

## \* Policy Evaluation

Given a policy  $\pi = \{\mu_0, \dots, \mu_{N-1}\}$ , the value function of  $\pi$  is defined as

$$J^\pi(x_0) = \mathbb{E}_{w_k; k=0,1,\dots,N-1} \left[ g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, \mu_k(x_k), w_k) \right]$$

So  $J^\pi: S_0 \rightarrow \mathbb{R}$  takes the total cost if policy  $\pi$  is applied at initial state  $x_0$ .

Corollary 1: For every initial state  $x_0$ , the value of policy  $\pi$ ,  $J^\pi(x_0)$  of the basic problem is equal to  $J_0(x_0)$  when the function is given by the last step of the following algorithm, which proceeds backward in time from period  $N-1$  to period 0.

$$J_N(x_N) = g_N(x_N)$$

$$J_k(x_k) = \mathbb{E}_{w_k} \left[ g_k(x_k, \mu_k(x_k), w_k) + J_{k+1}(f_k(x_k, \mu_k(x_k), w_k)) \right]$$

$$k = 0, 1, \dots, N-1$$

Proof: Just expand.

— This is policy evaluation algorithm.