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Article *in* Engineering Optimization · September 2017

DOI: 10.1080/0305215X.2017.1367392

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Taylor & Francis
Taylor & Francis Group

Engineering Optimization

ISSN: 0305-215X (Print) 1029-0273 (Online) Journal homepage: http://www.tandfonline.com/loi/geno20

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To cite this article: Dinh-Cong Du, Ho-Huu Vinh, Vo-Duy Trung, Ngo-Thi Hong Quyen & Nguyen-Thoi Trung (2017): Efficiency of Jaya algorithm for solving the optimization-based structural damage identification problem based on a hybrid objective function, Engineering Optimization, DOI: 10.1080/0305215X.2017.1367392

To link to this article: http://dx.doi.org/10.1080/0305215X.2017.1367392

Published online: 13 Sep 2017.

ENGINEERING OPTIMIZATION, 2017
https://doi.org/10.1080/0305215X.2017.1367392

Efficiency of Jaya algorithm for solving the optimization-based structural damage identification problem based on a hybrid objective function

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ABSTRACT

A Jaya algorithm was recently proposed for solving effectively both constrained and unconstrained optimization problems. In this article, the Jaya algorithm is further extended for solving the optimization-based damage identification problem. In the current optimization problem, the vector of design variables represents the damage extent of elements discretized by the finite element model, and a hybrid objective function is proposed by combining two different objective functions to determine the sites and extent of damage. The first one is based on the multiple damage location assurance criterion and the second one is based on modal flexibility change. The robustness and efficiency of the proposed damage detection method are verified through three specific structures. The obtained results indicate that even under relatively high noise level, the proposed method not only successfully detects and quantifies damage in engineering structures, but also shows better efficiency in terms of computational cost.

ARTICLE HISTORY
Received 5 May 2017
Accepted 13 July 2017

KEYWORDS
Jaya algorithm; damage detection; hybrid objective function; multiple damage location assurance criterion; flexibility matrix

1. Introduction

During the lifetime of a structure, damage can occur for various unforeseen reasons, such as ageing due to usage over time, effects of the environment and accidental events. Damage that leads to a decrease in structural stiffness may result in structural failure if the damage goes undetected. Therefore, monitoring and identifying the damage is necessary to guarantee the safe and serviceable conditions of existing structures. Early damage detection also helps to reduce maintenance costs and prolong the service life.

It is well known that the presence of damage in a structural system will induce changes in the modal parameters, which can be used as signals to infer the state of damage. Natural frequencies and mode shapes, the most common parameters, have been widely used for damage identification. Although natural frequencies are relatively easy to measure with high accuracy, the use of natural frequencies is limited by their low sensitivity to local damage. In comparison with natural frequencies, mode shapes can provide more information and are more sensitive to structural damage detection (Yan *et al*. 2007; Kim *et al*. 2003). However, these mode shapes are required at many locations on the structure and are sensitive to noise contamination. Besides, local changes in modal shapes are not

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recognizable in the case of low-level damage (Katunin 2015). To overcome these drawbacks of using natural frequencies or mode shapes, several variations, such as frequency response functions (Lee and Shin 2002; Rahmatalla, Eun, and Lee 2012), modal frequency curves (Zhang *et al.* 2014; Yang and Olutunde Oyadiji 2017), derivatives of mode shapes (Abdel Wahab and De Roeck 1999; Abdo 2012) and modal strain energy (Shi, Law, and Zhang 2000; Li, Fang, and Hu 2007; Li *et al.* 2016), have been developed by many researchers. Some approaches based on signal processing techniques using modal curvature have also been proposed in the literature (Qiao and Cao 2008; Cao *et al.* 2014; Rucka 2011). Of particular interest are the studies by Pandey and Biswas (1994, 1995), in which the authors first developed a damage detection and localization method based on changes in a flexibility matrix which combines the use of both natural frequencies and mode shapes. As pointed out by Jun Zhao (1999), this method proved to be more sensitive to structural defects than either the natural frequencies or the mode shapes. Since then, many researchers have applied and developed various approaches based on modal flexibility for the diagnosis and assessment of structural damage. For example, Li *et al.* (1999) proposed a flexibility approach for damage identification of cantilever-type structures using the data from dynamic modes. Stutz, Castello, and Rochinha (2005) presented a flexibility-based continuum method for damage detection. Masoumi, Jamshidi, and Bamdad (2015) formulated an objective function using a generalized flexibility matrix, then carrying out damage detection by solving a constrained optimization problem. Zare Hosseinzadeh *et al.* (2014) combined the use of a flexibility matrix using the data from only a few modes with an optimization algorithm for structural damage location and quantification. Perera, Ruiz, and Manzano (2007) introduced an objective function formulated by combining modal flexibility with another function.

One popular approach to determining the location and extent of structural damage is to solve inverse problems using optimization algorithms in which the objective function is defined in terms of discrepancies between the vibration data of the test models and those of the intact model (Friswell 2007; Majumdar *et al.* 2014). Over the past decade, various metaheuristic algorithms have been proposed for use in this approach, such as the genetic algorithm (GA) (Vakil-Baghmisheh *et al.* 2008; Liu and Jiao 2011), particle swarm optimization (PSO) (Yu and Chen 2010; Nanda, Maity, and Maiti 2014), ant colony optimization (ACO) (Majumdar, Maiti, and Maity 2012; Braun, Chiwiacowsky, and omez 2015), cuckoo search (CS) algorithm (Zare Hosseinzadeh *et al.* 2014; Xu, Liu, and Lu 2016) and evolutionary intelligence algorithms (Bagheri, Razeghi, and Ghodrati Amiri 2012; Stutz, Tenenbaum, and Corrêa 2015; Seyedpoor, Shahbandeh, and Yazdanpanah 2015; Masoumi, Jamshidi, and Bamdad 2015; Dinh-Cong *et al.* 2017). These algorithms have shown their effectiveness in damage identification. However, there are still many areas for further improvement, especially those relating to the computational cost of metaheuristic algorithms.

An advanced optimization algorithm called Jaya (a Sanskrit word meaning victory), first proposed by Venkata Rao (2016), is simple and easy to apply and does not require any algorithm-specific control parameters. The Jaya algorithm has been demonstrated to be capable of solving both constrained and unconstrained optimization problems, and its results have proved that this algorithm is more effective than other well-known optimization algorithms, *i.e.* GA, PSO, differential evolution (DE), ABC and teaching–learning-based optimization algorithms (Venkata Rao 2016). Since then, it has been rapidly applied and developed for numerous problems in various fields such as mechanical engineering (Abhishek *et al.* 2016; Rao *et al.* 2016), artificial neural network training (Suraj and Ghosh 2016), electrical engineering (Bhoye *et al.* 2016), structural optimization (Venkata Rao and Saroj 2017) and thermal system design (Rao and More 2017; Rao and Saroj 2017). Motivated by the success and potential application of the Jaya algorithm, this article tries to extend the algorithm for solving optimization-based damage identification problems.

In this study, a hybrid objective function is proposed by combining two different objective functions for determining the sites and extent of damage. The first one is based on the multiple damage location assurance criterion (MDLAC) and the second one is based on modal flexibility change. The advantage of this objective function is that only the first few modes are used for the identification of damage. The damage is represented by a reduction of the element stiffness matrix using a damage ratio, which is defined as design variables. The experimental data are simulated numerically by the standard finite element method. To investigate the performance of the proposed damage diagnosis method, three numerical examples are presented: a two-dimensional (2D) frame (one bay), a 21-bar planar truss and a four-storey frame (three bays). The influence of relatively high noise measurements on the accuracy of the proposed method is also considered in the examples. The damage detection results obtained by the proposed method are compared with those gained by other optimization algorithms such as the original DE and CS.

The rest of article is organized as follows. Section 2 presents the formulation of the problem under consideration, the Jaya algorithm, damage modelling and measurement noise. In Section 3, numerical simulations are studied to evaluate the performance of the proposed method. Finally, concluding remarks are given in Section 4.

2. Formulation of optimization-based damage identification problem

In the optimization-based damage identification problem, the damage identification process is generally achieved by minimizing an objective function which defines the difference between the measured and analytical modal parameters (*e.g.* natural frequency or mode shapes). To obtain the optimal solutions of the damage variables which are the damage ratios of the elements, the Jaya algorithm is used here as an efficient optimization tool for estimating the location and size of damage.

