MTH 9821 Numerical Methods for Finance – Homework 2

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1 Ch 8 #3

1.1 Subquestion i

The mid prices of the options are shown in Table 1.1.

Table 1.1: Mid Prices of Options

Strike	Mid Prices Call	Mid Prices Put
1450	432.8	9.55
1500	385.75	12.35
1550	339.55	16.05
1600	294.5	20.85
1675	229.7	30.55
1700	208.8	34.55
1750	168.8	44.45
1775	150.1	50.45
1800	131.7	57.3
1825	114.25	64.65
1850	97.7	73.05
1875	82.35	82.45
1900	68.05	93.3
1925	55.05	105.05
1975	33.65	133.55
2000	25.2	150.55
2050	13	188.3
2100	6.05	231.2

The OLS formula is

$$y = PVF - disc \times K$$

where $y = C - P$, C is the mid prices of the call options, P is the mid prices of the put options, and K is the strike price. By using OLS, we get

$$y = 1869.4031 - 0.9972K$$

Thus $PVF = 1869.4031$, $disc = 0.9972$.

1.2 Subquestion ii

The original Black-Scholes Formula is

$$C_{BS}(S, K, T, \sigma, r, q) = Se^{-qT}N(d_1) - Ke^{-rT}N(d_2)$$
$$P_{BS}(S, K, T, \sigma, r, q) = -Se^{-qT}N(-d_1) + Ke^{-rT}N(-d_2)$$

where S is the spot price of the index corresponding to these options, q is the dividend rate, r is the risk-free rate. The function $N(\cdot)$ is the standard normal cdf, and

$$d_1 = \frac{\log(S/K) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}$$
$$d_2 = \frac{\log(S/K) + (r - q - \sigma^2/2)T}{\sigma\sqrt{T}}$$

We can rewrite Black-Scholes formula using PVF and $disc$, then

$$C_{BS} = PVF \cdot N(d_1) - K \cdot disc \cdot N(d_2)$$
$$P_{BS} = -PVF \cdot N(-d_1) + K \cdot disc \cdot N(-d_2)$$

and

$$d_1 = \frac{\log\left(\frac{PVF}{K \cdot disc}\right)}{\sigma\sqrt{T}} + \frac{\sigma\sqrt{T}}{2}$$
$$d_2 = \frac{\log\left(\frac{PVF}{K \cdot disc}\right)}{\sigma\sqrt{T}} - \frac{\sigma\sqrt{T}}{2}$$

Let C_{BS} and P_{BS} be the mid prices of the call and put options. We can use Newton's method to calculate σ, which will be the implied volatility. The implied volatility is shown in Table 1.2.

Table 1.2: Implied Volatility

K	$\sigma^2_{\text{imp, c}}$	$\sigma^2_{\text{imp, p}}$
1450	0.0473548237418695	0.047763472746148
1500	0.0431902618976051	0.043488616065135
1550	0.039308491461635	0.0395996294773441
1600	0.0357431718481531	0.0359616052698727
1675	0.0307945166866716	0.0307075462798457
1700	0.0290614122800266	0.0289593032638681
1750	0.0258423104463723	0.0257818527826046
1775	0.0244890865947133	0.0242987325201529
1800	0.0229048005647355	0.0229011137326154
1825	0.0214286123535537	0.0213593275657909
1850	0.0199790343583373	0.0199229868497235
1875	0.0186480463469214	0.018513108797099
1900	0.0173158231345902	0.0172811093183057
1925	0.016049424826671	0.015938192074204
1975	0.0138591431080441	0.0137615241485985
2000	0.0128711121494107	0.0130099516032848
2050	0.0112692419430679	0.0114918512531617
2100	0.0101438878456743	0.0104521034753514

From Table 1.2 we can see that under the same strike price, the implied volatilities of calls and puts are very close.

