MARKETING MIX CONTRIBUTIONS

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Abstract

In this paper, two methods of calculating contributions for a multiplicative model are presented.The two methods Log-linear method and Mean value theorem are explained mathematically and the results obtained are compared.

This paper gives an overview of both the methods and recommends the best one to be used. For doing this, first a theoretical example is considered and then models are built on observed datasets from different verticals. In all the cases, the contributions are calculated using both Log-linear method and Mean value theorem and results are compared.

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1 INTRODUCTION

1.1 Contributions

The output of a given model is obtained by giving it some input parameters. For example, if we have a multiplicative model which predicts sales for spends in different marketing channels, if we change these input parameters(spends), obviously the output(sales) changes. This change has occurred due to change in multiple input factors(spends). So, given two different sets of inputs(spends) to a model, decomposition of the change in the output(sales) due to its individual variables is called contributions. How much does each marketing channel contribute to change in the sales value is called the contribution of that marketing channel.

Following are the two methods of calculating contributions that have been explained in this paper

1) Log-linear Method

2) Mean Value Theorem(MVT)

Both the methods have been explained mathematically and applied on a theoretical example. For the theoretical example, the paper elucidates each step involved in calculating the contributions and the computer has been used only when the calculation was not possible by hand. Then models are built on observed datasets from industry and contributions are calculated by both Log-linear and MVT methods

2 Log-linear Method

We shall first discuss the mathematics involved in this method and then pick an example and follow all the mathematical steps.

2.1 Mathematical Explaination of Log-linear Method

The idea behind this method is to change one variable at a time from a given set of variables and note the change in the output. Let us consider a model of two variables.

Example: Y represents sales and X_1, X_2represent marketing channels.

$$Y = f(X_1, X_2) \tag{1}$$

We first convert the multiplicative form into linear form by applying log on both sides.

$$y = log(Y) \tag{2}$$

$$f(x_1, x_2) = log(f(X_1, X_2)) \tag{3}$$

$$y = f(x_1, x_2) \tag{4}$$

This is the log linearized form of the original equation.

Let the minimum spends be (x_{min1}, x_{min2})

Let the current spends be (x_{curr1}, x_{curr2})

Then the change in $y(log(sales))$ due to the first variable (x_1) when it is changed from minimum to current is calculated by keeping the other variable at minimum spends

i.e

$$X_{chng1} = f(x_{curr1}, x_{min2}) - f(x_{min1}, x_{min2}) \tag{5}$$

Similarly,

$$X_{chng2} = f(x_{min1}, x_{curr2)}) - f(x_{min1}, x_{min2}) \tag{6}$$

Proportions of changes in output with change in single input is calculated by $X_{prop1} = \dfrac{X_{chng1}}{X_{chng1} + X_{chng2}}$ and $X_{prop2} = \dfrac{X_{chng2}}{X_{chng1} + X_{chng2}}$

Now the total change in sales value by changing all the parameters is calculated and split into proportions calculated above

$$\Delta Y = f(X_{curr1}, X_{curr2}) - f(X_{min1}, X_{min2}) \tag{7}$$

$$\triangle Y_{x1} = X_{prop1} * \triangle Y \tag{8}$$

$$\triangle Y_{x2} = X_{prop2} * \triangle Y \tag{9}$$

$$Y_{baseline} = f(X_{min1}, X_{min2}) \tag{10}$$

Now these terms combined with the baseline are split up into contributions

$$Total = Y_{baseline} + \triangle Y_{x1} + \triangle Y_{x2} \tag{11}$$

$$Baseline_{contrib} = \frac{Y_{baseline}}{Total} * 100 \tag{12}$$

$$X_{1contrib} = \frac{\triangle Y_{x1}}{Total} * 100 \tag{13}$$

$$X_{2contrib} = \frac{\triangle Y_{x2}}{Total} * 100 \tag{14}$$

2.2 Log-linear method applied on a theoretical example

Let us try to understand this method with an example,

Let there be two marketing channels Radio and TV. Spends on Radio is represented by(X_1) and spends on TV is represented by (X_2).

Let a model be defined as follows;

$$Y = 10 * X_1^{0.4} * X_2^{0.3} \tag{15}$$

where Y is the dependent variable giving us sales values for different spends.

Let the minimum spends be $10 and 15$ for Radio and TV respectively.

Let the current spends be $50 and 150$ for Radio and TV respectively.

$$X_{min1} = 10, X_{min2} = 15 \tag{16}$$

$$X_{curr1} = 50, X_{curr2} = 150 \tag{17}$$

We convert the multiplicative model into a linear model by applying log on both sides

$$log(Y) = log(10) + 0.4 * log(X_1) + 0.3 * log(X_2) \tag{18}$$

Let's see the change in the minimum value when first variable is changed from the minimum to current value;

$$X_{chng1} = f(x_{curr1}, x_{min2}) - f(x_{min1}, x_{min2}) = log(f(X_{curr1}, X_{min2})) - log(f(X_{min1}, X_{min2})) \tag{19}$$

$$= log(10 * 50^{0.4} * 15^{0.3}) - log(10 * 10^{0.4} * 15^{0.3}) = 0.6438 \tag{20}$$

Similarly for second variable,

$$X_{chng2} = f(x_{min1}, x_{curr2}) - f(x_{min1}, x_{min2}) = log(f(X_{min1}, X_{curr2})) - log(f(X_{min1}, X_{min2})) \tag{21}$$

$$= log(10 * 10^{0.4} * 150^{0.3}) - log(10 * 10^{0.4} * 15^{0.3}) = 0.6908 \tag{22}$$

Now we generate the proportions using these numbers,

$$X_{prop1} = \frac{X_{chng1}}{X_{chng1} + X_{chng2}} = \frac{0.6438}{0.6438 + 0.6908} = 0.4824 \tag{23}$$

$$X_{prop2} = \frac{X_{chng2}}{X_{chng1} + X_{chng2}} = \frac{0.6908}{0.6438 + 0.6908} = 0.5176 \tag{24}$$

Now we split the change in the output according to the above proportions

$$\Delta Y = f(X_{curr1}, X_{curr2}) - f(X_{min1}, X_{min2}) = 158.3871 \tag{25}$$

$$\triangle Y_{x1} = X_{prop1} * \triangle Y = 0.4824 * 158.3871 = 76.4045 \tag{26}$$

$$\triangle Y_{x2} = X_{prop2} * \triangle Y = 0.5176 * 158.3871 = 81.9826 \tag{27}$$

$$y_{baseline} = f(x_{min1}, x_{min2}) = 56.6014 \tag{28}$$

Now we combine these three terms and split them into their proportions

$$Total = Y_{baseline} + \triangle Y_{x1} + \triangle Y_{x2} = 214.9866 \tag{29}$$

$$Baseline_{contrib} = \frac{Y_{baseline}}{Total} * 100 = \boldsymbol{26.3276\%} \tag{30}$$

$$X_{1contrib} = \frac{\triangle Y_{x1}}{Total} * 100 = \boldsymbol{35.5389\%} \tag{31}$$

$$X_{2contrib} = \frac{\triangle Y_{x2}}{Total} * 100 = \boldsymbol{38.1335\%} \tag{32}$$

This analysis extends to more than two variables.

