# Tutorial 1

E2 202

Autumn 2022

**Exercise 1.**

1. Find a bijection between $\mathbb{N}$ and $\mathbb{Z}$.
2. Show that countable union of countable sets is countable.
3. Prove that $\mathbb{Q}$ is countable.

**Exercise 2.** Show that $[0, 1]$ is uncountable.

**Exercise 3.** If $\{A_i . i \in \mathbb{N}\}$ are subsets of $\mathcal{F}$, then show that $\cup_{i=1}^{\infty} A_i \in \mathcal{F}$ and $\cap_{i=1}^{N} A_i \in \mathcal{F}$.

**Exercise 4.** Show that if $\mathbb{P}(E_i) = 1$ for all $i \in \mathbb{N}$ then $\mathbb{P}(\cap_{i \in \mathbb{N}} E_i) = 1$.

**Exercise 5.** If $\mathcal{C}$ is an arbitrary collection of subsets of $\Omega$ and $\mathcal{H}$ is any $\sigma$-algebra such that $\mathcal{C} \subseteq \mathcal{H}$, then $\exists$ a $\sigma$-algebra $\sigma(\mathcal{C}) \subseteq \mathcal{H}$.

**Exercise 6.** Show that singleton sets in $\mathbb{R}$ are Borel-measurable sets.