This section briefly introduces the formulation of this problem, including a hybrid objective function, the Jaya algorithm, damage modelling and measurement noise.

2.1. Objective function

Many different objective functions have been introduced by previous research. Some typical objective functions used in this approach are the frequency error (Majumdar, Maiti, and Maity 2012), MDLAC (Guo and Li 2009; Messina, Williams, and Contursi 1998) and the flexibility matrix error (Stutz, Castello, and Rochinha 2005; Stutz, Tenenbaum, and Corrêa 2015). Within this context, the correct choice of suitable objective function is very important because it can significantly influence the success of any damage detection method.

The present work proposes a combination of two different objective functions to give an effective hybrid objective function for determining the sites and extent of damage. The first one is based on the MDLAC (natural frequency) and the second one is based on modal flexibility change.

The MDLAC, first introduced by Messina, Williams, and Contursi (1998), is an extension of the damage location assurance criterion (DLAC) to multi-damage identification. The advantage of MDLAC over DLAC is that the sensitivity matrix is incorporated into the correlation equation. The MDLAC function is expressed as follows:

$$\text{MDLAC}\,(\mathbf{x}) = \frac{|\Delta\mathbf{f}\delta\mathbf{f}(\mathbf{x})|^2}{(\Delta\mathbf{f}^{\mathrm{T}}\Delta\mathbf{f})(\delta\mathbf{f}^{\mathrm{T}}(\mathbf{x})\delta\mathbf{f}^{\mathrm{T}}(\mathbf{x}))}, \quad \mathbf{x} = (x_1, \ldots, x_n) \in [0, 1]^n \tag{1}$$

where $\mathbf{x}$ is the vector depicting the damage extent of n elements; and $\Delta\mathbf{f}$ and $\delta\mathbf{f}(\mathbf{x})$ are, respectively, the measured frequency change vector and the analytical frequency change vector, which are, respectively, defined by

$$\Delta\mathbf{f} = \frac{\mathbf{f}_{\mathrm{ud}} - \mathbf{f}_{\mathrm{exp}}}{\mathbf{f}_{\mathrm{ud}}}, \quad \delta\mathbf{f}(\mathbf{x}) = \frac{\mathbf{f}_{\mathrm{ud}} - \mathbf{f}_{\mathrm{ana}}(\mathbf{x})}{\mathbf{f}_{\mathrm{ud}}} \tag{2}$$

where $\mathbf{f}_{\mathrm{ud}}$ and $\mathbf{f}_{\mathrm{exp}}$ are denoted as the measured frequency vector of the undamaged (reference) and damaged structures, respectively; and $\mathbf{f}_{\mathrm{ana}}(\mathbf{x})$ is the analytical frequency vector corresponding to the damage variable vector $\mathbf{x}$.

It can be shown that the value MDLAC is between [0, 1], and MDLAC$(\mathbf{x}) = 1$ if $\mathbf{f}_{\mathrm{exp}} = \mathbf{f}_{\mathrm{ana}}(\mathbf{x})$ (*i.e.* the damage variable vector $\mathbf{x}$ represents exactly the deterioration of the structure). Nevertheless, it is necessary to point out that the function is very sensitive to damaged elements, while its sensitivity to healthy elements is low (Nobahari and Seyedpoor 2011). To tackle this deficiency, the MDLAC should be modified by combining it with another function.

As mentioned in Section 1, the modal flexibility is well known as a suitable parameter for predicting the location and extent of damage because of its high sensitivity to damage. The flexibility matrix is much more sensitive to damage than other parameters (*e.g.* natural frequency or mode shape) (Jun Zhao 1999). For this reason, a cost function based on this parameter is defined here, using the norm of the difference between the flexibility matrix of a modal test and the flexibility matrix of an analytical model. The function is written in the form:

$$\mathrm{F}(\mathbf{x}) = \frac{1}{nmod}\sum_{j}^{\mathrm{nc}}\left(\frac{\|\mathbf{F}_j^{\exp} - \mathbf{F}_j^{\mathrm{ana}}(\mathbf{x})\|_{\mathrm{Fro}}}{\|\mathbf{F}_j^{\exp}\|_{\mathrm{Fro}}}\right)^2, \quad \mathbf{x} = (x_1, \ldots, x_n) \in [0,1]^n \tag{3}$$

where $\|\bullet\|_{\mathrm{Fro}}$ is the Frobenius norm of a matrix; $\mathbf{F}_{\exp}$ is the flexibility matrix of the damage model and $\mathbf{F}_{\mathrm{ana}}(\mathbf{x})$ is the flexibility matrix of the analytical model with the vector of the damage parameters; *nmod* is the number of modes considered; and *nc* is the total number of column in the flexibility matrix, which is always equal to the total degrees of freedom (DOFs) of a structure.

By combining these two functions (Equations (1) and (3)), the hybrid objective function used for damage identification in this work is expressed as

$$f(\mathbf{x}) = \frac{1}{2}(1 - \mathrm{MDLAC}(\mathbf{x}) + \mathrm{F}(\mathbf{x})) \tag{4}$$

It should be noted that the minimum value of $f(\mathbf{x})$ is zero if the damage parameters are estimated exactly. To obtain the damage parameters, a robust optimization solver is applied to minimize the objective function. In the next section, the Jaya algorithm will be applied and presented in brief.

2.2. Jaya algorithm

The Jaya algorithm, proposed by Venkata Rao (2016), is a global search-based population method. This algorithm is based on the concept that it always tries to reach the best solution and to avoid failure solutions. Moreover, it is easy to implement as it requires only common controlling parameters (population size and number of generations). The details of this algorithm can be briefly summarized in four phases, as follows.

At the start of the algorithm, an initial population including *NP* individuals is generated randomly in the search space. Each individual in *NP* is a vector comprising *n* design variables $\mathbf{x}_j = (x_1, x_2, \ldots, x_n)$ and is created by

$$x_{j,i} = x_{j,i}^l + \mathrm{rand}[0,1] \times (x_{j,i}^u - x_{j,i}^l) \quad i = 1, 2, \ldots, NP; \quad j = 1, 2, \ldots, n \tag{5}$$

where x_j^l and x_j^u denote the lower and upper bounds of x_j, respectively; rand[0, 1] is uniformly distributed between 0 and 1; and *NP* is the population size.

Next, if $x_{j,i,G}$ is the value of the *j*th variable for the *i*th candidate during the *G*th iteration, then this value is used to generate a vector $\mathbf{x}'_{i,G}$ as

$$x'_{j,i,G} = x_{j,i,G} + r_{1,j,G} \times (x_{j,\mathrm{best},G} - |x_{j,i,G}|) - r_{2,j,G} \times (x_{j,\mathrm{worst},G} - |x_{j,i,G}|) \tag{6}$$

where $x_{j,\mathrm{best},G}$ and $x_{j,\mathrm{worst},G}$ are the values of the *j*th variable for the best candidate and the worst candidate, respectively; $r_{1,j,G}$ and $r_{2,j,G}$ are the random numbers in the range [0, 1]; the term $''r_{1,j,G} \times (x_{j,\mathrm{best},G} - |x_{j,i,G}|)''$ points out the tendency of the solution towards the best solution; and the term $''r_{2,j,G} \times (x_{j,\mathrm{worst},G} - |x_{j,i,G}|)''$ points out the tendency of the solution avoiding the worst solution.

The absolute value of the candidate solution $|x_{j,i,G}|$ helps to enhance the exploration ability of the algorithm.

Then, an operation considers the value of components $x'_{j,i,G}$ to reflect back to the allowable region if its values exceed the corresponding upper and lower bounds. This operation can be expressed as

$$x'_{j,i,G} = \begin{cases} 2x_j^l - x'_{j,i,G} & \text{if } x'_{j,i,G} < x_j^l \\ 2x_j^u - x'_{j,i,G} & \text{if } x'_{j,i,G} > x_j^u \\ x'_{j,i,G} & \text{otherwise} \end{cases} \tag{7}$$

Finally, based on the value of the objective function, the vector $\mathbf{x}'_{i,G}$ is compared to its counterpart target individual $\mathbf{x}_{i,G}$. If the vector $\mathbf{x}'_{i,G}$ has a lower functional value, it will survive to the next generation. Otherwise, the target vector $\mathbf{x}_{i,G}$ will be retained in the population.

$$\mathbf{x}_{i,G+1} = \begin{cases} \mathbf{x}'_{i,G} & \text{if } f(\mathbf{x}'_{i,G}) \le f(\mathbf{x}_{i,G}) \\ \mathbf{x}_{i,G} & \text{otherwise} \end{cases} \tag{8}$$

where f is the cost function that needs to be minimized and is given by Equation (4).