Simply put what we are doing is, we are linearizing a multiplicative model by applying log on both sides and then calculating the contributions in the log state which is nothing but the estimate multiplied by the change in the corresponding variable which is in turn calculated by taking the difference in the output by changing just one variable. Then we use these same proportions in the multiplicative model which is in dollar space and calculate the contributions. This method works under the assumption that the contributions (read proportions) found in the log space can be used directly on the multiplicative model which is in dollar space.

3 Mean Value Theorem

3.1 Mathematical background and explaination of Mean Value Theorem

The mean-value method is based on a well-known mathematical proof known as the ***Mean Value Theorem***. What this theorem implies is that for any continuous, smooth response curve defined on an interval of spends, there exists a spend value within that interval at which the slope of the curve is equal to the average slope of the curve in the interval. The primary reason for using this theorem in calculating contributions is, it allows us to draw a relationship between spends and lift – the average slope. The lift can be calculated as the average slope multiplied with the difference in spends.

Mean Value Theorem(referred to as MVT in future) : If a function is continuous and differential between $[a, b](b > a)$ then there exists a point c between a and b such that

$$f'(c) = \frac{f(b) - f(a)}{b - a} \tag{33}$$

We are interested in finding the $f(b) - f(a)$ which is the change in sales and this should be decomposed for different variables. So if we can find the value c for each variable, compute the derivative of the model at those points and multiply it with the change in spends, we shall get back the change in sales value because of MVT but in form of a vector having the same length as the number of variables.

$$\triangle Y = f'(c) * (b - a) \tag{34}$$

3.2 MVT method applied on a theoretical example

Let us again try to explain this with the same example as above,

Let the two variables be spends on Radio (X_1) and spends on TV (X_2).

Let a model be defined as follows;

$$Y = 10 * X_1^{0.4} * X_2^{0.3} \tag{35}$$

where y is the dependent variable giving us sales values for different spends.

Let the minimum spends be $10 and 15$ for radio and TV respectively.

Let the current spends be $50 and 150$ for radio and TV respectively.

$$(X_{min1} = 10, X_{min2} = 15) \Rightarrow a = (10, 15) \tag{36}$$

$$(X_{curr1} = 50, X_{curr2} = 150) \Rightarrow b = (50, 150) \tag{37}$$

From MVT,

$$f'(c) * (b - a) - (f(b) - f(a)) = 0 \tag{38}$$

We solve this equation for c

This step is performed using a computer,

$$c = (25.6536, 67.8310) \tag{39}$$

$$\Rightarrow f'(c) = (2.023, 0.5738) \tag{40}$$

Therefore the splitting of the the change in sales is done using this value of average slope in each dimension (variable) and multiplying it with the corresponding change in spends.

$$\triangle Y = f'(c) * (b - a) = (2.023, 0.5738) * ((50, 150) - (10, 15)) = (80.9206, 77.4667) \tag{41}$$

Now these change in sales are combined with the baseline value and the contributions are calculated.

$$Total = y_{baseline} + \triangle y_{x1} + \triangle y_{x2} = 214.9887 \tag{42}$$

$$Baseline_{contrib} = \frac{y_{baseline}}{Total} * 100 = \boldsymbol{26.3276\%} \tag{43}$$

$$X_{1contrib} = \frac{\triangle y_{x1}}{Total} * 100 = \boldsymbol{37.6395\%} \tag{44}$$

$$X_{2contrib} = \frac{\triangle y_{x2}}{Total} * 100 = \boldsymbol{36.0329\%} \quad (45)$$

As it is apparent that the contributions obtained by MVT method are different from those of the previous method. This method makes no assumptions as in the previous case. It calculates the average slope in each dimension(for each variable) and multiplies it with the change in spends for that variable.

From the next page onwards, a multiplicative model is built on a dataset and both the above described methods are used to calculate contributions. The calculation methods vary from the methdo used in the tool. In the MMx tool, the minimum spends is defined as the minimum of all the variables(columns) and hence baseline is defined as the sales value for these spends. But in the below calculations, the minimum spends is defined as the minimum of all the spends variables and the current value for all the non spends variables. As these variables are not under our control (for eg: gdp, unemployment rate etc). The value of these variable cannot be changed from a defined minimum spends to current spends and hence these should go into the baseline as the current values. This will result in the code giving no contributions for non spends variables but an increase the baseline contribution as their effect is being absorbed into the baseline.

4 Log-linear Method vs MVT Method on different models built on observed datasets

4.1 Linear Model

The following model is a linear model built on the Manor dataset which is an observed dataset from the retail industry. It has 7 independent variables

The functional form is

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + \beta_7 x_7 \tag{46}$$

Variable Name	Estimate
(Intercept)	-37825.9806
coupon_per_store	7.0296
direct_mail_per_store	202.1552
google_impressions_per_store	0.0145
newspaper_ads_per_store	0.3761
tv_trps_per_store	478.1461
avg_temp	3451.9265
gdp	31328.38

Table 1: Estimates of a linear model built on Manor

The contributions results by both the methods are:

Variable	Contribution(Log-linear Method)	Contribution(MVT- Method)	Difference
Baseline	74.19%	74.19%	0.00%
coupon_per_store	5.83%	5.76%	0.07%
direct_mail_per_store	7.99%	8.12%	-0.13%
google_impressions_per_store	3.72%	3.64%	0.08%
newspaper_ads_per_store	0.62%	0.61%	0.01%
tv_trps_per_store	7.65%	7.68%	-0.03%

Table 2: Contributions by Log-linear and MVT for a linear model built on Manor

Inference: There is hardly any difference in the contributions(which is due to the round off error in the MVT method while finding the derivative) results by both the methods in the linear case.

