Computer Vision & Image Processing
CSE 473 / 573

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Lecture 15
October 4, 2017
Single-view modeling

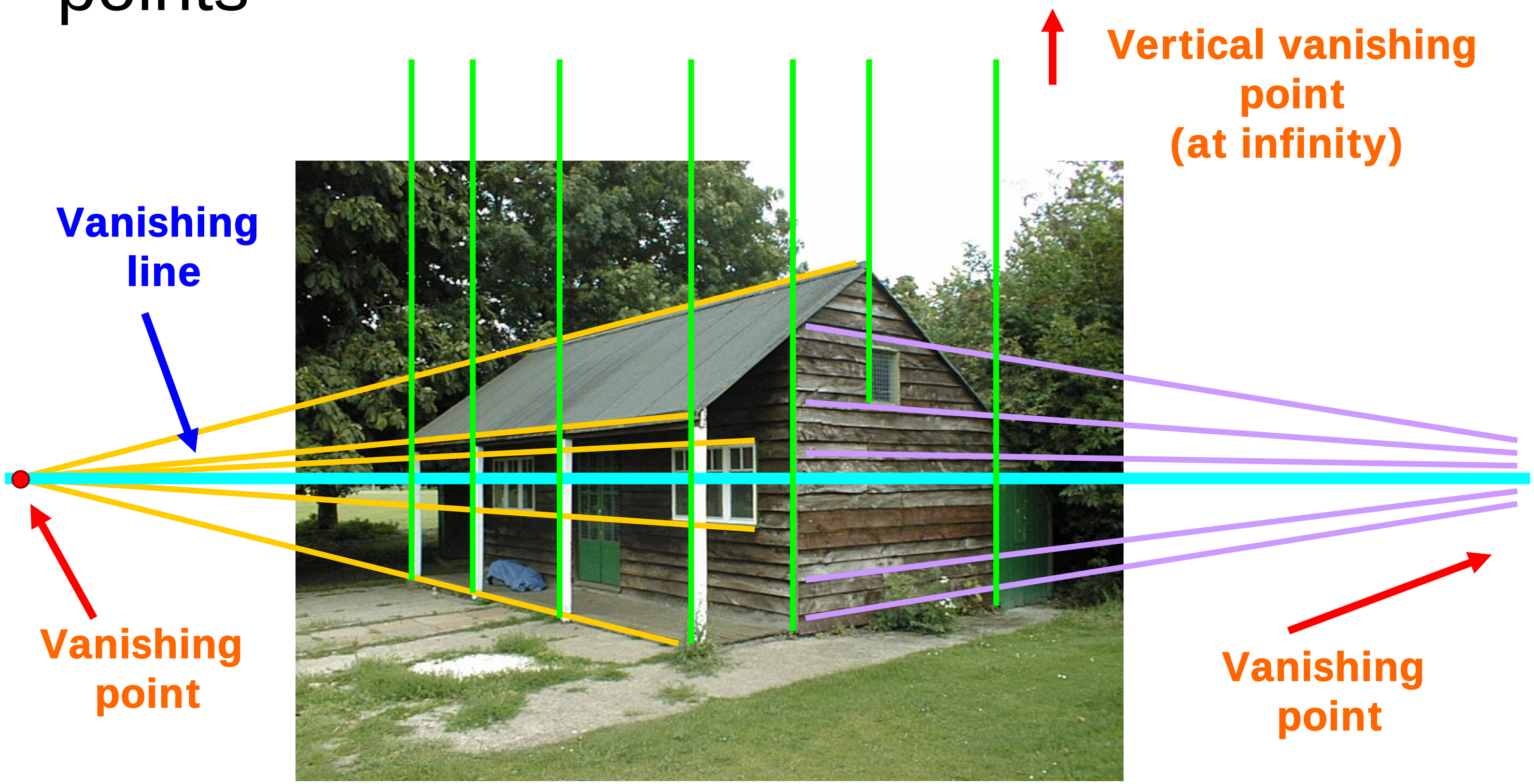
Single-view metrology

Magritte, *Personal Values*, 1952

Many slides from S. Seitz, D. Hoiem

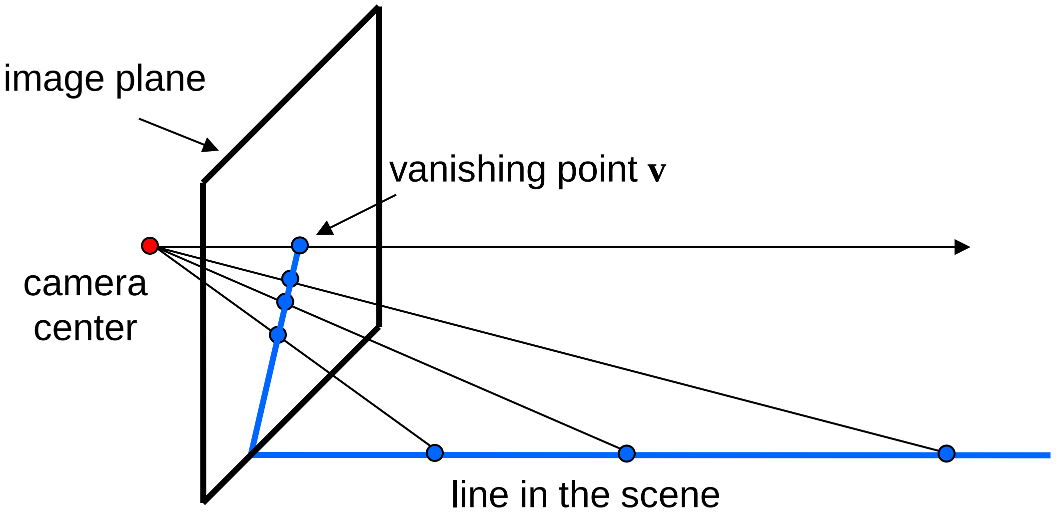
Camera calibration revisited

- What if world coordinates of reference 3D points are not known?
- We can use scene features such as vanishing points



