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CS 260
29th June 2015

Theory

Question 1

i. $f_1 = n^2 \; is \; O(n^2)$
$Let \; n_0 \; be \; 0 \; and \; c = 4$

By definition, $(f(n) \leq c * g(n))$
$n^2 \leq 4n^2 \; for \; n > n_0$

$f_1 = n^2 \; is \; \Omega(n^2)$
$Let \; n_0 \; be \; 0 \; and \; c = 1/10$

By definition, $(f(n) \geq c * g(n))$
$n^2 \geq \frac{1}{10} n^2 for \; n > n_0$

ii. $f_2 = n^2 + 1000n \; is \; O(n^2)$
$Let \; n_0 \; be \; 250 \; and \; c = 5$

By definition, $(f(n) \leq c * g(n))$
$n^2 + 1000n \leq 5n^2 for \; n > n_0$

$f_2 = n^2 + 1000n \; is \; \Omega(n^2)$
$Let \; n_0 \; be \; 0 \; and \; c = 1/10$

By definition, $(f(n) \geq c * g(n))$
$n^2 + 1000n \geq \frac{1}{10} n^2 for \; n > n_0$

iii. $f_3 = \begin{cases} n \; if \; n \; is \; odd \\ n^3 \; if \; n \; is \; even \end{cases} \; is \; O(n^3)$
$Let \; n_0 \; be \; 0,2,4,6,8 \ldots (even \; numbers) \; and \; c = 2$

By definition, $(f(n) \leq c * g(n))$
$n^3 \leq 2n^3 for \; n > n_0$

$f_3 = \begin{cases} n \; if \; n \; is \; odd \\ n^3 \; if \; n \; is \; even \end{cases} \; is \; \Omega(n^3)$

$Let\ n_0\ be\ 0,2,4,6,8\ldots(even\ numbers)\ \ and\ c = 1/10$

By definition, $(f(n)\ \geq\ c * g(n))$

$$n^3 \geq \frac{1}{10} n^3 for\ n > n_0$$

iv. $f_3 = \begin{cases} n\ if\ n \leq 100 \\ n^3\ if\ n > 100 \end{cases}\ is\ O(n^3)$

$Let\ n_0\ be\ 101\ and\ c = 2$

By definition, $(f(n)\ \leq\ c * g(n))$

$$n^3 \leq\ 2n^3 for\ n > n_0$$

$f_3 = \begin{cases} n\ if\ n \leq 100 \\ n^3\ if\ n > 100 \end{cases}\ is\ \Omega(n^3)$

$Let\ n_0\ be\ 101\ and\ c = 1/10$

By definition, $(f(n)\ \geq\ c * g(n))$

$$n^3 \geq \frac{1}{10} n^3 for\ n > n_0$$

Question 2

i. $O(n^3)$
ii. $O(n^3)$
iii. $O(n^2)$
iv. $O(n)$