4.2 Multiplicative Model

4.2.1 Manor Dataset

The following multiplicative model was built on Manor dataset which is an observed dataset from retail industry. It has non logged terms too so the functional form changes a little bit from the conventional multiplicative model.

The functional form is

$$y = A * (x_1)^{b1} * (x_2)^{b2} * (x_3)^{b3} * (x_4)^{b4} * (x_5)^{b5} * e^{(b_6(x_6)+b_7(x_7))} \tag{47}$$

Variable Name	Estimate
(Intercept)	10.759
coupon_per_store	0.072
direct_mail_per_store	0.028
google_impressions_per_store	0.064
newspaper_ads_per_store	0.003
tv_trps_per_store	0.014
avg_temp	0.007
gdp	0.024

Table 3: Estimates for a multiplicative model built on Manor

Contributions were calculated using both the methods discussed above. The results are printed below and a pie chart of contributions is plotted.

The contributions results by Log-linear method are:

Variable	Contribution
Baseline	64.07%
coupon_per_store	14.78%
direct_mail_per_store	9.76%
google_impressions_per_store	6.39%
newspaper_ads_per_store	0.34%
tv_trps_per_store	4.66%

Table 4: Contributions by Log-linear method for Manor

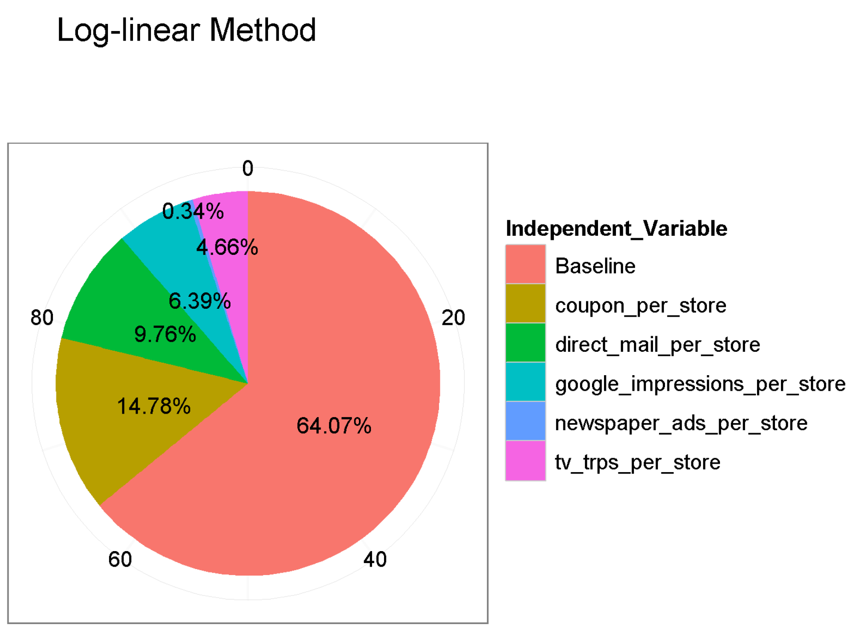

Figure 1: Contributions by Log-linear method for Manor

The contribution results by MVT method are:

Variable	Contribution
Baseline	64.07%
coupon_per_store	16.92%
direct_mail_per_store	7.17%
google_impressions_per_store	7.81%
newspaper_ads_per_store	0.15%
tv_trps_per_store	3.89%