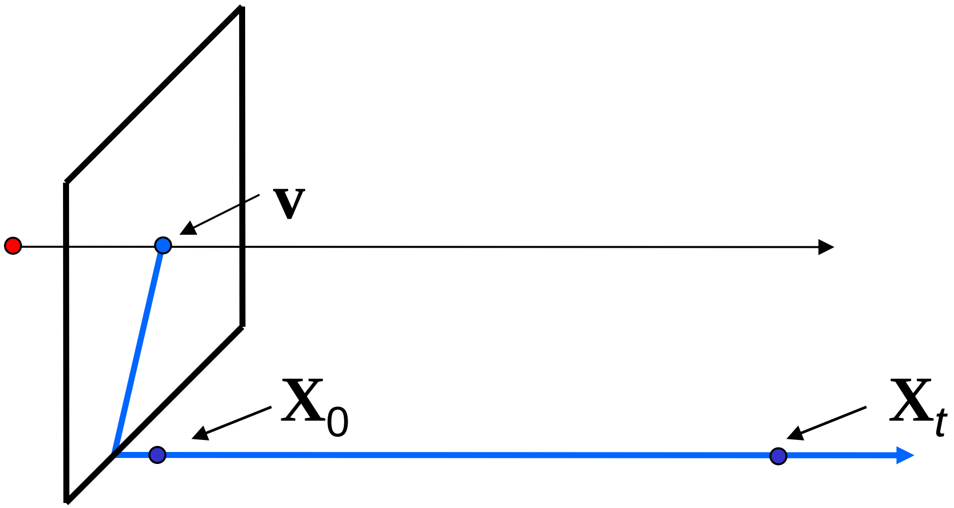
Slide from Efros, Photo from Criminisi

Recall: Vanishing points



- All lines having the same direction share the same vanishing point

Computing vanishing points

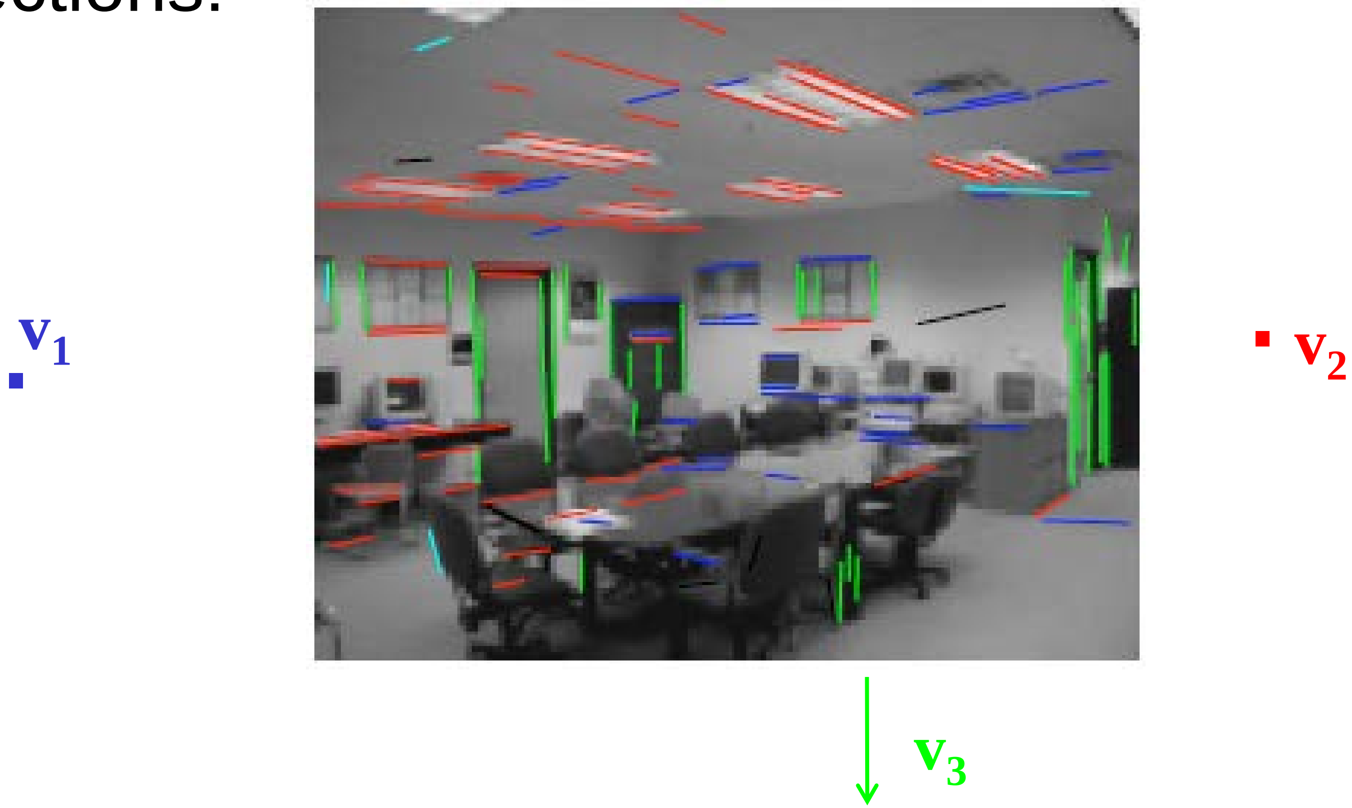


$$\mathbf{X}_t = \begin{bmatrix} x_0 + td_1 \\ y_0 + td_2 \\ z_0 + td_3 \\ 1 \end{bmatrix} = \begin{bmatrix} x_0/t + d_1 \\ y_0/t + d_2 \\ z_0/t + d_3 \\ 1/t \end{bmatrix} \qquad \mathbf{X}_\infty = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ 0 \end{bmatrix}$$

- $\mathbf{X}_\infty$ is a *point at infinity,* $\mathbf{v}$ is its projection: $\mathbf{v} = \mathbf{P}\mathbf{X}_\infty$
- The vanishing point depends only on *line direction*
- All lines having direction $\mathbf{d}$ intersect at $\mathbf{X}_\infty$

Calibration from vanishing points

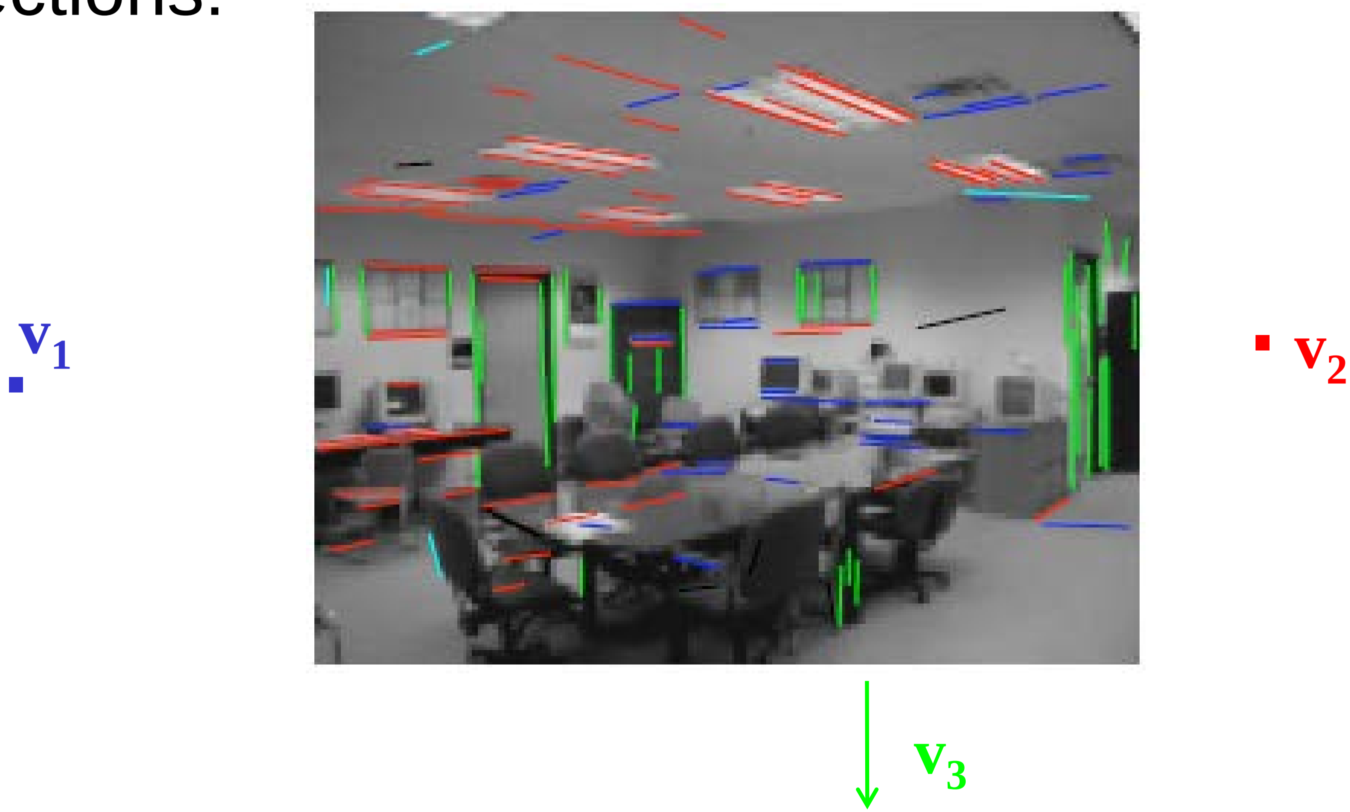
- Consider a scene with three orthogonal vanishing directions:



- Note: $\mathbf{v}_1$, $\mathbf{v}_2$ are *finite* vanishing points and $\mathbf{v}_3$ is an *infinite* vanishing point

Calibration from vanishing points

- Consider a scene with three orthogonal vanishing directions:



- We can align the world coordinate system with these directions

Calibration from vanishing points

$$\mathbf{P} = \begin{bmatrix} \boxed{\begin{matrix} * \\ * \\ * \end{matrix}} & \boxed{\begin{matrix} * \\ * \\ * \end{matrix}} & \boxed{\begin{matrix} * \\ * \\ * \end{matrix}} & \boxed{\begin{matrix} * \\ * \\ * \end{matrix}} \end{bmatrix} = \begin{bmatrix} \mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3 & \mathbf{p}_4 \end{bmatrix}$$

