CSE 573 / 473 Lectures:

Monday, Wednesday, Friday

3:00PM - 3:50PM 121 Cooke Hall

CSE 473 Recitation:

Wednesday

4:00PM – 4:50PM 6 Clemens Hall

Thursday

3:00PM – 3:50PM 106 Baldy Hall

Computer Vision & Image Processing CSE 473 / 573

Instructor - Kevin R. Keane, PhD

TAs - Radhakrishna Dasari, Yuhao Du, Niyazi Sorkunlu

Lecture 10

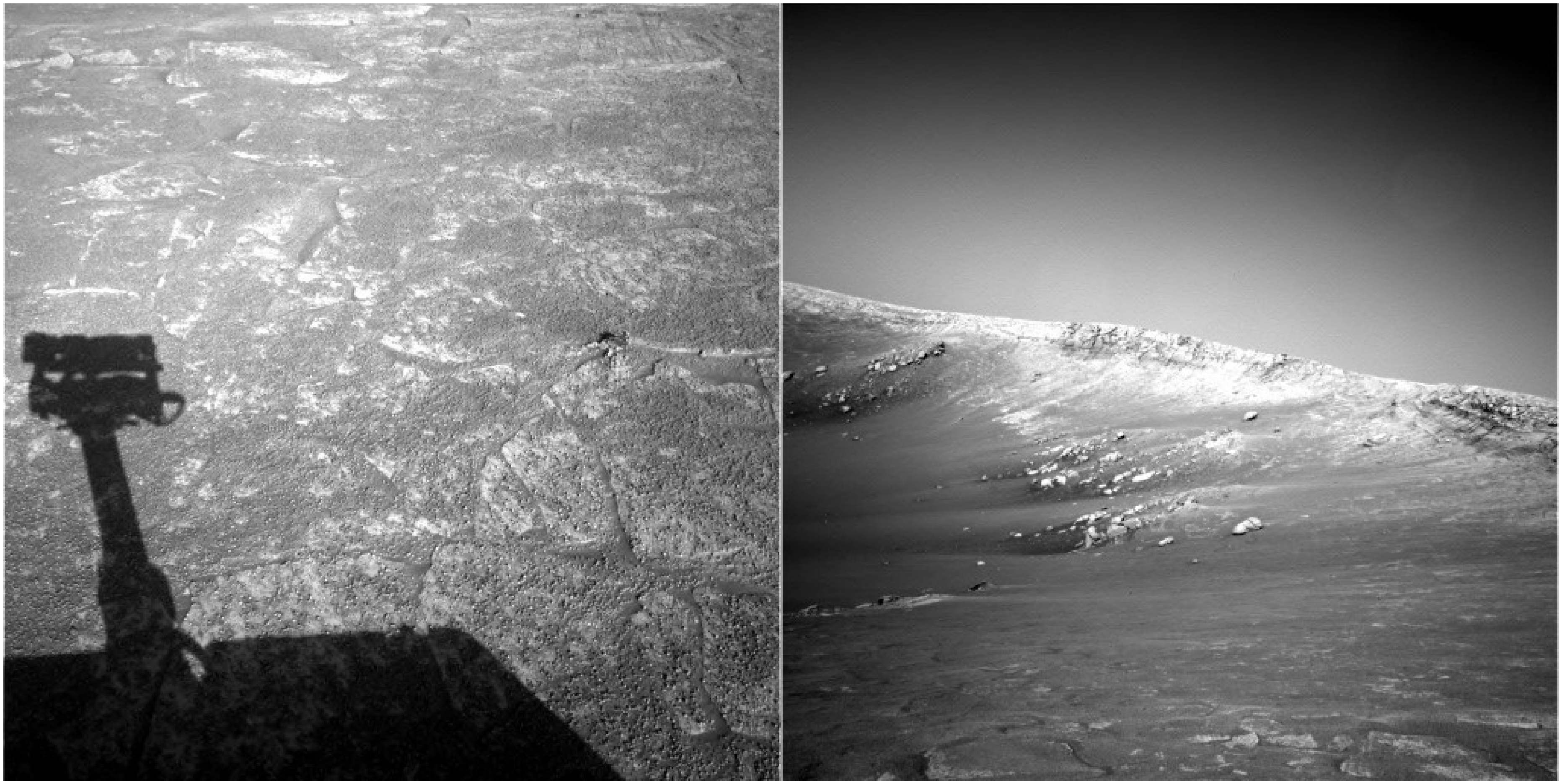
September 20, 2017

Local Features I

Schedule

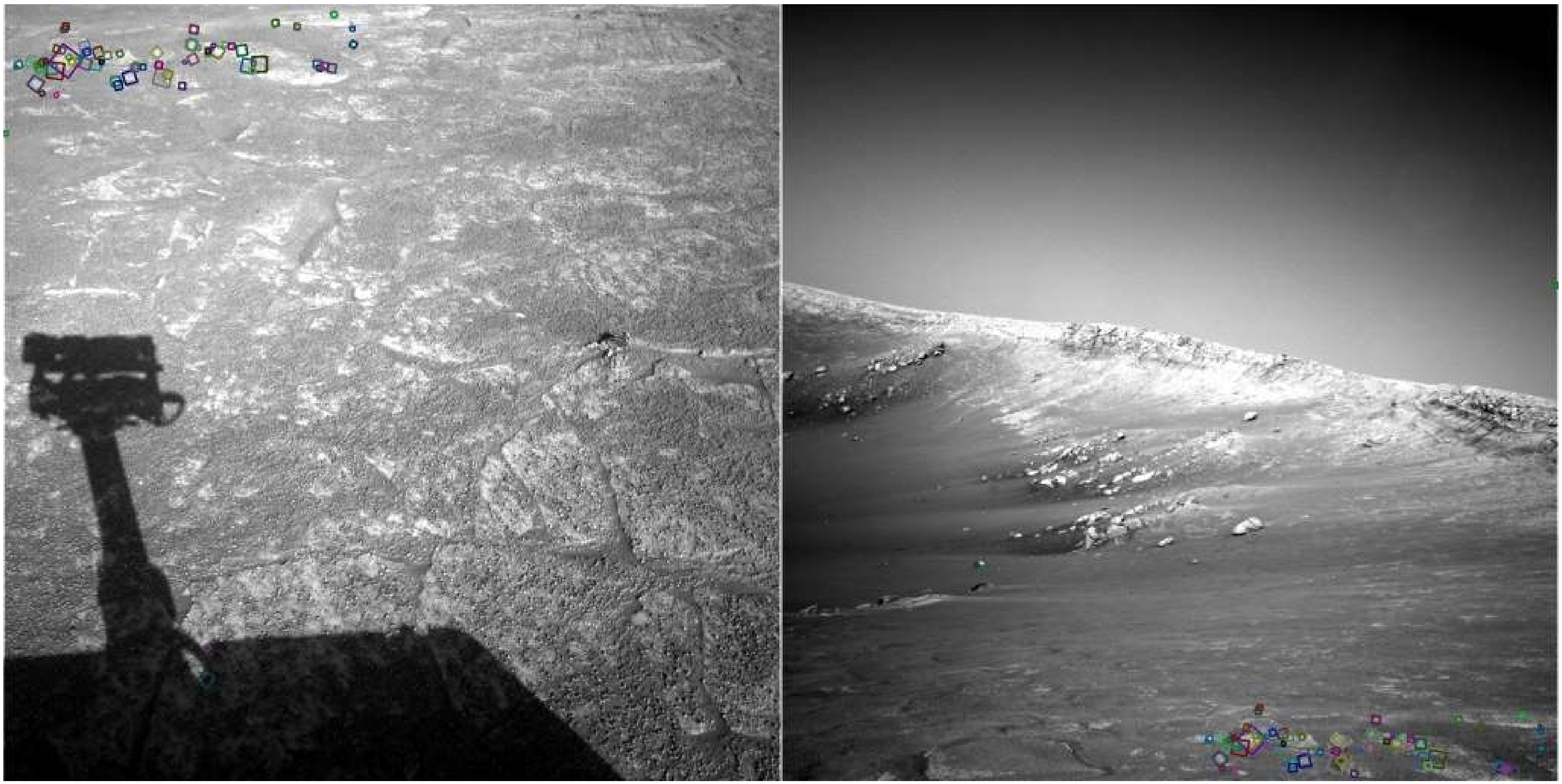
- Last class
  - Edges and pyramids
- Today
  - Local features
- Readings for today: Forsyth and Ponce Chapter 5

A hard feature matching problem



NASA Mars Rover images

Answer below (look for tiny colored squares...)

