EE658/758 Machine Learning in Engineering Assignment

1. Data Preprocessing

In [2]:
```
# Libraries
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from sklearn.model_selection import train_test_split # For Data Splitting
df = pd.read_csv('insurance.csv')
df.head()
```

Out[2]:

	Age	Gender	BMI	Children	Smoker	Region	Expenses
0	19	female	27.9	0	yes	southwest	16884.92
1	18	male	33.8	1	no	southeast	1725.55
2	28	male	33.0	3	no	southeast	4449.46
3	33	male	22.7	0	no	northwest	21984.47
4	32	male	28.9	0	no	northwest	3866.86

In [3]:
```
df.shape
```

Out[3]:
```
(1338, 7)
```

In [4]:
```
df.dtypes
```

Out[4]:
```
Age           int64
Gender       object
BMI         float64
Children      int64
Smoker       object
Region       object
Expenses    float64
dtype: object
```

In [5]:
```
df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 1338 entries, 0 to 1337
Data columns (total 7 columns):
 #   Column    Non-Null Count  Dtype
---  ------    --------------  -----
 0   Age       1338 non-null   int64
 1   Gender    1332 non-null   object
 2   BMI       1330 non-null   float64
 3   Children  1338 non-null   int64
 4   Smoker    1338 non-null   object
 5   Region    1336 non-null   object
 6   Expenses  1337 non-null   float64
dtypes: float64(2), int64(2), object(3)
memory usage: 73.3+ KB
```

In [6]:
```
df.describe()
```

Out[6]:

	Age	BMI	Children	Expenses
count	1338.000000	1330.000000	1338.000000	1337.000000
mean	39.207025	30.676917	1.136024	13273.306111
std	14.049960	6.094868	3.194662	12114.083012
min	18.000000	16.000000	-65.000000	1121.870000
25%	27.000000	26.300000	0.000000	4738.270000
50%	39.000000	30.400000	1.000000	9377.900000
75%	51.000000	34.700000	2.000000	16657.720000
max	64.000000	53.100000	70.000000	63770.430000

In [7]:

```
df.isnull().sum()
```

Out[7]:

```
Age         0
Gender      6
BMI         8
Children    0
Smoker      0
Region      2
Expenses    1
dtype: int64
```

In [8]:

```
df[df['Gender'].isnull()]
```

Out[8]:

	Age	Gender	BMI	Children	Smoker	Region	Expenses
425	45	NaN	24.3	5	no	southeast	9788.87
572	30	NaN	43.1	2	no	southeast	4753.64
729	41	NaN	36.1	1	no	southeast	6781.35
914	33	NaN	24.6	2	no	northwest	5257.51
1313	19	NaN	34.7	2	yes	southwest	36397.58
1334	18	NaN	31.9	0	no	northeast	2205.98

In [9]:

```
df[df['BMI'].isnull()]
```

Out[9]:

	Age	Gender	BMI	Children	Smoker	Region	Expenses
8	37	male	NaN	2	no	northeast	6406.41
283	55	female	NaN	1	no	northeast	11879.10
580	59	male	NaN	1	no	northeast	12913.99
769	38	female	NaN	2	no	northwest	6933.24
946	42	male	NaN	2	no	southwest	7160.09
1100	33	female	NaN	2	yes	northeast	16776.30
1235	26	male	NaN	0	no	northwest	2699.57
1335	18	female	NaN	0	no	southeast	1629.83

In [10]:

```
df[df['Expenses'].isnull()]
```

Out[10]:

	Age	Gender	BMI	Children	Smoker	Region	Expenses
810	46	female	30.8	3	no	southwest	NaN

In [11]:
```python
df[df['Region'].isnull()]
```

Out[11]:

	Age	Gender	BMI	Children	Smoker	Region	Expenses
652	48	female	31.1	0	no	NaN	8280.62
788	29	male	22.5	3	no	NaN	5209.58

In [12]:
```python
df.dropna(inplace=True)
```

In [13]:
```python
df.isnull().sum()
```

Out[13]:
```
Age         0
Gender      0
BMI         0
Children    0
Smoker      0
Region      0
Expenses    0
dtype: int64
```

In [14]:
```python
df[df.duplicated()]
```

Out[14]:

	Age	Gender	BMI	Children	Smoker	Region	Expenses
581	19	male	30.6	0	no	northwest	1639.56

In [15]:
```python
df[(df['Age'] == 19) & (df['Gender'] =='male') & (df['BMI'] == 30.6)]
```

Out[15]:

	Age	Gender	BMI	Children	Smoker	Region	Expenses
195	19	male	30.6	0	no	northwest	1639.56
581	19	male	30.6	0	no	northwest	1639.56

In [16]:
```python
df.drop_duplicates(inplace=True)
```

In [17]:
```python
df[df.duplicated()]
```

Out[17]:

Age	Gender	BMI	Children	Smoker	Region	Expenses

In [18]:
```python
df['Gender'].value_counts()
```

Out[18]:
```
Gender
male      668
female    652
Name: count, dtype: int64
```

In [19]:
```python
df['Smoker'].value_counts()
```

Out[19]:
```
Smoker
no     1048
yes     272
Name: count, dtype: int64
```

In [20]:
```python
df['Region'].value_counts()
```

Out[20]:
```
Region
southeast    359
southwest    322
northwest    321
northeast    318
Name: count, dtype: int64
```

Encoding

In [21]:
```python
gender_mapping = {'male': 0, 'female': 1}
smoker_mapping = {'no': 0, 'yes': 1}

df['Gender'] = df['Gender'].map(gender_mapping)
df['Smoker'] = df['Smoker'].map(smoker_mapping)
```

In [22]:
```python
df.head()
```

Out[22]:

	Age	Gender	BMI	Children	Smoker	Region	Expenses
0	19	1	27.9	0	1	southwest	16884.92
1	18	0	33.8	1	0	southeast	1725.55
2	28	0	33.0	3	0	southeast	4449.46
3	33	0	22.7	0	0	northwest	21984.47
4	32	0	28.9	0	0	northwest	3866.86

In [23]:

```
region_mapping = {'southeast': 1, 'southwest': 2, 'northwest': 3, 'northeast': 4}
df['Region'] = df['Region'].map(region_mapping)
```

In [24]:

```
df.head()
```

Out[24]:

	Age	Gender	BMI	Children	Smoker	Region	Expenses
0	19	1	27.9	0	1	2	16884.92
1	18	0	33.8	1	0	1	1725.55
2	28	0	33.0	3	0	1	4449.46
3	33	0	22.7	0	0	3	21984.47
4	32	0	28.9	0	0	3	3866.86

In [25]:

```
df.dtypes
```

Out[25]:

```
Age           int64
Gender        int64
BMI         float64
Children      int64
Smoker        int64
Region        int64
Expenses    float64
dtype: object
```

Min Max Scaling

In [26]:
```
from sklearn.preprocessing import MinMaxScaler
scaler = MinMaxScaler()
```

In [27]:
```
# Expenses column is Taregt COlumn
features_to_normalize = ['Age', 'Gender', 'BMI', 'Children', 'Smoker', 'Region']
features_to_normalize
```

Out[27]:
```
['Age', 'Gender', 'BMI', 'Children', 'Smoker', 'Region']
```

In [28]:
```
df[features_to_normalize] = scaler.fit_transform(df[features_to_normalize])
```

In [29]:
```
df.head()
```

Out[29]:

	Age	Gender	BMI	Children	Smoker	Region	Expenses
0	0.021739	1.0	0.320755	0.481481	1.0	0.333333	16884.92
1	0.000000	0.0	0.479784	0.488889	0.0	0.000000	1725.55
2	0.217391	0.0	0.458221	0.503704	0.0	0.000000	4449.46
3	0.326087	0.0	0.180593	0.481481	0.0	0.666667	21984.47
4	0.304348	0.0	0.347709	0.481481	0.0	0.666667	3866.86

2. Splitting the Data

In [30]:
```
X = df.drop('Expenses',axis=1) # Features
Y = df['Expenses']  # Labels
```

In [31]:
```
X.head()
```

Out[31]:

	Age	Gender	BMI	Children	Smoker	Region
0	0.021739	1.0	0.320755	0.481481	1.0	0.333333
1	0.000000	0.0	0.479784	0.488889	0.0	0.000000
2	0.217391	0.0	0.458221	0.503704	0.0	0.000000
3	0.326087	0.0	0.180593	0.481481	0.0	0.666667
4	0.304348	0.0	0.347709	0.481481	0.0	0.666667

In [32]:

```
Y.head()
```

Out[32]:

```
0    16884.92
1     1725.55
2     4449.46
3    21984.47
4     3866.86
Name: Expenses, dtype: float64
```

In [33]:

```
# Split Ratio ---> 75:25
# Training : Testing
# Random_State for reproducibility
X_train, X_test, y_train, y_test = train_test_split(X, Y, test_size=0.25, random_state=42)
```

In [34]:

```
print("X_train:\n", X_train.head(1))
print("\nX_test:\n", X_test.head(1))
print("\ny_train:\n", y_train.head(1))
print("\ny_test:\n", y_test.head(1))
```

```
X_train:
          Age  Gender       BMI  Children  Smoker    Region
587  0.347826     1.0  0.382749  0.488889     1.0  0.666667

X_test:
         Age  Gender       BMI  Children  Smoker  Region
685  0.76087     0.0  0.280323  0.496296     0.0     1.0

y_train:
 587    43943.88
Name: Expenses, dtype: float64

y_test:
 685    11244.38
Name: Expenses, dtype: float64
```

3. Gradient Descent Implementation

In [35]:
```python
X_train.head()
```

Out[35]:

	Age	Gender	BMI	Children	Smoker	Region
587	0.347826	1.0	0.382749	0.488889	1.0	0.666667
356	0.608696	0.0	0.752022	0.503704	0.0	0.000000
992	0.695652	1.0	0.420485	0.496296	0.0	0.333333
464	0.021739	0.0	0.247978	0.481481	0.0	0.666667
950	0.847826	0.0	0.061995	0.481481	0.0	1.000000

In [36]:
```python
y_train.head()
```

Out[36]:
```
587    43943.88
356     8944.12
992    10118.42
464     1632.04
950    11534.87
Name: Expenses, dtype: float64
```

