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CS 260 – Assignment #2
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Written Problems

Question 1

a) Let $f(n) = 1$ && $g(n) = 17$
$Then\ there\ exists\ a\ c\ such\ that\ cf(n) > g(n)\ for\ any\ n > n_0$
$thus\ by\ choosing\ c = 20\ and\ n_0 = 0, the\ statement\ holds\ \boldsymbol{true}.$

b) Let $f(n) = n^2$ && $g(n) = \frac{n(n-1)}{2}$
$Then\ there\ exists\ a\ c\ such\ that\ cf(n) > g(n)\ for\ any\ n > n_0$
$thus\ by\ choosing\ c = 2\ and\ n_0 = 0, the\ statement\ holds\ \boldsymbol{true}.$

c) Let $f(n) = n^3$ && $g(n) = \max(n^3, 10n^2)$
$Then\ there\ exists\ a\ c\ such\ that\ cf(n) > g(n)\ for\ any\ n > n_0$
$thus\ by\ choosing\ c = 2\ and\ n_0 = 10, the\ statement\ holds\ \boldsymbol{true}.$

d) By taking the order of the most significant term of the summation formula,
when k = 0

$$\sum_{i=1}^{n} i^0 = n\ and\ is\ O(n)\ and\ \Omega(n)$$

when k = 1

$$\sum_{i=1}^{n} i^1 = \frac{n(n+1)}{2}\ and\ is\ O(n^2)\ and\ \Omega(n^2)$$

when k = 2

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{2}\ and\ is\ O(n^3)\ and\ \Omega(n^3)$$

when k = 3

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2\ and\ is\ O(n^4)\ and\ \Omega(n^4)$$

Thus

$$\sum_{i=1}^{n} i^k\ is\ O(n^{k+1})\ and\ \Omega(n^{k+1})\ holds\ \boldsymbol{true}$$

e) This is because the most significant term controls the rate of increase of the function, thus the order of the most significant term is taken as the function of the big O and big Omega notation. In other words, a k^{th} degree polynomial would be $O(n^k)$ and $\Omega(n^k)$.

Question 2

This is ordered in increasing order.
(h) – $(1/3)^n$
(d) – log(log n)
(c) – log n
(e) – $\log^2 n$
(b) - $\sqrt{n}$
(j) – 17
(g) - $\sqrt{n}\log^2 n$
(f) – n/log n
(a) – n
(i) – $(3/2)^n$

Question 3

a) T(1) = 1;
T(2) = 1 + T(1) + T(1) = 1 + 1 + 1 = 3
T(4) = 1 + T(2) + T(2) = 1 + 3 + 3 = 7
T(8) = 1 + T(4) + T(4) = 1 + 7 + 7 = 15
T(16) = 1 + T(8) + T(8) = 1 + 15 + 15 = 31

Thus here we see the relationship between n and the time taken.
Where

$$T(n) = 2n - 1$$

b) Thus the big O for T(n) is

$$T(n) = O(n) \text{ and } \Omega(n)$$

Question 4

This will not work as the node is being deleted after the key is found which causes the linked list to be broken. The NEXT() function in the following line will not work as the node exists no more. The node cannot be simply deleted.

To fix the situation, the previous node needs to be connected to the next node and after that is done, the node with the selected key can be deleted without affecting the entire linked list.

Question 5

$$NEXT = \frac{n(n+1)(2n+1) + 3n(n+1)}{4} + n$$

$$END = \frac{n(n+1)}{2} + n$$

$$FIRST = \frac{n(n+1)}{2} + 1$$