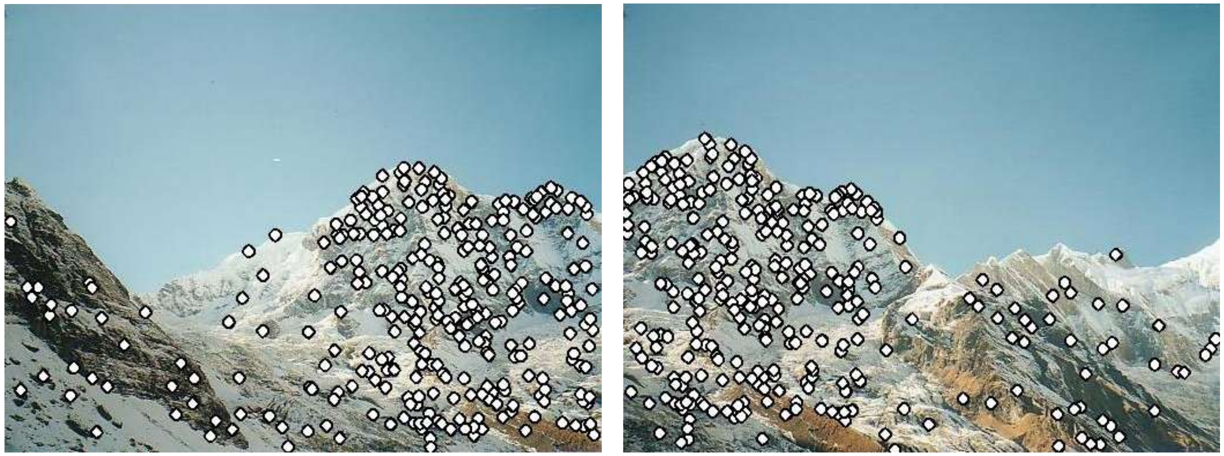


NASA Mars Rover images
with SIFT feature matches
Figure by Noah Snavely

Image neighborhoods

- We want to find patches that are "worth representing"
  - to match from image to image
  - to represent textures
  - to represent objects
- Requirements
  - Covariant to translation, rotation, scale
    - i.e. if the image is translated, rotated, scaled, so are the neighborhoods
    - important to ensure that the representation of the patch is stable
  - Localizable in translation, rotation, scale
    - we can estimate the position, orientation and size of the patch and get the answer about right
- Methods exist for richer sets of requirements

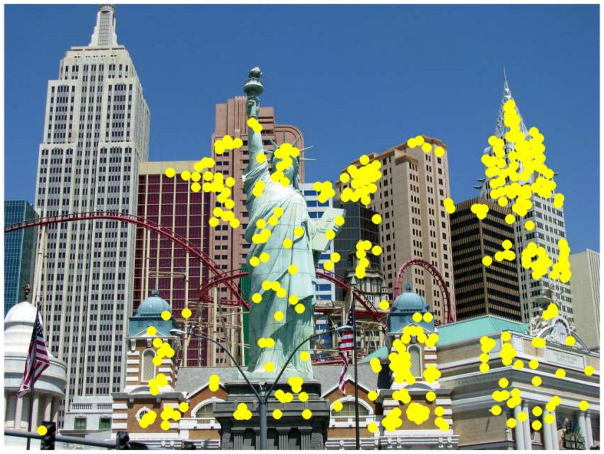
Characteristics of good features



- Repeatability
  - The same feature can be found in several images despite geometric and photometric transformations
- Saliency
  - Each feature has a distinctive description
- Compactness and efficiency
  - Many fewer features than image pixels
- Locality
  - A feature occupies a relatively small area of the image; robust to clutter and occlusion

Applications

- Feature points are used for:
  - Image alignment
  - 3D reconstruction
  - Motion tracking
  - Robot navigation
  - Indexing and database retrieval
  - Object recognition



Overview

- Corners (Harris Detector)

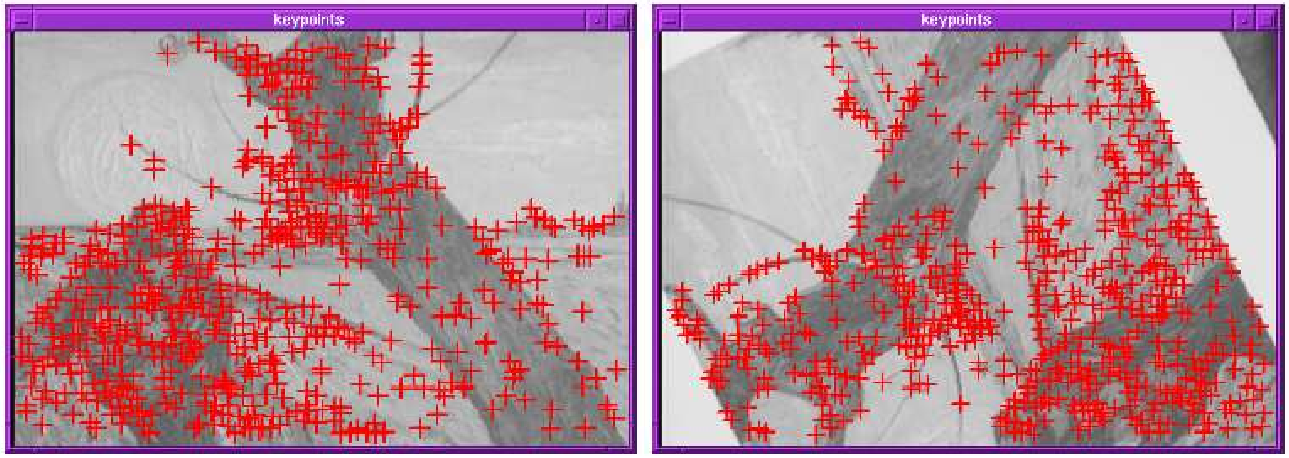
- Blobs

- Descriptors

Overview

- Corners (Harris Detector)
- Blobs
- Descriptors

Finding Corners



- Key property: in the region around a corner, image gradient has two or more dominant directions
- Corners are repeatable and distinctive

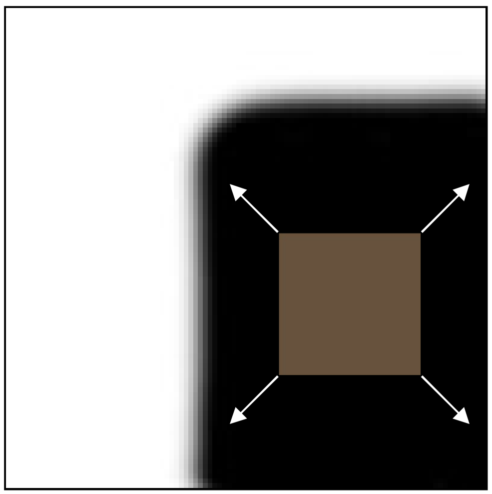
C.Harris and M.Stephens. "A Combined Corner and Edge Detector."
Proceedings of the 4th Alvey Vision Conference: pages 147--151.