Table 5: Contributions by MVT method for Manor

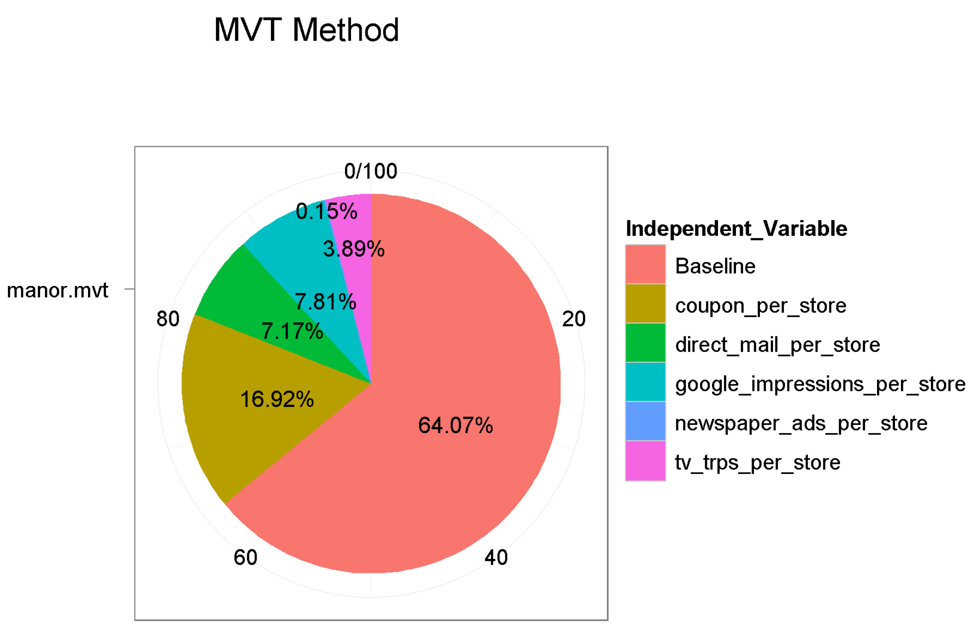

Figure 2: Contributions by MVT method for Manor

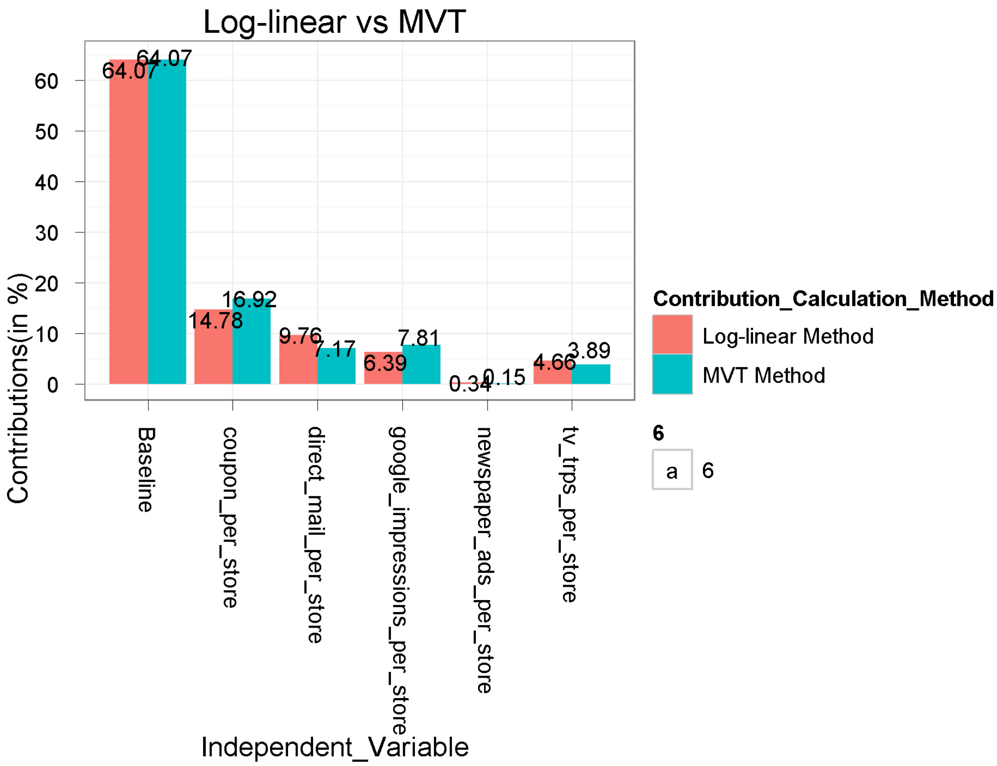

Figure 3: Contributions by Log-linear and MVT methods for Manor

Both the results do not match. Below is a table with both the results and the difference.Also a bar graph of both the results is shown to high lite the difference.

Variable	Log-linear Method	MVT Method	Difference
Baseline	64.07%	64.07%	0.00%
coupon_per_store	14.78%	16.92%	-2.14%
direct_mail_per_store	9.76%	7.17%	2.59%
google_impressions_per_store	6.39%	7.81%	-1.42%
newspaper_ads_per_store	0.34%	0.15%	0.19%
tv_trps_per_store	4.66%	3.89%	0.77%

Table 6: Contributions by both Log-linear and MVT methods for Manor

Inference: There is a significant difference in the contribution results calculated by both the methods for a multiplicative model.

4.2.2 Dell - BHM

The following multiplicative model was built on Dell - BHM dataset.

The functional form is

$$y = A * (x_1)^{b1} * (x_2)^{b2} * (x_3)^{b3} * (x_4)^{b4} * (x_5)^{b5} * (x_6)^{b6} * (x_7)^{b7} * (x_8)^{b8} * (x_9)^{b9} \tag{48}$$

Variable	Estimate
Intercept	11.2302
pub_incentive_cost	0.1747
print_spend_con_adstk3	0.0561
dell_ds_ms_hpb_idc_pub_10	0.5564
ple_print_spend	0.0122
cci	0.2065
total_domore_spend_adstk2	0.0068
total_ple_ooh_adstk4	0.0192
pub_seasonality	0.4712
sku_on_discount_pub	0.4854

Table 7: Estimates for multiplicative model built on DELL-BHM

We shall calculate contributions using both the methods discussed above. The results are printed below and a pie chart of contributions is plotted.

The contributions results by Log-linear method are:

Variable	Contributions
Baseline	40.97%
pub_incentive_cost	17.36%
print_spend_con_adstk3	15.98%
dell_ds_ms_hpb_idc_pub_10	4.14%
ple_print_spend	8.22%
total_domore_spend_adstk2	0.88%
total_ple_ooh_adstk4	12.44%

Table 8: Contributions by Log-linear method for DELL-BHM

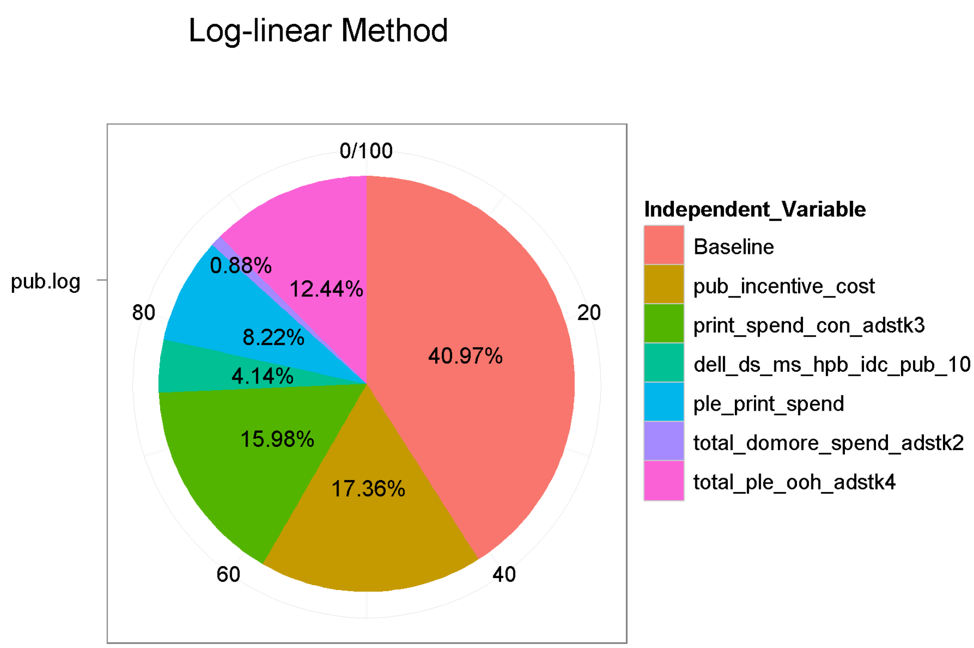

Figure 4: Contributions by Log-linear method for DELL-BHM

The contribution results by MVT method are:

Variable	Contributions
Baseline	40.97%
pub_incentive_cost	24.78%
print_spend_con_adstk3	18.24%
dell_ds_ms_hpb_idc_pub_10	4.56%
ple_print_spend	4.14%
total_domore_spend_adstk2	0.40%
total_ple_ooh_adstk4	6.91%