2.3. Damage modelling and measurement noise

To simulate damage in the structure, the element stiffness matrix of certain elements will be adjusted. If one assumes that the damage ratio of the eth element is a^e within the range [0, 1], where 0 implies a healthy element and 1 means that the element loses its stiffness completely, then the global stiffness matrix of damage structures can be expressed as

$$\mathbf{K} = \sum_{e=1}^{Ne} (1 - a^e)\mathbf{K}^e \tag{9}$$

where Ne is the number of elements and $\mathbf{K}^e$ is the stiffness matrix of the eth element.

In practice, the effect of measurement noise on damage identification is unavoidable. Hence, to obtain a more realistic model, natural frequencies and mode shapes are polluted by adding a random

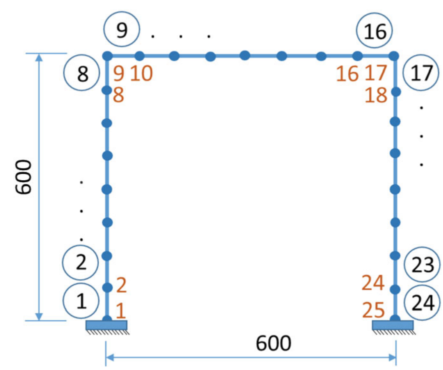


Figure 1. Finite element model of a two-dimensional frame with 24 elements.

error, given by (Wang *et al.* 2012; Vo-Duy *et al.* 2016)

$$\omega_i^{\text{noise}} = (1 + (2\,\text{rand} - 1)\eta_f)\omega_i, \phi_{ji}^{\text{noise}} = (1 + (2\,\text{rand} - 1)\eta_m)|\phi_{ji}|, \quad i = (1, 2, \ldots, nmod) \quad (10)$$

in which ω_i^{noise} is the ith natural frequency contaminated by noise and ϕ_{ji}^{noise} is the jth component of the ith mode shape vector contaminated by noise; η_f and η_m are the level of the additional noise of frequency and mode shape, respectively.

3. Numerical examples

In this section, three examples, namely a 24-element 2D frame (one bay), a 21-bar planar truss and a four-storey frame (three bays), are presented to investigate the performance of the proposed method

Table 1. Two different damage cases induced in the 24-element two-dimensional frame.

Case	1		2	
Element no.	13	1	12	17
Damage ratio	0.2	0.20	0.3	0.20

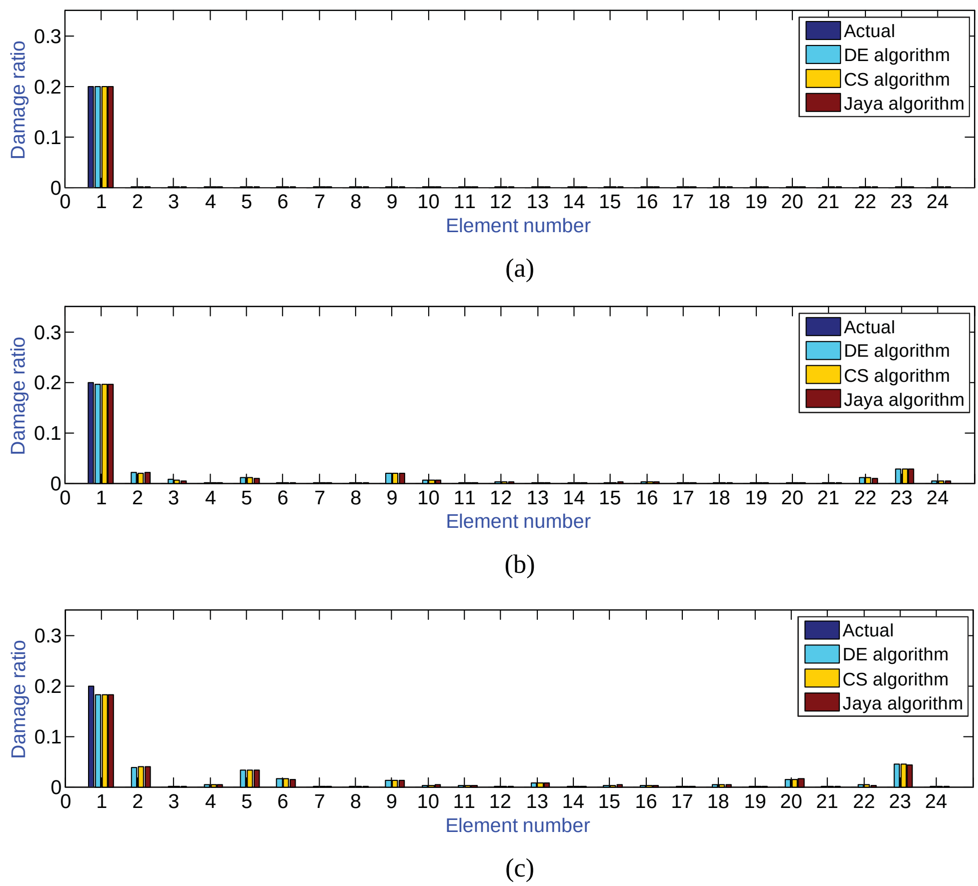


Figure 2. Comparison of obtained damage identification results from different optimization algorithms for case 1 of the frame with noise levels: (a) noise-free; (b) noise level 1; (c) noise level 2. DE = differential evolution; CS = cuckoo search.

for identifying single and multiple structural damage. The damage detection results obtained by the proposed method are compared with those gained by the DE and CS algorithms. Free vibration analysis of structures is implemented by the standard finite element method using MATLAB®. For each of these examples, two different damage scenarios are considered, with and without measurement noise. Here, the effect of the magnitude of noise levels on the accuracy of the proposed method for damage identification is examined from medium level (0.3% noise in natural frequency and 5.0% noise in mode shape; named noise level 1) to relatively high level (0.5% noise in natural frequency and 8.0% noise in mode shape; noise level 2). It is assumed that only the first five natural frequencies and corresponding mode shapes are used in these examples.

The parameters of the Jaya algorithm in all examples are given by: population size (NP) = 50, maximum integration = 3000, random number (r) = [0, 1] and stop criterion = 10^{-6}. Note that the population size, maximum integration and stop criterion for all algorithms are the same; the remaining parameter settings for the DE and CS algorithms are taken from Vo-Duy *et al.* (2016) and Yang and Deb (2009), respectively. To ensure fairness in the comparison of the robustness of the considered algorithms, 10 independent runs of these algorithms are performed for each test scenario. The average results are plotted in the figures, while the statistical results of damage assessment are listed in the tables.