Finding image neighborhoods - I

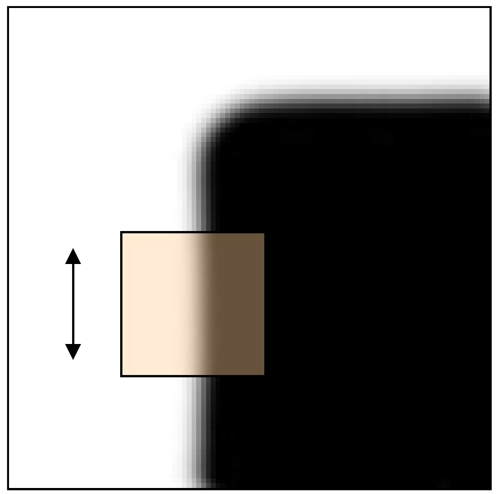
- Corner finding strategy
  - Find centers
  - At each center, estimate scale
  - Now from center, scale, estimate orientation

Corner Detection: Basic Idea

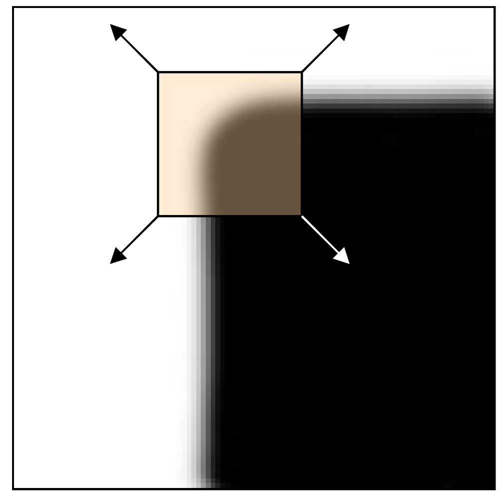
- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity



“flat” region: no change in all directions



“edge”: no change along the edge direction



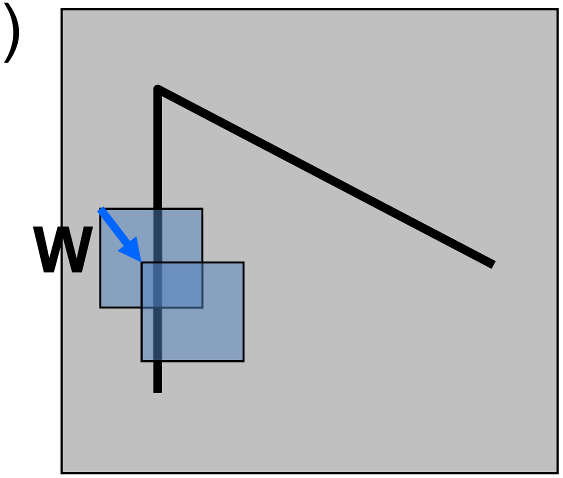
“corner”: significant change in all directions

Source: A. Efros

Feature detection: the math

Consider shifting the window **W** by (u,v)

- how do the pixels in **W** change?
- compare each pixel before and after by summing up the squared differences (SSD)
- this defines an SSD "error" of *E(u,v)*:



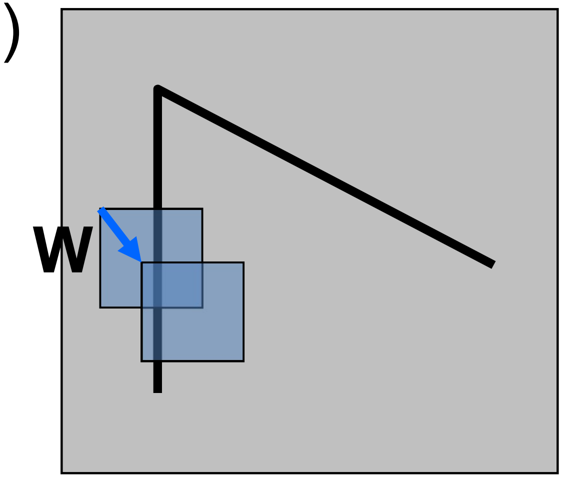
$$E(u,v) = \sum_{(x,y)\in W} \left[I(x+u,y+v) - I(x,y)\right]^2$$

Source: S. Seitz

Feature detection: the math

Consider shifting the window **W** by (u,v)

- how do the pixels in **W** change?
- compare each pixel before and after by summing up the squared differences (SSD)
- this defines an SSD "error" of *E(u,v)*:



$$E(u,v) = \sum_{(x,y)\in W} \left[I(x+u,y+v) - I(x,y)\right]^2$$

Source: S. Seitz

Small motion assumption

Taylor Series expansion of I:

$$I(x+u, y+v) = I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \text{higher order terms}$$

If the motion (u,v) is small, then first order approx is good

$$I(x+u, y+v) \approx I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v$$

$$\approx I(x, y) + [I_x \;\; I_y] \begin{bmatrix} u \\ v \end{bmatrix}$$

shorthand: $I_x = \frac{\partial I}{\partial x}$

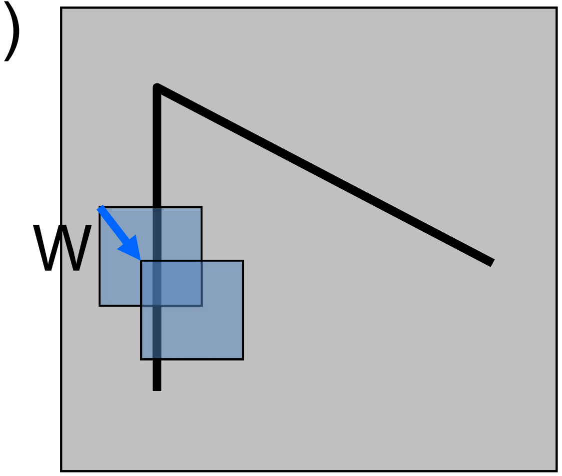
Plugging this into the formula on the previous slide…

Source: S. Seitz

Feature detection: the math

Consider shifting the window W by (u,v)

- how do the pixels in W change?
- compare each pixel before and after by summing up the squared differences
- this defines an "error" of E(u,v):



$$E(u,v) = \sum_{(x,y)\in W} [I(x+u,y+v) - I(x,y)]^2$$

$$\approx \sum_{(x,y)\in W} \left[I(x,y) + [I_x \;\; I_y] \begin{bmatrix} u \\ v \end{bmatrix} - I(x,y)\right]^2$$

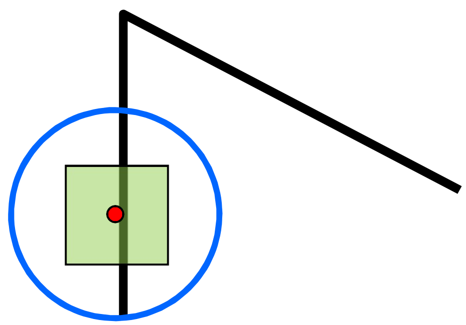
$$\approx \sum_{(x,y)\in W} \left[[I_x \;\; I_y] \begin{bmatrix} u \\ v \end{bmatrix}\right]^2$$