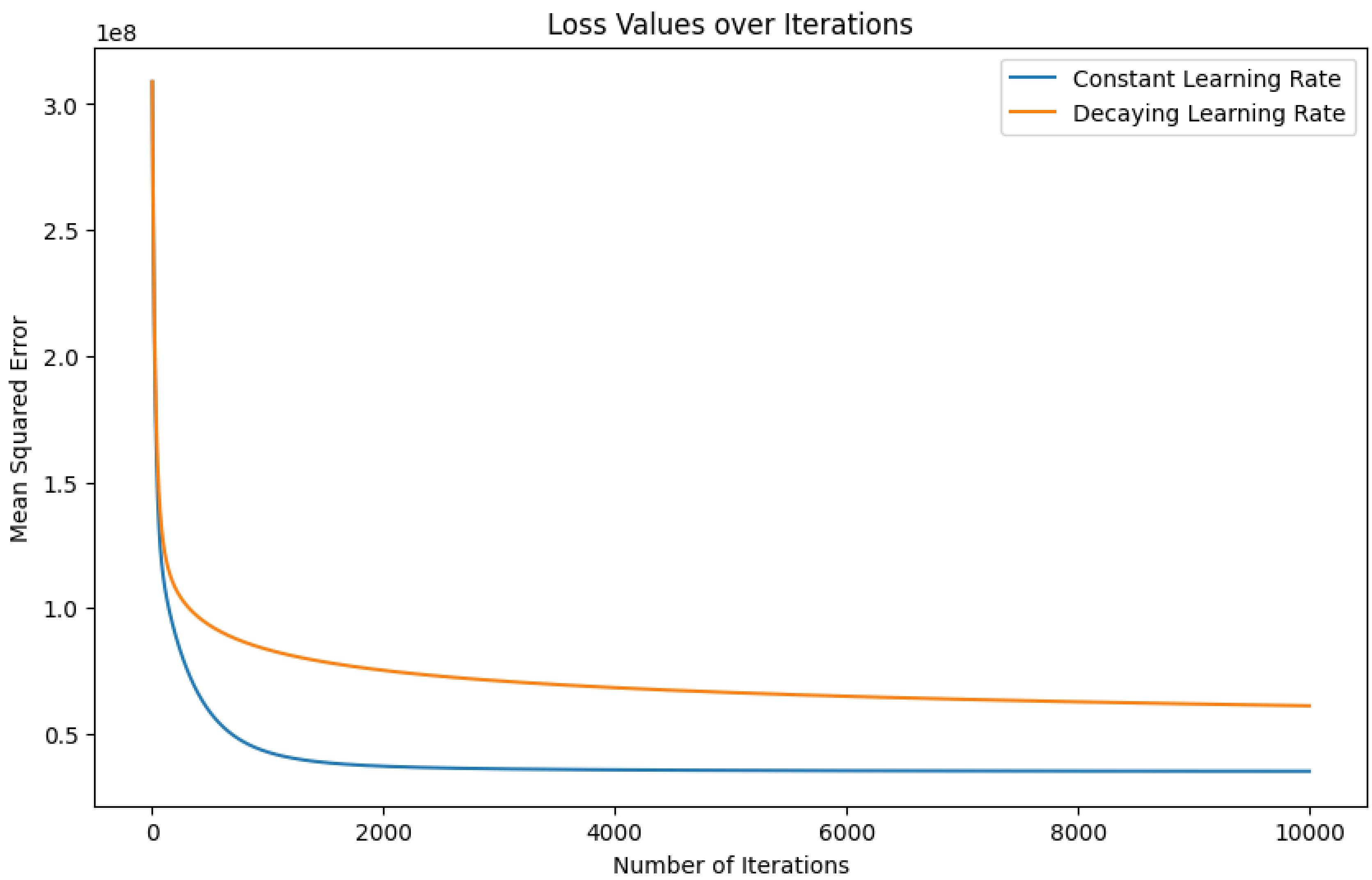
In [37]:
```
from sklearn.metrics import mean_squared_error
def gradient_descent(X_train, y_train, learning_rate=0.01, n_iterations=10000, decay_rate=None):
    if len(X_train.shape) == 1: # For 1D Conversion
        X_train = X_train.reshape(-1, 1)

    if len(y_train.shape) == 1: # For 1D Conversion since ytrain is seris so using numpy here
        y_train = y_train.to_numpy().reshape(-1, 1)

    # X bias term
    X_b = np.c_[np.ones((X_train.shape[0], 1)), X_train]

    # Data points and features
    m, n = X_train.shape[0],X_train.shape[0]

    # Weight matrix (W^T)
    W = np.random.randn(X_train.shape[1] + 1, 1)

    loss_values = []

    for iteration in range(n_iterations):
        gradients = 1/m * X_b.T.dot(X_b.dot(W) - y_train)

        if decay_rate:
            current_learning_rate = learning_rate / (1 + decay_rate * iteration)
        else:
            current_learning_rate = learning_rate

        W = W - current_learning_rate * gradients
        predictions = X_b.dot(W)
        loss_values.append(mean_squared_error(y_train, predictions))

        #result_string = f"Coefficient and Intercept of the Model: {W}\nLoss Values:\n{loss_values}"
        #print(result_string)
```

```
gradient_descent(X_train, y_train) # With Constant_Rate
```

In [38]:
```
# Learning Rate Analysis with 0.01
gradient_descent(X_train, y_train, decay_rate=0.01)
```

In [39]:
```
# Plot for Consant & Decay Learning Rates
def gradient_descent(X_train, y_train, learning_rate=0.01, n_iterations=10000, decay_rate=None):
    if len(X_train.shape) == 1: # For 1D Conversion
        X_train = X_train.reshape(-1, 1)

    if len(y_train.shape) == 1: # For 1D Conversion since ytrain is seris so using numpy here
        y_train = y_train.to_numpy().reshape(-1, 1)

    # X bias term
    X_b = np.c_[np.ones((X_train.shape[0], 1)), X_train]

    # Data points and features
    m, n = X_train.shape[0], X_train.shape[1]

    # Weight matrix (W^T)
    W_constant = np.random.randn(n + 1, 1)
    W_decaying = np.random.randn(n + 1, 1)

    loss_values_constant = []
    loss_values_decaying = []

    for iteration in range(n_iterations):

        gradients_constant = 1/m * X_b.T.dot(X_b.dot(W_constant) - y_train)
        W_constant = W_constant - learning_rate * gradients_constant
        predictions_constant = X_b.dot(W_constant)
        loss_values_constant.append(mean_squared_error(y_train, predictions_constant))

        gradients_decaying = 1/m * X_b.T.dot(X_b.dot(W_decaying) - y_train)
        if decay_rate:
            current_learning_rate_decaying = learning_rate / (1 + decay_rate * iteration)
        else:
            current_learning_rate_decaying = learning_rate
        W_decaying = W_decaying - current_learning_rate_decaying * gradients_decaying
```

```
        predictions_decaying = X_b.dot(W_decaying)
        loss_values_decaying.append(mean_squared_error(y_train, predictions_decaying))

    plt.figure(figsize=(10, 6))
    plt.plot(range(1, n_iterations + 1), loss_values_constant, label='Constant Learning Rate')
    plt.plot(range(1, n_iterations + 1), loss_values_decaying, label='Decaying Learning Rate')
    plt.xlabel('Number of Iterations')
    plt.ylabel('Mean Squared Error')
    plt.title('Loss Values over Iterations')
    plt.legend()
    plt.show()

    return W_constant, loss_values_constant, W_decaying, loss_values_decaying

W_constant, loss_values_constant, W_decaying, loss_values_decaying = gradient_descent(X_train, y_train, decay_rate=0.01)
```



