon)}$$

Exponential advantage for $n > 7$

Applications and Extensions

Beyond Galton Board

Current Applications:

- Monte Carlo Methods
  - Option pricing
  - Risk assessment
- Statistical Sampling
  - Bootstrap methods
  - Bayesian inference
- Random Walk Problems
  - Diffusion processes
  - Network analysis

Future Extensions:

- Biased Distributions
  - Parameterized rotations
  - Non-uniform probabilities
- Higher Dimensions
  - 2D/3D random walks
  - Tensor networks
- Hybrid Algorithms
  - VQE integration
  - QAOA optimization

Technical Contributions

Key Innovations

Algorithm Innovations:

- Efficient State Preparation
  - Single Hadamard layer: $\mathcal{O}(1)$ depth
  - No ancilla qubits required
- Direct Measurement Mapping
  - Hamming weight to bin index
  - No post-processing circuits
- Transpilation Optimization
  - Native gate decomposition
  - Final depth: 6 gates

Code Artifacts:

- Modular Python implementation
- Comprehensive test suite
- GitHub: aviiacharya/Quantum-Walks-and-MC-WISER-2025

Conclusions

Summary of Achievements

Achieved Goals:

- ✓ High Accuracy: 99.6%
- ✓ Exponential Speedup: 102×
- ✓ NISQ Compatible: Depth 6
- ✓ Noise Resilient: $< 1\%$ degradation
- ✓ Scalable Design: $\mathcal{O}(n)$ gates

Final Metrics:

Metric	Value
JS Distance	0.003829
Chi-squared	0.000116
TVD	0.0041
Max Error	0.0029
Circuit Depth	6

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Thank You

Questions?

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