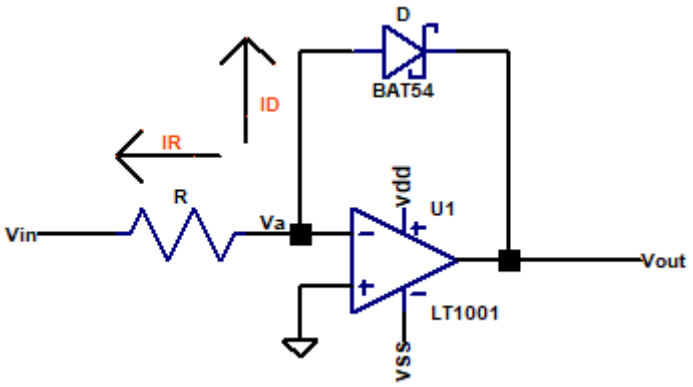
פרויקט מסכם קורס תכנון תת מערכות ספרתיות ואנלוגיות

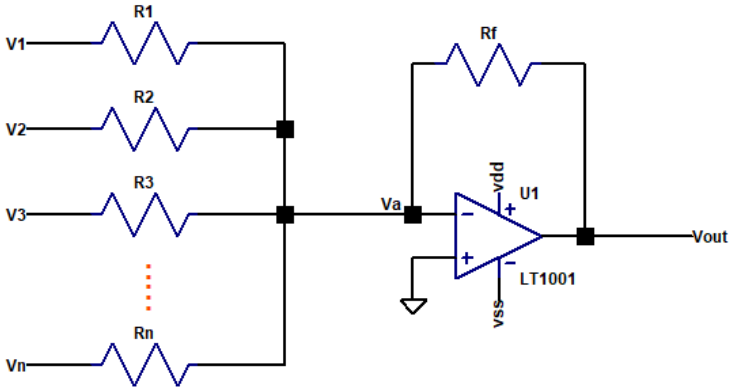
מגישים: אבי שבל ודניאל שקד

Stage 1: log amplifier:



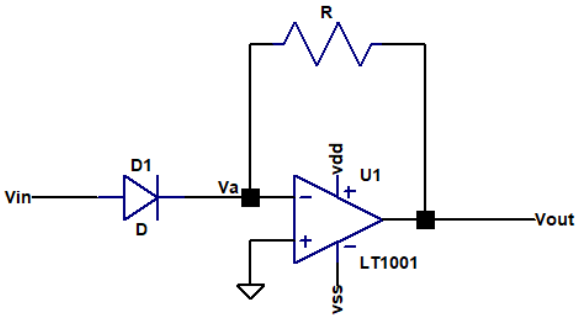
$$V_{out} = N \cdot V_T \cdot Ln(\frac{V_{in}}{-I_s R} + 1) \quad , \quad I_D = -I_R = I_s \cdot (e^{\frac{V_a}{N \cdot V_t}} - 1)$$

Stage 2: Inverting sum amplifier:



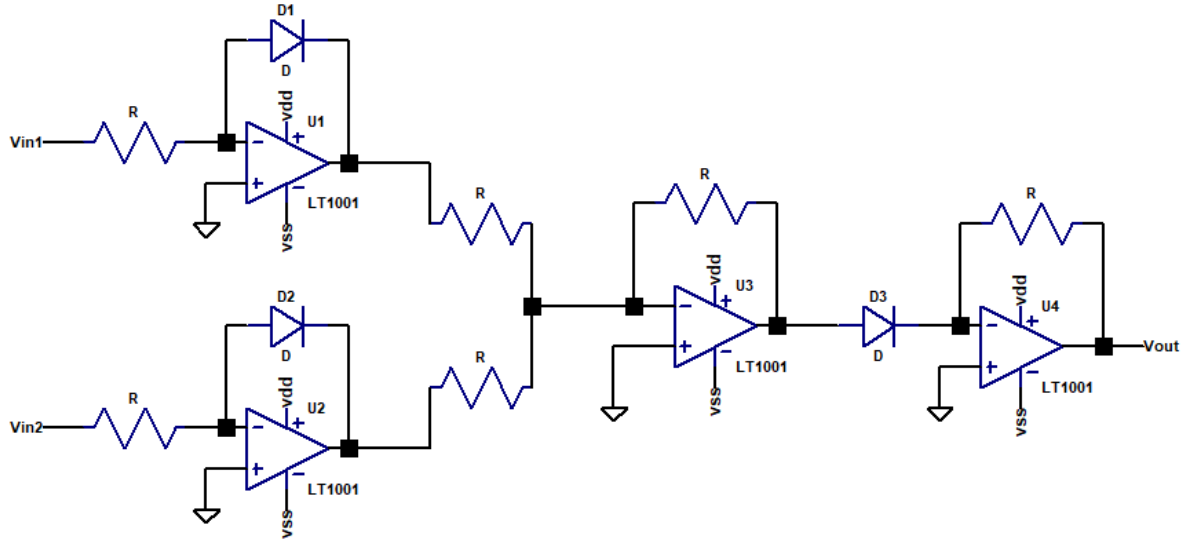
$$V_{out} = -R_f \cdot \sum_{i=1}^{n} \frac{V_i}{R_i} = -R_f \cdot \sum_{i=1}^{n} I_i$$

