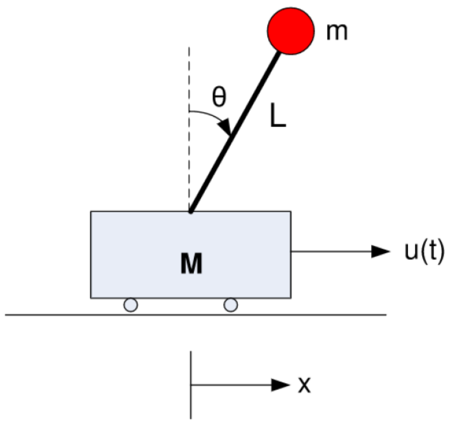
Eigenvalues and Stability Criteria

Experiment 1

Aim:

To comment on stability of system by using given model.

Problem Statement:



Consider a cart pendulum system. State variables include angle of the pendulum θ, angular velocity $\dot{\theta}$, horizontal displacement x, and velocity $\dot{x}$. Mathematical Model of this system in state space form:

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{g}{L}(\frac{m}{M}+1) & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ \frac{1}{ML} \end{bmatrix} u$$

Find eigenvalues of matrix A and comment on behaviour of this system when input u=0, and

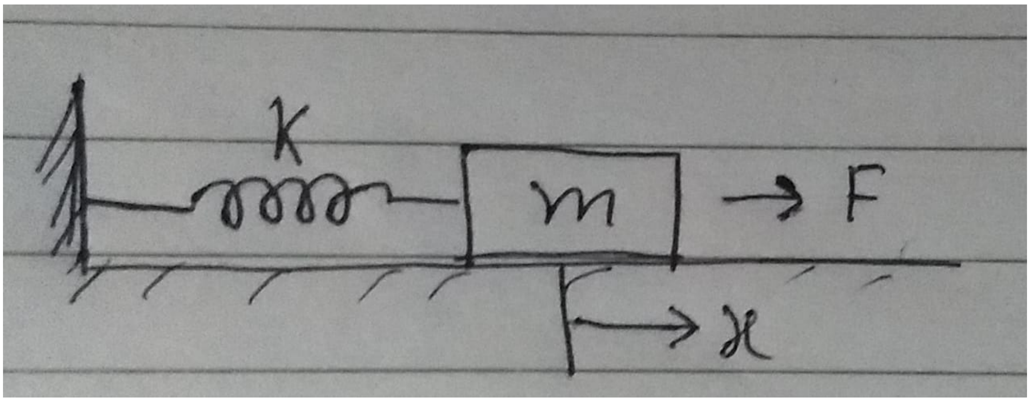
$\theta = 0$.

Experiment 2

Aim:

To find range of K matrix for which system is stable

Problem Statement:



Consider a spring mass system. A variable force F is applied on the mass. Model of this system can be given as

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} F$$

Let a matrix $K = \begin{bmatrix} K_1 & K_2 \end{bmatrix}$. Let $F = -Kx$.

As $\dot{x} = Ax + Bu$, $u = F$, and $F = -Kx$,

$$\begin{aligned} \dot{x} &= Ax + B(-Kx) \\ &= (A - BK)x \end{aligned}$$

Now this new coefficient of x, $A - BK$, can be considered the new A matrix of this system. To find stability of the system with this input, apply the stability criteria on the new matrix, $A' = A - BK$.

Find the range of K_1 and K_2 for which the system is stable.