Shape Optimization of Toroidal Sweeps

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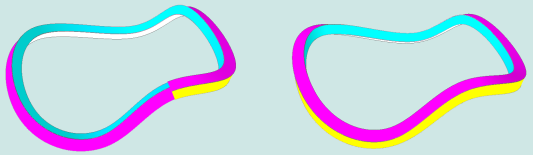
Objective

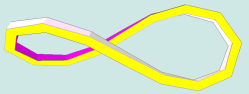
- Minimize energy of a curve incurred by bending and twisting
- Evert a torus in a minimal energy fashion

Trading Off Bending and Twisting



Sweeping out a curve in space causes start and end orientations to be mismatched. The amount of mismatch is the twist. Rotating curve by this amount gradually across the sweep matches up the endpoints.

A planar figure-8 with no twist, but more bending than a circle.

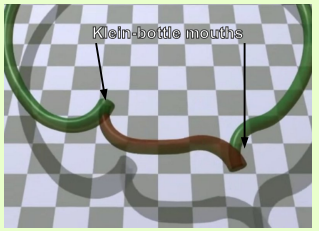


Unfolding the figure-8.

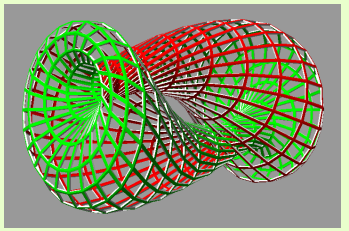
Fully unfolded into a circle with 360° of accumulated twist from the unfolding.

Torus Eversion

- Turn an torus inside-out by deforming it into two Klein-bottle mouths attached to each other
- Additionally, minimize energy along the way

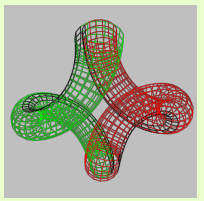


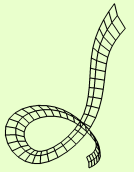
Torus eversion, rendered by Chéritat



Visualizing the Klein-bottle mouths

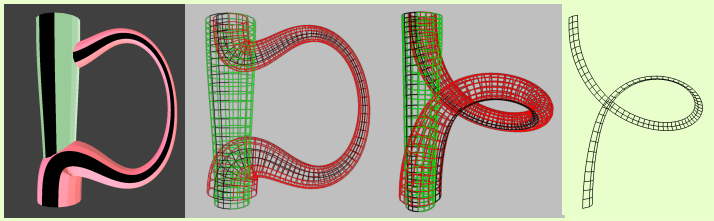
A halfway point: First approach. Model two arms as separate, non-closed space curves, connected by end caps (not shown). Minimize bending and twist of each arm separately. Both arms incur 180° twist for a total of 360° twist across both arms.

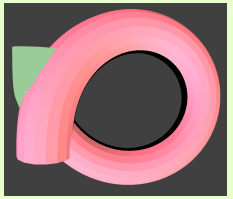


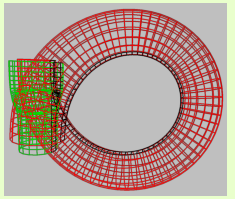


The stripe on the right arm is shown here. If an ant were to walk on the stripe, it would end up rotating 180° around the Z-axis.

Straightening out one arm: By allowing the green arm to straighten and the end caps to come closer together, the red arm starts incurring the full 360° of twist. Starting with the configuration on the left, the red arm folds up, incurring 270° of twist in the last two images.





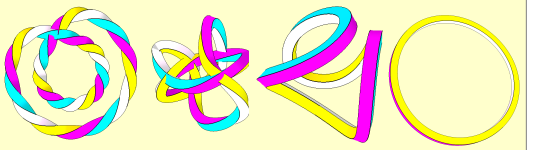


Collapsing the straight arm: As the end caps move closer together, the green arm becomes shorter and the red arm absorbs all 360° degrees of twist. Now, the green arm can be fully collapsed and the red arm can be unfolded into a torus with no twist.

Gradient Descent

- Surfaces represented as polylines with cross-sections
- Optimization via Gradient Descent
  - Vary the coordinates of each control point (parameters), see if energy decreases
  - Change all parameters according to amount by which energy decreases
  - Susceptible to local minima in energy functional

A highly twisted and bent curve unfolds into a circle with no twist, using gradient descent.

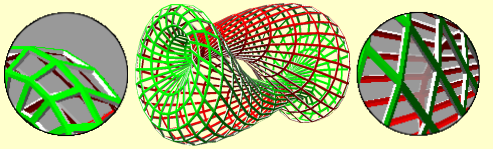


Energy Functionals

- **Bending:** Compute angle between two struts and take deviation from collinearity
- **Twisting:** Using rotation minimizing frames,
  - Closed Curves: compare beginning and end orientations
  - Non-closed curves: forward project normal vectors and compare to a priori expected ending orientation
- **Length Constraints:** Bound length of struts by treating them like stretched springs
- Can weigh these parts to trade-off between them

Future Work

- Fully gridded parameterization of surface
  - Requires many control points, so many parameters over which to optimize



Bending calculated as deviation of adjacent patches from co-planarity

Integrate these over entire surface area

Twisting calculated based on deviation of edges from right angles.