- $\mathbf{p}_1 = \mathbf{P}(1,0,0,0)^T$ – the vanishing point in the x direction
- Similarly, $\mathbf{p}_2$ and $\mathbf{p}_3$ are the vanishing points in the y and z directions
- $\mathbf{p}_4 = \mathbf{P}(0,0,0,1)^T$ – projection of the origin of the world coordinate system
- Problem: we can only know the four columns up to independent scale factors, additional constraints needed to solve for them

Calibration from vanishing points

- Let us align the world coordinate system with three orthogonal vanishing directions in the scene:

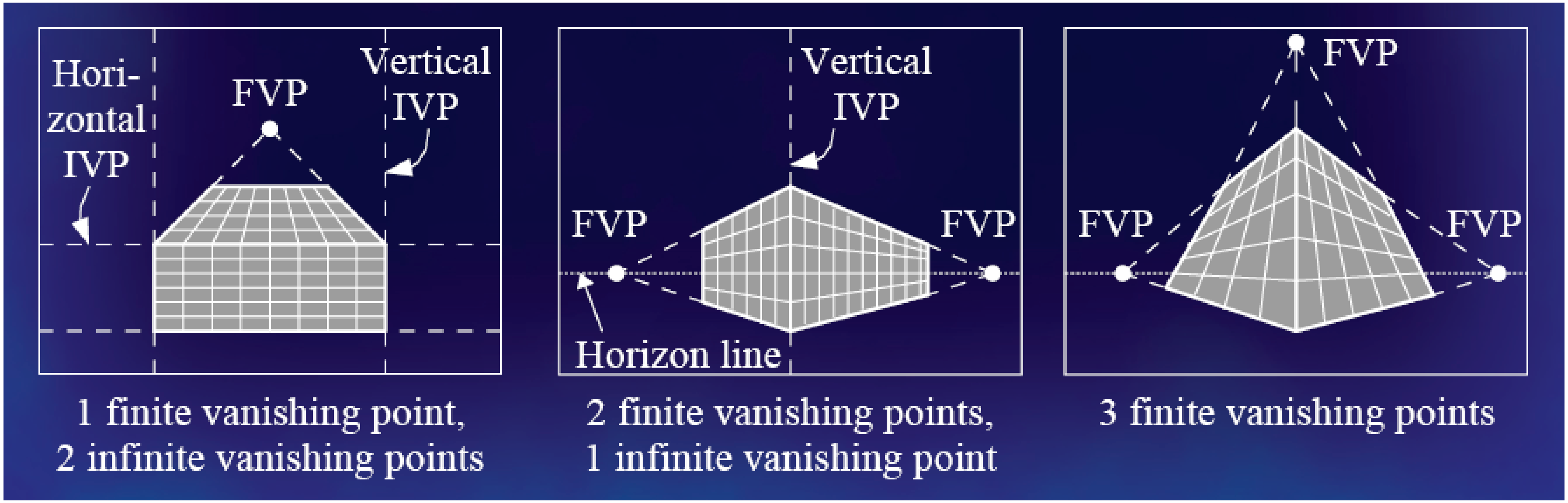
$$\mathbf{e_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{e_2} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{e_3} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad \lambda_i \mathbf{v}_i = \mathbf{K}[\mathbf{R} \mid \mathbf{t}] \begin{bmatrix} \mathbf{e}_i \\ 0 \end{bmatrix} = \mathbf{K}\mathbf{R}\mathbf{e}_i$$

$$\mathbf{e}_i = \lambda_i \mathbf{R}^T \mathbf{K}^{-1} \mathbf{v}_i, \quad \mathbf{e}_i^T \mathbf{e}_j = 0$$

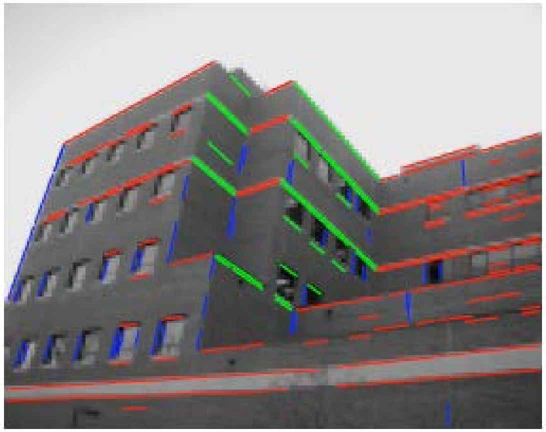
$$\mathbf{v}_i^T \mathbf{K}^{-T} \mathbf{R} \mathbf{R}^T \mathbf{K}^{-1} \mathbf{v}_j = \mathbf{v}_i^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{v}_j = 0$$

- Each pair of vanishing points gives us a constraint on the focal length and principal point

Calibration from vanishing points



Cannot recover focal length, principal point is the third vanishing point



Can solve for focal length, principal point

Rotation from vanishing points

$$\lambda_i \mathbf{v}_i = \mathbf{K}\left[\mathbf{R} \mid \mathbf{t}\right]\begin{bmatrix} \mathbf{e}_i \\ 0 \end{bmatrix} = \mathbf{K}\mathbf{R}\mathbf{e}_i$$

$$\lambda_1 \mathbf{K}^{-1}\mathbf{v}_1 = \mathbf{R}\mathbf{e}_1 = [\ \mathbf{r}_1 \quad \mathbf{r}_2 \quad \mathbf{r}_3\ ]\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \mathbf{r}_1$$

Thus, $\lambda_i \mathbf{K}^{-1}\mathbf{v}_i = \mathbf{r}_i$.

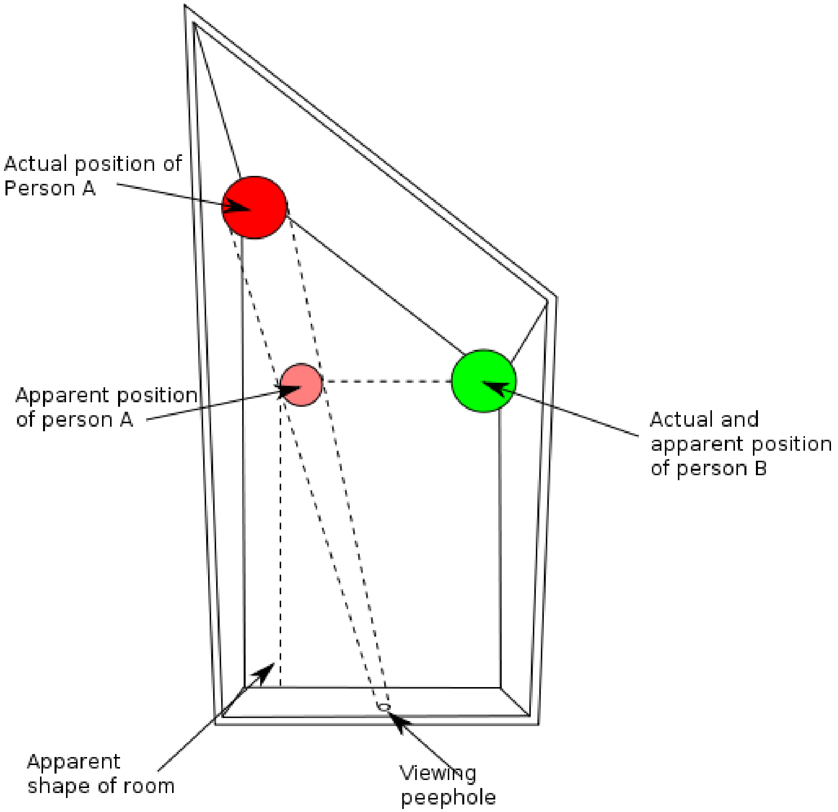
Get λ_i by using the constraint $\|\mathbf{r}_i\|^2=1$.