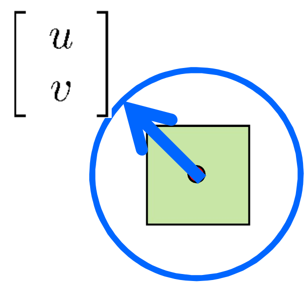
Source: S. Seitz

Feature detection: the math

This can be rewritten:

$$E(u,v) = \sum_{(x,y)\in W} [u\ v] \underbrace{\begin{bmatrix} I_x^2 & I_x I_y \\ I_y I_x & I_y^2 \end{bmatrix}}_{H} \begin{bmatrix} u \\ v \end{bmatrix}$$





For the example above

- You can move the center of the green window to anywhere on the blue unit circle
- Which directions will result in the largest and smallest E values?
- We can find these directions by looking at the eigenvectors of ***H***

Source: S. Seitz

Quick eigenvalue/eigenvector review

The **eigenvectors** of a matrix **A** are the vectors **x** that satisfy:

$$Ax = \lambda x$$

The scalar λ is the **eigenvalue** corresponding to **x**

- The eigenvalues are found by solving:

$$det(A - \lambda I) = 0$$

- In our case, ***A** is **H*** and is a 2x2 matrix, so we have: $det \begin{bmatrix} h_{11} - \lambda & h_{12} \\ h_{21} & h_{22} - \lambda \end{bmatrix} = 0$

- The solution:

$$\lambda_{\pm} = \frac{1}{2}\left[(h_{11} + h_{22}) \pm \sqrt{4h_{12}h_{21} + (h_{11} - h_{22})^2}\right]$$

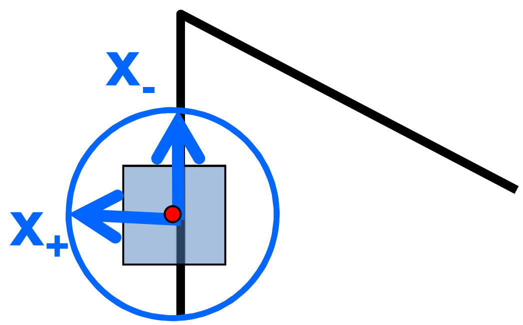
Once you know λ, you find **x** by solving $\begin{bmatrix} h_{11} - \lambda & h_{12} \\ h_{21} & h_{22} - \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

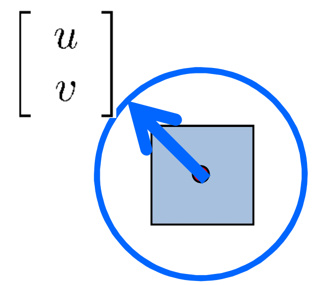
Source: S. Seitz

Feature detection: the math

This can be rewritten:

$$E(u,v) = \sum_{(x,y)\in W} [u \; v] \underbrace{\begin{bmatrix} I_x^2 & I_x I_y \\ I_y I_x & I_y^2 \end{bmatrix}}_{H} \begin{bmatrix} u \\ v \end{bmatrix}$$





Eigenvalues and eigenvectors of H

- Define shifts with the smallest and largest change (E value)
- x_+ = direction of **largest** increase in E.
- λ_+ = amount of increase in direction x_+
- x_- = direction of **smallest** increase in E.
- λ- = amount of increase in direction x_+

$$\begin{aligned} Hx_+ &= \lambda_+ x_+ \\ Hx_- &= \lambda_- x_- \end{aligned}$$

Source: S. Seitz

Feature detection: the math

How are λ_+, $\mathbf{x}_+$, λ_-, and $\mathbf{x}_+$ relevant for feature detection?

- What's our feature scoring function?

Source: S. Seitz

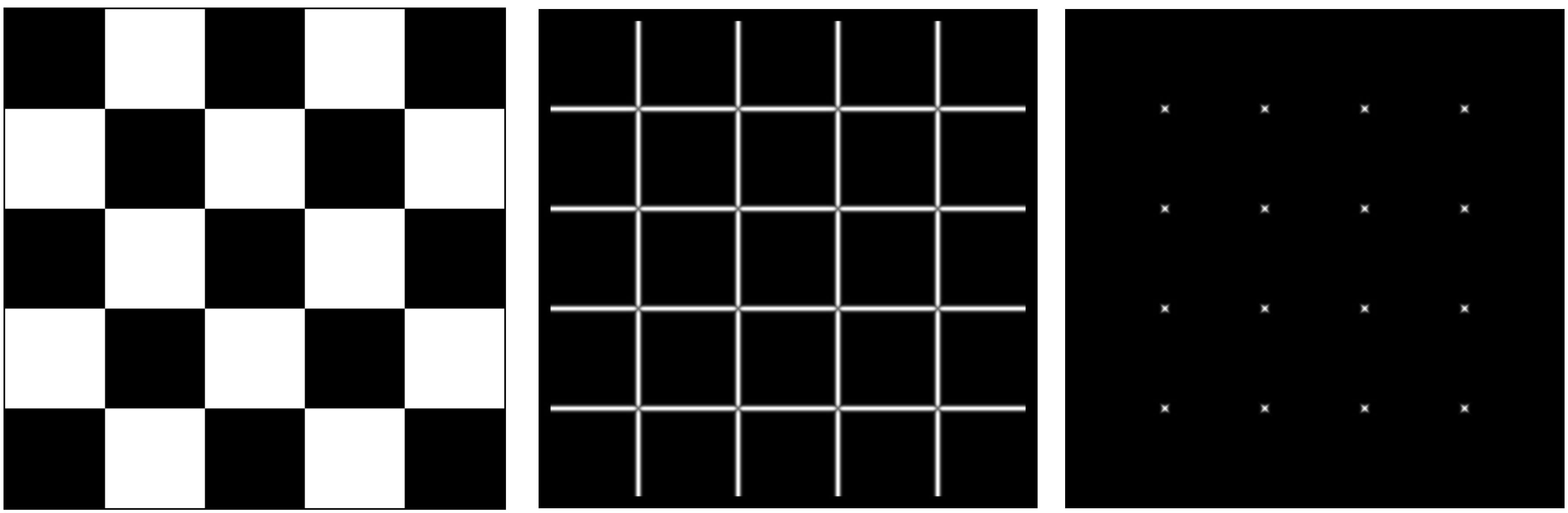
Feature detection: the math

How are λ_+, $\mathbf{x}_+$, λ_-, and $\mathbf{x}_+$ relevant for feature detection?

- What's our feature scoring function?

