ReinforcementLearning Notes

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1 Introduction

Reinforcement Learning is also known as

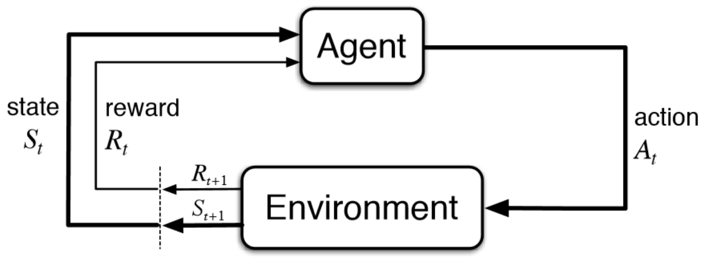
- optimal control
- Approximate Dynamic programming
- Neuro-Dynamic Programming

Reinforcement Learning is an area of machine learning inspired by the behavioural psychology concerned with how software agents ought to take actions in an environment so as to maximise some notion of cumulative reward.
Animal psychology

- negative rewards - pain and hunger
- positive reinforcements - pleasure and food
- reinforcements used to train animals

Applying the similar philosophy to software agents

2 Definition



Reinforcement Leaning Application Areas :

- Game Playing
- Operations Research
- Elevator Scheduling
- controls
- spoken dialouge systems
- data center energy consumption
- self managing networks
- autonomous vehicles
- computational finance

3 Markov Decision Process

Definition

- State $s \in S$
- Action $a \in A$
- reward $r \in \mathbb{R}$
- Transition model $Pr(s_{t+1}|s_t, a_t)$
- Reward model $Pr(r|s_{t+1}, s_t, a_t)$
- Discount Factor $\gamma \in [0, 1]$
- Horizon T

Goal is to find a policy $\pi(a|s)$ that maximises the expectation of discounted return.

How RL differs from MDP solutions

- No Transition Model
- No Reward Model

$$J(\pi) = \mathbb{E}_\pi \left[\sum_{t=0}^{T} \gamma^t r_t \right]$$

However, we still solve the MDP problem using RL by interacting with the environment by learning the transition and reward models or directly learning the policy.

Types of RL algorithms

- Model Based- if we try to learn the transition and reward models
- Model Free - here, we don't learn any model dynamics. No transition and reward models. Below are the types of model free RL algorithms
- Value Based- if we try to learn the value function $V(s)$ of the state or value function of state-action pair $Q(s,a)$.
- Policy Based- if we try to learn the policy $\pi(a|s) - \pi(s,a)$ directly.
- Policy Gradient- if we try to learn the policy $\pi(a|s) - \pi(s,a)$ directly using gradient ascent.
- Actor Critic - contains both policy $\pi(a|s) - \pi(s,a)$ and value function $V(s) - Q(s,a)$.

4 Model Free Evaluation

Monte Carlo Evaluation

$$\begin{aligned}
V^{\pi}(s) &= \mathbb{E}_{\pi}\left[\sum_{t=0}^{T} \gamma^t r_t\right] \\
&\approx \frac{1}{n(s)} \sum_{n(s)}^{k=1} \left[\sum_{t=0}^{T} \gamma^t r_t\right] && \text{(sample approximation)} \\
&= \frac{1}{n(s)} \sum_{n(s)}^{k=1} G_k && \text{discounted sum of reward is defined as } G_k
\end{aligned}$$

Temporal Difference Learning

$$\begin{aligned}
V^{\pi}(s) &= \mathbb{E}_{\pi}[r|s] + \gamma \sum_{s_{t+1}} Pr[s_{t+1}|s_t] V^{\pi}(s_{t+1}) \\
&\approx r + \gamma V^{\pi}(s_{t+1}) && \text{(one sample approximation)}
\end{aligned}$$

5 Monte Carlo Evaluation

Monte Carlo Evaluation

$$G_k = \sum_t \gamma^t r_t^{(k)}$$

G_k is a discounted sum of rewards in one episode or trajectory.
Approximate value function

$$\begin{aligned}
V_n^\pi(s) &\approx \frac{1}{n(s)} \sum_{k=1}^{n(s)} G_k \\
&= \frac{1}{n(s)} \left[G_{n(s)} + \sum_{k=1}^{n(s)-1} G_k \right] \\
&= \frac{1}{n(s)} \left[G_{n(s)} + (n(s)-1) \frac{1}{n(s)-1} \sum_{k=1}^{n(s)-1} G_k \right] \\
&= \frac{1}{n(s)} \left[G_{n(s)} + (n(s)-1) V_{n-1}^\pi(s) \right] \\
&= V_{n-1}^\pi(s) + \frac{1}{n(s)} \left[G_{n(s)} - V_{n-1}^\pi(s) \right] \\
&= V_{n-1}^\pi(s) + \alpha \left[G_{n(s)} - V_{n-1}^\pi(s) \right] \qquad \text{where } \alpha = \tfrac{1}{n(s)}
\end{aligned}$$

Incremental update step

$$V_n^\pi(s) \leftarrow V_{n-1}^\pi(s) + \alpha_n \left[G_{n(s)} - V_{n-1}^\pi(s) \right]$$

iterate over each sample trajectory and do the incremental update of the value function.