Calibration from vanishing points: Summary

- Solve for K (focal length, principal point) using three orthogonal vanishing points
- Get rotation directly from vanishing points once calibration matrix is known

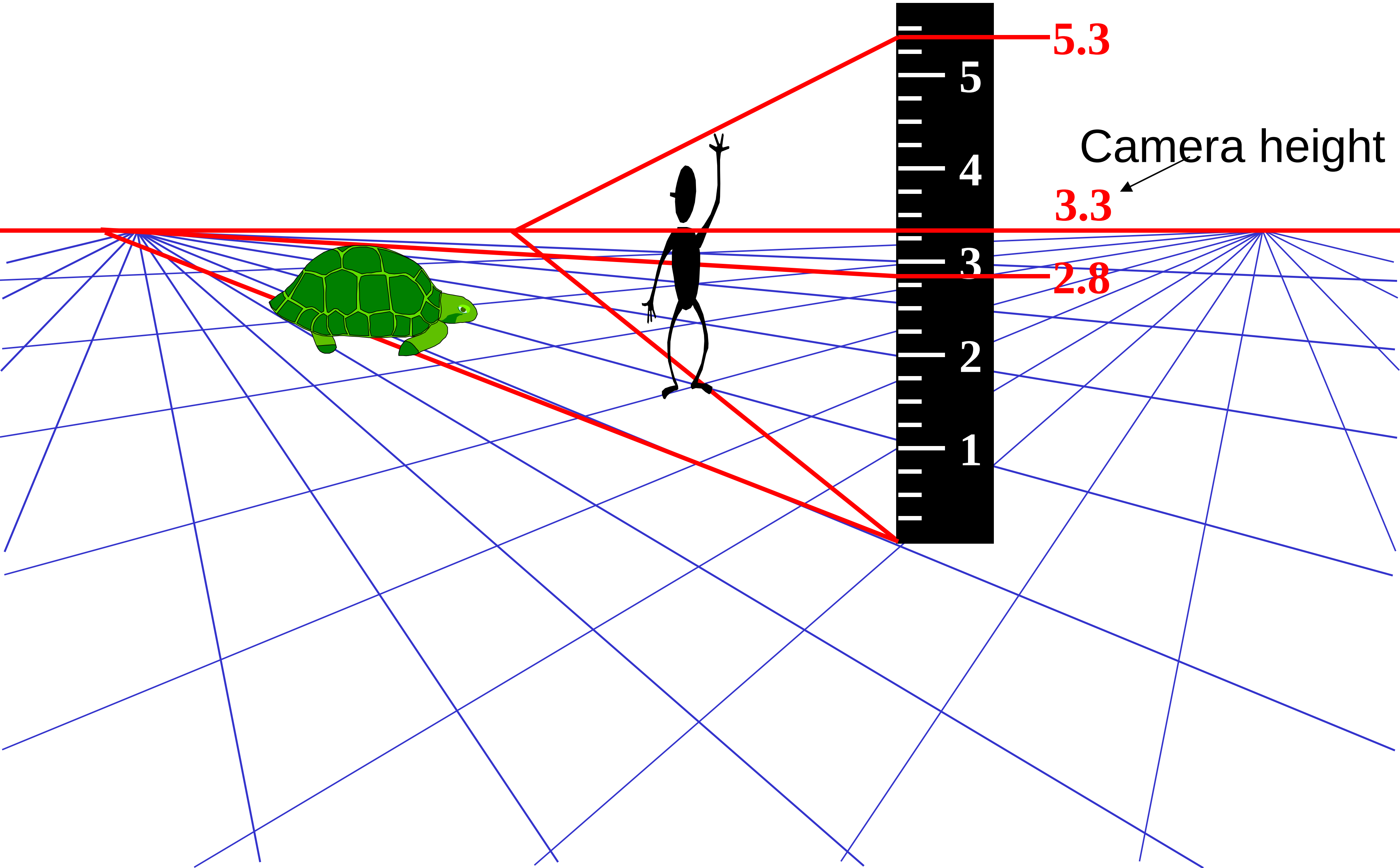
- Advantages
  - No need for calibration chart, 2D-3D correspondences
  - Could be completely automatic
- Disadvantages
  - Only applies to certain kinds of scenes
  - Inaccuracies in computation of vanishing points
  - Problems due to infinite vanishing points

Making measurements from a single image

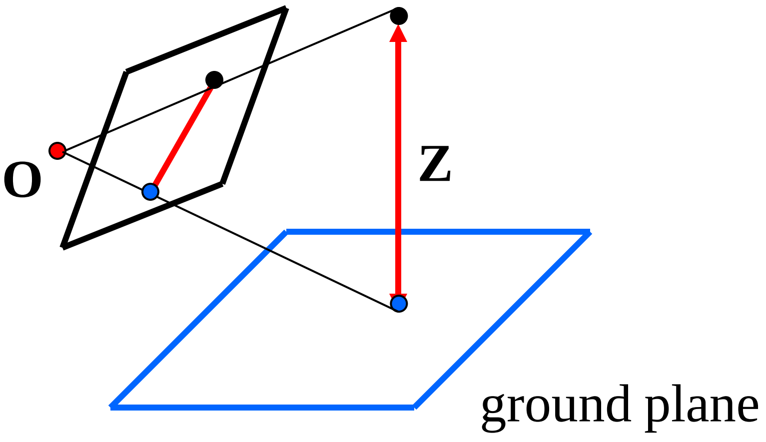


http://en.wikipedia.org/wiki/Ames_room

Recall: Measuring height



Measuring height without a ruler



Compute Z from image measurements

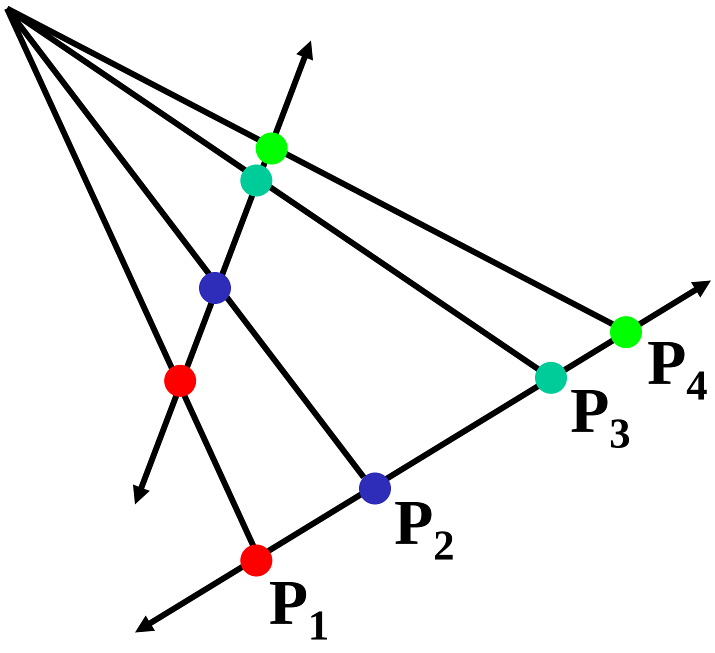
- Need more than vanishing points to do this

Projective invariant

- We need to use a *projective invariant*: a quantity that does not change under projective transformations (including perspective projection)
  - What are some invariants for similarity, affine transformations?