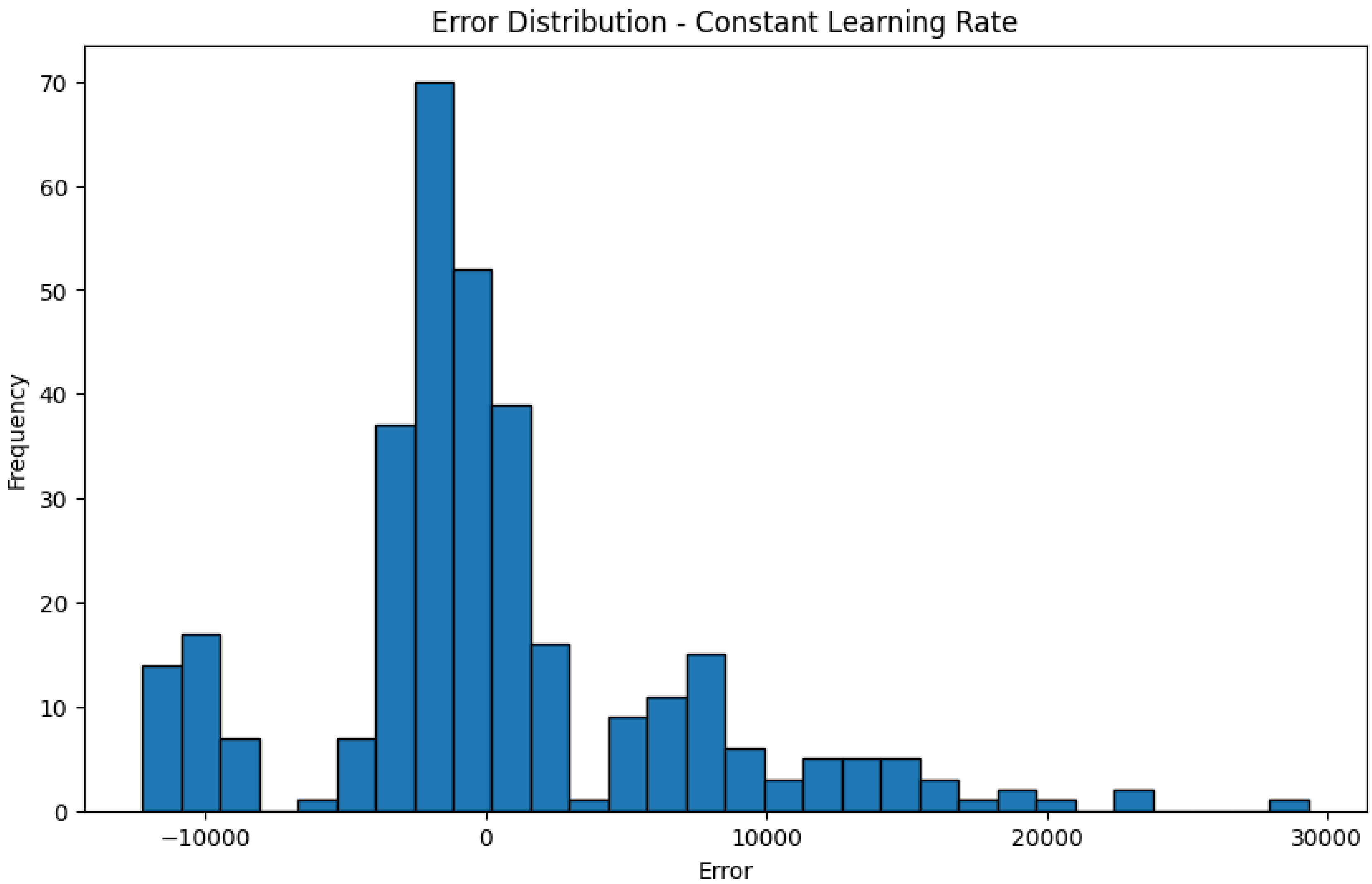
4. Model Evaluation

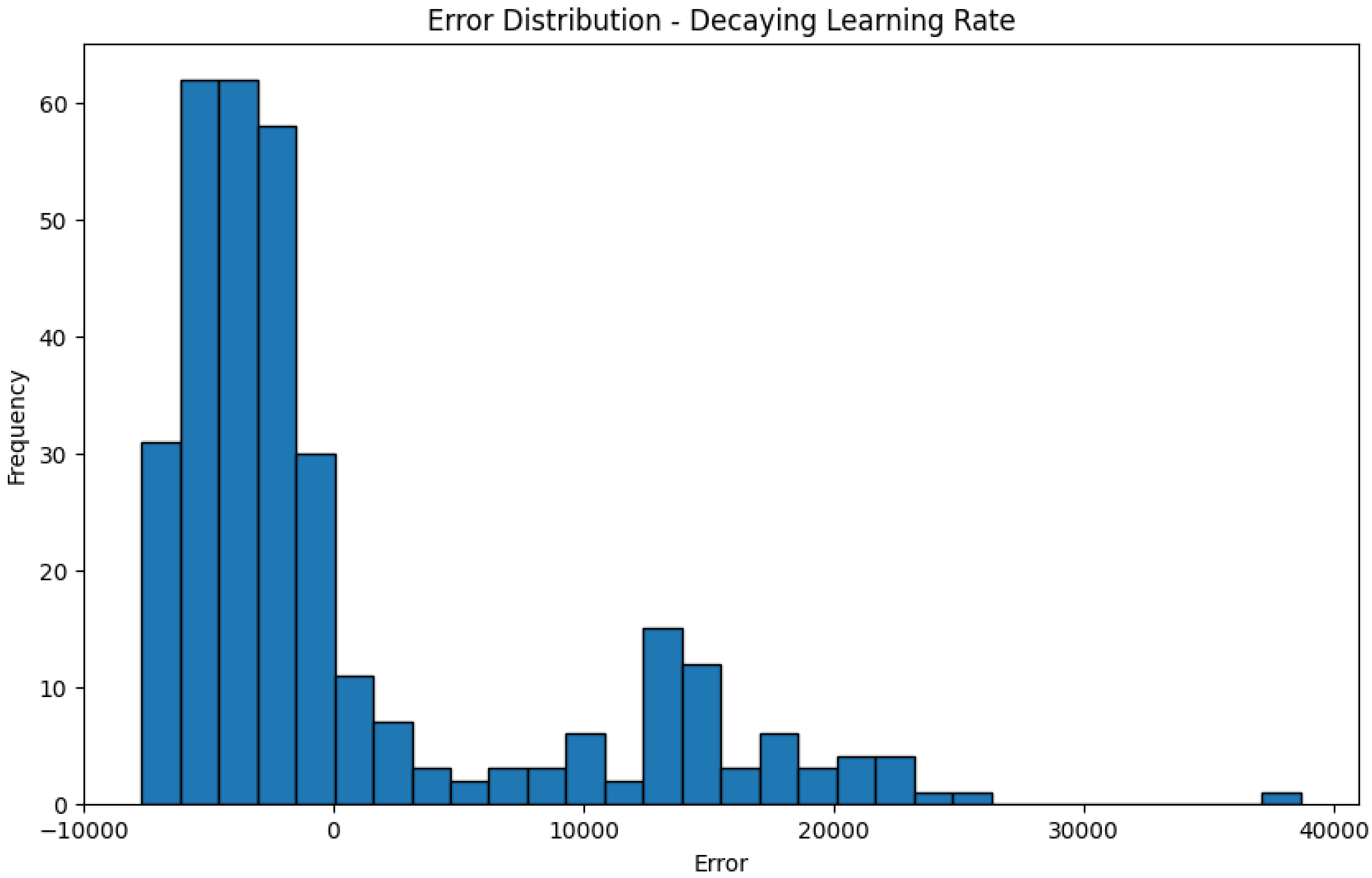
In [40]:

```
from sklearn.metrics import mean_absolute_error, mean_squared_error
def evaluate_model(X_test, y_test, W_constant, W_decaying):
    if len(X_test.shape) == 1: # For 1D Conversion
        X_test = X_test.reshape(-1, 1)
```

```python
if len(y_test.shape) == 1: # For 1D Conversion since ytrain is seris so using numpy here
    y_test = y_test.to_numpy().reshape(-1, 1)

X_test_b = np.c_[np.ones((X_test.shape[0], 1)), X_test]


predictions_test_constant = X_test_b.dot(W_constant)
predictions_test_decaying = X_test_b.dot(W_decaying)

# Computing MAE and MSE for both constant and decaying learning rates
mae_constant = mean_absolute_error(y_test, predictions_test_constant)
mse_constant = mean_squared_error(y_test, predictions_test_constant)

mae_decaying = mean_absolute_error(y_test, predictions_test_decaying)
mse_decaying = mean_squared_error(y_test, predictions_test_decaying)

# Histogram For Consstant
error_distribution_constant = y_test - predictions_test_constant
plt.figure(figsize=(10, 6))
plt.hist(error_distribution_constant, bins=30, edgecolor='black')
plt.title('Error Distribution - Constant Learning Rate')
plt.xlabel('Error')
plt.ylabel('Frequency')
plt.show()

# Histogram For Decying Rate
error_distribution_decaying = y_test - predictions_test_decaying
plt.figure(figsize=(10, 6))
plt.hist(error_distribution_decaying, bins=30, edgecolor='black')
plt.title('Error Distribution - Decaying Learning Rate')
plt.xlabel('Error')
plt.ylabel('Frequency')
plt.show()

print("Constant Learning Rate:")
print(f"MAE: {mae_constant:.2f}")
print(f"MSE: {mse_constant:.2f}")

print("\nDecaying Learning Rate:")
print(f"MAE: {mae_decaying:.2f}")
```

```python
    print(f"MSE: {mse_decaying:.2f}")

# Example usage
evaluate_model(X_test, y_test, W_constant, W_decaying)
```