Table 9: Contributions by MVT method for DELL-BHM

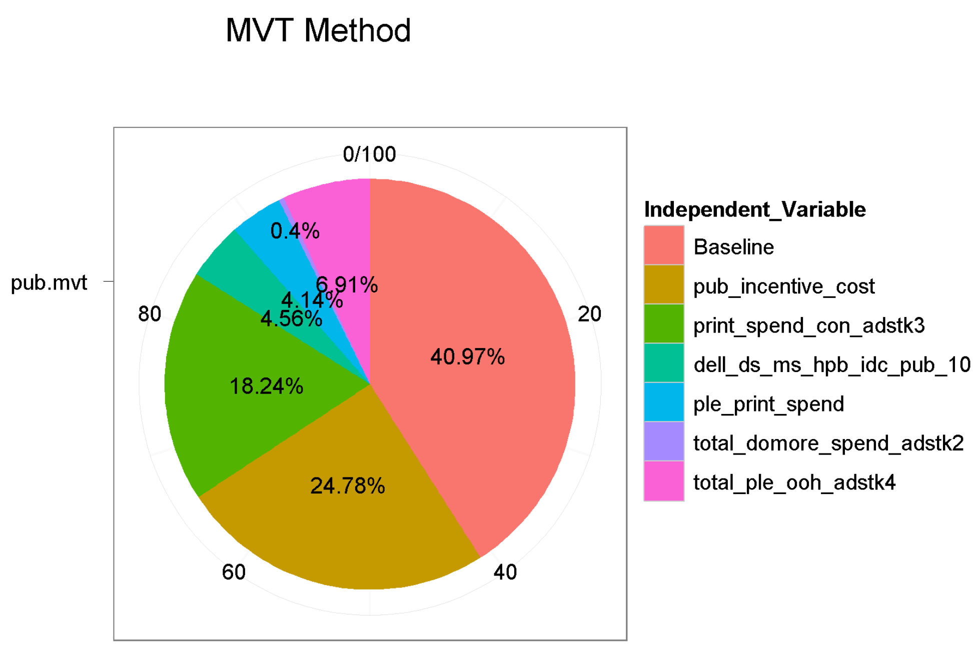

Figure 5: Contributions by MVT method for DELL-BHM

Inference: Uniroot function (which is used to solved for c in the equation) works only for equations having odd number of roots. This method needs some rework to account for cases with even number of roots. In this example for some set of current spends the uniroot function failed to get the correct solution for c.

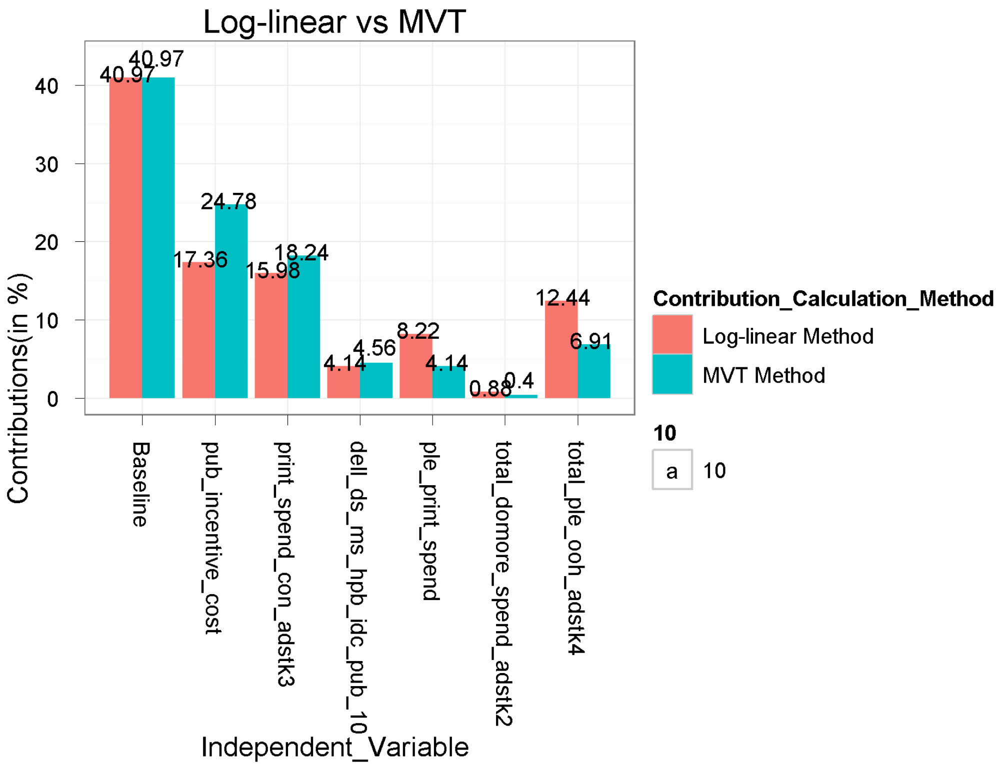

Figure 6: Contributions by both Log-linear and MVT method for DELL-BHM

Both the results do not match. Below is a table with both the results and the difference.Also a bar graph of both the results is shown to high lite the difference.

Variable	Log-linear Method	MVT Method	Difference
Baseline	40.97%	40.97%	0.00%
pub_incentive_cost	17.36%	24.78%	-7.42%
print_spend_con_adstk3	15.98%	18.24%	-2.26%
dell_ds_ms_hpb_idc_pub_10	4.14%	4.56%	-0.42%
ple_print_spend	8.22%	4.14%	4.08%
total_domore_spend_adstk2	0.88%	0.40%	0.48%
total_ple_ooh_adstk4	12.44%	6.91%	5.53%

Table 10: Contributions by both Log-linear and MVT method for DELL-BHM

4.2.3 Allstate

The following multiplicative model was built on Allstate dataset. It has 44 independent variables.