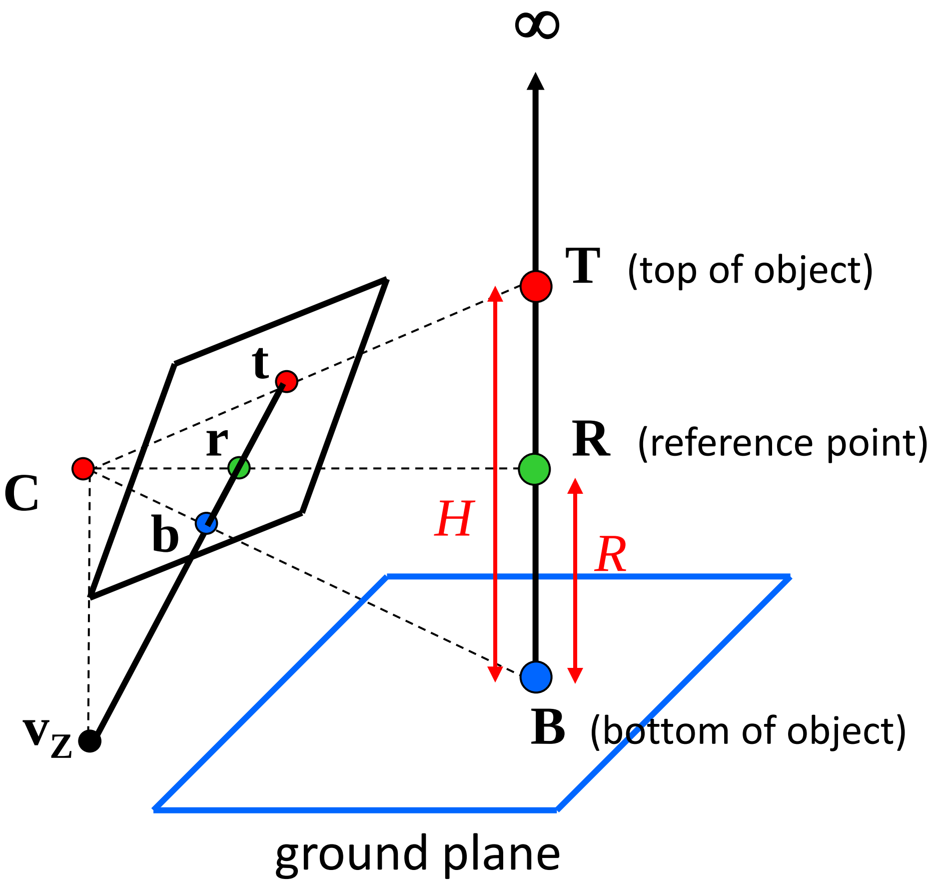
Projective invariant

- We need to use a *projective invariant*: a quantity that does not change under projective transformations (including perspective projection)
- The *cross-ratio* of four points:



$$\frac{\left\|\mathbf{P}_3 - \mathbf{P}_1\right\| \left\|\mathbf{P}_4 - \mathbf{P}_2\right\|}{\left\|\mathbf{P}_3 - \mathbf{P}_2\right\| \left\|\mathbf{P}_4 - \mathbf{P}_1\right\|}$$

Measuring height



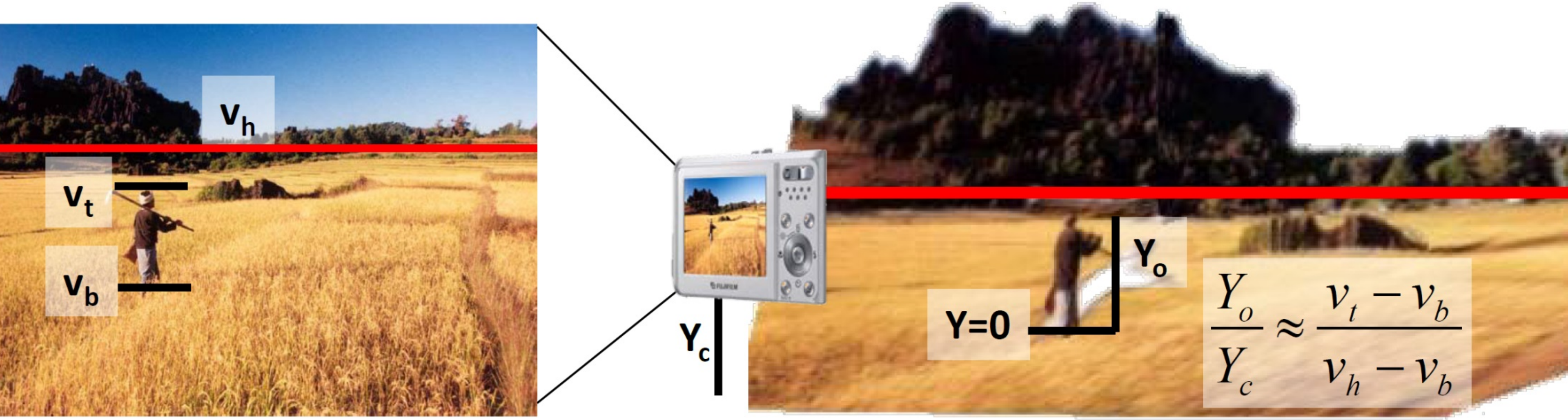
$$\frac{\|\mathbf{T}-\mathbf{B}\|\,\|\infty-\mathbf{R}\|}{\|\mathbf{R}-\mathbf{B}\|\,\|\infty-\mathbf{T}\|}=\frac{H}{R}$$

scene cross ratio

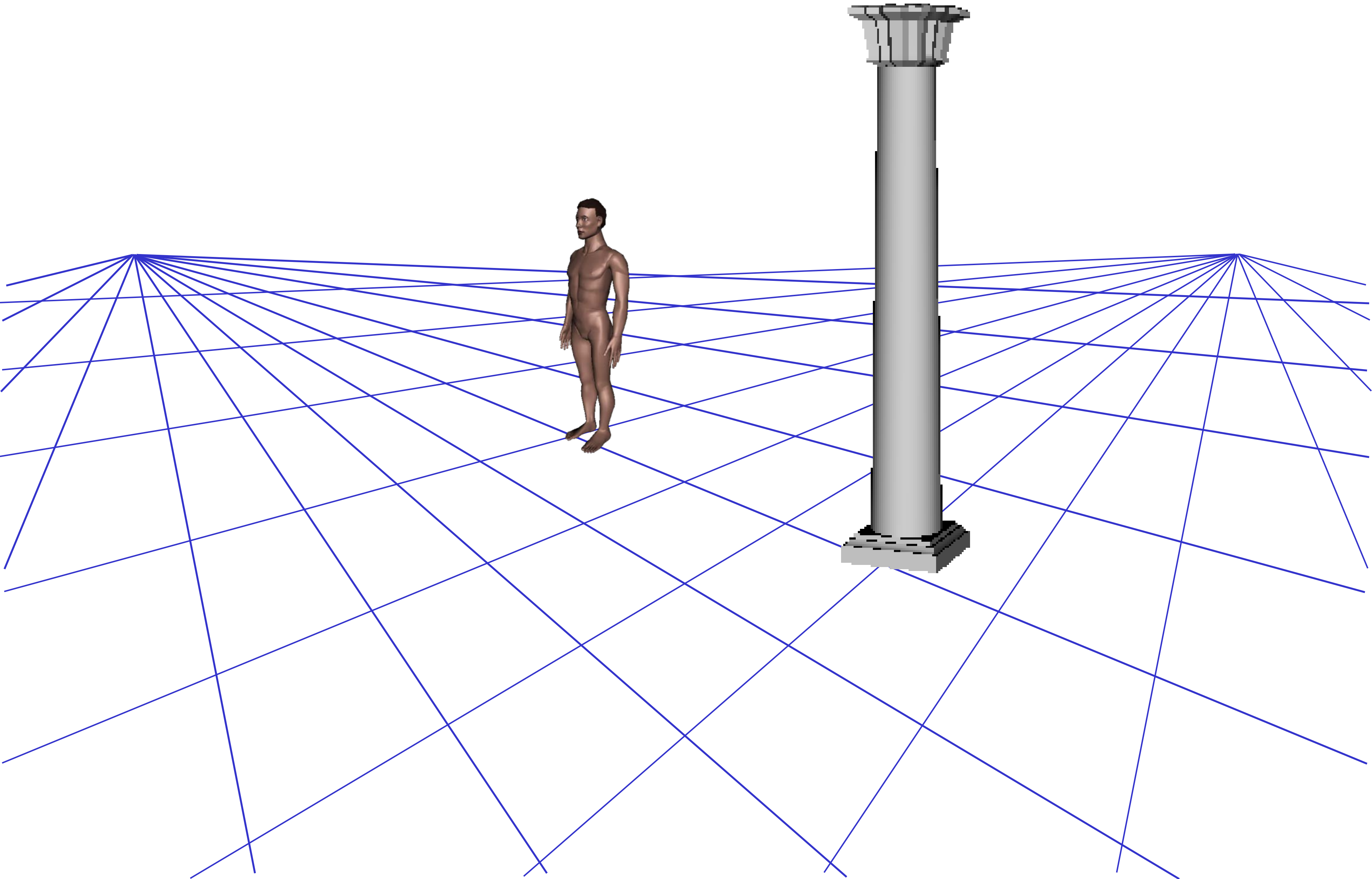
$$\frac{\|\mathbf{t}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{r}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{t}\|}=\frac{H}{R}$$