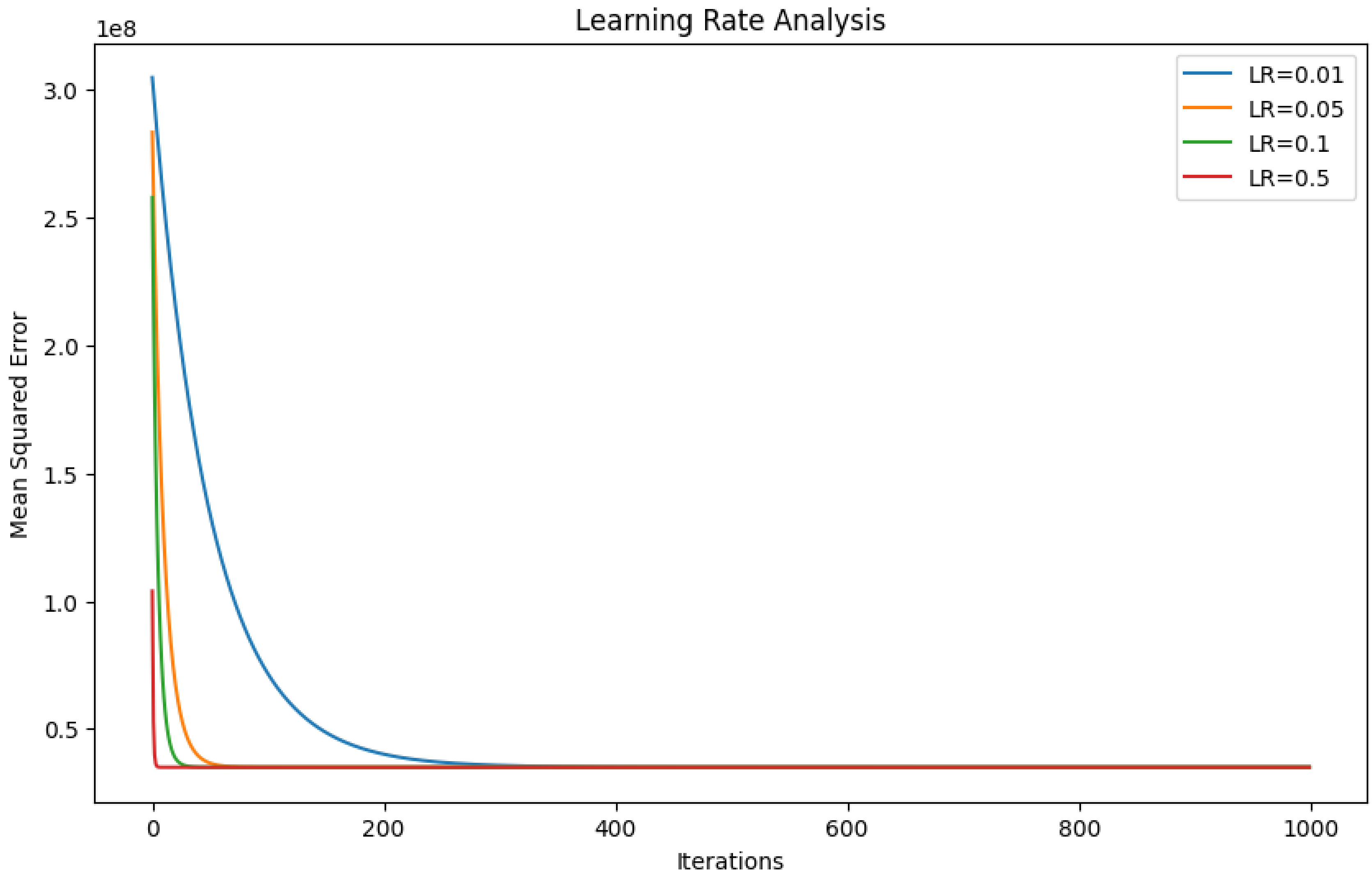



```
Constant Learning Rate:
MAE: 4543.38
MSE: 43794979.02

Decaying Learning Rate:
MAE: 5988.32
MSE: 65776966.98
```

5. Learning Rate Analysis

In [41]:
```python
from sklearn.preprocessing import StandardScaler

def learning_rate_analysis(X, y, learning_rates, n_iterations=1000):

    # Convert y to a NumPy array if it's a Pandas Series
    if isinstance(y, pd.Series):
        y = y.to_numpy()

    # 2d to 1D Conversion
    if len(X.shape) == 1:
        X = X.reshape(-1, 1)
    if len(y.shape) == 1:
        y = y.reshape(-1, 1)

    X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2, random_state=42)

    scaler = StandardScaler()
    X_train_scaled = scaler.fit_transform(X_train)
    X_test_scaled = scaler.transform(X_test)

    W = np.random.randn(X_train_scaled.shape[1] + 1, 1)

    X_train_b = np.c_[np.ones((X_train_scaled.shape[0], 1)), X_train_scaled]

    loss_values = []

    for learning_rate in learning_rates:
        W_current = W.copy()

        local_loss_values = []

        for iteration in range(n_iterations):
            gradients = 1/X_train_b.shape[0] * X_train_b.T.dot(X_train_b.dot(W_current) - y_train)
            W_current = W_current - learning_rate * gradients

            predictions = X_train_b.dot(W_current)
```

```
            local_loss_values.append(mean_squared_error(y_train, predictions))

        loss_values.append(local_loss_values)

    plt.figure(figsize=(10, 6))
    for i, learning_rate in enumerate(learning_rates):
        plt.plot(range(n_iterations), loss_values[i], label=f'LR={learning_rate}')

    plt.xlabel('Iterations')
    plt.ylabel('Mean Squared Error')
    plt.title('Learning Rate Analysis')
    plt.legend()
    plt.show()

    return loss_values

learning_rates_to_analyze = [0.01, 0.05, 0.1, 0.5]
loss_values = learning_rate_analysis(X, Y, learning_rates_to_analyze)
```