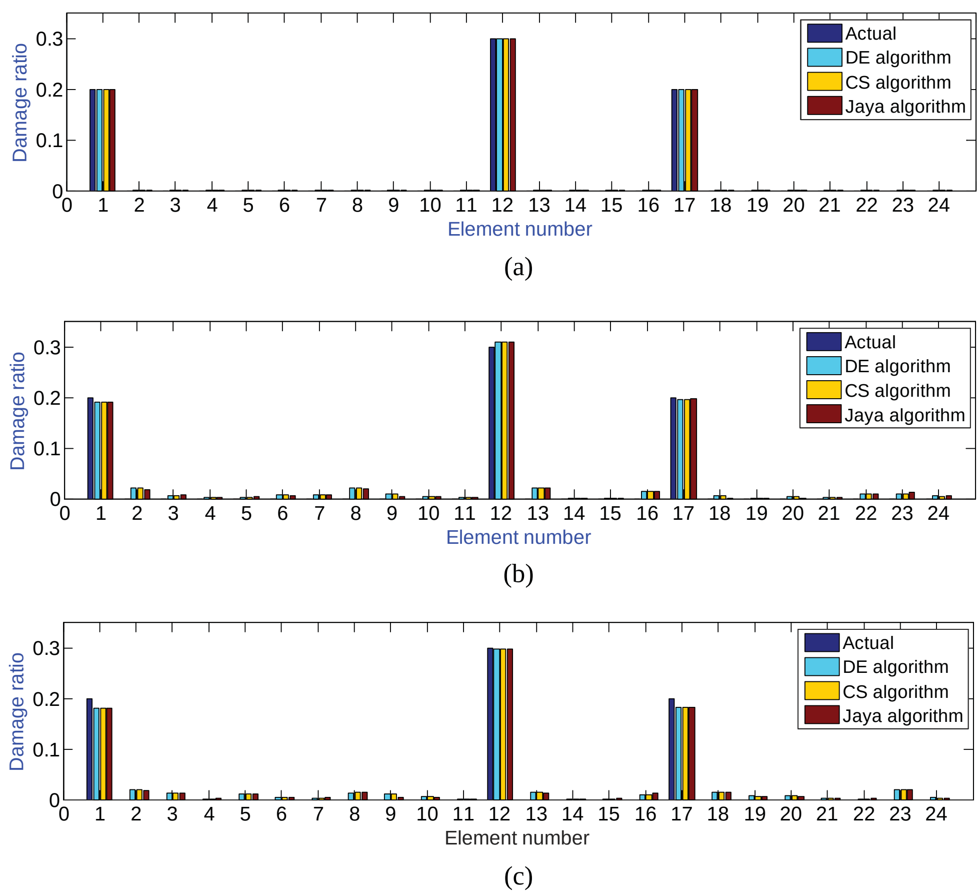


Figure 3. Comparison of obtained damage identification results from different optimization algorithms for case 2 of the frame with noise levels: (a) noise-free; (b) noise level 1; (c) noise level 2. DE = differential evolution; CS = cuckoo search.

Table 2. Statistical results of damage assessment from different optimization algorithms for both scenarios of the frame (one bay) with noise levels.

Case	Noise level	Actual location	DE algorithm Avg. value	DE algorithm Std. dev.	DE algorithm Avg. NSA	CS algorithm Avg. value	CS algorithm Std. dev.	CS algorithm Avg. NSA	Jaya algorithm Avg. value	Jaya algorithm Std. dev.	Jaya algorithm Avg. NSA
1	0%	α^1	0.2001	0.0002	54,380	0.2000	0.0000	30,780	0.1999	0.0002	5,765
	5%	α^1	0.1957	0.0084	74,660	0.1959	0.0085	42,635	0.1959	0.0085	8,020
	8%	α^1	0.1834	0.0263	84,070	0.1831	0.0262	47,210	0.1829	0.0264	8,795
2	0%	α^1	0.1992	0.0004	49,980	0.1999	0.0002	31,765	0.1994	0.0003	7,850
		α^{12}	0.3002	0.0004		0.3000	0.0002		0.3002	0.0004	
		α^{17}	0.1994	0.0006		0.1998	0.0002		0.1996	0.0007	
	5%	α^1	0.1916	0.0100	69,870	0.1914	0.0099	40,550	0.1916	0.0103	8,405
		α^{12}	0.3095	0.0203		0.3096	0.0203		0.3097	0.0217	
		α^{17}	0.1971	0.0361		0.1966	0.0360		0.1972	0.0344	
	8%	α^1	0.1810	0.0204	68,675	0.1812	0.0204	41,630	0.1811	0.0205	8,910
		α^{12}	0.2989	0.0204		0.2988	0.0203		0.2978	0.0207	
		α^{17}	0.1822	0.0168		0.1819	0.0170		0.1827	0.0168	

Note: DE = differential evolution; CS = cuckoo search; Avg. value = average value of damage ratio with respect to *f*; Std. dev. = standard deviation with respect to *f*; Avg. NSA = average number of structural analyses.

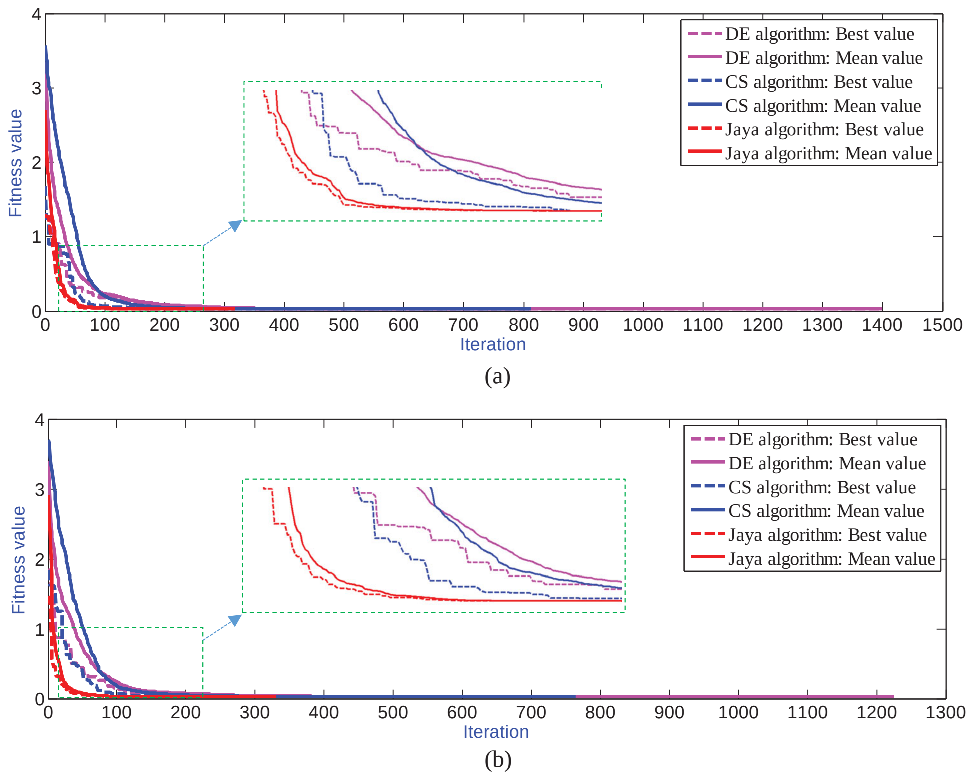


Figure 4. Comparison of convergence history from different optimization algorithms for each scenario of the frame (one bay) with noise level 2: (a) for case 1; (b) for case 2. DE = differential evolution; CS = cuckoo search.

3.1. A 24-element 2D frame (one bay)

A 2D frame structure is considered in the first numerical example for evaluating the performance of the proposed damage identification method. The finite element model of this frame consists of 24 elements with 25 nodes as described in Figure 1. The beam and two columns of the frame have the same cross-sectional area of $A = 9 \times 10^{-4}$ m^2. The Young's modulus and mass density of the frame are $E = 71$ GPa and $\rho = 2210$ kg/m^3, respectively. Two different damage scenarios, including single and multiple damage cases, are examined. The details of damaged elements and their damage ratios for both scenarios are summarized in Table 1.

To illustrate the accuracy and efficiency of the proposed method, a comparison between the present Jaya algorithm, DE algorithm and CS algorithm is conducted for both scenarios and at different noise levels. The average values of the damage ratio of all elements obtained by the three algorithms for scenarios 1 and 2 are presented in Figures 2 and 3, respectively. In noise-free conditions (Figures 2(a) and 3(a)), all the local damage is detected correctly by these three algorithms for both scenarios, element 13 in case 1 and elements 1, 12 and 17 in case 2. In noise-contaminated conditions (Figures 2(b, c) and 3(b, c)), all three algorithms can successfully determine the actual site of damage with negligible false alarms. In addition, these figures show that: (1) the averages of the identified results from the three algorithms are very similar; and (2) an increase in measurement noise leads to a decrease in the precision of identification.