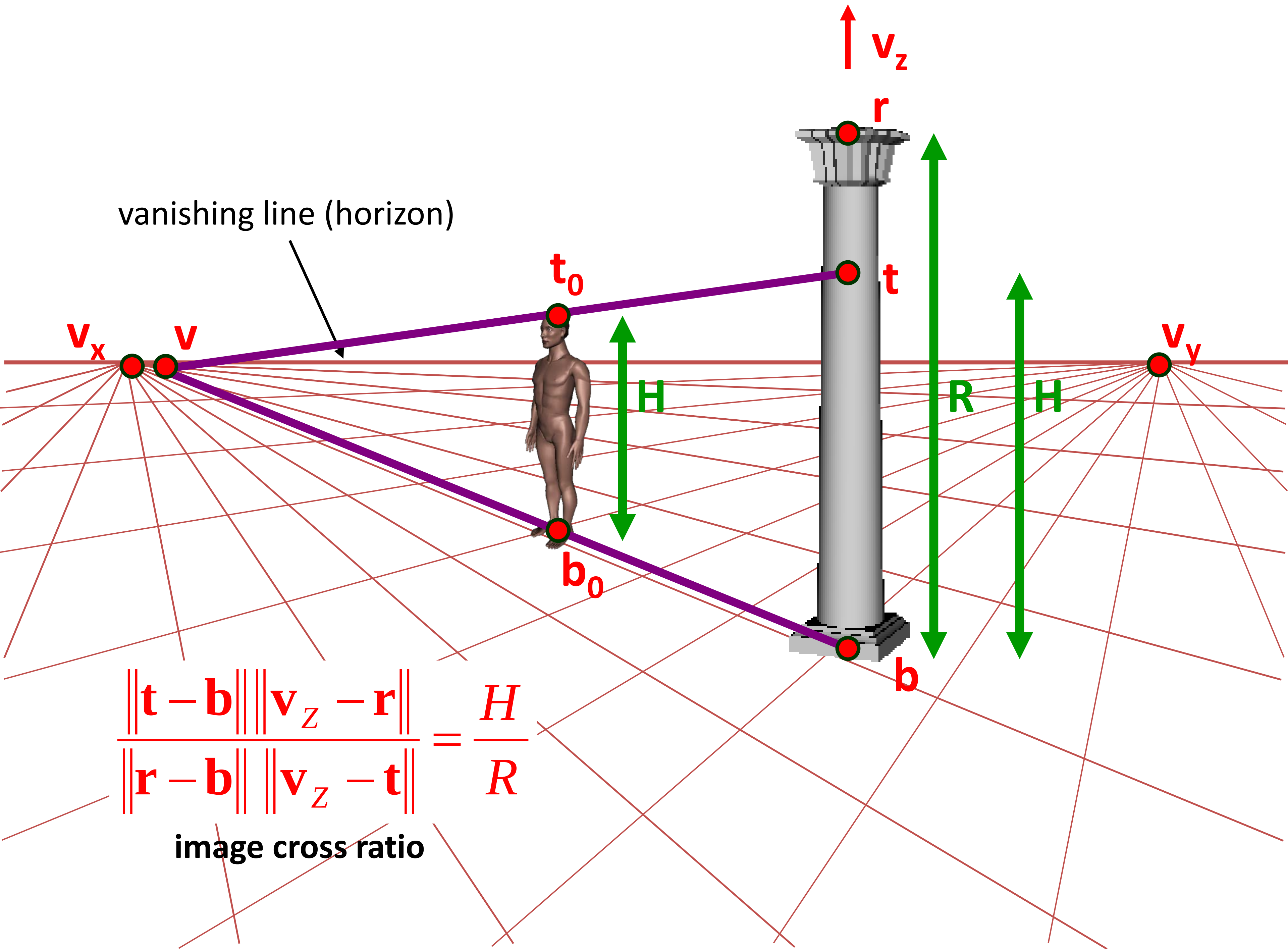
image cross ratio

Hoiem and Savarese figure 2.3



Measuring height without a ruler





$$\frac{\|\mathbf{t}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{r}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{t}\|}=\frac{H}{R}$$

image cross ratio

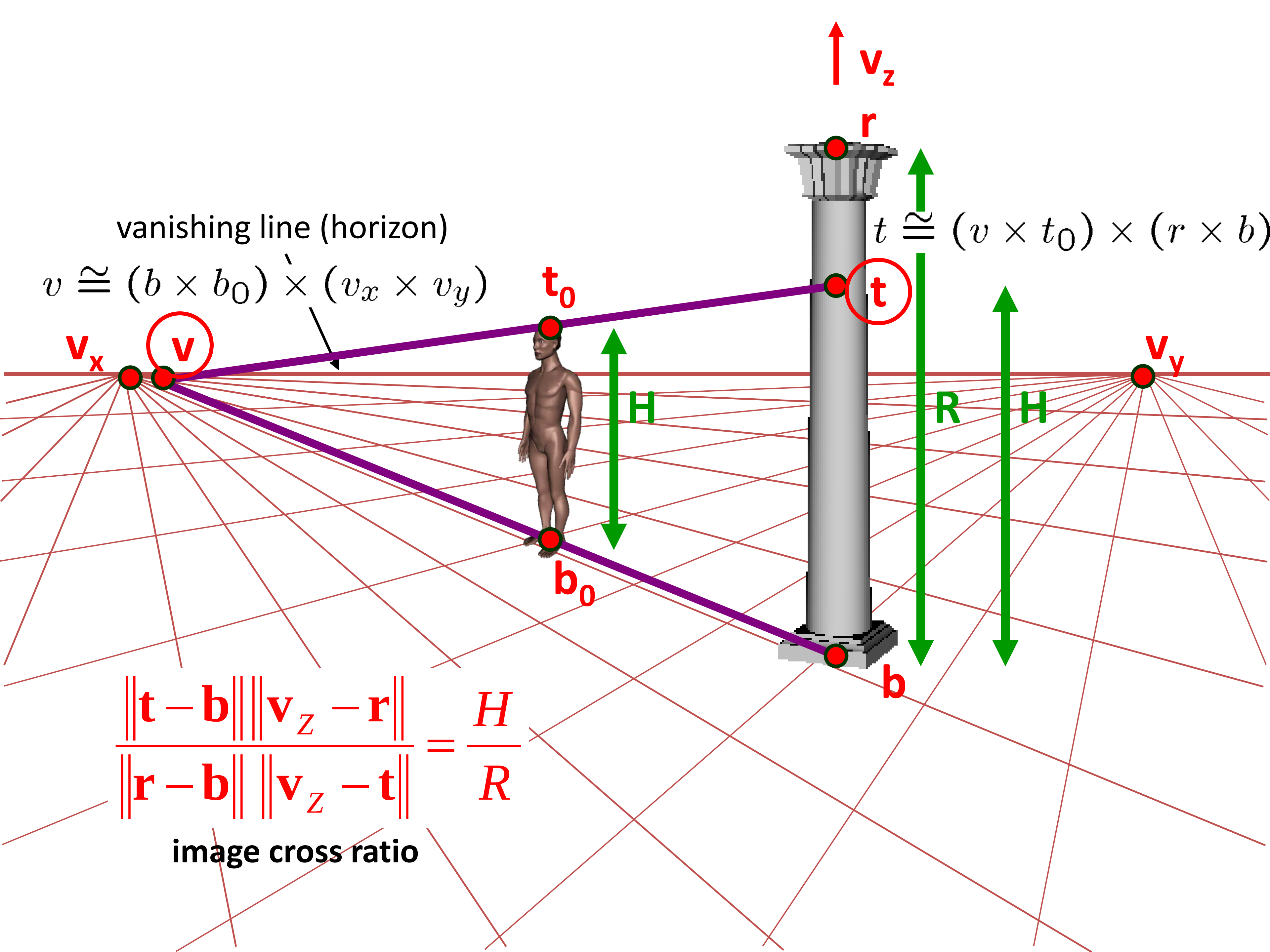
2D lines in homogeneous coordinates

- Line equation: $ax + by + c = 0$

$$\mathbf{l}^T \mathbf{x} = 0 \quad \text{where} \quad \mathbf{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

