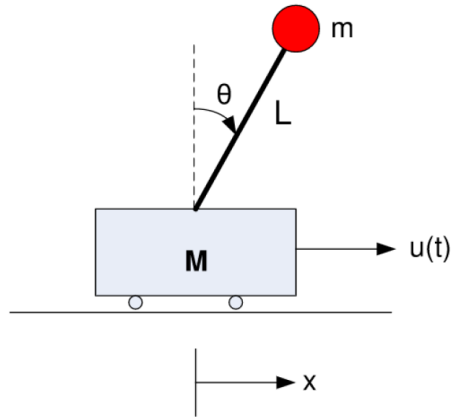# Eigenvalues and Stability Criteria

Solution to Experiment 1

Aim:

To comment on stability of system by using given model.

Problem Statement:

Consider a cart pendulum system. State variables include angle of the pendulum θ, angular velocity $\dot{\theta}$, horizontal displacement x, and velocity $\dot{x}$. Mathematical Model of this system in state space form:

$$Model$$

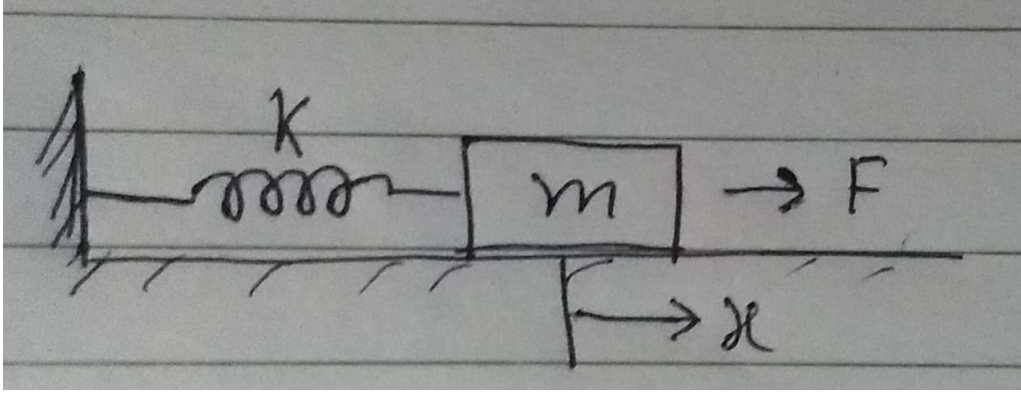Find eigenvalues of matrix A and comment on behaviour of this system when input u=0.

  i. For $\theta = 0$

 ii. For $\theta = \pi$

Aim:

To find range of K matrix for which system is stable

Problem Statement:

Consider a spring mass system. A variable force F is applied on the mass. Model of this system can be given as

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} F$$

Let a matrix $K = \begin{bmatrix} K_1 & K_2 \end{bmatrix}$. Let $F = -Kx$.

As $\dot{x} = Ax + Bu$, $u = F$, and $F = -Kx$,

$$\dot{x} = Ax + B(-Kx)$$
$$= (A - BK)x$$

Now this new coefficient of x, $A - BK$, can be considered the new A matrix of this system. To find stability of the system with this input, apply the stability criteria on the new matrix, $A' = A - BK$.

Find the range of K_1 and K_2 for which the system is stable.

Solution

According to the theory given in problem statement, eigenvalues of $A - BK$ should be negative.

Here, $A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$, $K = \begin{bmatrix} K_1 & K_2 \end{bmatrix}$.

So, $BK = \begin{bmatrix} 0 & 0 \\ \frac{K_1}{m} & \frac{K_2}{m} \end{bmatrix}$

$$A - BK = A' = \begin{bmatrix} 0 & 1 \\ \frac{-k-K_1}{m} & -\frac{K_2}{m} \end{bmatrix}$$

To find eigenvalues of A',

$$\begin{bmatrix} 0 - \lambda & 1 \\ \frac{-k-K_1}{m} & -\frac{K_2}{m} - \lambda \end{bmatrix} = 0$$

$$\therefore \lambda^2 + \frac{K_2}{m}\lambda + \frac{k + K_1}{m} = 0$$

Let λ_1 and λ_2 be the solutions of this equation. For both of these to be negative,

$$\lambda_1 + \lambda_2 < 0$$
$$\lambda_1 \lambda_2 > 0$$

So,

$$\frac{-K_2}{m} < 0 \qquad\qquad \Rightarrow K_2 > 0$$
$$\frac{k + K_1}{m} > 0 \qquad\qquad \Rightarrow K_1 > -k$$