For further investigation, the statistical results of damage assessment including the average, standard deviation and average number of structural analyses from the three algorithms for both scenarios are provided in Table 2. It can be clearly seen that even under the high level of measurement noise, the three algorithms can find the degree of damaged elements with satisfactory precision. In particular, for both scenarios without noise, the mean errors between the obtained average damage ratios and the actual damage ratios by the DE, CS and Jaya algorithms are 0.20%, 0.06% and 0.16%, respectively, while those for both cases with noise level 2 are 7.26%, 7.38% and 7.41%, respectively. In addition, the

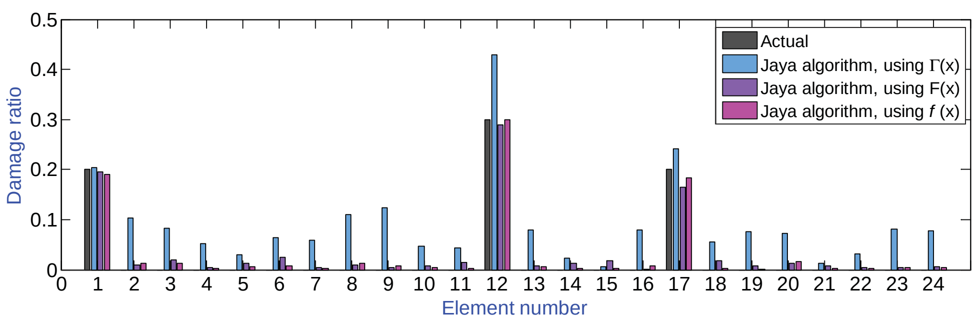


Figure 5. Comparison of obtained damage identification results from the Jaya algorithm using three different objective functions.

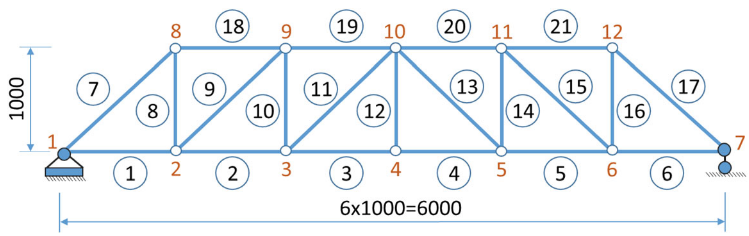


Figure 6. Finite element model of a 21-bar planar truss.

Table 3. Three different damage cases induced in the 21-bar planar truss.

Case	1	2		
Element no.	8	1	9	12
Damage ratio	0.25	0.3	0.4	0.3

standard deviation of predicted results, which signifies the variation in the mean values of damage extent, is relatively small and the values are quite close to each other. However, regarding the computational cost, the Jaya algorithm requires the lowest number of structural analyses among these three algorithms. Consequently, the proposed method can find the optimal solution to the problem efficiently and quickly. In addition, the number of structural analyses increases with increasing noise intensity.

The convergence history of the best run for each damage scenario in the case with noise level 2 obtained using the three optimization algorithms is compared in Figure 4. It can be seen that the convergence speed of the Jaya algorithm is much faster than that of the other two algorithms. With reference to Figure 4(a), the Jaya algorithm stops reaching optimal results after around 330 iterations, while to obtain the same optimal results, the DE and CS algorithms need around 760 and 1220 iterations, respectively.

In this example, to confirm the superiority of the proposed hybrid objective function, two other functions are also examined here for comparison. The first one is defined based on modal flexibility change, as in Equation (3). The second function is formulated by combining MDLAC and the change of mode shapes before and after damage, which can be written as follows:

$$\Gamma(\mathbf{x}) = \frac{1}{2}(1 - \mathrm{MDLAC}(\mathbf{x}) + \Phi(\mathbf{x})), \quad \mathbf{x} = (x_1, \ldots, x_n) \in [0, 1]^n \tag{11}$$

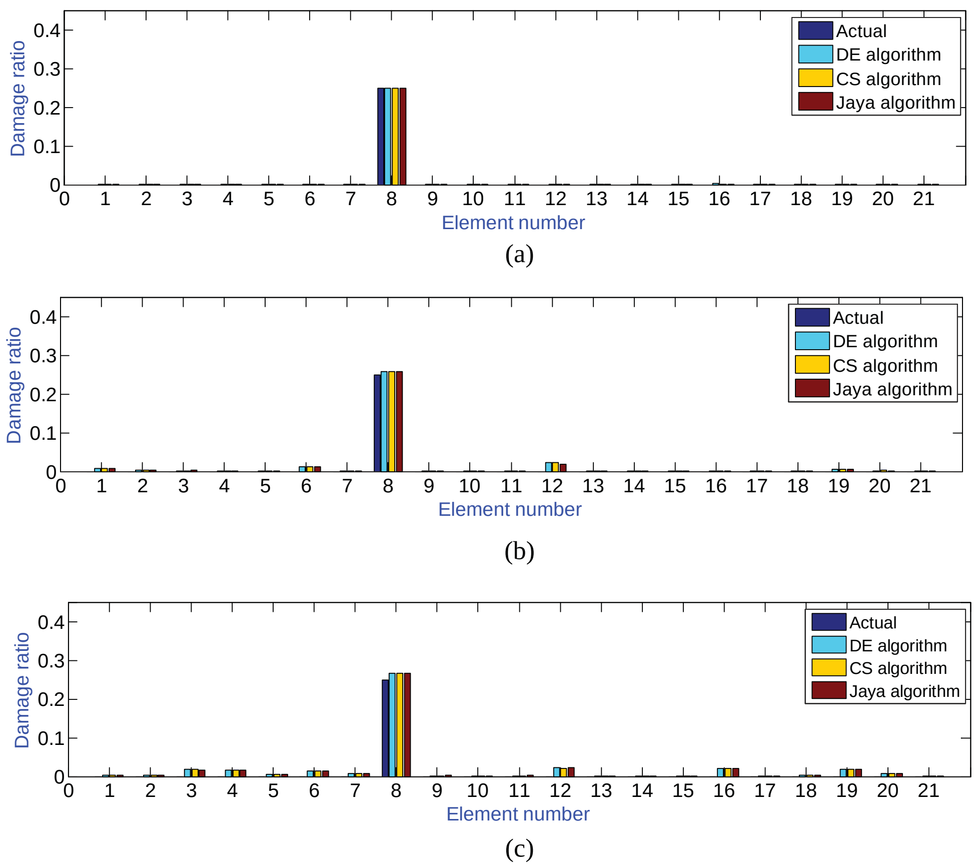


Figure 7. Comparison of obtained damage identification results from different optimization algorithms for case 1 of the truss with noise levels: (a) noise-free; (b) noise level 1; (c) noise level 2. DE = differential evolution; CS = cuckoo search.

where

$$\Phi(\mathbf{x}) = \frac{1}{nmod} \sum_{j}^{nm} \left(\frac{||\Phi_j^{\text{exp}} - \Phi_j^{\text{ana}}(\mathbf{x})||_{\text{Fro}}}{||\Phi_j^{\text{exp}}||_{\text{Fro}}} \right)^2 \tag{12}$$

where Φ_j^* is the jth mode shape vector.

The first five natural frequencies and corresponding mode shapes are employed in the comparison. The final identification results with noise level 2 from the Jaya algorithm using three different objective functions ($\mathrm{F}(\mathbf{x})$, $\Gamma(\mathbf{x})$ and $f(\mathbf{x})$) are shown in Figure 5. Although elements 1, 12 and 17 can be detected by the $\Gamma(\mathbf{x})$ function, there are many false alarms in its prediction. In contrast, with the $\mathrm{F}(\mathbf{x})$ and $f(\mathbf{x})$ functions, the exact location of the damage can be obtained with negligible false results. It is also clear that the results obtained using the proposed hybrid objective function are more accurate than those using the $\mathrm{F}(\mathbf{x})$ function.

