Shape Optimization of Toroidal Sweeps

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EECS

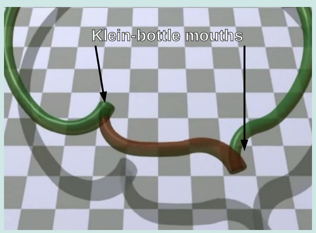
Smooth Transformations

- Homotopy:smooth transformation from one surface to another without introducing sharp creases or tears
- Want to visualize such transformations without specifying intermediate states, achieved by an optimization process over some energy functional

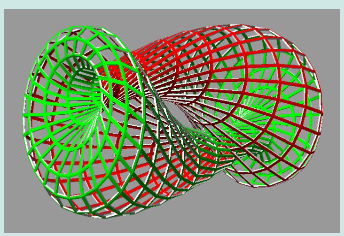
- Everting a torus is a homotopy
- **Want to visualize eversion process proceeding *spontaneously* from some halfway point**

Torus Eversion

- Turn an torus inside-out by deforming it into two Klein-bottle mouths attached to each other
- Proceed from symmetrical halfway point by minimizing energy. Can we find such an energy functional?



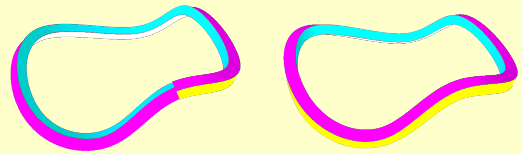
Torus eversion, rendered by Chéritat

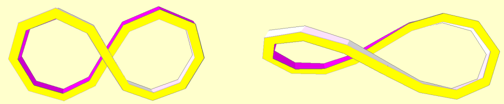


A possible halfway point in the eversion process.

Bending and Twisting

Sweeping out a curve in space causes start and end orientations to be mismatched. The amount of mismatch is the twist. Rotating curve by this amount gradually across the sweep matches up the endpoints.





A planar figure-8 with no twist, but more bending than a circle

Unfolding the figure-8

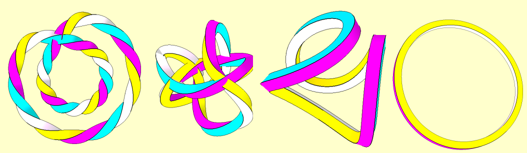
Same homotopy class

Fully unfolded into a circle with 360° of accumulated twist from the unfolding

Gradient Descent

- Surfaces represented as polylines with circular cross-sections
- Optimization via Gradient Descent
  - Vary the coordinates of each control point (parameters), revert change if energy does not decrease
  - Iterate, then stop when no decrease possible
  - Susceptible to local minima in energy functional

A highly twisted and bent curve unfolds into a circle with no twist, using gradient descent.

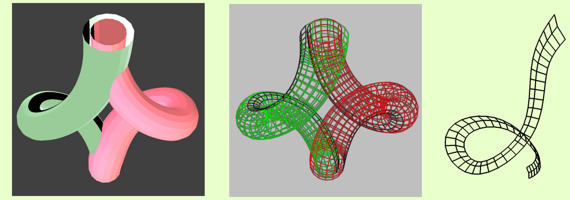


Energy Functionals

- **Bending:** Compute angle between two struts and take deviation from collinearity
- **Twisting:** Using rotation minimizing frames,
  - Closed Curves: compare beginning and end orientations
  - Non-closed curves: forward project normal vectors and compare to a priori expected ending orientation
- Can weigh these parts to trade-off between them

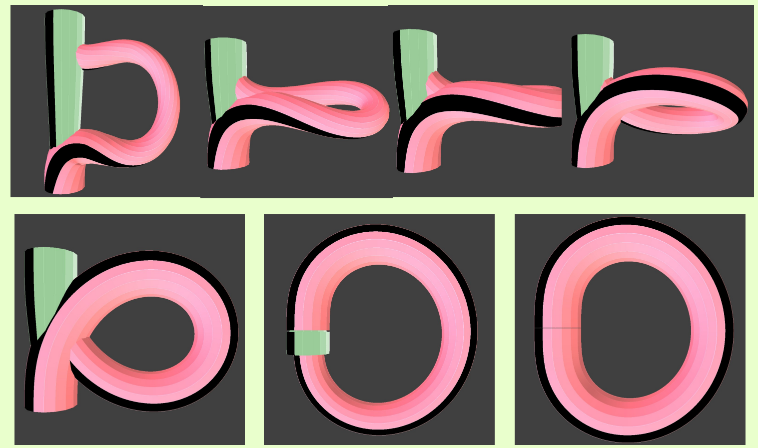
Optimizing the Eversion Process

A halfway point: First approach. Model two arms as separate, non-closed space curves, connected by end caps (not shown). Minimize bending and twist of each arm separately. Both arms incur 180° twist for a total of 360° twist across both arms.



The stripe on the right arm is shown here. If an ant were to walk on the stripe, it would end up rotating 180°.

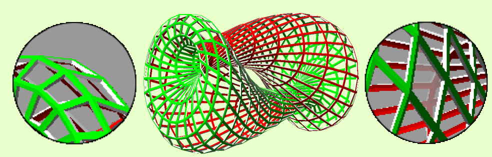
Straightening out one arm: By allowing the green arm to straighten and the end caps to come closer together, the red arm is forced to incur all 360° of twist. The twist is then removed by folding the red arm until it is untwisted.



Collapsing the straight arm: Finally, the green arm is shortened until it completely collapses and only the inverted torus remains.

Future Work

- Fully gridded parameterization of surface
  - Requires many control points, so many parameters over which to optimize



Bending calculated as deviation of adjacent patches from co-planarity

Integrate these over entire surface area

Twisting calculated based on deviation of edges from right angles.