6 Temporal Difference Learning

Temporal Difference Learning approximate value function

$$V^\pi(s) \approx r + \gamma V^\pi(s_{t+1})$$

Incremental update step

$$V_n^\pi(s) \leftarrow V_{n-1}^\pi(s) + \alpha_n \left[r + V_{n-1}^\pi(s_{t+1}) - V_{n-1}^\pi(s) \right]$$

7 Dynamic Programming

we compute the state value function $V^\pi(s)$ for the policy π. This is called the *policy evaluation* problem. We can write the Bellman equation for the state value function as

$$\begin{aligned}V^{\pi}(s) &= \mathbb{E}_{\pi}\left[\sum_{t=0}^{\infty}\gamma^{t} r_{t+1} \mid s_t = s\right] \\ &= \mathbb{E}_{\pi}\left[G_t \mid s_t = s\right] \\ &= \mathbb{E}_{\pi}\left[R_{t+1} + \gamma G_{t+1} \mid s_t = s\right] \\ &= \mathbb{E}_{\pi}\left[R_{t+1} + \gamma V^{\pi}(s_{t+1}) \mid s_t = s\right] \\ &= \sum_{a \in \mathcal{A}} \pi(a \mid s) \sum_{s', r} p(s', r \mid s, a)\left[r + \gamma V^{\pi}(s')\right]\end{aligned} \tag{1}$$

If enviroment dynamics are known, then $V^{\pi}(s)$ is a simultaneous linear equations in $|S|$ unknowns. Iterative solutions are most suitable. Assume v_0 as the initial approximation for the state value function. Then update rule is given by the bellman equation

$$v_{k+1}(s) = \sum_{a \in \mathcal{A}} \pi(a \mid s) \sum_{s', r} p(s', r \mid s, a)\left[r + \gamma v_k(s')\right] \tag{2}$$

This iterative approach converges to $V^{\pi}(s)$. The policy evaluation problem is solved when $v_k(s) = V^{\pi}(s)$ for all $s \in S$.

Iterative Policy Evaluation Algorithm

Algorithm 1 Iterative Policy Evaluation

```
1: Input policy $\pi$, initial approximation $v_0(s) \forall s \in S$,
2: Set discount factor $\gamma$, $k = 0$, $V(terminal) = 0$
3: $v_k(s) \leftarrow v_0(s)$ for all $s \in S$
4: while $|v_k - v_{k-1}| < \theta$ do
5:     $v_{k+1}(s) \leftarrow \sum_{a \in \mathcal{A}} \pi(a \mid s) \sum_{s',r} p(s', r \mid s, a)\left[r + \gamma v_k(s')\right]$ for all $s \in S$
6:     $k \leftarrow k + 1$
7: end while
8: return $v_k(s)$ for all $s \in S$
```

State-Action Value Function

The state-action value function $Q^{\pi}(s, a)$ is defined as the expected return starting from state s, taking action a, and then following policy π thereafter. We can write the Bellman equation for the state-action value function as

$$
\begin{aligned}
Q^{\pi}(s,a) &= \mathbb{E}_{\pi}\left[\sum_{t=0}^{\infty}\gamma^{t} r_{t+1} \mid s_t = s, a_t = a\right] \\
&= \mathbb{E}_{\pi}\left[G_t \mid s_t = s, a_t = a\right] \\
&= \mathbb{E}_{\pi}\left[R_{t+1} + \gamma G_{t+1} \mid s_t = s, a_t = a\right] \\
&= \mathbb{E}_{\pi}\left[R_{t+1} + \gamma Q^{\pi}(s_{t+1}, a_{t+1}) \mid s_t = s, a_t = a\right] \\
&= \sum_{s',r} p(s', r \mid s, a)\left[r + \gamma \sum_{a'} \pi(a' \mid s') Q^{\pi}(s', a')\right] \\
&= \mathbb{E}_{\pi}\left[R_{t+1} + \gamma Q^{\pi}(s_{t+1}, a_{t+1}) \mid s_t = s, a_t = a\right] \\
&= \sum_{s',r} p(s', r \mid s, a)\left[r + \gamma V^{\pi}(s')\right]
\end{aligned}
\tag{3}
$$

policy improvement theorem helps us in updating the policy once we found out the value function of a policy by policy evaluation. The theorem states that if we have a policy π and a value function $V^{\pi}(s)$, then there exists a policy π' such that $V^{\pi'}(s) \geq V^{\pi}(s)$ for all $s \in S$. This means that we can always improve the value of a policy by following a greedy policy with respect to the value function. The greedy policy with respect to the value function is defined as

$$\pi'(s) = \arg\max_{a \in \mathcal{A}} Q^{\pi}(s,a) \tag{4}$$

which leads to $V^{\pi'}(s) \geq V^{\pi}(s)$ for all $s \in S$.