The functional form is

$$y = A * (x_1)^{b1} * (x_2)^{b2} * (x_3)^{b3} ......43 times \tag{49}$$

Variable	Estimate	Vairable	Estimate
(Intercept)	8.0537	am_htv_yca_h_impr_st_adstock50	0.0129
lag1_am_ntv_uncls_g_impr_st	0.0012	am_htv_vp_h_impr_st_adstock50	0.0088
lag2_am_ntv_ml_g_impr_st	0.0024	am_htv_sptsp_h_impr_st_adstock50	0.0081
lag1_am_ntv_vp_g_impr_st	0.0034	am_ntv_sptsp_g_impr_st_adstock_75	0.0009
relative_radio_impr	0.0395	am_ntv_spl_g_impr_st_adstock_75	0.0099
relative_ntv_impr	0.1038	am_ntv_drtv_g_impr_st_adstock_75	0.0120
relative_lclprint_impr	0.015	am_ntv_bb_g_impr_st_adstock_75	0.0006
am_radio_ych_g_st_adstock50	0.0068	am_ntv_afd_g_impr_st_adstock_75	0.0021
am_radio_yca_g_st_adstock50	0.0031	flag_florida_2009	1.1740
am_radio_local_g_spnd_st_adstock50	0.0044	jan_flag	0.0958
am_radio_afd_g_spnd_st_adstock50	0.0031	feb_flag	0.0787
am_lclnews_sptsp_g_spnd_st_adtsock50	0.0138	ci_average_std	0.2052
organic_clk_tot	0.0011	week_twoooosix	-7.5606
am_ooh_local_g_spnd_st_adstock50	0.0023	bau_index3_nonstd	0.2531
am_ooh_grn_g_spnd_st_adstock50	0.0345	aug_flag	0.0808
am_ntv_rldp_g_impr_st_adstock_75	0.0024	flag_for_twothousandten	0.0036
am_ooh_ddc_g_spnd_st_adstock50	0.0105	pi_a_as	0.5914
am_ooh_av_g_spnd_st_adstock50	0.0018	thanks_giving_flag	-0.2939
tag_prospect	0.001	christmas_flag	-0.4842
coopspend_total	0.0073	pi_a_an	0.0000
am_htv_afd_h_impr_st_adstock50	0.0044	econ_bankruptcy_st_index_new	-0.0437
am_htv_av_h_impr_st_adstock50	0.0055	jtime_trend	-0.2097

Table 11: Estimates of multiplicative model built on Allstate dataset

The contribution results by Log-linear method are:

Variable	Contributions
Baseline	65.26%
lag1_am_ntv_uncls_g_impr_st	0.65%
lag2_am_ntv_ml_g_impr_st	0.02%
lag1_am_ntv_vp_g_impr_st	0.08%
relative_radio_impr	2.07%
relative_ntv_impr	1.72%
relative_lclprint_impr	0.68%
am_radio_yca_g_st_adstock50	0.20%
am_radio_local_g_spnd_st_adstock50	1.58%
organic_clk_tot	0.02%
am_ooh_local_g_spnd_st_adstock50	1.04%
am_ntv_rldp_g_impr_st_adstock_75	1.18%
am_ooh_av_g_spnd_st_adstock50	0.44%
tag_prospect	0.46%
coopspend_total	3.91%
am_htv_afd_h_impr_st_adstock50	1.89%
am_htv_av_h_impr_st_adstock50	0.57%
am_htv_yca_h_impr_st_adstock50	6.50%
am_htv_vp_h_impr_st_adstock50	0.25%
am_htv_sptsp_h_impr_st_adstock50	0.49%
am_ntv_sptsp_g_impr_st_adstock_75	0.28%
am_ntv_spl_g_impr_st_adstock_75	0.42%
am_ntv_drtv_g_impr_st_adstock_75	8.87%
am_ntv_bb_g_impr_st_adstock_75	0.12%
am_ntv_afd_g_impr_st_adstock_75	1.29%

Table 12: Contributions by Log-linear method for Allstate

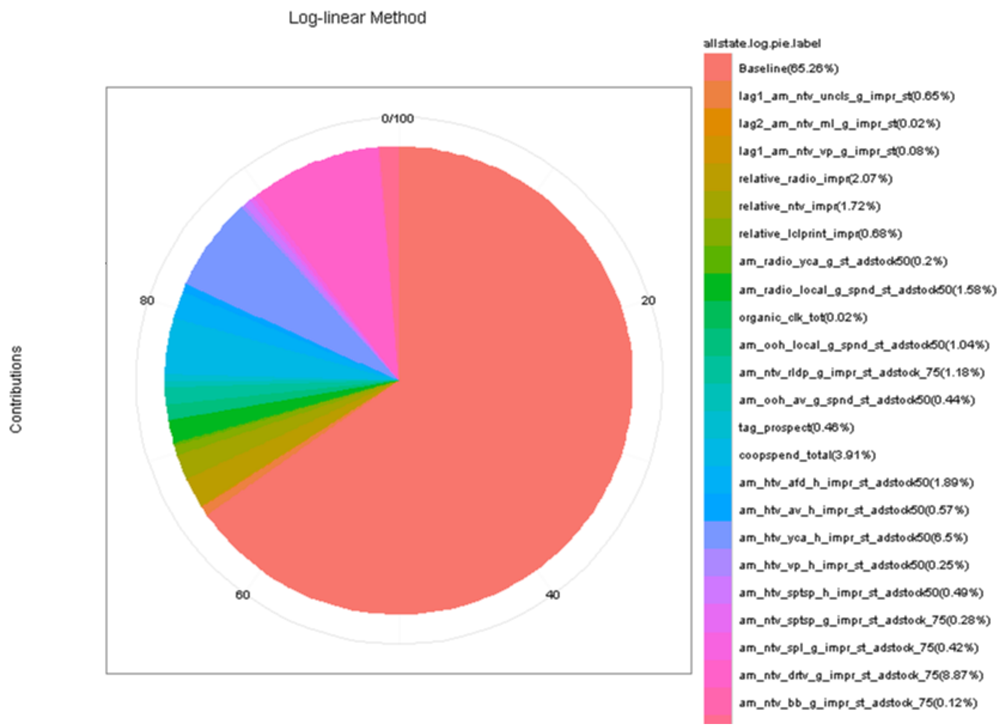

Figure 7: Contributions by Log-linear method for Allstate

The contributions by MVT mehtod are:

Variable	Contributions
Baseline	65.26%
lag1_am_ntv_uncls_g_impr_st	0.74%
lag2_am_ntv_ml_g_impr_st	0.02%
lag1_am_ntv_vp_g_impr_st	0.08%
relative_radio_impr	2.80%
relative_ntv_impr	2.03%
relative_lclprint_impr	0.90%
am_radio_yca_g_st_adstock50	0.20%
am_radio_local_g_spnd_st_adstock50	1.41%
organic_clk_tot	0.02%
am_ooh_local_g_spnd_st_adstock50	0.99%
am_ntv_rldp_g_impr_st_adstock_75	1.31%
am_ooh_av_g_spnd_st_adstock50	0.37%
tag_prospect	0.45%
coopspend_total	3.97%
am_htv_afd_h_impr_st_adstock50	1.88%
am_htv_av_h_impr_st_adstock50	0.61%
am_htv_yca_h_impr_st_adstock50	6.50%
am_htv_vp_h_impr_st_adstock50	0.22%
am_htv_sptsp_h_impr_st_adstock50	0.69%
am_ntv_sptsp_g_impr_st_adstock_75	0.32%
am_ntv_spl_g_impr_st_adstock_75	0.63%
am_ntv_drtv_g_impr_st_adstock_75	7.34%
am_ntv_bb_g_impr_st_adstock_75	0.15%
am_ntv_afd_g_impr_st_adstock_75	1.10%

