Eigenvalues in Control Systems

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1 Eigenvectors and Eigenvalues

Any vector in n dimensional space can be scaled or rotated by a certain amount by multiplying a matrix to it. Every matrix curves the space in a specific way. Vectors which, when multiplied by a matrix, only scale in length without changing their direction, are called as *eigenvectors* of that matrix. An n*n matrix has minimum 0 and maximum n unit eigenvectors. Every vector which can be obtained by scaling this unit vector can be considered as eigenvector of that matrix. The scalar by which eigenvectors are scaled is called as eigenvalue corresponding to that eigenvector.

$$AV = \lambda V \tag{1}$$

where

- A = n*n matrix
- V = eigenvector of A
- λ = eigenvalue of A corresponding to V

To find eigenvalues,

$$|A - \lambda I| = 0 \tag{2}$$

Roots of this n degree equation, λ, are considered eigenvalues of the matrix A. To find corresponding eigenvectors, equation 1 is used.

2 Eigenvalues in State Space Equations

Consider the state space equation, $\dot{x} = Ax + Bu$. To find transfer function of this system, take Laplace transform of this.

$$\begin{aligned} sX(s) &= AX(s) + BU(s) \\ (sI - A)X(s) &= BU(s) \\ X(s) &= (sI - A)^{-1}BU(s) \end{aligned}$$

If $X(s)$ is output, output to input ratio,

$$\frac{X(s)}{U(s)} = (sI - A)^{-1}B \tag{3}$$

Looking at this equation, it can be said that values of s for which the term $(sI - A)$ becomes 0, can be called the poles of the system. Now, if this term is compared with equation 2, it can be said that the term will become zero at eigenvalues of A.

Considering these things, we conclude that *eigenvalues of A matrix from state space equation represent the poles of the system.* Every conclusion that can be drawn using poles, can be drawn by the eigenvalues of the matrix A.