State Space Equations

In a state-space system, the internal state of the system is explicitly accounted for by an equation known as the **state equation**. The system output is given in terms of a combination of the current system state, and the current system input, through the **output equation**. These two equations form a system of equations known collectively as **state-space equations**. The state-space is the vector space that consists of all the possible internal states of the system.

Consider the continuous linear time-invariant form of the state-space equations:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

This first equation is called the state equation and this second equation is called the output equation. The state-Space representation revolves around the **state vector X** and how the state vector or the derivative of the state variable changes, it is the **linear combination** of the current state and the linear combination of Inputs.

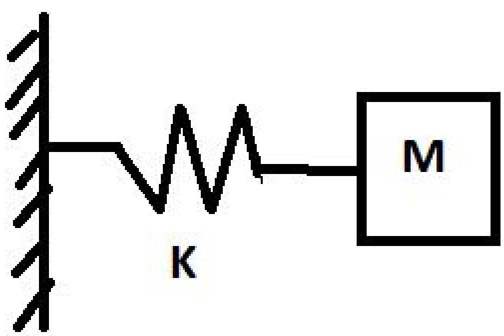
The State equation has two matrices A and B, the A matrix tells how each state is connected to each other and the B matrix describes out the Inputs enters into the system or how inputs react to the states. Whereas in output equation, y is the output vector or the part which we interested in knowing, there are two matrices used in this equation the C matrix tells the system how the states are combined to give the output and whereas D matrix s used to bypass the input from the system to show up in the output, most of the time this output is fed back to the system to close up the feedback loop.

State variables

State variables are a set of variables that can fully describe the system and by fully describe it means they can give us enough information to predict the future behavior of our system. The state variables represent values from inside the system, that can change over time. In an electric circuit, for instance, the node voltages or the mesh currents can be state variables. In a mechanical system, the forces applied by springs, gravity, and dashpots can be state variables. We denote the input variables with *u*, the output variables with *y*, and the state variables with *x*. In essence, we have the following relationship:

$$y = f(x,u)$$

To understand this let's take our example of the spring-mass system and let's find out what states we require and for that consider where will this block be in 1 second?



We obviously want the spring constant and the mass of the block, also not forgetting any external input to the system, but this would tell only the part of the story of the system as you have no idea about the initial conditions of the system.
We would want to know if the mass is currently moving and how much force is applied by the spring, and if we knew these to initial conditions we would be able to predict the future behavior of the system. For that let's consider the 1st question "is the mass currently moving?", to know this we would need the velocity of the block so we can say we need velocity to predict the future of the system therefore velocity is a state variable. Now coming to the 2nd question "What is the force by the spring?" this can be derived easily i.e Spring constant times the distance it is stretched, we need distance or position of the mass to predict the future state, that means the position of the mass is a state variable. So the minimum number of the variable we need to fully describe the system is 2:- the velocity of the mass, and the position of the mass.