Table 13: Contributions by MVT method for Allstate

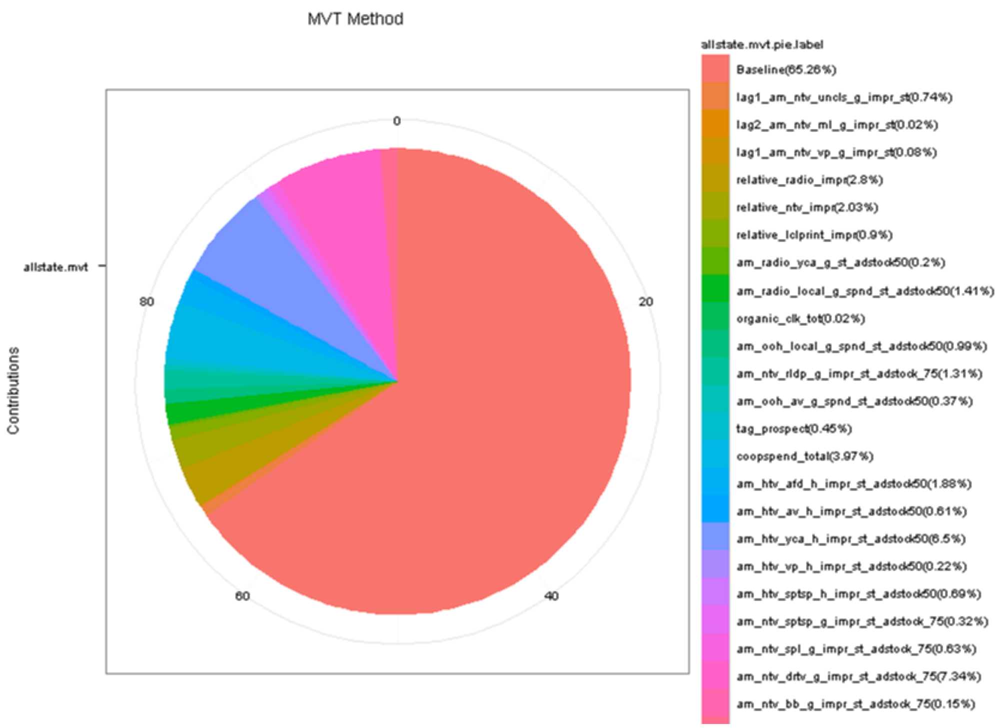

Figure 8: Contributions by MVT method for Allstate

Both the results do not match. Below is a table with both the results and the difference.Also a bar graph of both the results is shown to high lite the difference.

Variable	Log-linear Method	MVT Method	Difference
Baseline	65.26%	65.26%	0.00%
lag1_am_ntv_uncls_g_impr_st	0.65%	0.74%	-0.10%
lag2_am_ntv_ml_g_impr_st	0.02%	0.02%	-0.01%
lag1_am_ntv_vp_g_impr_st	0.08%	0.08%	0.01%
relative_radio_impr	2.07%	2.80%	-0.73%
relative_ntv_impr	1.72%	2.03%	-0.31%
relative_lclprint_impr	0.68%	0.90%	-0.22%
am_radio_yca_g_st_adstock50	0.20%	0.20%	0.00%
am_radio_local_g_spnd_st_adstock50	1.58%	1.41%	0.17%
organic_clk_tot	0.02%	0.02%	0.00%
am_ooh_local_g_spnd_st_adstock50	1.04%	0.99%	0.05%
am_ntv_rldp_g_impr_st_adstock_75	1.18%	1.31%	-0.13%
am_ooh_av_g_spnd_st_adstock50	0.44%	0.37%	0.07%
tag_prospect	0.46%	0.45%	0.02%
coopspend_total	3.91%	3.97%	-0.06%
am_htv_afd_h_impr_st_adstock50	1.89%	1.88%	0.00%
am_htv_av_h_impr_st_adstock50	0.57%	0.61%	-0.04%
am_htv_yca_h_impr_st_adstock50	6.50%	6.50%	0.01%
am_htv_vp_h_impr_st_adstock50	0.25%	0.22%	0.03%
am_htv_sptsp_h_impr_st_adstock50	0.49%	0.69%	-0.19%
am_ntv_sptsp_g_impr_st_adstock_75	0.28%	0.32%	-0.04%
am_ntv_spl_g_impr_st_adstock_75	0.42%	0.63%	-0.21%
am_ntv_drtv_g_impr_st_adstock_75	8.87%	7.34%	1.53%
am_ntv_bb_g_impr_st_adstock_75	0.12%	0.15%	-0.03%
am_ntv_afd_g_impr_st_adstock_75	1.29%	1.10%	0.19%

Table 14: Contributions by both Log-linear and MVT on Allstate

Inference: The difference is not very noticeable in this case as the baseline has ”65%” contributions and there are too many variables hence the numbers itseld are too small to vary. But still, there is significant difference for few variables as seen in the table.