Want *E(u,v)* to be ***large*** for small shifts in ***all*** directions

- the *minimum* of *E(u,v)* should be large, over all unit vectors [u v]
- this minimum is given by the smaller eigenvalue (λ_-) of ***H***

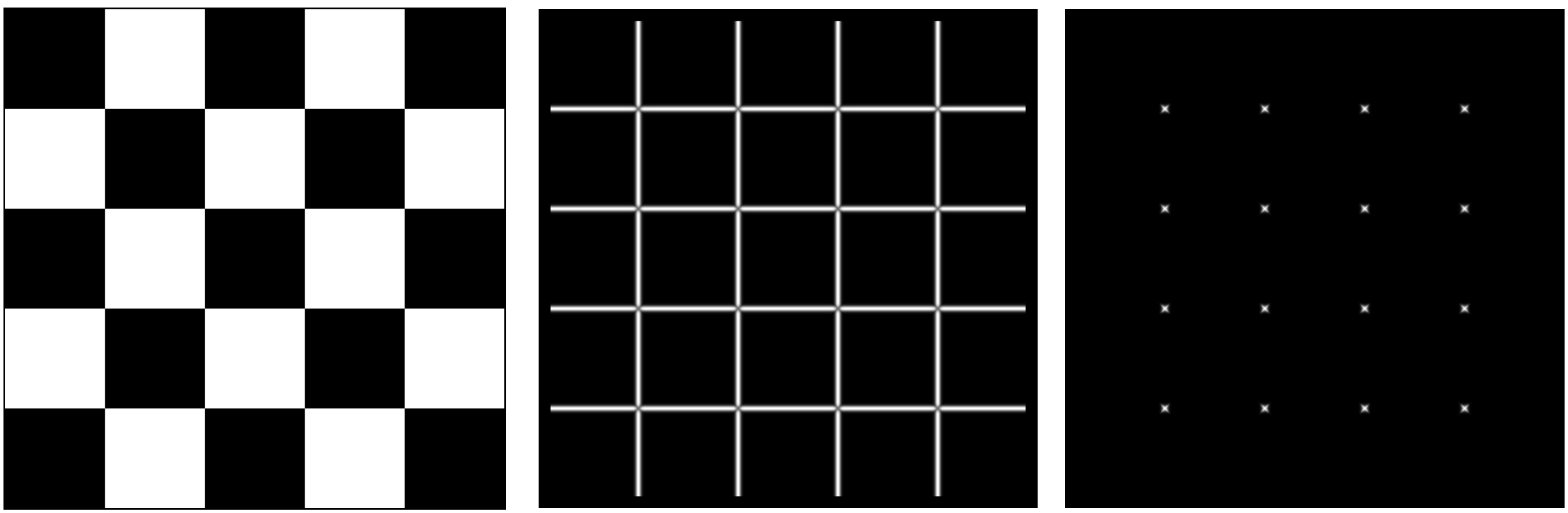


Source: S. Seitz I λ_+ λ_-

Feature detection summary

Here's what you do

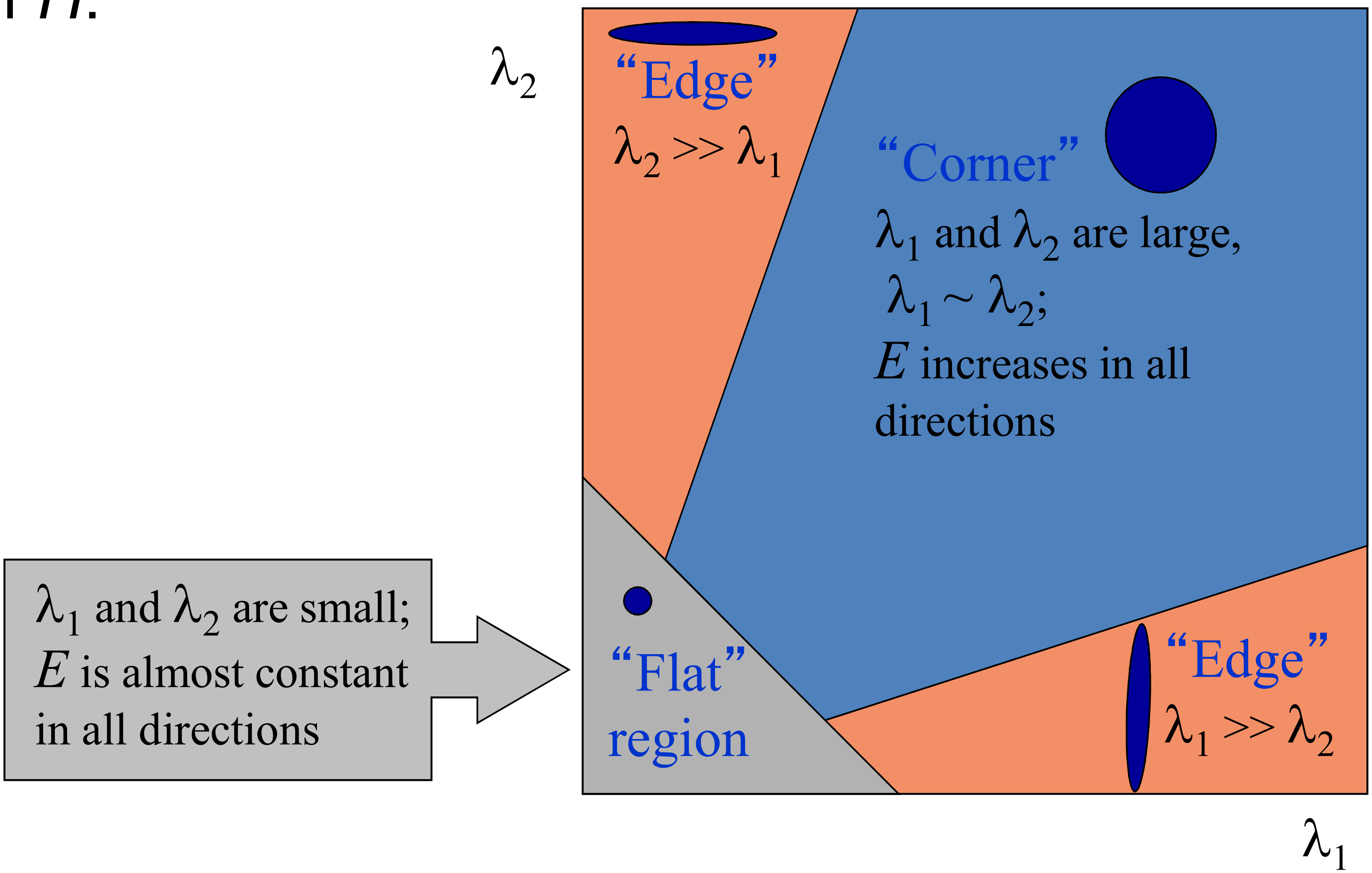
- Compute the gradient at each point in the image
- Create the ***H*** matrix from the entries in the gradient
- Compute the eigenvalues.
- Find points with large response (λ_- > threshold)
- Choose those points where λ_- is a local maximum as features



Source: S. Seitz I λ_+ λ_-

Interpreting the eigenvalues

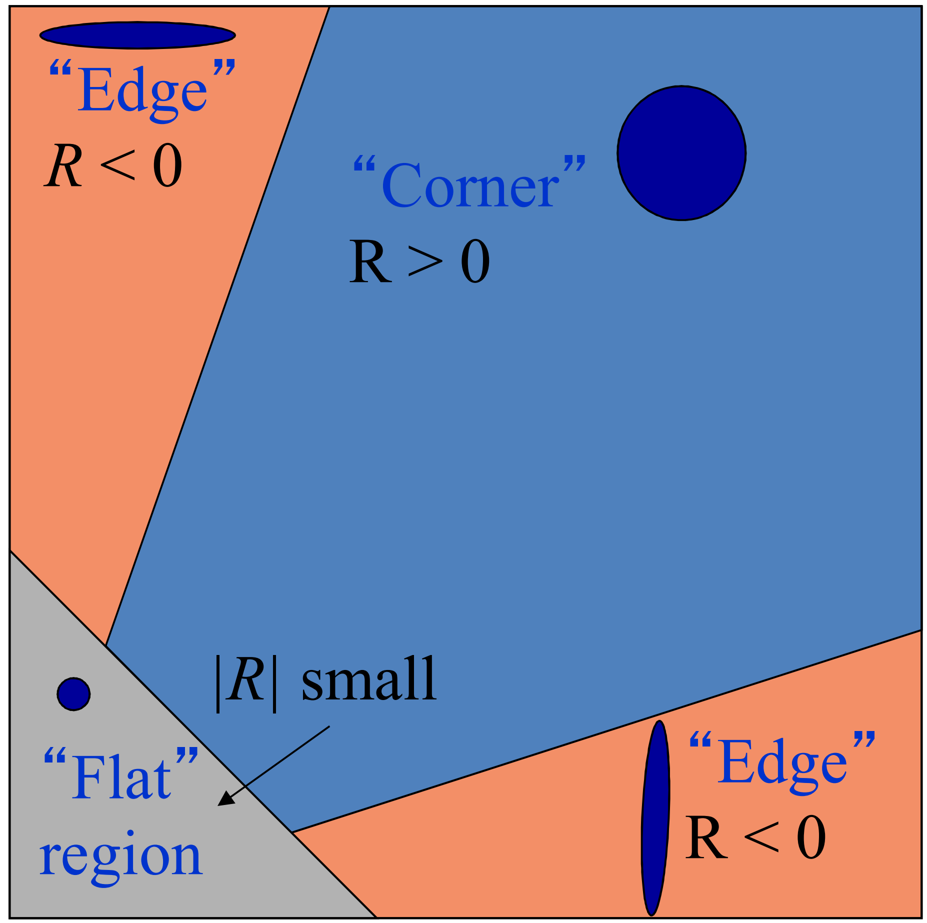
Classification of image points using eigenvalues of *H*:



Corner response function

$$R = \det(H) - \alpha\,\mathrm{trace}(H)^2 = \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

α: constant (0.04 to 0.06)



Harris detector: Steps

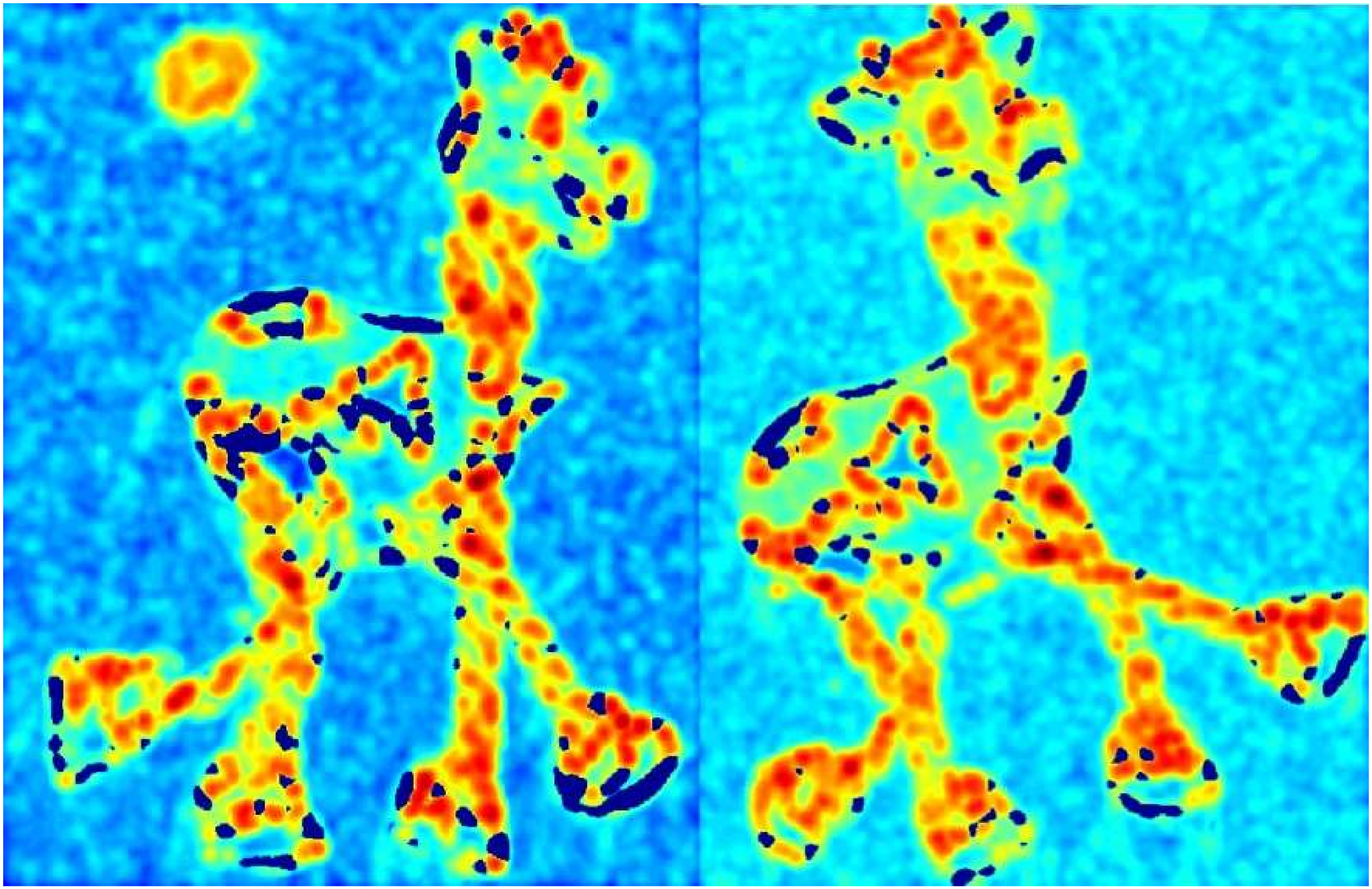
1. Compute Gaussian derivatives at each pixel
2. Compute second moment matrix *H* in a Gaussian window around each pixel
3. Compute corner response function *R*
4. Threshold *R*
5. Find local maxima of response function (nonmaximum suppression)

C.Harris and M.Stephens. "A Combined Corner and Edge Detector.“ *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

