AM5600: Computational Methods in Mechanics (July-Nov. 2018)

Assignment #4

Due: At the beginning of class on Sep. 25, 2018

1. Find the order of convergence (R) for secant method. If p is the exact root, then:
$$\lim_{N \to \infty} \frac{|p - x_{N+1}|}{|p - x_N|^R} = K$$
   Where, K is an asymptotic constant.

2. Let $g(x) = x\cos(x)$. Solve $x = g(x)$ and find all the fixed points of $g(x)$. Can fixed-point iteration be used to find all the solutions? Why?

3. Let $f(x) = (x + 1)^3$. Find the roots of $f(x)$ using Newton Raphson with an initial guess of $x_0 = -0.9$ till the true error $\left(\varepsilon_t = \left|\frac{p - x_i}{p}\right|\right)$ falls below 10^{-3} (where, p is the exact root and x_i is the approximation of the root on the ith iteration). Find the order convergence (R). Does the method has quadratic convergence $(R = 2)$ and justify your choice?

4. Denote the intervals that arise from bisection method by $[a_0, b_0], [a_1, b_1], \ldots, [a_n, b_n]$

   a. Show that $a_0 \leq a_1 \leq a_n \leq \ldots \leq b_n \leq \ldots \leq b_1 \leq b_0$

   b. Prove: $b_n - a_n = (b_0 - a_0)/2^n$

   c. Finally, if $c_n = (a_n + b_n)/2$ then
   $$\lim_{n \to \infty} a_n = \lim_{n \to \infty} c_n = \lim_{n \to \infty} b_n$$

5. Find the minimum value of the function $f(x) = x^2 - 2\sin x, x > 0$, which occurs for $x \in [0.5, 1.0]$ using Regula-Falsi method correct up to 3 decimals.

6. Solve the simultaneous equations:
   $$x^2 y + y - 4 = 0$$
   $$y^2 = x^3$$
   using the Newton-Raphson method taking the initial guess as $x_0 = 0.45$ and $y_0 = 0.3$. Compute the values of x_3 and y_3 (3 iterations) correct to three decimal places.

AM5801/AM5810: Computational Lab (optional for students crediting AM5600)

Due: At the end of lab on Oct. 3, 2018

I. Write the MATLAB codes for finding the multiple roots using Bisection and Regula-Falsi method for:

$$f(x) = \sin(10x) + \cos(3x), x \in [3,6]$$

Firstly, an incremental search algorithm would be required to locate approximate location of the various roots. To elaborate, discretize $x \in [3,6]$ within a fine resolution and utilize $f(x_i)f(x_{i-1}) < 0$ root lies between x_{i-1} and x_i. Plot the true error vs iteration for both the methods using a semi-log plot. Finally, find the order of convergence (R) for both the methods (refer Q1 for details).

II. Write the MATLAB codes for finding the root using Newton-Raphson and Secant method for:

$$f(x) = (x - 1)(\exp(x - 1) - 1)$$

Plot the true error vs iteration for both the methods using a semi-log plot. Find the order of convergence (R) for both the methods (refer Q1 for details). Can we utilize bracketing methods to find the roots for $f(x)$?