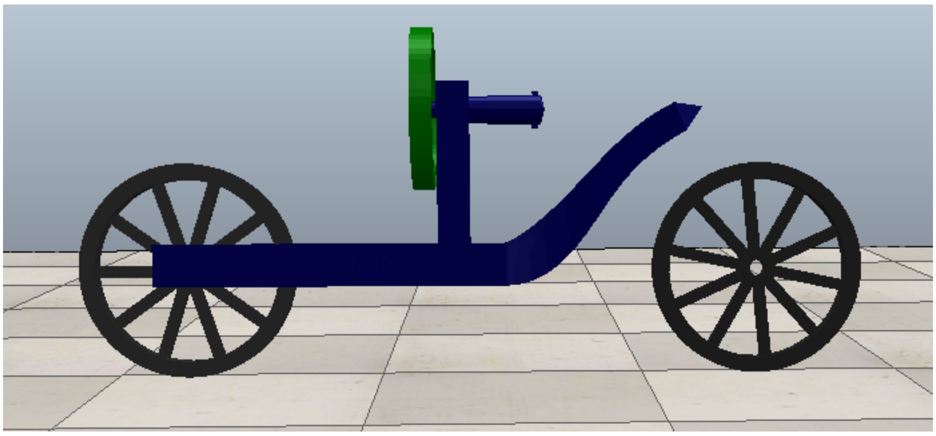
PID:17/Shreya Rastogi May 2020

Kinetic and Potential energy of a system

1 Aim

To find out the Kinetic energy and Potential energy of a system. Using these two quantities, find out the Lagrangian of the system and the Euler Lagrange equation.

2 Solution



Since the Cyclebot system is based on Reaction wheel balanced Inverted Pendulum system, we are going to find out the kinetic energy and potential energy of the latter.

List of symbols:

- m_1 : mass of pendulum
- L_1 : Distance between pivot and COM of pendulum
- I_1 : Moment of inertia of pendulum about it's COM
- m_2 : mass of reaction wheel
- L_2 : Distance between pivot and COM of reaction wheel
- I_2 : Moment of inertia of reaction wheel about it's COM
- $\theta, \dot{\theta}$: Tilt angle and angular velocity of bicycle
- $\phi, \dot{\phi}$: Angular position and angular velocity of reaction wheel
- T_r : Torque provided to the reaction wheel

Total Kinetic Energy:

$$KE = \frac{1}{2}(m_1 L_1^2 + m_2 L_2^2 + I_1)\dot{\theta}^2 + \frac{1}{2}(I_2)(\dot{\theta} + \dot{\phi})^2 \tag{1}$$

$$= \frac{1}{2}(m_1 L_1^2 + m_2 L_2^2 + I_1 + I_2)\dot{\theta}^2 + I_2\dot{\theta}\dot{\phi} + \frac{1}{2}I_2\dot{\phi}^2 \tag{2}$$

Total Potential Energy:

$$PE = (m_1 L_1 + m_2 L_2)gCos\theta \tag{3}$$

Euler Lagrange equation of system will be derived using

$$\frac{d}{dt}\frac{\partial u}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \tau_i \tag{4}$$

τ_i is the external force corresponding to generalized coordinates q_i, and

$$L(q, \dot{q}) = KE(q, \dot{q}) - PE(q, \dot{q}) \tag{5}$$

where

L : Lagrangian Operator

KE : Kinetic energy of the system

PE : Potential energy of the system

Here we have θ and ϕ as generalized coordinates

$$\therefore\ \mathbf{L} = \textbf{KE - PE} \tag{6}$$

$$= \frac{1}{2}(m_1 L_1^2 + m_2 L_2^2 + I_1 + I_2)\dot{\theta}^2 + I_2\dot{\theta}\dot{\phi} + \frac{1}{2}I_2\dot{\phi}^2 - (m_1 L_1 + m_2 L_2)gCos\theta \tag{7}$$

We know,

$$\frac{d}{dt}\frac{\partial u}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = 0 \tag{8}$$

Using (7) in (8)

$$(m_1 L_1^2 + m_2 L_2^2 + I_1 + I_2)\ddot{\theta} + I_2\ddot{\phi} - (m_1 L_1 + m_2 L_2)gSin\theta = 0 \tag{9}$$

We also know,

$$\frac{d}{dt}\frac{\partial u}{\partial \dot{\phi}} - \frac{\partial L}{\partial \phi} = T_r \tag{10}$$

Using (7) in (10)

$$I_2\ddot{\theta} + I_2\ddot{\phi} = T_r \tag{11}$$

Hence the Euler Lagrange equations of this system are:

$$(m_1L_1^2 + m_2L_2^2 + I_1 + I_2)\ddot{\theta} + I_2\ddot{\phi} - (m_1L_1 + m_2L_2)gSin\theta = 0 \tag{12}$$

$$I_2\ddot{\theta} + I_2\ddot{\phi} = T_r \tag{13}$$

Solving these equations for $\ddot{\theta}$ and $\ddot{\phi}$, we get these equations:

$$\ddot{\theta} = \frac{(m_1L_1 + m_2L_2)g}{m_1L_1^2 + m_2L_2^2 + I_1}sin\theta - \frac{1}{m_1L_1^2 + m_2L_2^2 + I_1}T \tag{14}$$

$$\ddot{\phi} = -\frac{(m_1L_1 + m_2L_2)g}{m_1L_1^2 + m_2L_2^2 + I_1}sin\theta + \frac{m_1L_1^2 + m_2L_2^2 + I_1 + I_2}{I_2(m_1L_1^2 + m_2L_2^2 + I_1)}T \tag{15}$$

Hence, this is how we find out the kinetic energy, the potential energy and the Lagrangian of a system. Using the Lagrangian operator, we go to find out the Euler Lagrange equations of the system.