Harris Detector: Steps

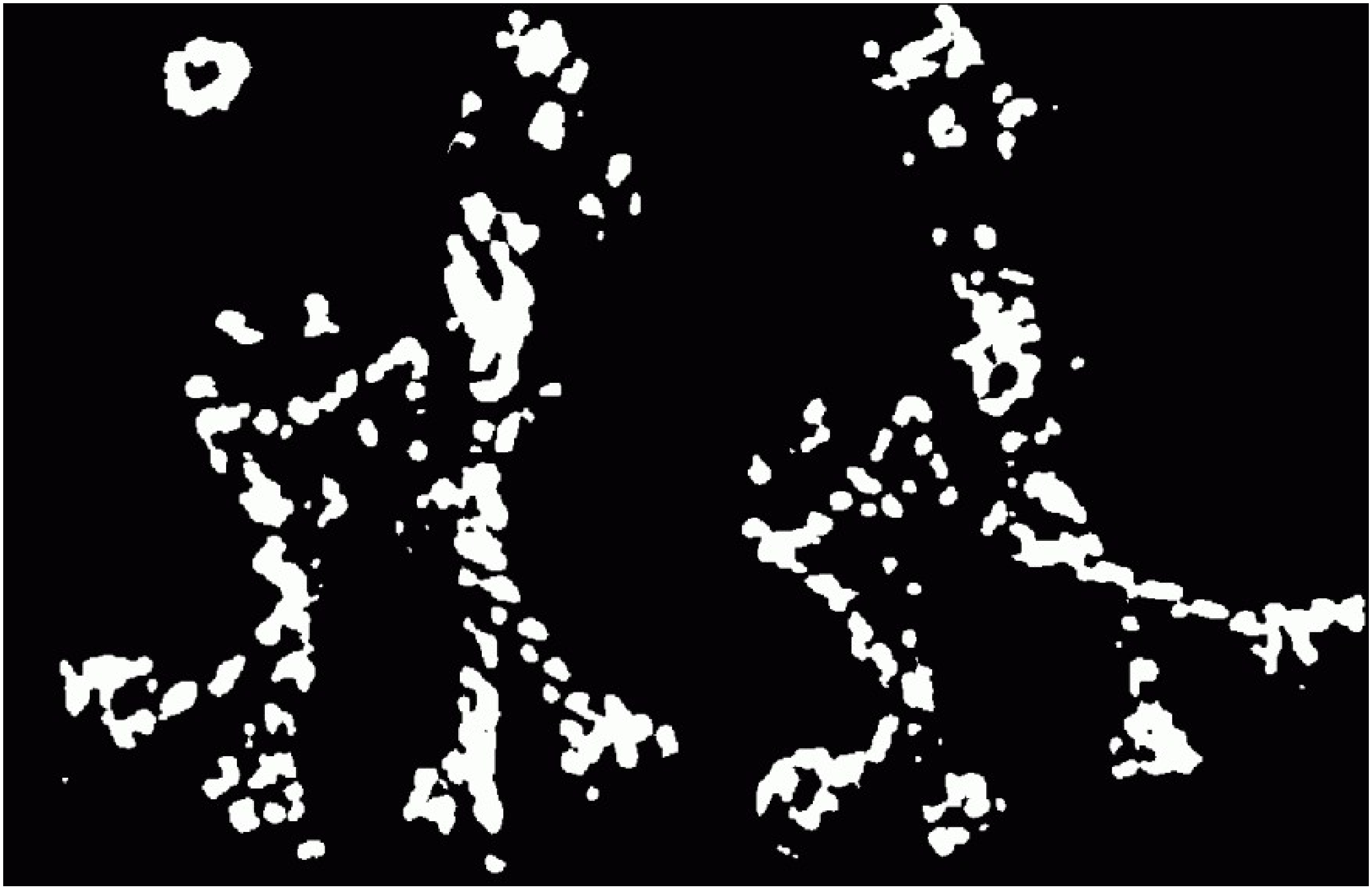
Harris Detector: Steps

Compute corner response R



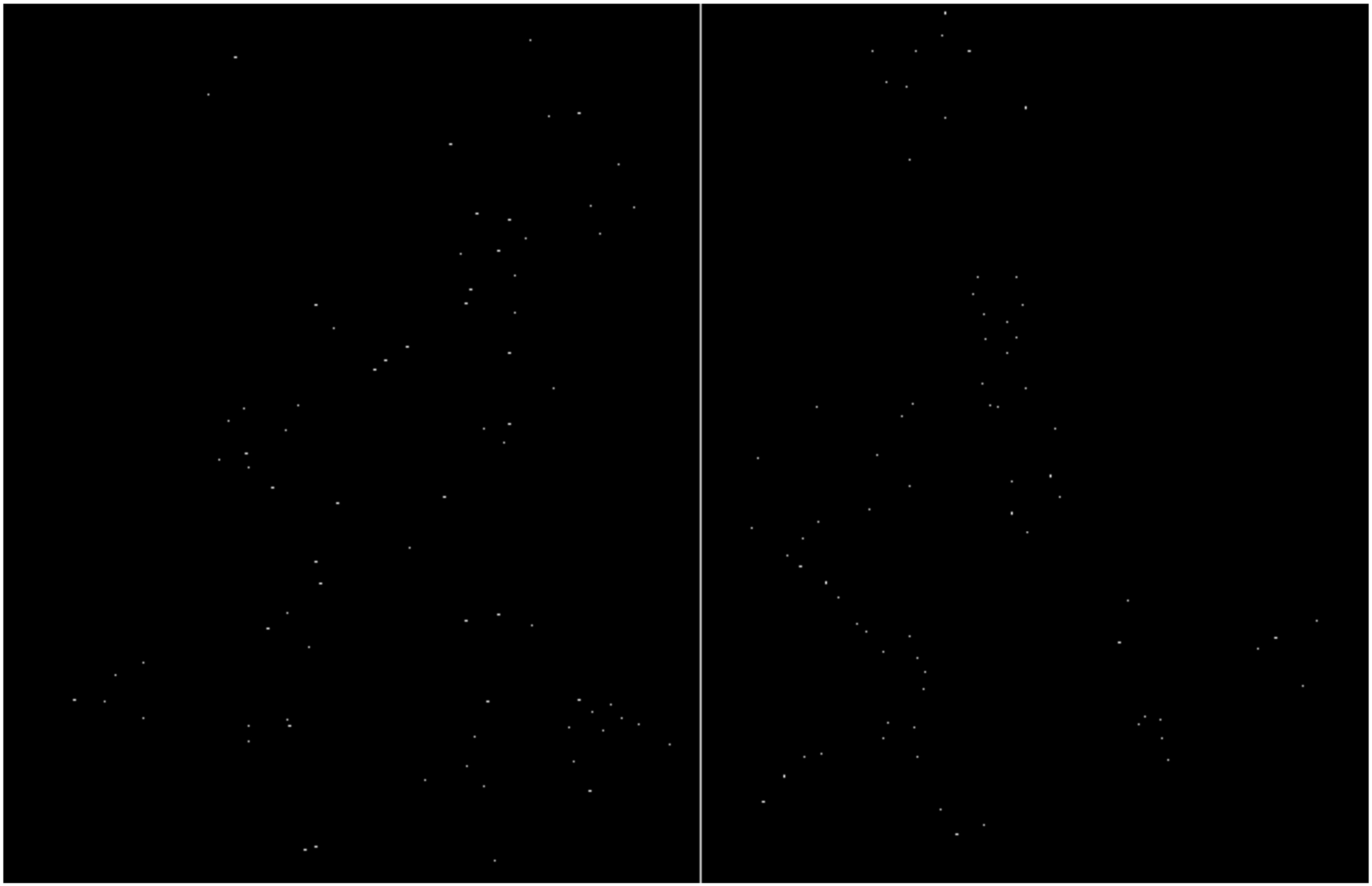
Harris Detector: Steps

Find points with large corner response: R>threshold



Harris Detector: Steps

Take only the points of local maxima of R

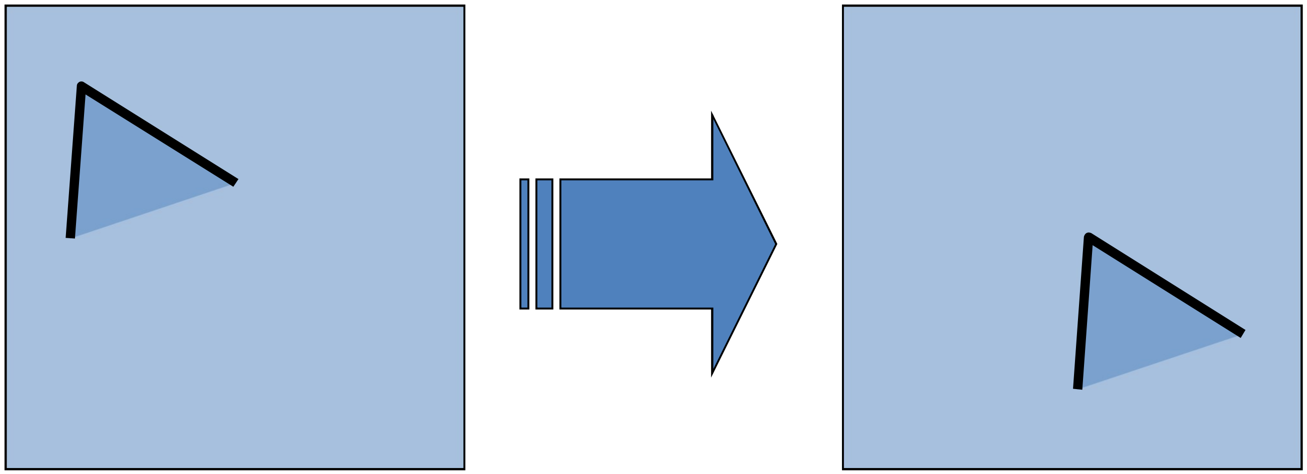


Harris Detector: Steps

Invariance and covariance

- We want features to be *invariant* to photometric transformations and *covariant* to geometric transformations
  - **Invariance:** image is transformed and features do not change
  - **Covariance:** if we have two transformed versions of the same image, features should be detected in corresponding locations

