AM5600: Computational Methods in Mechanics (July-Nov. 2018)

Assignment #7

Due: At the beginning of class on Nov. 12, 2018

1. For the wave equation $u_{tt}(x,t) = 9u_{xx}(x,t)$, what relationship between h and k must occur to produce the following finite difference equation:

$$u_{i,j+1} = u_{i+1,j} + u_{i-1,j} - u_{i,j-1}$$

2. Solve $u_{xx} + u_{yy} = -4u$ over $\mathcal{R} = \{(x,y): 0 \le x \le 1, 0 \le y \le 1\}$ with the boundary values

$$u(x,y) = \cos(2x) + \sin(2y)$$

3. Determine the system of equations for implementing the Laplace nine-point difference equation $O(h^4)$ accurate scheme for all the unknowns for $u(x,y)$ in a 4x4 grid assuming the boundary values for $u(x,y)$ are known (Dirchlet boundary conditions).

4. For the wave equation $u_{tt}(x,t) = 4u_{xx}(x,t)$, can it be solved numerically using finite difference for $h = 0.03$ and $k = 0.02$?

5. Consider steady heat diffusion in the unit square $0 \le x \le 1, 0 \le y \le 1$. Let $T(0,y) = T(x,0) = 100$ and $T_x(1,y) = T_y(1,y) = 0$. Solve this problem using the 5-point Laplace stencil $h = k = 0.025$ by hand using the Liebmann and Gauss elimination method.

6. Assume a solid plate of thickness, $L = 1\ cm$ and thermal diffusivity, $\alpha = 0.01\ cm^2/s$ and the heat transfer is governed by $T_t(x,t) = \alpha T_{xx}(x,t)$. The plate is heated to an initial temperature distribution, $T(x,0)$ (refer below) after which the source was turned off.

$$T(x,0) = 200x \quad 0 \le x \le 0.5$$

$$T(x,0) = 200(1-x) \quad 0.5 \le x \le 1$$

where, T is in °C. The temperature of the two faces of the plate is held at 0°C at all times. Find $T(x,t)$ at $t = 0.2$ for $h = k = 0.1$ using the forward time centered-space (FTCS) method.

AM5801/AM5810: Computational Lab (optional for students crediting AM5600)

Due: At the end of lab on Nov. 7, 2018

I. Solve Problem 5 by developing a numerical scheme for 5- and 9- point finite difference schemes. Utilize both the Leibmann and Gauss Seidel methods. Solve the problem for several different step sizes. Compare computational cost by evaluating run time and number of function evaluations. Use *surf* and *contour* for visualization of results.

II. Solve the heat equation $T_t(x,t) = \alpha T_{xx}(x,t)$, for $0 \le x \le 1, 0 \le t \le 0.1$, with initial conditions $T(x,0) = \sin(\pi x) + \sin(2\pi x)$ for $0 \le x \le 1$ *and* $t = 0$ and the boundary conditions are:

$$T(0,t) = 0 \quad 0 \le t \le 0.1$$

$$T(1,t) = 0 \quad 0 \le t \le 0.1$$

utilize several different values for h and k and $r = 0.75$ *and* 1. Develop the Crank-Nicholson and FTCS method.

III. Solve the heat equation $u_{tt}(x,t) = u_{xx}(x,t)$, for $0 \le x \le 1, 0 \le t \le 1$ with the following initial and boundary conditions:

$$u(0,t) = 0 \text{ and } u(1,t) = 0 \text{ for } 0 \le t \le 1$$

$$u(x,0) = \sin(\pi x) \text{ and } u_t(x,0) = 0 \text{ for } 0 \le x \le 1$$

Choose different combinations of h and k and plot the solutions using *surf* and *contour*.