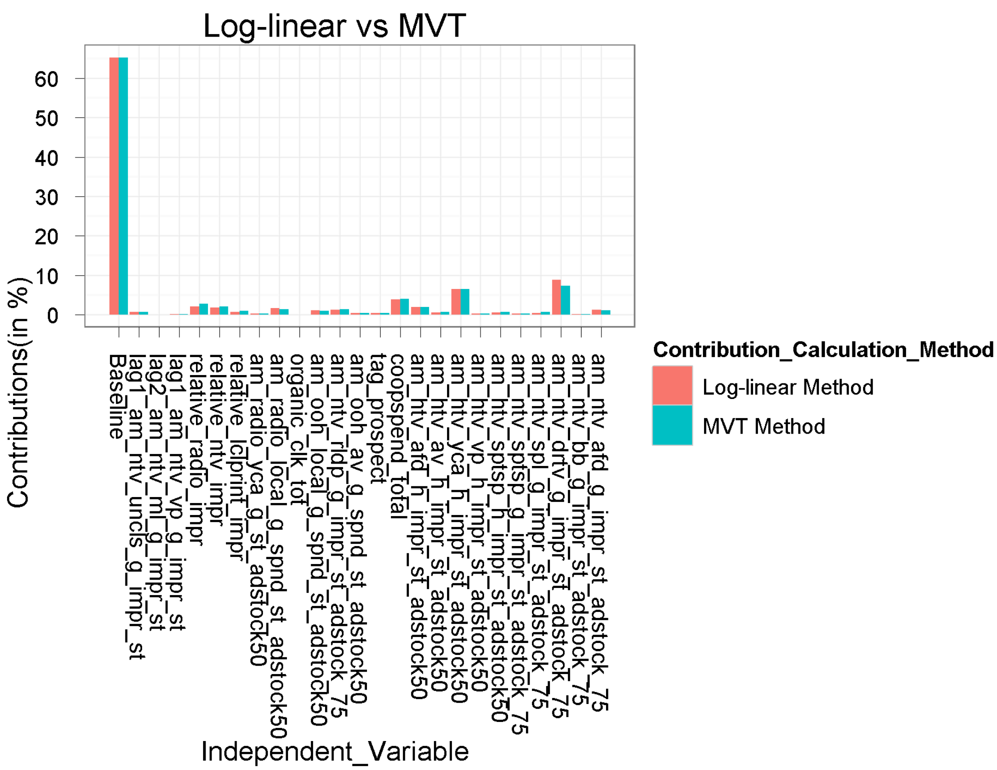

Figure 9: Contributions by both Log-linear and MVT on Allstate

5 CONCLUSION

The two methods for calculating contributions discussed in this paper are

1) Log-linear Method

2) Mean Value Theorem Method

In the first method, we first convert the model to a linear model by taking a logarithm on both sides. Then we calculate the change in the output for the change in each variable, from the baseline value. In effect we are multiplying each variable's estimate with the difference in spends of its corresponding variable. This is the contribution in a linear case. The same proportions are used in the original multiplicative model and the contributions are calculated. We are ***neglecting*** the effect of the other terms while calculating the difference due to one term. When we take the difference in output in the linear case, for change in each variable, rest of them get cancelled out. But in the multiplicative model, subtracting the output for change in each variable will still have rest of the variables. Another major issue is the assumption that the contribution in log space and dollar space remain the same. Which is incorrect. Consider the following example

$$24 = 2 * 3 * 4 \tag{50}$$

The percentage of 2 in 24 is 8.33%

$$log(24) = log(2) + log(3) + log(4) \tag{51}$$

The percentage of $log(2)$ in $log(24)$ is 21.81%

Hence the assumption that the proportions remain the same in the log space and the dollar space is not valid.

In the second method, we calculate the average slope for each variable and then multiply it with the change in the corresponding variable. This is achieved by taking a partial derivative for each variable at the point where the slope of the tangent at that point equals the average slope. This point is calculated using the mean value theorem equation. This method does not neglect the effect of other terms and does not make any assumptions while calculating contributions. The only condition for this method is that the model should be continuous and differentiable between minimum spends and current spends.

Hence Mean Value Theorem method is the appropriate approach to calculate the contributions when the functional form is known.

6 FEW QUESTIONS ANSWERED

- Q1. Why is there a difference in the variation in the results by both the methods for different datasets? A. The absolute mean difference was calculated between the results obtained by both the methods for increasing difference between current spends and minimum spends. For example if we have (a_1, b_1, c_1) as contribution results by log-linear method and (a_2, b_2, c_2) as contribution results by MVT method. The absolute mean difference would be $mean(abs(a_1 - a_2), abs(b_1 - b_2), abs(a_3 - b_3))$. The present current spends was multiplied by a factor of 1.2 to get current spends for every next iteration and the present minimum spends was added 0.2 to get minimum spends for every next iteration. This was done for 100 iterations. Following is a plot between the absolute mean difference and the difference in current and minimum spends.

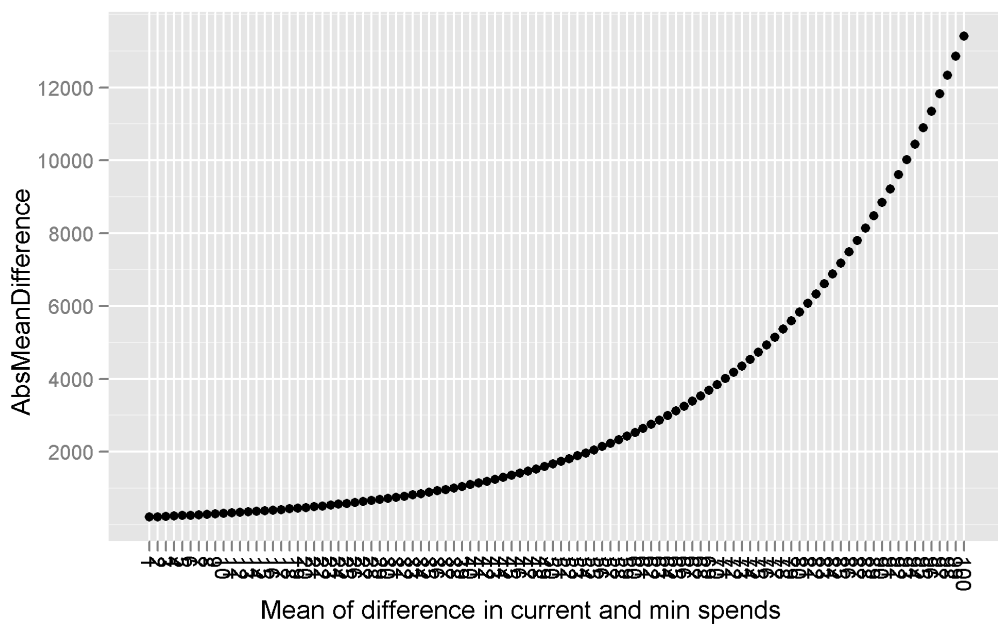

Figure 10: Increasing difference between current and minimum spends vs Absolute mean differnce in contributions by log-linear and mvt method

- Q2. Do both the methods give the same ranking order for the variables i.e the highest contributor by both the methods will be same and so on? A. For answering the above question, the same thing as above was done but this time, instead of calculating the mean absolute difference ,the results were sorted in decreasing order and the order of the indexes of the results by both the methods was compared. The order was same for both the methods for all the 100 iterations.

7 NEXT STEPS

- Uniroot needs to be modified or a new method to be found so that the equation can be solved for any number of roots.
- The definition of minimum spends and hence the baseline needs to be discussed. What value should be chosen as minimum spends for non-spends variables needs to be decided.
- If the decision is to keep current values as minimum spends for non-spends variables then a method for splitting up the baseline contribution into the non-spends variables needs to be found.

8 REFERENCES

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5) ggplot help - "http://had.co.nz/ggplot2"