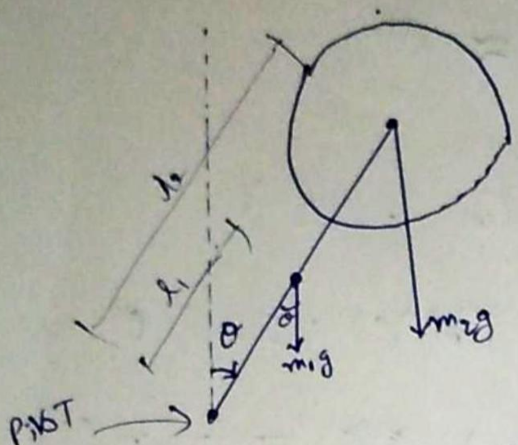
Modeling Inverted pendulum (Reaction wheel)

$m_1 \rightarrow$ mass of pole
$m_2 \rightarrow$ mass of disc



Here. Torque generated on pole is

$\tau_p =$ Torque by pole mass
$\tau_d =$ Torque by disc mass.
$\tau_m =$ counter Torque generated by motor.
$I =$ Total inertia around pivot
$\alpha =$ ~~acetated~~ accelaratim of body

$$\left.\begin{array}{l} \tau_p = m_1 g l_1 \sin\theta \\ \tau_d = m_2 g l_2 \sin\theta \\ \tau_m = \tau_m \\ I\alpha = \text{resultant torque on body.} \end{array}\right\}$$

$$\tau_p + \tau_d + \tau_m = I\alpha$$

$$\therefore m_1 g_1 l_1 \sin\theta + m_2 g_2 l_2 \sin\theta + \tau_m = I\ddot{\theta}$$

$$\therefore (m_1 g_1 l_1 + m_2 g_2 l_2)\sin\theta + \tau_m = I\ddot{\theta} \quad \text{--- } 1$$

taking laplase --- (without linearizatim)

$$(m_1 g_1 l_1 + m_2 g_2 l_2)\frac{1}{s^2+1}\theta(s) + \tau_m(s) = I s^2 \theta(s)$$

$$\therefore \frac{\theta(s)}{\tau_m(s)} = \frac{s^2+1}{Is^4 + Is^2 - (W_{PL} + W_{DL})} \quad \text{--- } 2$$

$$\boxed{W_{PL} = m_1 g_1 l_1, \quad W_{DL} = m_2 g l_2}$$

Now if we linearize eqⁿ (I)

θ is very small and around 0

so--- $\sin\theta \approx \theta$

So...

$$(m_1 g_1 l_1 + m_2 g_2 l_2)\,\theta + \tau_m = I\ddot{\theta}$$

$$(m_1 g_1 l_1 + m_2 g l_2)\,\theta(s) + \tau_m(s) = I s^2 \theta(s)$$

$$\therefore\ \left((m_1 g l_1 + m_2 g l_2) - I s^2\right)\theta(s) = -\tau_m(s)$$

$$\therefore\ \cancel{\frac{\theta(s)}{\tau_m(s)} = \frac{-1}{m g l_1 + m g l_2}}$$

$$\boxed{\frac{\theta(s)}{\tau_m(s)} = \frac{-1}{(m_1 g l_1 + m_2 g l_2) - I s^2}}$$

↳ Tf. with linearization.

where $I = (I_{cp} + m_1 d_1^2) + (I_{cD} + m_2 l_2^2)$

$I_{cp} \Rightarrow$ Inertia of pol around its center of mass

$I_{cD} \Rightarrow$ Inertia of disc around its center of mass.