6. Scikit-learn Implementation

In [42]:

```
from sklearn.linear_model import LinearRegression
def scikit_learn_regression(X, y):
    X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2, random_state=42)
```

```python
    scaler = StandardScaler()
    X_train_scaled = scaler.fit_transform(X_train)
    X_test_scaled = scaler.transform(X_test)

    model = LinearRegression()

    model.fit(X_train_scaled, y_train)

    predictions = model.predict(X_test_scaled)

    # MSE & MAE Comparsion
    mae = mean_absolute_error(y_test, predictions)
    mse = mean_squared_error(y_test, predictions)

    return mae, mse

mae, mse = scikit_learn_regression(X, Y)
print("MAE:", mae)
print("MSE:", mse)
```

```
MAE: 4743.839512329654
MSE: 47119179.47700916
```

7. Normal Equation Implementation

In [43]:

```python
def normal_equation_regression(X, y):
    X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2, random_state=42)

    scaler = StandardScaler()
    X_train_scaled = scaler.fit_transform(X_train)
    X_test_scaled = scaler.transform(X_test)

    X_train_b = np.c_[np.ones((X_train_scaled.shape[0], 1)), X_train_scaled]

    theta = np.linalg.inv(X_train_b.T.dot(X_train_b)).dot(X_train_b.T).dot(y_train)

    X_test_b = np.c_[np.ones((X_test_scaled.shape[0], 1)), X_test_scaled]

    predictions = X_test_b.dot(theta)
```

```
    mae = mean_absolute_error(y_test, predictions)
    mse = mean_squared_error(y_test, predictions)

    return mae, mse

mae_normal, mse_normal = normal_equation_regression(X, Y)
print("MAE (Normal Equation):", mae_normal)
print("MSE (Normal Equation):", mse_normal)
```

```
MAE (Normal Equation): 4743.839512329654
MSE (Normal Equation): 47119179.477009155
```

Observation 💡 💡 💡 :

- MAE and MSE are almost Same for Linear Regression Model and Normal Equation because both use Linear Regression Model

In [44]:
```
# MAE & MSE Calcualtion for Gradient_Descent
def gradient_descent(X, y, learning_rate=0.01, n_iterations=1000, decay_rate=None):
    if len(X.shape) == 1:  # For 1D Conversion
        X = X.reshape(-1, 1)

    if len(y.shape) == 1:  # For 1D Conversion since ytrain is series so using numpy here
        y = y.to_numpy().reshape(-1, 1)

    X_b = np.c_[np.ones((X.shape[0], 1)), X]

    m, n = X.shape[0], X.shape[1]

    W = np.random.randn(n + 1, 1)

    loss_values = []

    for iteration in range(n_iterations):
        gradients = 1/m * X_b.T.dot(X_b.dot(W) - y)

        if decay_rate:
            current_learning_rate = learning_rate / (1 + decay_rate * iteration)
        else:
            current_learning_rate = learning_rate
```

```python
        W = W - current_learning_rate * gradients
        predictions = X_b.dot(W)
        loss_values.append(mean_squared_error(y, predictions))

    return W, loss_values

scaler = StandardScaler()
X_train_scaled = scaler.fit_transform(X_train)
X_test_scaled = scaler.transform(X_test)

W_gradient_descent, loss_values_gradient_descent = gradient_descent(X_train_scaled, y_train)
mae_gradient_descent = mean_absolute_error(y_test, np.c_[np.ones((X_test_scaled.shape[0], 1)), X_test_scaled].dot(W_gradient_d
mse_gradient_descent = mean_squared_error(y_test, np.c_[np.ones((X_test_scaled.shape[0], 1)), X_test_scaled].dot(W_gradient_de
print(f"Gradient Descent - MAE: {mae_gradient_descent:.4f}, MSE: {mse_gradient_descent:.4f}")
```

```
Gradient Descent - MAE: 4618.5731, MSE: 45290930.4499
```

In [45]:

```python
import time
np.random.seed(42)
X = np.random.rand(1000, 5)
theta_true = np.random.rand(6, 1)
X_b = np.c_[np.ones((1000, 1)), X]
y = X_b.dot(theta_true) + np.random.randn(1000, 1) * 0.1  # Adding noise for realism
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2, random_state=42)


scaler = StandardScaler()
X_train_scaled = scaler.fit_transform(X_train)
X_test_scaled = scaler.transform(X_test)

# Computational efficiency for Gradient Descent
start_time = time.time()
W_gradient_descent, _ = gradient_descent(X_train_scaled, y_train)
end_time = time.time()
mae_gradient_descent = mean_absolute_error(y_test, np.c_[np.ones((X_test_scaled.shape[0], 1)), X_test_scaled].dot(W_gradient_d
mse_gradient_descent = mean_squared_error(y_test, np.c_[np.ones((X_test_scaled.shape[0], 1)), X_test_scaled].dot(W_gradient_de
ct_gradient = end_time-start_time
print(f"Gradient Descent - Time: {ct_gradient:.4f}s, MAE: {mae_gradient_descent:.4f}, MSE: {mse_gradient_descent:.4f}")

# For Normal Equation
start_time = time.time()
```

```python
mae_normal, mse_normal = normal_equation_regression(X_train, y_train)
end_time = time.time()
ct_normal = end_time-start_time
print(f"Normal Equation - Time: {ct_normal:.4f}s, MAE: {mae_normal:.4f}, MSE: {mse_normal:.4f}")

# For scikit-learn Linear Regression
start_time = time.time()
mae_scikit_learn, mse_scikit_learn = scikit_learn_regression(X_train, y_train)
end_time = time.time()
ct_scikit_learn = end_time - start_time
print(f"Scikit-learn Linear Regression - Time: {ct_scikit_learn:.4f}s, MAE: {mae_scikit_learn:.4f}, MSE: {mse_scikit_learn:.4f
```

```
Gradient Descent - Time: 0.7779s, MAE: 0.0897, MSE: 0.0120
Normal Equation - Time: 0.0000s, MAE: 0.0860, MSE: 0.0118
Scikit-learn Linear Regression - Time: 0.0105s, MAE: 0.0860, MSE: 0.0118
```