Stage 3: exponential amplifier:



$$V_{out} = I_s \cdot R \cdot (e^{\frac{V_{in}}{N \cdot V_t}} - 1)$$

Circuit: analog voltage multiplier:



$$V_{out} = I_s \cdot R \cdot \left(e^{\frac{N \cdot V_t \left(Ln\left(\frac{V_{in1}}{I_s \cdot R}+1\right)+Ln\left(\frac{V_{in2}}{I_s \cdot R}+1\right)\right)}{N \cdot V_t}} - 1 \right) = I_s \cdot R \cdot \left(e^{\left(Ln\left(\frac{V_{in1}}{I_s \cdot R}+1\right)+Ln\left(\frac{V_{in2}}{I_s \cdot R}+1\right)\right)} - 1 \right)$$

$$\rightarrow \boldsymbol{V_{in} \cong I_s \cdot R} \rightarrow V_{out} = I_s \cdot R \cdot \left(e^{Ln\left(\frac{V_{in1} \cdot V_{in2}}{(I_s \cdot R)^2}+\frac{V_{in1}}{I_s \cdot R}+\frac{V_{in2}}{I_s \cdot R}+1\right)} - 1 \right)$$

$$= I_s \cdot R \cdot \left(\frac{V_{in1} \cdot V_{in2}}{(I_s \cdot R)^2} + \frac{V_{in1}}{I_s \cdot R} + \frac{V_{in2}}{I_s \cdot R} + 1 - 1 \right) = \frac{V_{in1} \cdot V_{in2}}{I_s \cdot R} + V_{in1} + V_{in2}$$

$$\rightarrow \boldsymbol{V_{in} \gg I_s \cdot R} \rightarrow V_{out} \approx \frac{V_{in1} \cdot V_{in2}}{I_s \cdot R} - 1$$

LT1001 Characteristics:

- Supply Voltage ±22V MAX
- Input Voltage ±22V MAX
- Differential Input Voltage ±30V
- Input Offset Voltage 0-15uV
- Input Offset Current 0.3-2.0nA
- Input Bias current ±2.0nA
- Large Signal Voltage Gain 450-800 V/mV ≈113-118db
- CMRR Min 114db, Typical 126 db
- PSRR Min 110db, Typical 123db
- Gain-Bandwidth 400-800 MHz
- Slew Rate 0.25 V/us

BAT54 Diode Characteristics:

- N coeff = 1
- $I_s = 0.1uA$
- $R_s = 2.2\ \Omega$

Calibration:

We want to calibrate our device to achieve accurate multiplication. We set R=1k and we get $I_s R = 100uV = 0.1mV$, this value is our base unit. Working with voltages around our base voltages implies:

$$\boldsymbol{V_{in} \cong I_s \cdot R} \rightarrow V_{out} = \frac{V_{in1} \cdot V_{in2}}{I_s \cdot R} + V_{in1} + V_{in2}$$

To simplify, a multiplication of Vin1*Vin2= -100uV – (100uV+100uV)=-300uV for Vin1=Vin2=100uV.

This means that our Error is E= Vin1 + Vin2.