2 Ch 8 #7

2.1 Subquestion i

By using OLS regression, we get

$$T_{3,LR} = 0.0123 + 0.1272T_2 + 0.3340T_5 + 0.5298T_{10}$$

The approximation error

$$\text{error}_{LR} = ||T_3 - T_{3,LR}|| = 0.04301298$$

2.2 Subquestion ii

By using linear interpolation, we get $T_{3,linear_interp} = [4.650000, 4.770000, 4.773333, 4.783333, 4.783333, 4.796667, 4.780000, 4.786667, 4.810000\ 4.790000, 4.803333, 4.800000, 4.783333, 4.760000, 4.763333]$.

Thus the approximation error

$$\text{error}_{linear_interp} = ||T_3 - T_{3,linear_interp}|| = 0.2066129$$

2.3 Subquestion iii

By using cubic spline interpolation, we get $T_{3,cubic_interp} = [4.641333, 4.762000, 4.765889, 4.780889, 4.781222, 4.789111, 4.773667, 4.781778, 4.805667, 4.786000, 4.799556, 4.795333, 4.779556, 4.756333, 4.758222]$.

Thus the approximation error

$$\text{error}_{cubic_interp} = ||T_3 - T_{3,cubic_interp}|| = 0.1864353$$

2.4 Subquestion iv

The OLS regression has the least approximation error, and the linear interpolation has the approximation error. That is because OLS estimator is BLUE. It has very little residual, while linear interpolation is very rough.

3 Ch 8 #8

3.1 Subquestion i

The weekly percentage returns of these stocks are listed in Table 3.1.

3.2 Subquestion ii

By using OLS regression, we get

$$\begin{aligned} JPM = -0.003222 + 0.755635\ GS - 0.064294\ MS + 0.302755\ BAC + 0.227142\ RBS \\ - 0.094029\ CS - 0.398494\ UBS + 0.147026\ RY + 0.010758\ BCS \end{aligned}$$

where those tickers represent the weekly returns of the corresponding tickers.

The approximation error is 0.1319426.

3.3 Subquestion iii

By using OLS regression, we get

$$JPM = -0.001105 + 0.614013\ GS - 0.067123\ MS + 0.251915\ BAC$$

where those tickers represent the weekly returns of the corresponding tickers.

The approximation error is 0.1514875.

3.4 Subquestion iv

$$\begin{aligned} JPM = 8.83982 + 0.05480\ GS - 0.06069\ MS + 1.46197\ BAC + 0.93938\ RBS \\ 0.85709\ CS - 2.24728\ UBS + 0.28110\ RY - 0.07965\ BCS \end{aligned}$$

where those tickers represent the prices of the corresponding tickers.

The approximation error is 6.443954. Compared to subquestion ii, the approximation error is much larger.