In [46]:
```python
mae_gradient_descent,mae_normal,mae_scikit_learn
```

Out[46]:
```
(0.089650517747645, 0.086048081464194, 0.08604808146419404)
```

In [47]:
```python
mse_gradient_descent,mse_normal,mse_scikit_learn
```

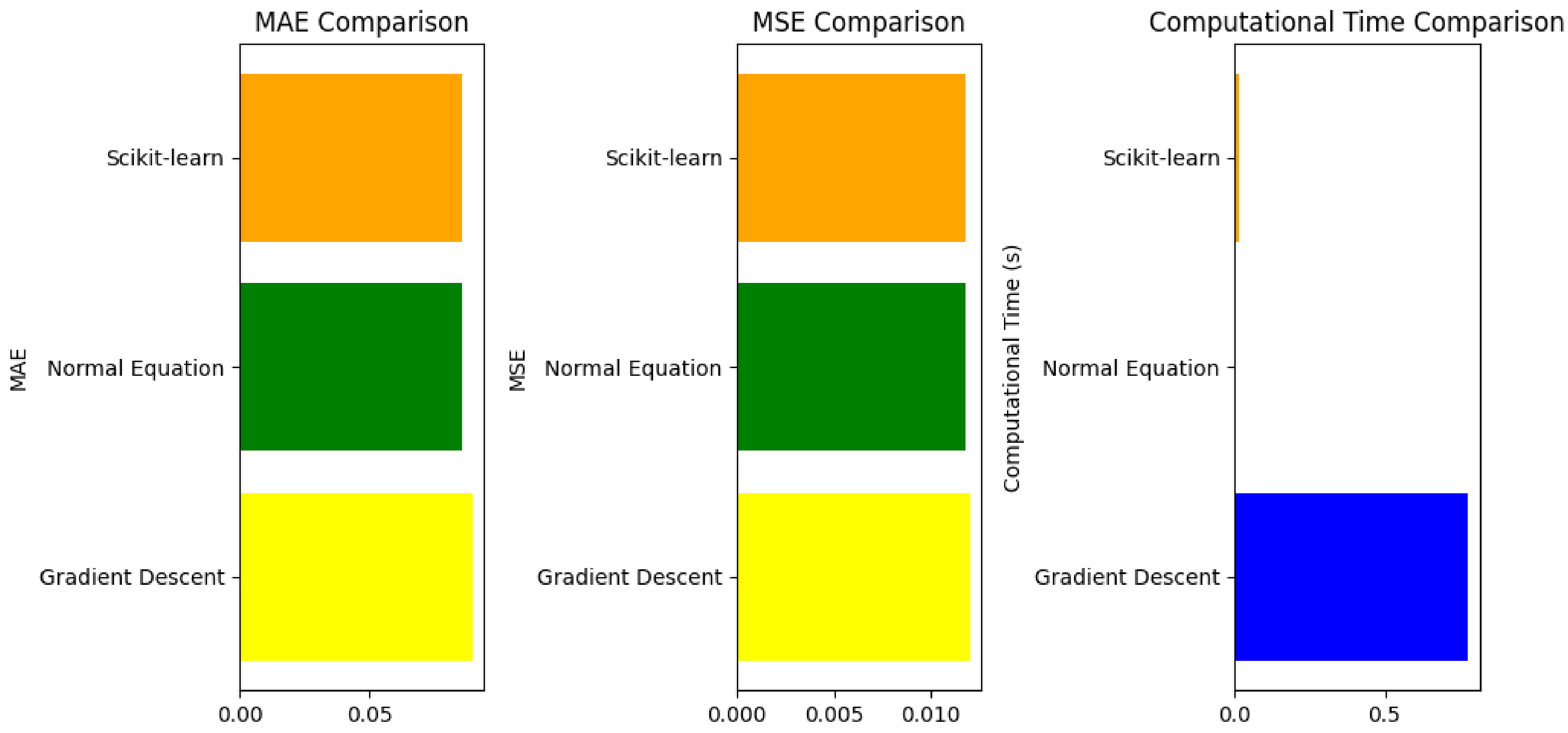
Out[47]:
```
(0.012018391740325483, 0.011786053686983744, 0.011786053686983738)
```

In [48]:
```python
ct_gradient,ct_normal,ct_scikit_learn
```

Out[48]:
```
(0.7779483795166016, 0.0, 0.010514736175537109)
```

In [51]:
```python
import matplotlib.pyplot as plt

# Values
mae_values = [mae_gradient_descent, mae_normal, mae_scikit_learn]
mse_values = [mse_gradient_descent, mse_normal, mse_scikit_learn]
ct_values = [ct_gradient, ct_normal, ct_scikit_learn]

# Types
methods = ['Gradient Descent', 'Normal Equation', 'Scikit-learn']

# MAE plot
plt.figure(figsize=(10, 5))
```

```
plt.subplot(1, 3, 1)
plt.barh(methods, mae_values, color=['yellow', 'green', 'orange'])
plt.title('MAE Comparison')
plt.ylabel('MAE')

# MSE Plot
plt.subplot(1, 3, 2)
plt.barh(methods, mse_values, color=['yellow', 'green', 'orange'])
plt.title('MSE Comparison')
plt.ylabel('MSE')

# Computational Time Plot
plt.subplot(1, 3, 3)
plt.barh(methods, ct_values, color=['blue', 'green', 'orange'])
plt.title('Computational Time Comparison')
plt.ylabel('Computational Time (s)')

plt.tight_layout()
plt.show()
```


MAE Comparison
MSE Comparison
Computational Time Comparison
MAE
MSE
Computational Time (s)
Scikit-learn
Normal Equation
Gradient Descent
0.00
0.05
0.000
0.005
0.010
0.0
0.5