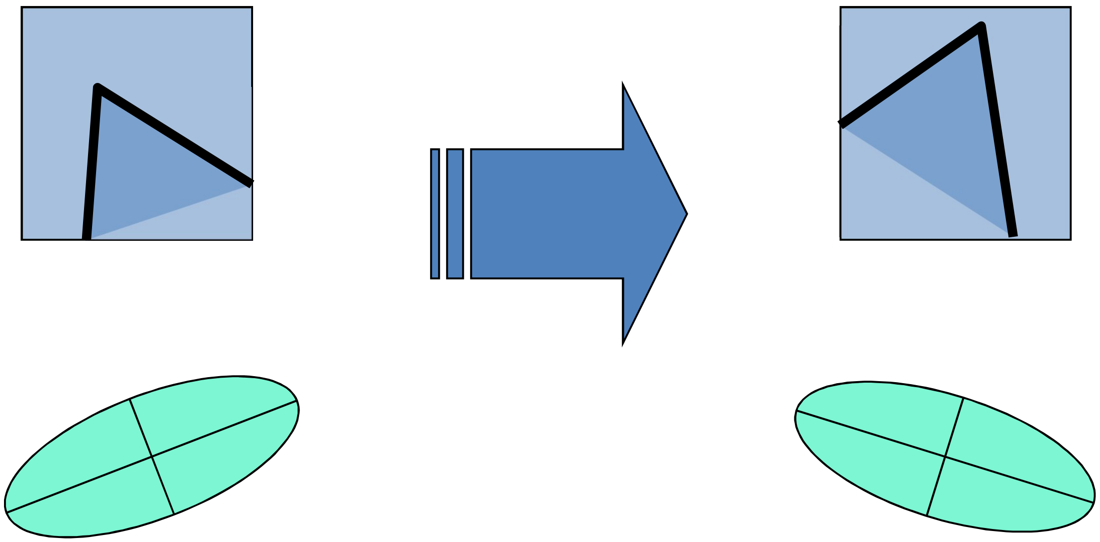
Image translation



- Derivatives and window function are shift-invariant

Corner location is covariant w.r.t. translation

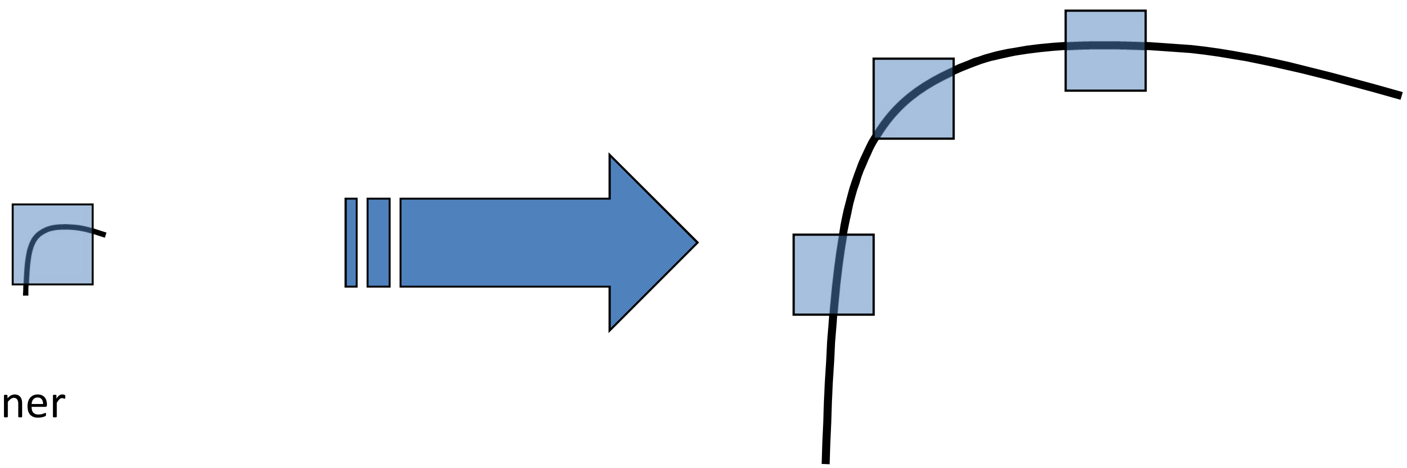
Image rotation



Second moment ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner location is covariant w.r.t. rotation

Scaling



Corner

All points will be classified as edges

Corner location is not covariant to scaling!

Harris corner detector summary

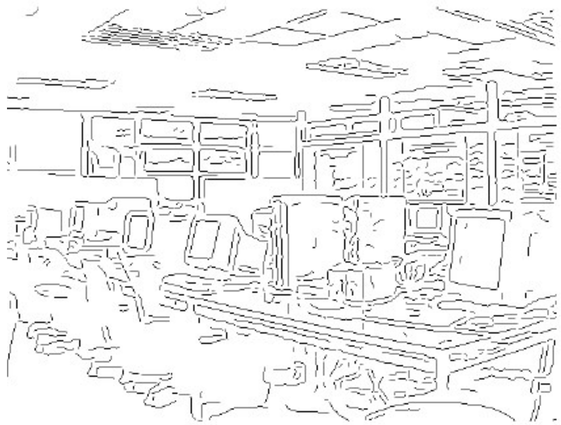
- Good corners
  - High contrast
  - Sharp change in edge orientation
- Image features at good corners
  - Large gradients that change direction sharply
    - Will have 2 large eigenvalues
- Compute matrix H by summing over window

$$\mathcal{H} = \sum_{window} \{(\nabla I)(\nabla I)^T\}$$

$$\approx \sum_{window} \left\{ \begin{array}{cc} (\frac{\partial G_\sigma}{\partial x} * *\mathcal{I})(\frac{\partial G_\sigma}{\partial x} * *\mathcal{I}) & (\frac{\partial G_\sigma}{\partial x} * *\mathcal{I})(\frac{\partial G_\sigma}{\partial y} * *\mathcal{I}) \\ (\frac{\partial G_\sigma}{\partial x} * *\mathcal{I})(\frac{\partial G_\sigma}{\partial y} * *\mathcal{I}) & (\frac{\partial G_\sigma}{\partial y} * *\mathcal{I})(\frac{\partial G_\sigma}{\partial y} * *\mathcal{I}) \end{array} \right\}$$

What about internal structure?

- Edges & Corners convey boundary information



- What about interior texture of the object?

Next class

- More on local features next class
- Readings for next lecture:
  - Same as today
- Readings for today:
  - Forsyth and Ponce 5; Szeliski 3.1-3.3

Slide Credits

- David A. Forsyth - UIUC
- Fei Fei Li - Stanford
- Svetlana Lazebnik – UIUC
- Rob Fergus – NYU

Questions