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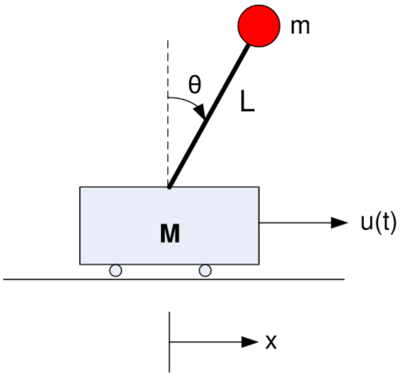
Controllability and Observability

1 Solution



Controllability measures the ability of a particular actuator configuration to control all the states of the system. A system is said to be completely state controllable if it is possible to transfer the system state from an initial state X(t_o) to any other desired state X(t_d) in specified time by a control vector $U(t)$.

The test for controllability can be done both mathematically and through code. Mathematically the CO matrix, that is, the controllability matrix is expressed by:

$$\mathbf{CO} = \begin{bmatrix} B & AB & A^2B & ... & A^{n-1}B \end{bmatrix}$$

where A is the state matrix and B is the input matrix of the system.

- We will be using *Octave* functions to show whether this system is controllable and observable.

For controllability we use the function:

$$\mathbf{CO} = \mathbf{ctrb}(\mathbf{A}, \mathbf{B})$$

it calculates the controllability matrix of the state-space LTI object system.
The system is controllable if CO has full **rank** n.
Note : The rank of a matrix is the number of linearly independent rows or columns and equals the dimension of the row and column space.

Octave Output for Controllability

```
>> m = 1;
>> M = 5;
>> L = 2;
>> g = 9.8;
>> A = [ 0 1 0 0;
         0 0 -1.96 0;
         0 0 0 1;
         0 0 5.88 0];
>> B = [0; 0.2; 0; -0.1];
>> CO = ctrb(A,B)
CO =

   0.00000   0.20000   0.00000   0.19600
   0.20000   0.00000   0.19600   0.00000
   0.00000  -0.10000   0.00000  -0.58800
  -0.10000   0.00000  -0.58800   0.00000

>> rank(CO)
ans =  4
```

This shows that this system is controllable. The system has a full rank n, which is equal to 4, which is equal to the number of states of the system.

Observability : The system is completely observable if the state x can be determined from the knowledge of u(input) and y(output) over a finite time segment.

The test for observability can be done both mathematically and through code. Mathematically the Ob matrix, that is, the observability matrix is expressed by:

$$\mathbf{Ob} = \begin{bmatrix} C \\ CA \\ CA^2 \\ \dots \\ CA^{n-1} \end{bmatrix}$$

where A is the state matrix and C is the output matrix of the system.
For observability we use the function:

$$\mathbf{Ob} = \mathbf{obsv}(\mathbf{A}, \mathbf{C})$$

it calculates the obserability matrix of the state-space LTI object system.
The system is observable if Ob has full **rank** n.

Octave Output for observability

```
>> m = 1;
>> M = 5;
>> L = 2;
>> g = 9.8;
>> A = [ 0 1 0 0;
         0 0 -1.96 0;
         0 0 0 1;
         0 0 5.88 0];
>> C = [1 0 0 0;
        0 1 0 0;
        0 0 1 0;
        0 0 0 1];
>> Ob = obsv(A,C);
>> rank(Ob)
ans =  4
```

This shows that this system is obserable. The system has a full rank n, which is equal to 4, which is equal to the number of states of the system.