3.2. A 21-bar planar truss

The second example involves a 21-bar planar truss, as referred to in Masoumi, Jamshidi, and Bamdad (2015). The geometric information of the truss is shown in Figure 6. The mass density

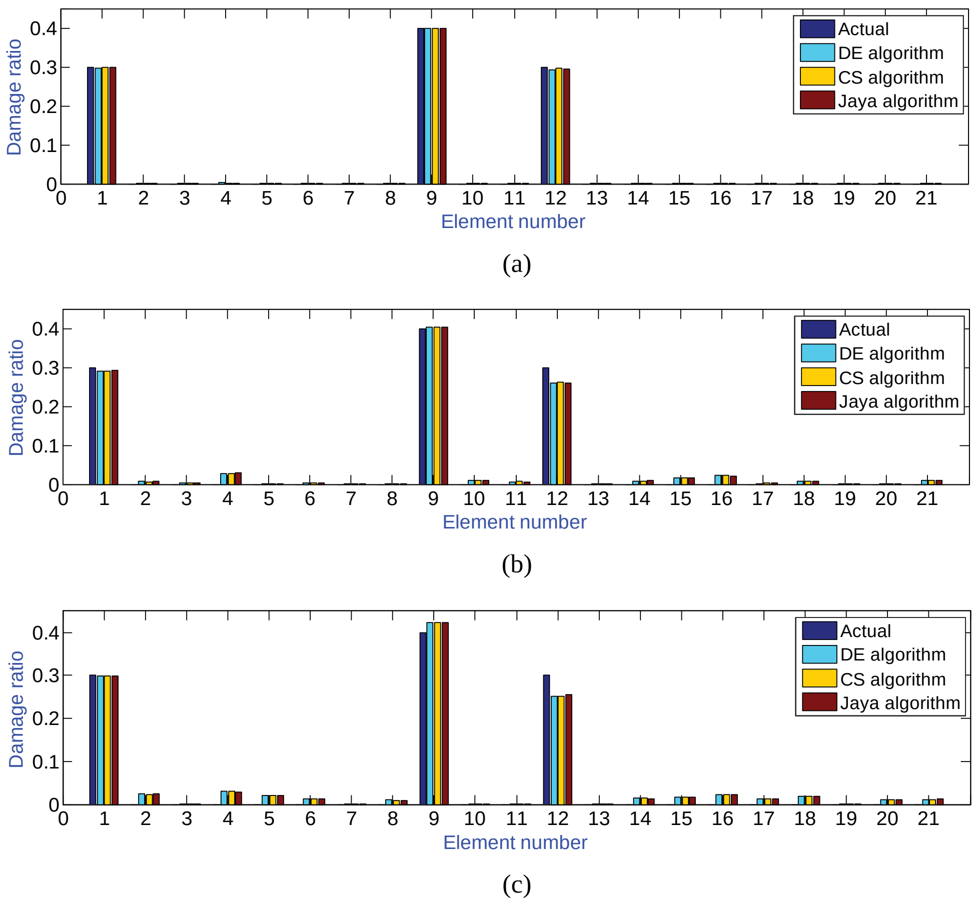


Figure 8. Comparison of obtained damage identification results from different optimization algorithms for case 2 of the frame with noise levels: (a) noise-free; (b) noise level 1; (c) noise level 2. DE = differential evolution; CS = cuckoo search.

Table 4. Statistical results of damage assessment in the truss for both scenarios in the case without noise and with noise levels.

			DE algorithm			CS algorithm			Jaya algorithm		
Case	Noise level	Actual location	Avg. value	Std. dev.	Avg. NSA	Avg. value	Std. dev.	Avg. NSA	Avg. value	Std. dev.	Avg. NSA
1	0%	α^8	0.2500	0.0015	49,710	0.2500	0.0000	40,350	0.2505	0.0018	9,035
	5%	α^8	0.2590	0.0423	77,140	0.2586	0.0433	50,975	0.2586	0.0442	14,145
	8%	α^8	0.2680	0.0973	78,270	0.2669	0.0973	57,230	0.2678	0.0976	14,410
2	0%	α^1	0.2992	0.0003	38,210	0.2998	0.0003	29,840	0.2995	0.0002	8,360
		α^9	0.4004	0.0005		0.4001	0.0002		0.4000	0.0003	
		α^{12}	0.2944	0.0024		0.2984	0.0011		0.2969	0.0017	
	5%	α^1	0.2926	0.0073	48,640	0.2927	0.0075	34,010	0.2927	0.0077	10,945
		α^9	0.4060	0.0133		0.4060	0.0132		0.4060	0.0128	
		α^{12}	0.2614	0.0489		0.2623	0.0485		0.2603	0.0513	
	8%	α^1	0.2984	0.0191	52,060	0.2985	0.0188	35,910	0.2984	0.0188	12,135
		α^9	0.4231	0.0204		0.4234	0.0203		0.4236	0.0203	
		α^{12}	0.2524	0.0789		0.2515	0.0808		0.2559	0.0799	

Note: DE = differential evolution; CS = cuckoo search; Avg. value = average value of damage ratio with respect to *f*; Std. dev. = standard deviation with respect to *f;* Avg. NSA = average number of structural analyses.

Table 5. Two different damage cases induced in the four-storey structure.

Case	1	2		
Element no.	1	1	8	21
Damage ratio	0.25	0.2	0.3	0.2

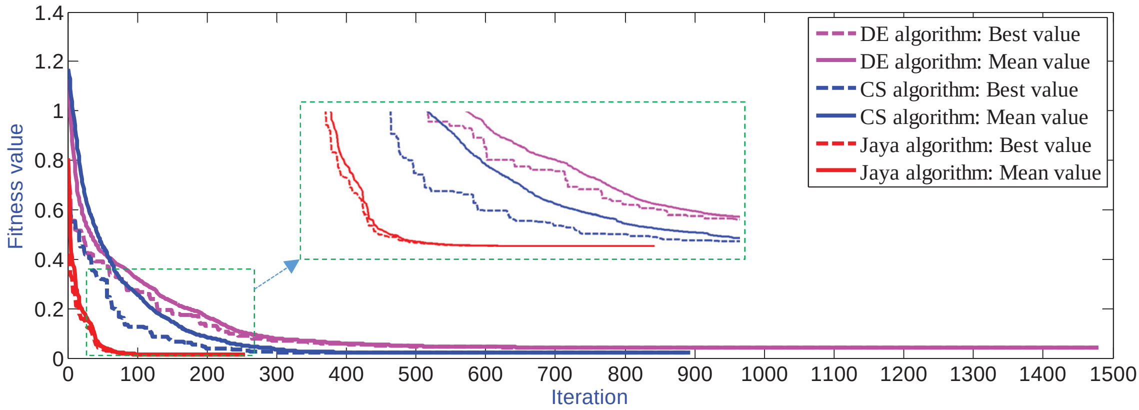
Table 6. Statistical results of damage assessment in the four-storey structure for both scenarios in the case without noise and with noise levels.

			DE algorithm			CS algorithm			Jaya algorithm		
Case	Noise level	Actual location	Avg. value	Std. dev.	Avg. NSA	Avg. value	Std. dev.	Avg. NSA	Avg. value	Std. dev.	Avg. NSA
1	0%	α^{12}	0.2495	0.0002	107,930	0.2498	0.0001	58,350	0.2496	0.0001	19,585
	5%	α^{12}	0.2273	0.0206	143,735	0.2273	0.0206	77,340	0.2278	0.0203	28,240
	8%	α^{12}	0.2207	0.0322	160,775	0.2208	0.0321	80,000	0.2231	0.0300	29,640
2	0%	α^1	0.1995	0.0002	107,445	0.1998	0.0001	55,055	0.1997	0.0002	20,840
		α^8	0.2993	0.0002		0.2998	0.0001		0.2996	0.0002	
		α^{21}	0.1999	0.002		0.1999	0.0001		0.1999	0.0001	
	5%	α^1	0.1978	0.0140	143,280	0.1977	0.0139	69,310	0.1978	0.0138	29,400
		α^8	0.2721	0.0184		0.2722	0.0184		0.2722	0.0182	
		α^{21}	0.1964	0.0180		0.1964	0.0182		0.1964	0.0181	
	8%	α^1	0.1899	0.0268	158,805	0.1899	0.0268	73,780	0.1899	0.0268	31,455
		α^8	0.2531	0.0377		0.2531	0.0378		0.2533	0.0377	
		α^{21}	0.1769	0.0402		0.1770	0.0402		0.1771	0.0403	