Using this python Code:

```python
import numpy as np
N = 1
V_T = 0.026  # Thermal voltage in volts
I_S = 1e-7  # Saturation current in amps
Vin1=0.1e-3
Vin2=0.1e-3
R=1000
def log_amp(Vin,R):
    return -N * V_T * np.log( Vin / ( I_S * R) + 1 )
def sum_amp(V,R,Rf):
    Vout=0
    for i in range(len(V)):
        Vout+= V[i]/ R
    return -Rf * Vout
def exp_amp(Vin,R):
    return I_S * R * ( np.exp( Vin / ( N * V_T ) ) -1 )
def multiplier_amp(Vin1,Vin2,R):
    return exp_amp(sum_amp([log_amp(Vin1,R),log_amp(Vin2,R)],R,R),R)

print(f'Vin / IsR = {Vin1 / (I_S * R)}, the scale of 1 is {(I_S * R)}')
print(f'Vout={multiplier_amp(Vin1,Vin2,R)}')
```

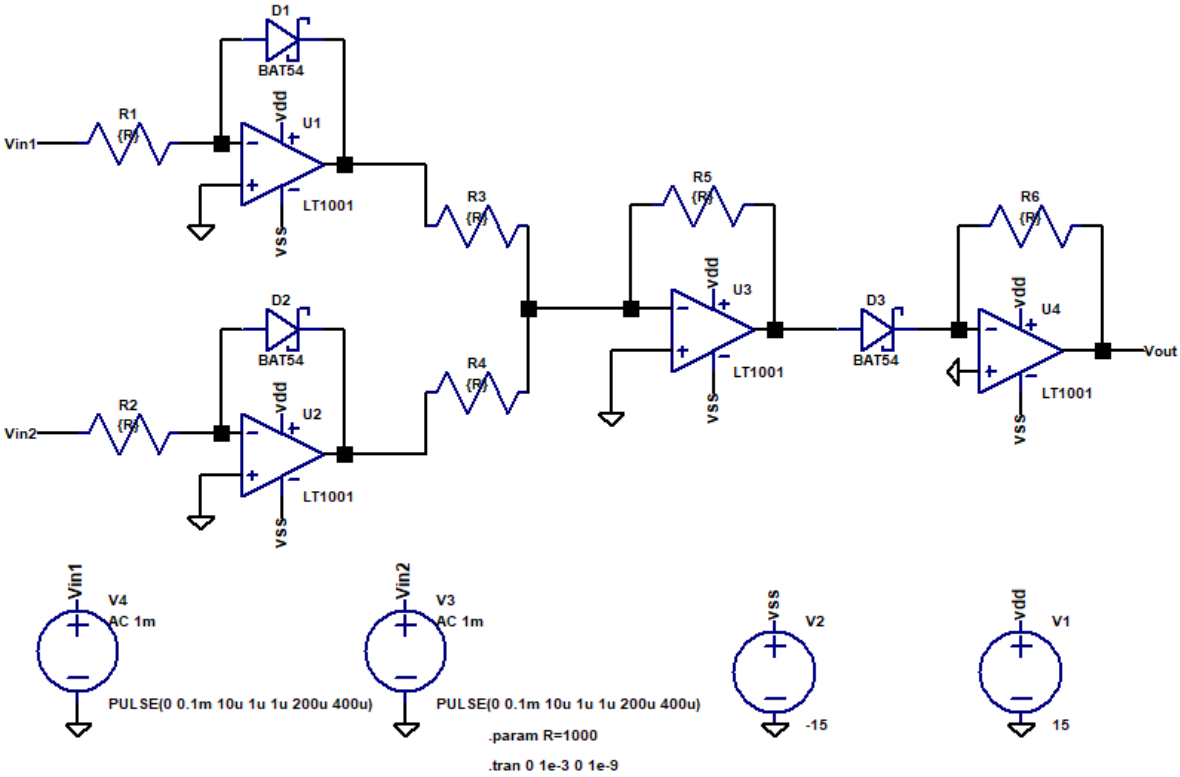
Output:

Vin / IsR = 1.0000000000000002, the scale of 1 is 9.999999999999999e-05

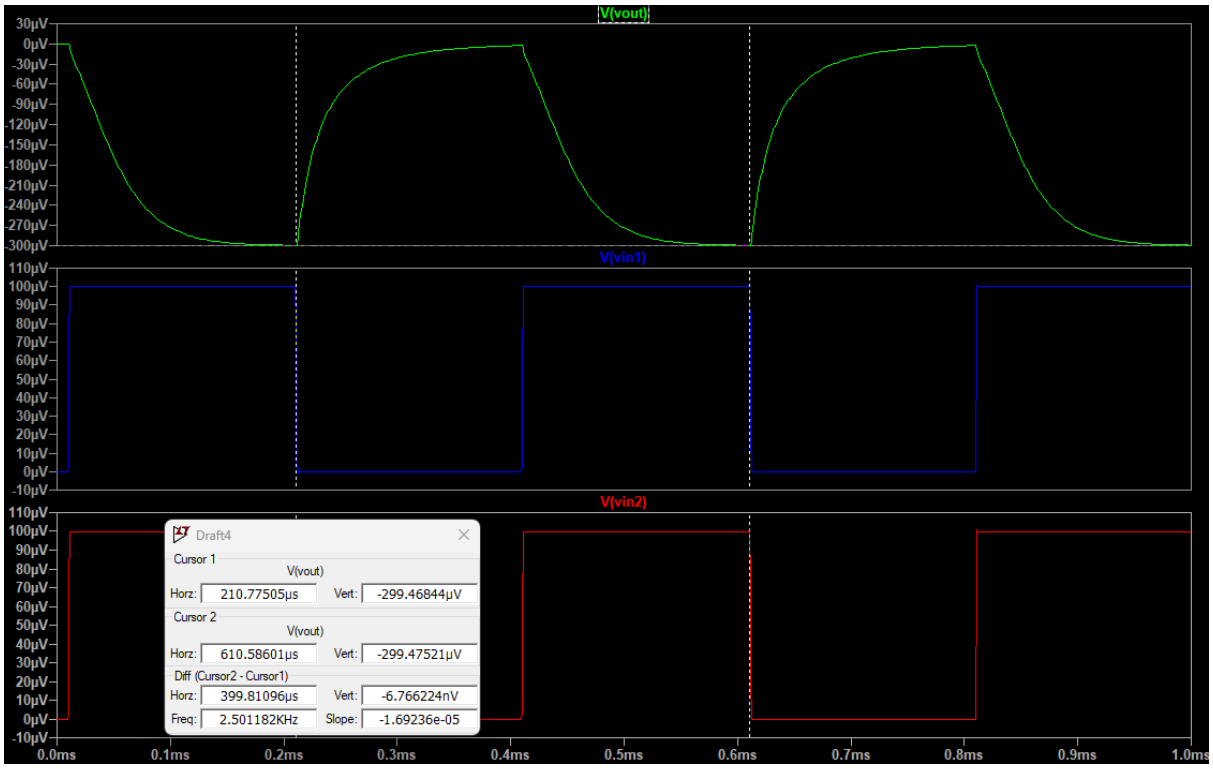
Vout=0.00029999999999999987

Vin1*Vin2 / (I_S * R) + Vin1 + Vin2 = 0.00030000000000000003

Simulation Setup:

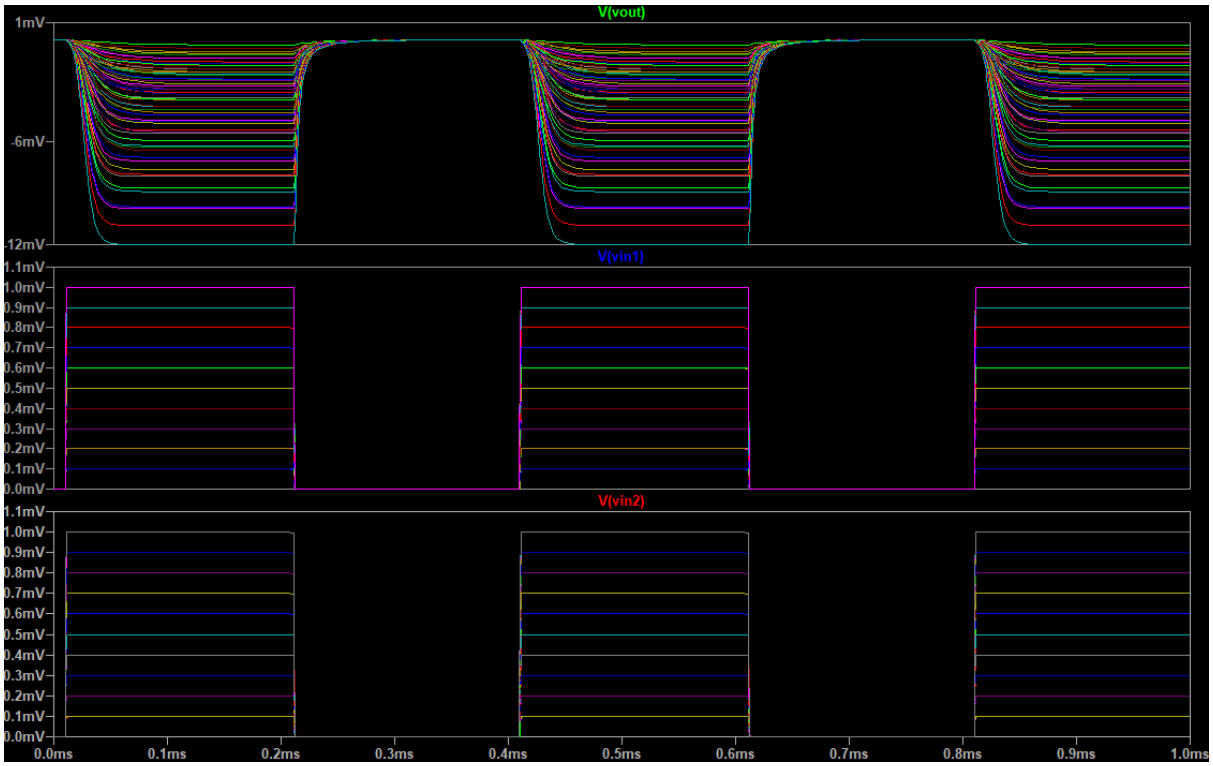


Result:



Prediction Error = |0.000299987 – 0.00029947521| / |0.00029947521| *100= **0.1749%**

We will use .step param and .meas function in lt spice to show and calculate the error for various inputs:



Vin1=0.0001, Vin2=0.0001, result = 0.0003,sim = 0.000299482, error % = 0.173

Vin1=0.0002, Vin2=0.0001, result = 0.0005,sim = 0.000499503, error % = 0.0995

Vin1=0.0003, Vin2=0.0001, result = 0.0007,sim = 0.000699354, error % = 0.0924

Vin1=0.0004, Vin2=0.0001, result = 0.0009,sim = 0.00089919, error % = 0.0901

Vin1=0.0005, Vin2=0.0001, result = 0.0011,sim = 0.00109903, error % = 0.0883

Vin1=0.0006, Vin2=0.0001, result = 0.0013,sim = 0.00129887, error % = 0.087

Vin1=0.0007, Vin2=0.0001, result = 0.0015,sim = 0.00149871, error % = 0.0861

Vin1=0.0008, Vin2=0.0001, result = 0.0017,sim = 0.00169854, error % = 0.086

Vin1=0.0009, Vin2=0.0001, result = 0.0019,sim = 0.00189837, error % = 0.0859

Vin1=0.001, Vin2=0.0001, result = 0.0021,sim = 0.00209819, error % = 0.0863

Vin1=0.0001, Vin2=0.0002, result = 0.0005,sim = 0.000499503, error % = 0.0995

Vin1=0.0002, Vin2=0.0002, result = 0.0008,sim = 0.000799677, error % = 0.0404

Vin1=0.0003, Vin2=0.0002, result = 0.0011,sim = 0.00109958, error % = 0.0382

Vin1=0.0004, Vin2=0.0002, result = 0.0014,sim = 0.00139947, error % = 0.0379

Vin1=0.0005, Vin2=0.0002, result = 0.0017,sim = 0.00169935, error % = 0.0382

Vin1=0.0006, Vin2=0.0002, result = 0.002,sim = 0.00199922, error % = 0.039

Vin1=0.0007, Vin2=0.0002, result = 0.0023,sim = 0.00229908, error % = 0.04

Vin1=0.0008, Vin2=0.0002, result = 0.0026,sim = 0.00259894, error % = 0.0408

Vin1=0.0009, Vin2=0.0002, result = 0.0029,sim = 0.00289878, error % = 0.0421

Vin1=0.001, Vin2=0.0002, result = 0.0032,sim = 0.00319861, error % = 0.0435

Vin1=0.0001, Vin2=0.0003, result = 0.0007,sim = 0.000699354, error % = 0.0924

Vin1=0.0002, Vin2=0.0003, result = 0.0011,sim = 0.00109958, error % = 0.0382

Vin1=0.0003, Vin2=0.0003, result = 0.0015,sim = 0.00149945, error % = 0.0367

Vin1=0.0004, Vin2=0.0003, result = 0.0019,sim = 0.00189928, error % = 0.0379

Vin1=0.0005, Vin2=0.0003, result = 0.0023,sim = 0.0022991, error % = 0.0391

Vin1=0.0006, Vin2=0.0003, result = 0.0027,sim = 0.0026989, error % = 0.0408

Vin1=0.0007, Vin2=0.0003, result = 0.0031,sim = 0.00309868, error % = 0.0426

Vin1=0.0008, Vin2=0.0003, result = 0.0035,sim = 0.00349844, error % = 0.0446

Vin1=0.0009, Vin2=0.0003, result = 0.0039,sim = 0.00389818, error % = 0.0467

Vin1=0.001, Vin2=0.0003, result = 0.0043,sim = 0.0042979, error % = 0.0489

Vin1=0.0001, Vin2=0.0004, result = 0.0009,sim = 0.00089919, error % = 0.0901

Vin1=0.0002, Vin2=0.0004, result = 0.0014,sim = 0.00139947, error % = 0.0379

Vin1=0.0003, Vin2=0.0004, result = 0.0019,sim = 0.00189928, error % = 0.0379

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Vin1=0.0006, Vin2=0.0004, result = 0.0034,sim = 0.00339847, error % = 0.045

Vin1=0.0007, Vin2=0.0004, result = 0.0039,sim = 0.00389814, error % = 0.0477

Vin1=0.0008, Vin2=0.0004, result = 0.0044,sim = 0.00439778, error % = 0.0505

Vin1=0.0009, Vin2=0.0004, result = 0.0049,sim = 0.00489738, error % = 0.0535

Vin1=0.001, Vin2=0.0004, result = 0.0054,sim = 0.00539695, error % = 0.0565

Vin1=0.0001, Vin2=0.0005, result = 0.0011,sim = 0.00109903, error % = 0.0883

Vin1=0.0002, Vin2=0.0005, result = 0.0017,sim = 0.00169935, error % = 0.0382

Vin1=0.0003, Vin2=0.0005, result = 0.0023,sim = 0.0022991, error % = 0.0391

Vin1=0.0004, Vin2=0.0005, result = 0.0029,sim = 0.00289877, error % = 0.0424

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Vin1=0.0007, Vin2=0.0005, result = 0.0047,sim = 0.00469752, error % = 0.0528

Vin1=0.0008, Vin2=0.0005, result = 0.0053,sim = 0.005297, error % = 0.0566

Vin1=0.0009, Vin2=0.0005, result = 0.0059,sim = 0.00589644, error % = 0.0604

Vin1=0.001, Vin2=0.0005, result = 0.0065,sim = 0.00649583, error % = 0.0642

Vin1=0.0001, Vin2=0.0006, result = 0.0013,sim = 0.00129887, error % = 0.087

Vin1=0.0002, Vin2=0.0006, result = 0.002,sim = 0.00199922, error % = 0.039

Vin1=0.0003, Vin2=0.0006, result = 0.0027,sim = 0.0026989, error % = 0.0408

Vin1=0.0004, Vin2=0.0006, result = 0.0034,sim = 0.00339847, error % = 0.045

Vin1=0.0005, Vin2=0.0006, result = 0.0041,sim = 0.00409798, error % = 0.0493

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Vin1=0.0009, Vin2=0.0006, result = 0.0069,sim = 0.00689535, error % = 0.0674

Vin1=0.001, Vin2=0.0006, result = 0.0076,sim = 0.00759452, error % = 0.0722

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Vin1=0.0003, Vin2=0.0007, result = 0.0031,sim = 0.00309868, error % = 0.0426

