AM5600: Computational Methods in Mechanics (July-Nov. 2018)

Assignment #1

Due: At the beginning of class on Aug 21, 2018

1. Find the number(s) c referred to in the intermediate value theorem for each function over the interval indicated and for the given value of L.
   i. $f(x) = -x^2 + 2x + 3, x \in [-1,0]\ using\ L = 2.$
   ii. $f(x) = \sqrt{x^2 - 5x - 2}, x \in [6,8]\ using\ L = 3.$
2. Find the number c referred in the mean value theorem for integrals for $f(x) = x\cos x$ defined in the interval $[0, 3\pi/2]$.
3. Convert the following numbers to their binary form; the subscript denotes the basis
   a. $(49)_{10}$ d. $(10.2)_{10}$
   b. $(750)_8$ e. $(620)_8$
   c. $(0.5)_{10}$ f. $(600.3)_8$
4. Convert the following binary numbers to decimal form; the subscript denotes the basis
   a. $(101101)_2$ c. $(100101)_2$
   b. $(110.01)_2$ d. $(0.01001)_2$
5. If $y = x^{1/2}$, show that
   a. the relative error in y will be 1/2 of the relative error in x.
   b. Hence, estimate $(100.5)^{1/2}$ and the absolute and relative error.
6. Given the Taylor expressions for h < 1:

$$\frac{1}{1-h} = 1 + h + h^2 + h^3 + O(h^4)$$

$$cosh = 1 - \frac{h^2}{2!} + \frac{h^4}{4!} + O(h^6)$$

   Determine the order of approximation for their sum and product respectively.

AM5801/AM5810: Computational Lab (optional for students crediting AM5600)

Due: At the end of lab on Aug 13, 2018

1. Write a generalized code to convert any number N_a (where a is the base) to binary system. Verify the algorithm by solving the Q3 in the above section. (10 pts.)
2. Consider the quadratic equation $ax^2 + bx + c = 0$ such that $a \neq 0$ and $b^2 - 4ac > 0$. The roots of the equations can be computed using the formula: (10 pts.)

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \qquad \text{or} \qquad x_1, x_2 = \frac{2c}{b \pm \sqrt{b^2-4ac}}$$

   Construct a MATLAB algorithm that will accurately compute the roots of a quadratic equation for all situations and test it for the following equations.
   a. $x^2 - 1000.001x + 1 = 0$ d. $x^2 - 1000000.000001x + 1 = 0$
   b. $x^2 - 10000.0001x + 1 = 0$ e. $x^2 - 5000.002x + 10 = 0$
   c. $x^2 - 100000.00001x + 1 = 0$
3. For the following infinite series expansion: (15 pts.)

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + O(x^4)$$

Starting from the zeroth order expansion $\left(\frac{1}{1-x} = 1\right)$ include additional terms in the expansions and evaluate the expression at $x = 0.1$, until the relative error falls below 10^{-5}.

4. Using the Taylor series expansion approximate up to the second order approximation $\left(h^2 \frac{f''(x)}{2!}\right)$, find $cos(\pi/3)$ by using $h = \pi/3 , \pi/6, .. , \pi/48 , \pi/96$. Plot the absolute error vs. step size (h) on a log-log plot and comment on its slope. (15 pts.)