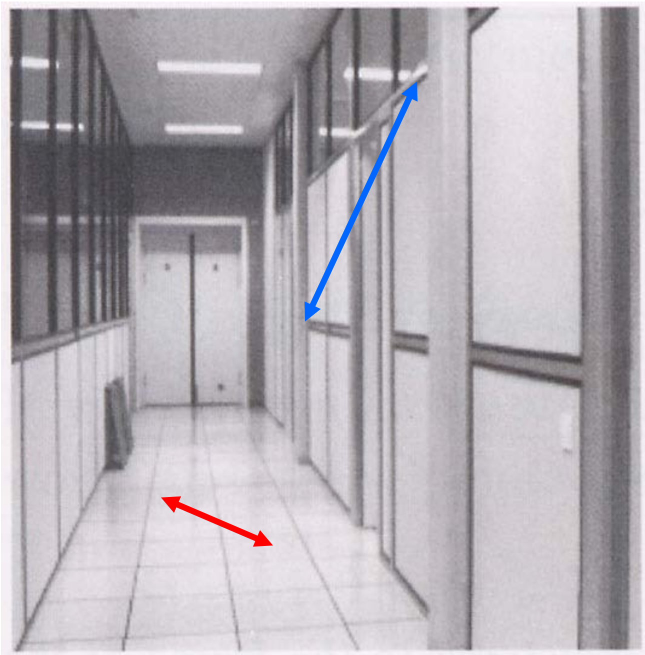
- Line passing through two points: $\mathbf{l} = \mathbf{x}_1 \times \mathbf{x}_2$

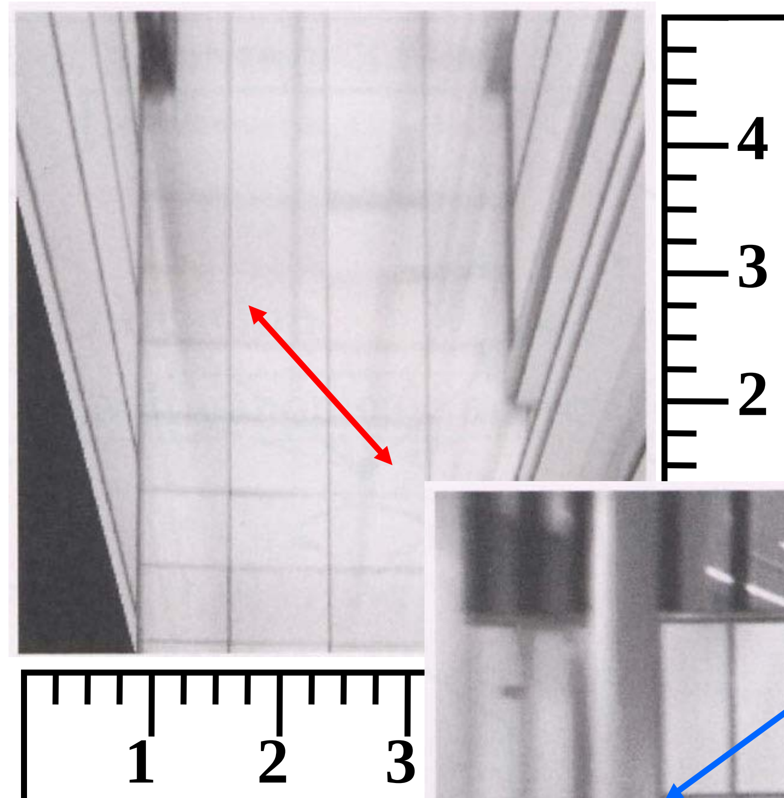
- Intersection of two lines: $\mathbf{x} = \mathbf{l}_1 \times \mathbf{l}_2$
  - What is the intersection of two parallel lines?

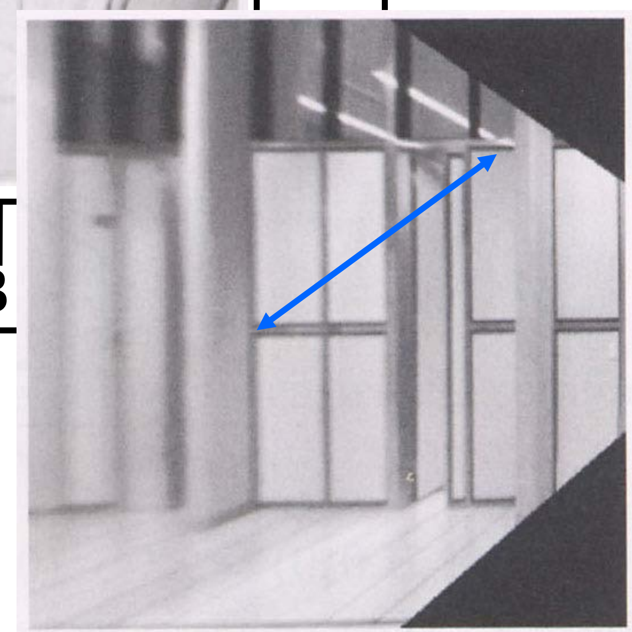


$\mathbf{v_z}$
$\mathbf{r}$
vanishing line (horizon)
$t \cong (v \times t_0) \times (r \times b)$
$v \cong (b \times b_0) \times (v_x \times v_y)$
$\mathbf{t_0}$
$\mathbf{t}$
$\mathbf{v_x}$
$\mathbf{v}$
$\mathbf{v_y}$
H
R
H
$\mathbf{b_0}$
$\mathbf{b}$
$\frac{\|\mathbf{t}-\mathbf{b}\| \|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{r}-\mathbf{b}\| \|\mathbf{v}_Z-\mathbf{t}\|} = \frac{H}{R}$
image cross ratio

Measurements on planes



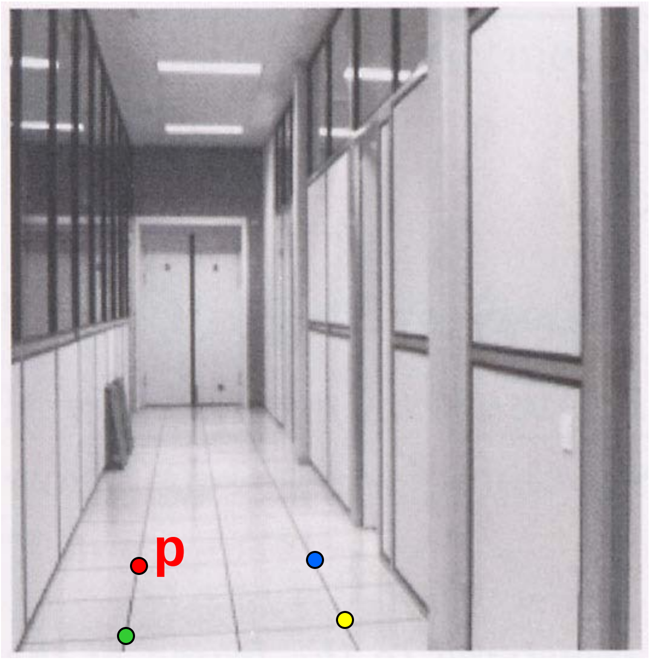


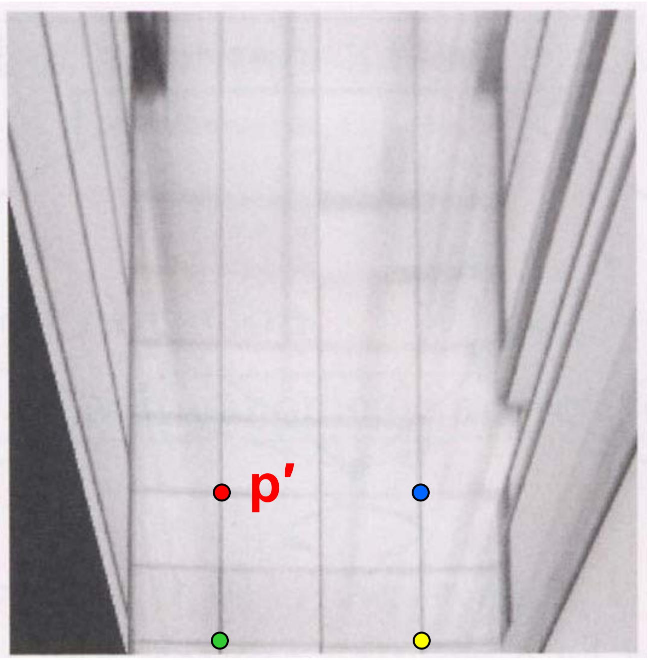


Approach: unwarp then measure

What kind of warp is this?