Table 3.1: Weekly returns of some stocks

date	**JPM**	**GS**	**MS**	**BAC**	**RBS**	**CS**	**UBS**	**RY**	**BCS**
2012-01-23	0.0359	0.13	0.119	0.103	0.183	0.183	0.178	0.0391	0.139
2012-01-30	0.0289	0.0515	0.0944	0.0758	0.0436	0.0441	0.0438	0.0248	0.0718
2012-02-06	-0.0176	-0.029	-0.0317	0.0294	-0.0352	-0.0768	-0.0523	-0.00343	-0.019
2012-02-13	0.0231	0.0157	-0.0256	-0.00622	0.00912	0.0422	0.0305	-0.00134	0.0704
2012-02-20	0	0	0	0	0	0	0	0	0
2012-02-27	0.0561	0.0381	-0.0152	0.015	-0.00226	0.017	-0.0324	0.0689	0.0316
2012-03-05	0.00981	-0.0223	-0.0261	-0.00986	-0.0668	-0.0303	-0.0284	0.00501	-0.0625
2012-03-12	0.0862	0.0481	0.063	0.217	0.091	0.116	0.0719	0.0214	0.0667
2012-03-19	0.0133	0.0264	0.0407	0.00512	-0.00222	-0.014	-0.014	-0.00872	-0.0212
2012-03-26	0.0181	-0.0143	-0.0337	-0.0275	-0.0145	-0.0238	-0.0149	-0.000352	-0.0421
2012-04-02	-0.0294	-0.0513	-0.0636	-0.0356	-0.0939	-0.0655	-0.0655	-0.0137	-0.0827
2012-04-09	-0.0254	-0.0246	-0.0608	-0.0597	-0.0187	-0.0342	-0.0477	-0.0212	-0.0218
2012-04-16	-0.0113	-0.023	0.0117	-0.037	-0.0267	0.0133	0.00162	0.0283	0.00446
2012-04-23	0.0145	0.0175	-0.0276	-0.0132	0.0392	-0.0549	0.0161	0.0186	0.0636
2012-04-30	-0.0368	-0.0474	-0.0557	-0.062	-0.00881	-0.0883	-0.0214	-0.0506	-0.0709
2012-05-07	-0.115	-0.063	-0.0659	-0.0246	-0.0698	-0.0277	-0.0114	-0.0229	-0.0404
2012-05-14	-0.094	-0.065	-0.107	-0.0704	-0.145	-0.0707	-0.0755	-0.0606	-0.134
2012-05-21	0.000303	0.0127	-0.00752	0.0186	0.0399	0.00715	0.0266	-0.0356	0.0243
2012-05-28	0	0	0	0	0	0	0	0	0
2012-06-04	0.00546	-0.0176	0.0348	0.0589	0.0706	0.0264	0.013	0.00352	0.0431
2012-06-11	0.0401	0.0118	0.0425	0.045	0.122	-0.0697	0.0239	0.0243	0.0649
2012-06-18	0.0273	-0.0212	-0.0105	0.00507	-0.0268	-0.00425	-0.00334	0.0101	-0.00713
2012-06-25	-0.00734	0.0238	0.0312	0.0303	-0.108	-0.0245	-0.0201	0.0102	-0.183
2012-07-02	-0.0432	-0.00409	-0.0303	-0.0636	-0.0779	-0.0262	-0.0589	0.0124	-0.00293
2012-07-09	0.0642	0.0205	-0.00639	0.0209	0.0319	-0.0213	-0.0299	-0.00234	-0.00294
2012-07-16	-0.0603	-0.0335	-0.0907	-0.096	-0.0124	-0.0287	-0.0505	0	-0.0324
2012-07-23	0.0883	0.0795	0.0597	0.034	0.0814	0.0543	0.0798	0.00665	0.066
2012-07-30	-0.0216	-0.00652	0.0215	0.0164	-0.0145	-0.0364	-0.0173	-0.000194	0.0105
2012-08-06	0.0243	0.0202	0.0602	0.0418	0.0338	0.0221	0.0121	0.00369	0.0858
2012-08-13	0.000272	0.00565	-0.00137	0.0336	0.0355	0.0381	0.0128	0.0557	0.0477
2012-08-20	0.00518	0.0131	-0.00206	0.02	-0.0274	0.0526	0.0145	-0.00623	-0.0174
2012-08-27	-0.000813	0.0117	0.0302	-0.0209	0.0141	0.0026	-0.00446	0.033	-0.0194
2012-09-03	0	0	0	0	0	0	0	0	0
2012-09-10	0.119	0.148	0.216	0.197	0.253	0.2	0.209	0.0318	0.273
2012-09-17	-0.0167	-0.0382	-0.0636	-0.0461	-0.0111	-0.00995	-0.0415	-0.00658	-0.0263
2012-09-24	-0.00986	-0.026	-0.0199	-0.0307	-0.0662	-0.0756	-0.0573	0	-0.0381
2012-10-01	0.0381	0.0495	0.0454	0.0555	0.0168	0.0671	0.0509	0.0223	0.0454
2012-10-08	-0.00216	0.00746	-0.0109	-0.0215	0.0213	-0.0062	-0.0148	-0.0123	0.0193
2012-10-15	0.0168	0.0285	0.0127	0.0351	0.0347	0.0473	0.0349	0.0162	0.0088