Policy Improvement Algorithm

Assuming a deterministic policy $\pi(s) = a$

Algorithm 2 Policy Improvement

```
1: Input policy $\pi$, value function $v(s)$ for all $s \in S$,
2: Set discount factor $\gamma$
3: Evaluate the policy $\pi$ to get $v(s)$ for all $s \in S$ using Algorithm 1 Policy
   Evaluation.
4: Improve the policy by following a greedy policy with respect to the value
   function.$\pi \leftarrow \arg\max_{a \in \mathcal{A}} Q^{\pi}(s,a)$ for all $s \in S$
```

However, policy improvement algorithm involves policy evaluation. So, we need to repeat the policy evaluation and policy improvement until the policy converges. So, Value iteration algorithm updates the value function by acting greedily with respect to the value function and then improves the policy by following a greedy policy with respect to the value function. The algorithm is given below.

Value Iteration Algorithm

Assuming a deterministic policy $\pi(s) = a$

Algorithm 3 Value Iteration

```
1: Input policy $\pi$, initial approximation $v_0(s) \forall s \in S$,
2: Set discount factor $\gamma$, $k = 0$, $V(terminal) = 0$
3: $v_k(s) \leftarrow v_0(s)$ for all $s \in S$
4: while $|v_k - v_{k-1}| < \theta$ do
5:     $v_{k+1}(s) \leftarrow \max_{a \in \mathcal{A}} \sum_{s',r} p(s', r \mid s, a) [r + \gamma v_k(s')]$ for all $s \in S$
6:     $k \leftarrow k + 1$
7: end while
8: return $v_k(s)$ for all $s \in S$
```

8 DQN Learning

Algorithm 4 DQN Learning- Gradient Learning

```
Initialise a Q network with parameters $\theta$
start with state $s_t$
while True do
    take action $a_t$
    observe next state $s_{t+1}$ and $R_{t+1}$
    calculate the gradient
        $$\theta \leftarrow \theta + \alpha \bigtriangledown_\theta [R_{t+1} + \gamma max_a Q_\theta(s_{t+1}, a) - Q_\theta(s_t, a_t)]$$
    $s_t \leftarrow s_{t+1}$
end while
```

Algorithm 5 DQN Learning- Experience Replay Learning

```
Initialise a Q network with parameters $\theta$
start with state $s_t$
Initialise a replay buffer $D$
while True do
    take action $a_t$
    observe next state $s_{t+1}$ and $R_{t+1}$
    save the transition $(s_t, a_t, R_{t+1}, s_{t+1})$ in $D$
    while some epochs do
        sample a mini-batch $N$ from the replay buffer $D$
        calculate the gradient
            $\theta \leftarrow \theta + \alpha \bigtriangledown_\theta \frac{1}{N} \sum_{i=1}^{N} \left[R^i_{t+1} + \gamma max_{a^i} Q_\theta(s^i_{t+1}, a^i) - Q_\theta(s^i_t, a^i_t)\right]$
    end while
    $s_t \leftarrow s_{t+1}$
end while
```

Algorithm 6 DQN Learning- Experience Replay Learning with Target network

```
Initialise a Q network with parameters $\theta$ and Target $Q$ network with parameters $\theta'$
$\theta' \leftarrow \theta$
start with state $s_t$
Initialise a replay buffer $D$
while True do
    take action $a_t$
    observe next state $s_{t+1}$ and $R_{t+1}$
    save the transition $(s_t, a_t, R_{t+1}, s_{t+1})$ in $D$
    while some epochs do
        sample a mini-batch $N$ from the replay buffer $D$
        calculate the gradient
            $\theta \leftarrow \theta + \alpha \bigtriangledown_\theta \frac{1}{N} \sum_{i=1}^{N} \left[R^i_{t+1} + \gamma max_{a^i} Q_{\theta'}(s^i_{t+1}, a^i) - Q_\theta(s^i_t, a^i_t)\right]$
    end while
    $\theta' \leftarrow \theta$
    $s_t \leftarrow s_{t+1}$
end while
```

9 Policy Gradient Algorithms

Policy Gradient Theorem
you have a stochastic policy $\pi(a|s)$.

$$\bigtriangledown v_\pi(s) = \bigtriangledown \left[\sum_a \pi(a|s) Q_\pi(s,a) \right]$$

$$\bigtriangledown J(\theta) \propto \sum_s \mu(s) \sum_s Q_\pi(s,a) \bigtriangledown (\pi_\theta(a|s)$$

$$\bigtriangledown J(\theta) = \mathbb{E}_\pi \left[G_t \bigtriangledown ln\pi(a|s) \right]$$