Image rectification



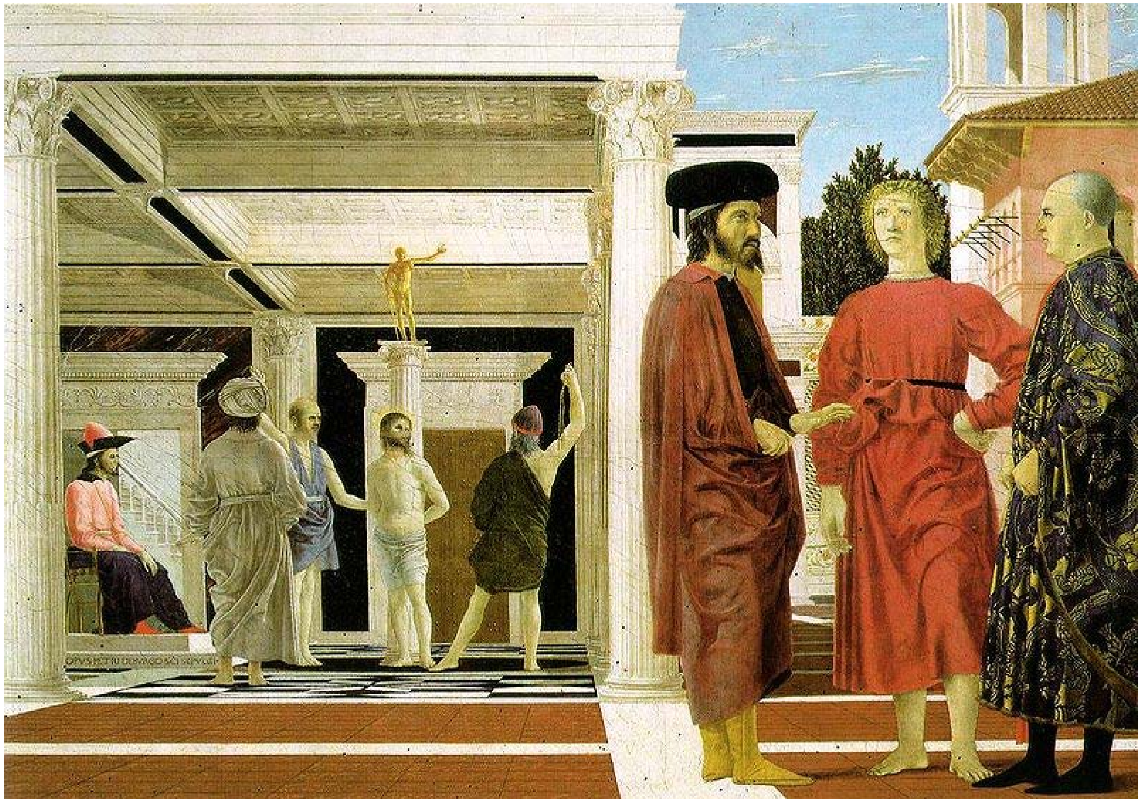


To unwarp (rectify) an image

- solve for homography **H** given **p** and **p′**

– how many points are necessary to solve for **H**?

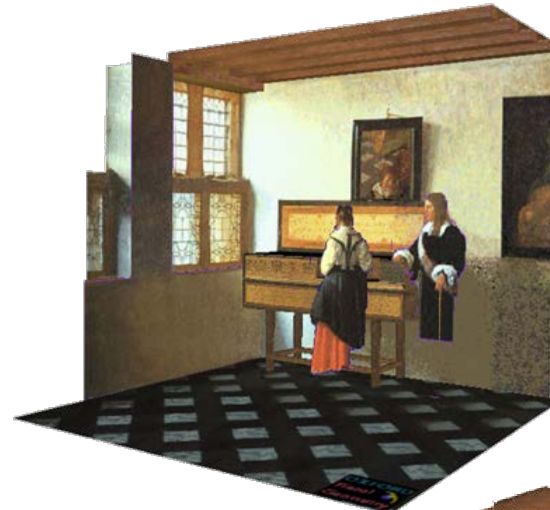
Image rectification: example



Piero della Francesca*, Flagellation,* ca. 1455

Application: 3D modeling from a single image

J. Vermeer, *Music Lesson*, 1662



A. Criminisi, M. Kemp, and A. Zisserman, [Bringing Pictorial Space to Life: computer techniques for the analysis of paintings](), *Proc. Computers and the History of Art*, 2002

http://research.microsoft.com/en-us/um/people/antcrim/ACriminisi_3D_Museum.wmv

Application: 3D modeling from a single image

D. Hoiem, A.A. Efros, and M. Hebert, "Automatic Photo Pop-up", SIGGRAPH 2005.

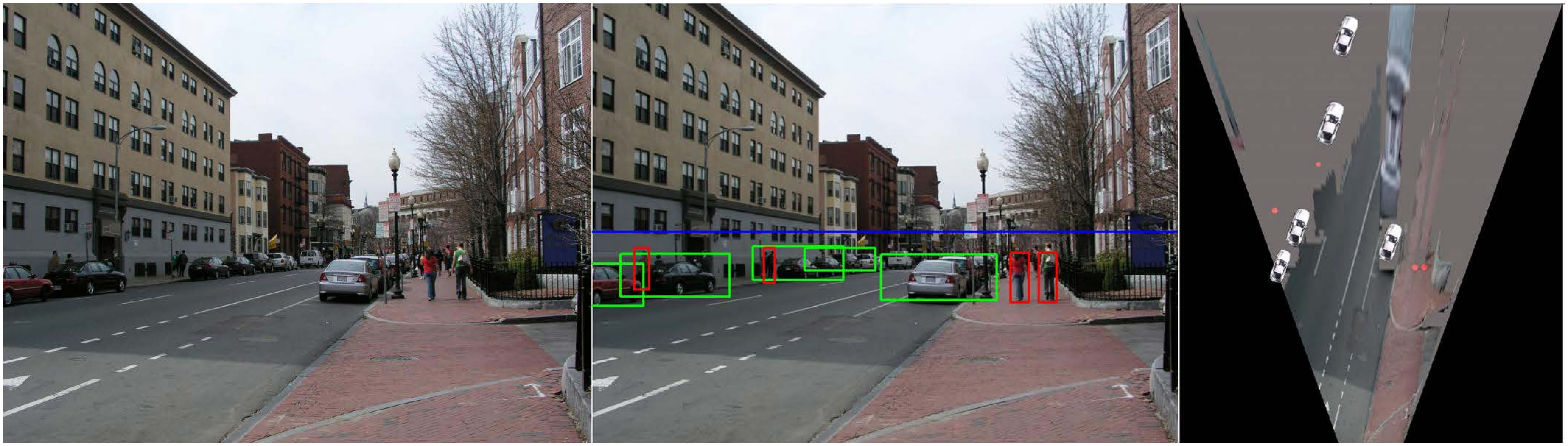
http://dhoiem.cs.illinois.edu/projects/popup/popup_movie_450_250.mp4

Application: Image editing

Inserting synthetic objects into images:
http://vimeo.com/28962540

K. Karsch and V. Hedau and D. Forsyth and D. Hoiem, "Rendering Synthetic Objects into Legacy Photographs," *SIGGRAPH Asia* 2011

Application: Object recognition



D. Hoiem, A.A. Efros, and M. Hebert, "Putting Objects in Perspective", CVPR 2006

Slide credits

Svetlana Lazebnik