Vin1=0.0004, Vin2=0.0007, result = 0.0039,sim = 0.00389814, error % = 0.0477

Vin1=0.0005, Vin2=0.0007, result = 0.0047,sim = 0.00469752, error % = 0.0528

Vin1=0.0006, Vin2=0.0007, result = 0.0055,sim = 0.0054968, error % = 0.0582

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Vin1=0.0009, Vin2=0.0007, result = 0.0079,sim = 0.00789412, error % = 0.0745

Vin1=0.001, Vin2=0.0007, result = 0.0087,sim = 0.00869304, error % = 0.0801

Vin1=0.0001, Vin2=0.0008, result = 0.0017,sim = 0.00169854, error % = 0.086

Vin1=0.0002, Vin2=0.0008, result = 0.0026,sim = 0.00259894, error % = 0.0408

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Vin1=0.0004, Vin2=0.0008, result = 0.0044,sim = 0.00439778, error % = 0.0505

Vin1=0.0005, Vin2=0.0008, result = 0.0053,sim = 0.005297, error % = 0.0566

Vin1=0.0006, Vin2=0.0008, result = 0.0062,sim = 0.00619611, error % = 0.0628

Vin1=0.0007, Vin2=0.0008, result = 0.0071,sim = 0.00709511, error % = 0.0689

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Vin1=0.0001, Vin2=0.0009, result = 0.0019,sim = 0.00189837, error % = 0.0859