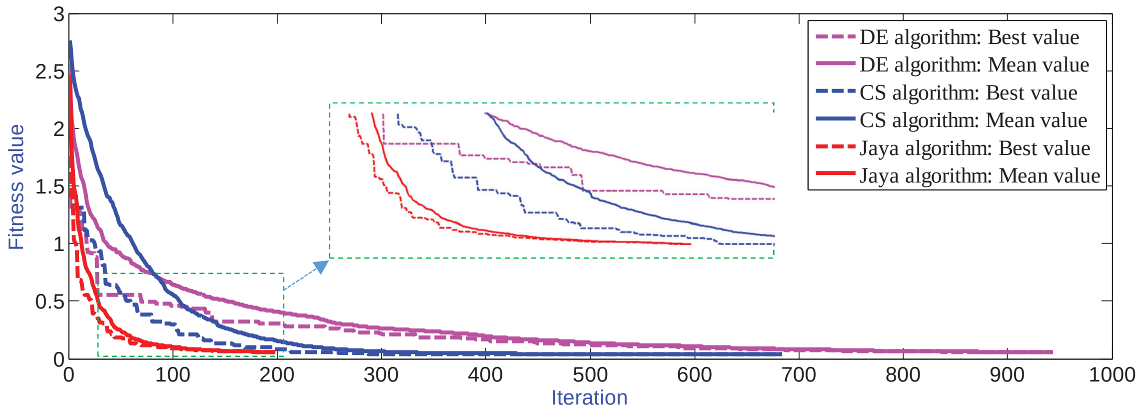
Note: DE = differential evolution; CS = cuckoo search; Avg. value = average value of damage ratio with respect to *f*; Std. dev. = standard deviation with respect to *f;* Avg. NSA = average number of structural analyses.

and Young's modulus for all truss members are 7800 kg/m^3 and 200 GPa, respectively. The cross-sectional areas of all the bars are divided into three groups: (1) $A_1 = \ldots A_6 = 15 \times 10^{-4}$ m^2; (2) $A_7 = \ldots = A_{17} = 9 \times 10^{-4}$ m^2; and (3) $A_{18} = \ldots = A_{21} = 12 \times 10^{-4}$ m^2. Two damage scenarios, consisting of one and three damaged locations, are considered, as given in Table 3.

For the sake of comparison, the DE, CS and Jaya algorithms are applied to calculate the degree of damaged elements in this truss structure. Figures 7 and 8 show the average values of the damage ratio of all elements obtained by the three algorithms for scenarios 1 and 2, respectively. It is observed that for both scenarios, the three algorithms can precisely locate the actual sites of single and multiple



(a)



(b)

Figure 9. Comparison of convergence history from different optimization algorithms for each scenario of the truss with noise level 2: (a) for case 1; (b) for case 2. DE = differential evolution; CS = cuckoo search.

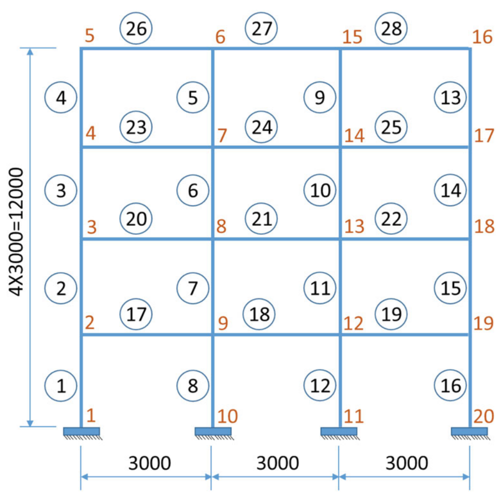


Figure 10. Finite element model of a four-storey structure.

damage in the truss structure, even with relatively high noise levels. Also from these figures, it is clear that all of them have the same degree of accuracy for damage identification.

For further investigation, statistical results including the average, standard deviation and average number of structural analyses for these three algorithms for both scenarios are presented in Table 4. Again, the results show that the damage ratios are detected by all algorithms with small error ratios. More specifically, for both cases without noise, the mean error between the obtained average damage ratios and the actual damage ratios by the DE, CS and Jaya algorithms are 0.37%, 0.10% and 0.29%, respectively, while those for both cases with noise level 2 are 7.30%, 7.14% and 7.08%, respectively. The standard deviation values of the damage assessment results from the three algorithms are relatively small (except for element 8 in case 1 and element 12 in case 2 when considering noise level 2). Moreover, the number of structural analyses needed to reach the optimal solution increases with the increase in the level of measurement noise. However, the results show that the Jaya algorithm always requires far fewer structural analyses than both the DE and CS algorithms. The comparison illustrates the accuracy and robustness of the proposed method for the damage identification problem.

Figure 9 shows the comparison of the convergence histories of the three optimization algorithms for each scenario in the case with measurement noise level 2. Again, the figure indicates that even with the addition of noise, the convergence speed of the proposed algorithm is considerably faster than that of the other two algorithms.

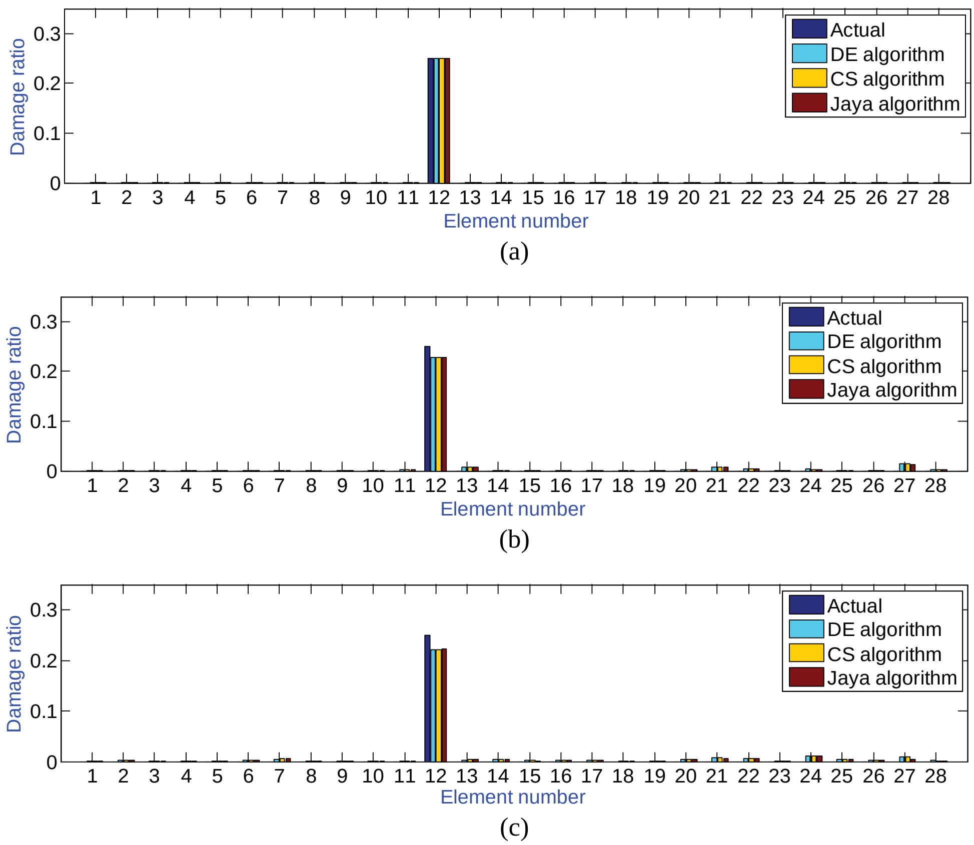


Figure 11. Comparison of obtained damage identification results from different optimization algorithms for case 1 of the frame with noise levels: (a) noise-free; (b) noise level 1; (c) noise level 2. DE = differential evolution; CS = cuckoo search.

3.3. A four-storey structure (three bays)

The last example considers a four-storey (three-bay) steel 2D frame with 28 elements with 20 nodes and three DOFs in each node (Mohan, Maiti, and Maity 2013), as shown in Figure 10. All members of the frame have the same cross-section of $A = 0.04\,\mathrm{m}^2$. The modulus of elasticity and the mass density of the material are $E = 200$ GPa and $\rho = 7860\,\mathrm{kg/m}^3$, respectively. Two damage cases involving single and multiple damage locations are listed in Table 5. A comparison is carried out to evaluate the performance of the proposed method.

In general, the requirement of the mode shape data at all DOFs of the finite element model restricts its real application because incomplete or only translational mode shape values are available from experimental tests. Hence, in the last example, it is assumed that only translational DOFs are selected as measured DOFs. To do this, the improved reduction system method developed by O'Callahan (1989) is applied to the numerical model to condense the mass and stiffness matrices of the structure. The improved reduction system method has been used successfully in damage detection procedures, as shown in a few studies (Zare Hosseinzadeh *et al*. 2014; Kourehli 2016).