Vin1=0.0002, Vin2=0.0009, result = 0.0029,sim = 0.00289878, error % = 0.0421

Vin1=0.0003, Vin2=0.0009, result = 0.0039,sim = 0.00389818, error % = 0.0467

Vin1=0.0004, Vin2=0.0009, result = 0.0049,sim = 0.00489738, error % = 0.0535

Vin1=0.0005, Vin2=0.0009, result = 0.0059,sim = 0.00589644, error % = 0.0604

Vin1=0.0006, Vin2=0.0009, result = 0.0069,sim = 0.00689535, error % = 0.0674

Vin1=0.0007, Vin2=0.0009, result = 0.0079,sim = 0.00789412, error % = 0.0745

Vin1=0.0008, Vin2=0.0009, result = 0.0089,sim = 0.00889274, error % = 0.0816

Vin1=0.0009, Vin2=0.0009, result = 0.0099,sim = 0.00989121, error % = 0.0889

Vin1=0.001, Vin2=0.0009, result = 0.0109,sim = 0.0108895, error % = 0.0964

Vin1=0.0001, Vin2=0.001, result = 0.0021,sim = 0.00209819, error % = 0.0863

Vin1=0.0002, Vin2=0.001, result = 0.0032,sim = 0.00319861, error % = 0.0435

Vin1=0.0003, Vin2=0.001, result = 0.0043,sim = 0.0042979, error % = 0.0489

Vin1=0.0004, Vin2=0.001, result = 0.0054,sim = 0.00539695, error % = 0.0565

Vin1=0.0005, Vin2=0.001, result = 0.0065,sim = 0.00649583, error % = 0.0642

Vin1=0.0006, Vin2=0.001, result = 0.0076,sim = 0.00759452, error % = 0.0722

Vin1=0.0007, Vin2=0.001, result = 0.0087,sim = 0.00869304, error % = 0.0801

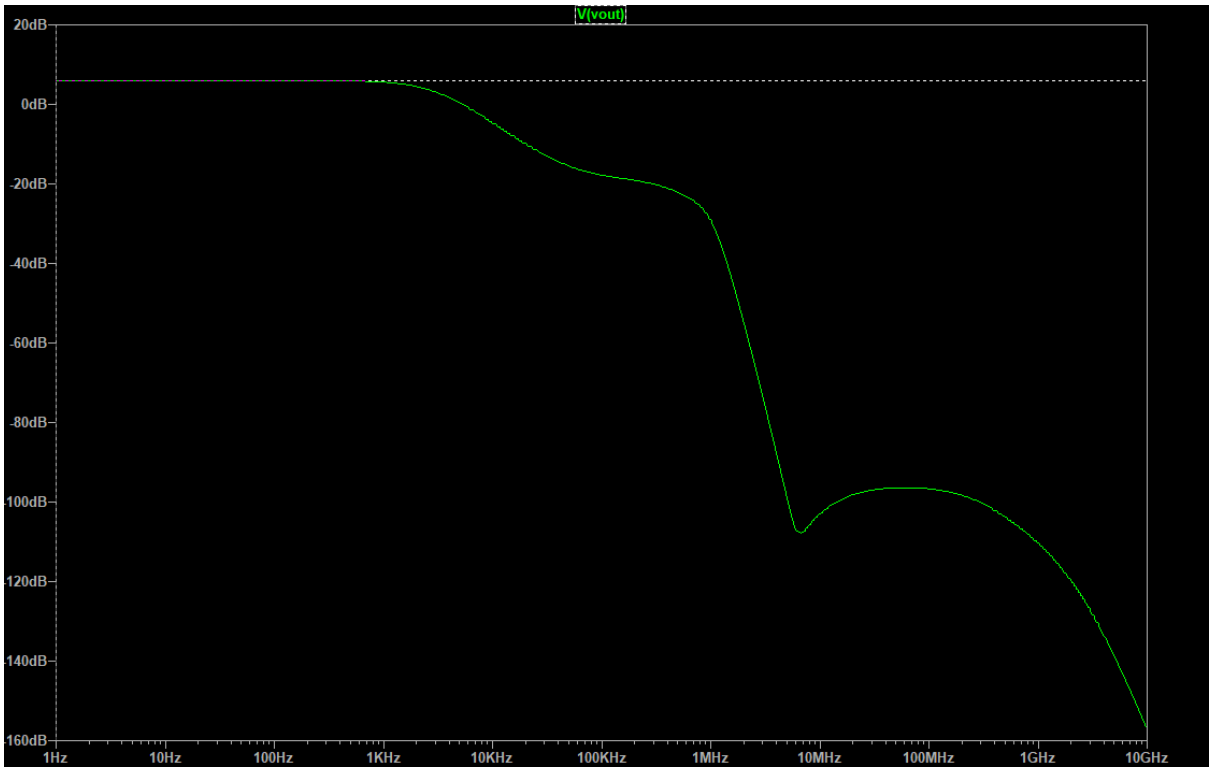
Vin1=0.0008, Vin2=0.001, result = 0.0098,sim = 0.00979138, error % = 0.088

Vin1=0.0009, Vin2=0.001, result = 0.0109,sim = 0.0108895, error % = 0.0964

Vin1=0.001, Vin2=0.001, result = 0.012,sim = 0.0119875, error % = 0.104

We were able to achieve a good precision with our device. Adding a final stage to subtract (Vin1 + Vin2) from Vout will set the device to be exactly $\frac{V_{in1} \cdot V_{in2}}{I_s \cdot R}$.

As for multiplication of small signals, we can look at the AC analysis , and set our results to our preferred scale:



The error produced by the device will be affected by the gain/frequency. So we are not limited by the bandwidth outside the gain bandwidth, only by the error caused by frequency shift.

As for the amplitudes, we need to take in account our calibration setup:

$$\rightarrow \boldsymbol{V_{in} \cong I_s \cdot R} \rightarrow V_{out} = I_s \cdot R \cdot \left(e^{Ln\left(\frac{V_{in1} \cdot V_{in2}}{(I_s \cdot R)^2} + \frac{V_{in1}}{I_s \cdot R} + \frac{V_{in2}}{I_s \cdot R} + 1\right)} - 1 \right)$$

$$= I_s \cdot R \cdot \left(\frac{V_{in1} \cdot V_{in2}}{(I_s \cdot R)^2} + \frac{V_{in1}}{I_s \cdot R} + \frac{V_{in2}}{I_s \cdot R} + 1 - 1 \right) = \frac{V_{in1} \cdot V_{in2}}{I_s \cdot R} + V_{in1} + V_{in2}$$

$$\rightarrow \boldsymbol{V_{in} \gg I_s \cdot R} \rightarrow V_{out} \approx \frac{V_{in1} \cdot V_{in2}}{I_s \cdot R} - 1$$

Setting the DC offset voltage to meet the condition: $\rightarrow \boldsymbol{V_{in} \gg I_s \cdot R}$ reduce error but in exchange for power due to Gain-Bandwidth tradeoff. Also must take in account the voltage offsets of the amplifiers.

The power dissipation of the device is $V_{dd} * I_{dd} = 5V * 9mA = 45mW$. This value match precisely with the power dissipation provided by the datasheet of LT1001.