Figures 11 and 12 show the average values of the damage ratio of all elements obtained by the three algorithms for scenarios 1 and 2, respectively. The graphical results show that: (1) the three algorithms can correctly detect the damaged members in the frame, even under noisy conditions and with incomplete modal data; (2) the magnitude of errors in the identified damage severity increases with the increase in the level of measurement noise; and (3) all algorithms give approximately the same accuracy for damage prediction with the considered noise levels.

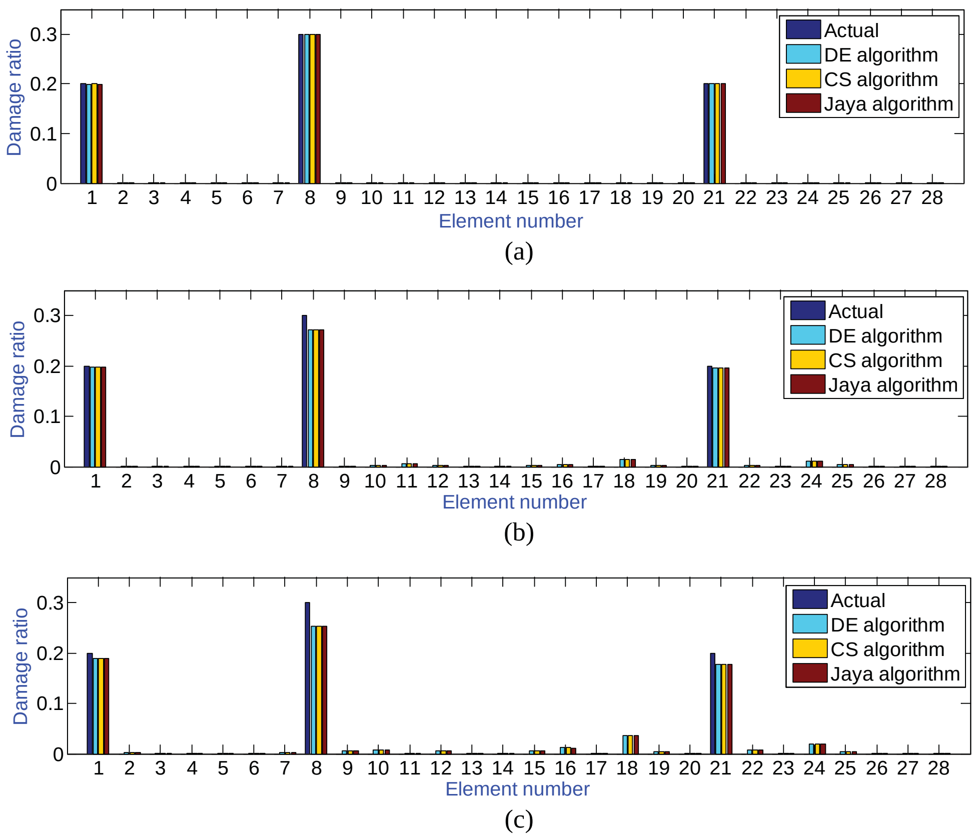


Figure 12. Comparison of obtained damage identification results from different optimization algorithms for case 2 of the frame with noise levels: (a) noise-free; (b) noise level 1; (c) noise level 2. DE = differential evolution; CS = cuckoo search.

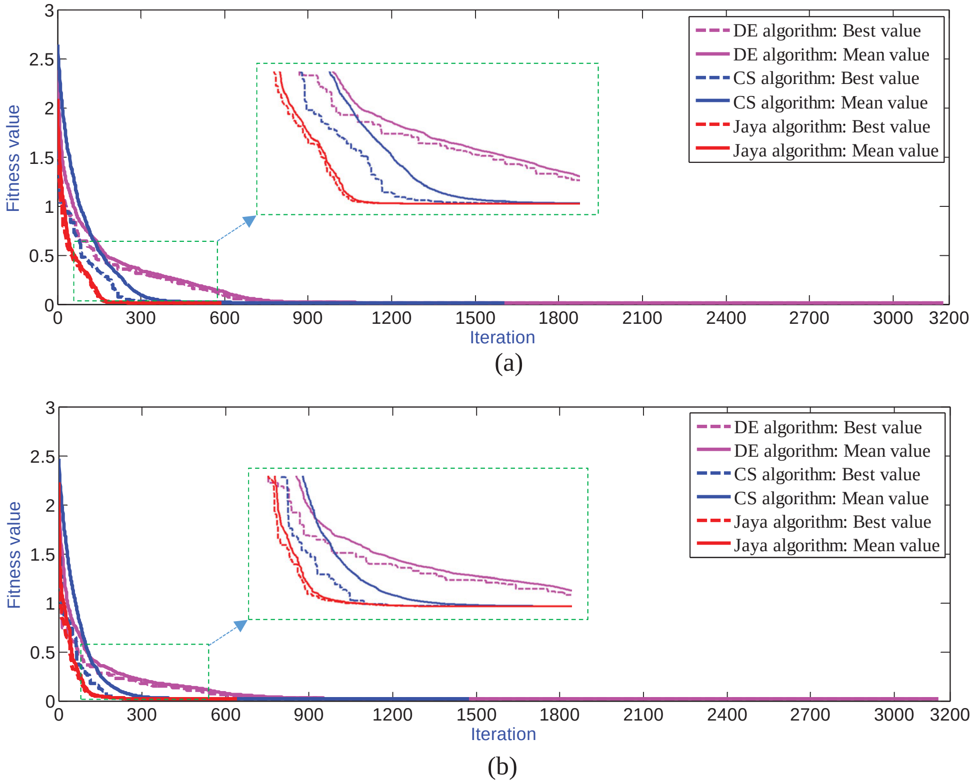


Figure 13. Comparison of convergence history from different optimization algorithms for each scenario of the four-storey structure with noise level 2: (a) for case 1; (b) for case 2. DE = differential evolution; CS = cuckoo search.

Table 6 reports the statistical results comprising the average, standard deviation and average number of structural analyses from these three algorithms for both scenarios. The statistical results show that: (1) the average damage ratio for all elements is determined by the three algorithms with satisfactory accuracy; (2) the standard deviation values of the damage assessment results are relatively small (except for noise level 2) and quite close to each other; and (3) in terms of the computational cost, the Jaya algorithm is more efficient because it always requires far fewer structural analyses than the other two algorithms. These results again confirm the efficiency and robustness of the proposed method in detecting the locations and extent of damage in the structure.

A comparison of the convergence velocity of the three optimization algorithms for each scenario in the case of noise level 2 is shown in Figure 13. Again, it is easy to see that the Jaya algorithm has the best convergence speed among these three algorithms.

4. Conclusions

In this study, the Jaya algorithm is extended to solve the optimization-based damage identification problem. The damage assessment is defined by solving the optimization problem in which the objective function is formulated by combining the MDLAC with criteria based on the difference between the flexibility matrix of the test models and the flexibility matrix of the intact model. To evaluate the performance of the proposed damage identification method, three numerical examples of various structures with single and multiple damage scenarios are examined. The effect of measurement noise levels on the accuracy of the proposed method is also investigated. Based on the obtained results, some conclusions can be drawn:

- In comparison with two other objective functions, the present hybrid objective function is more effective for detecting the location and degree of damage.
- The Jaya algorithm and two other algorithms (DE and CS) can offer quantitative evaluation of the location and level of damage with satisfactory accuracy, and also provide the same level of precision even under the relatively high noise levels. However, the Jaya algorithm always has the best convergence speed, as well as requiring the lowest number of structural analyses among these three algorithms.
- Despite the efficiency and robustness of the proposed method for structural damage assessment in the numerical simulations, it should be pointed out that some aspects of the method could not be covered in the present study. First, the simulation of noise cannot properly represent real conditions, although a relatively high level of noise of mode shapes was considered in the simulated scenarios of the damaged structure to imply considerable modelling errors. Therefore, modelling errors related to geometric and/or material properties should be further investigated. Secondly, in the present work, the structural damage model is simple, whereas in reality, many kinds of local damage are much more complicated. Consequently, further investigation of the use of complicated damage models such as the crack model is necessary to examine the general applicability of the proposed method.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This research is funded by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 107.02-2017.08.

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