AM5600: Computational Methods in Mechanics (July-Nov. 2018)

Assignment #3

Due: At the beginning of class on Sep 18, 2018

1. Let **A** be a $m \times n$ matrix and **B** be a $n \times p$ matrix. Prove that $(AB)' = B'A'$.
2. Prove that $\det(AB) = \det(A)\det(B)$ for any two square matrices of the same size.
3. Show that the triangular factorization is unique. In other words, If **A** is non-singular then **L** and **U** are unique upper and lower triangular matrices.
4. Determine if **A** is a diagonally dominant matrix (perform pivoting if necessary):

$$A = \begin{bmatrix} 3.02 & -1.05 & 2.53 \\ 4.33 & 0.56 & -1.78 \\ -0.83 & -0.54 & 1.47 \end{bmatrix}$$

   Similarly, Condition Number of **A** ($Cond[A] = \|A\|.\|A^{-1}\|$, where $\|A\| = \max_{1\le i\le n} \sum_{i=1}^{n} |a_{ij}|$ quantify the condition number for **A**. Can we solve **AX = b** to obtain unique solutions by using any of the iterative schemes?
5. a) Prove that $Cond[A] \geq 1$ for any square matrix. Are there any exceptions to this? b) Find a 2×2 matrix A which has $Cond[A] = 289$?
6. Find the solution to the following system of equations:

$$x_1 + x_2 = 7$$
$$2x_1 + 3x_2 - x_3 = 9$$
$$4x_2 + 2x_3 - 3x_4 = 10$$
$$2x_3 - 4x_4 = 12$$

7. $A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ is singular and almost diagonally dominant matrix for $AX = 0$
   a.) Demonstrate what happens when Jacobi method with these initial guesses [1,1], [1,-1] and [-1,1].
   b.) Repeat part (a) with Gauss Seidel method.
   c.) Now change $a_{12} = a_{21} = -1.99$ and now repeat part (a).

AM5801/AM5810: Computational Lab (optional for students crediting AM5600)

Due: At the end of lab on Sep 19, 2018

I. Write a MATLAB code using the Jacobi iterative method for solving the following system of equations:

$$4x_1 + x_2 - x_3 = 13$$
$$x_1 - 5x_2 - x_3 = -8$$
$$2x_1 - x_2 - 6x_3 = -2$$

II. Solve the following equation using Gauss Seidel iterative method:

$$x_1 + (1/2)x_2 + (1/3)x_3 + (1/4)x_4 = (25/12)$$
$$(1/2)x_1 + (1/3)x_2 + (1/4)x_3 + (1/5)x_4 = (77/60)$$
$$(1/3)x_1 + (1/4)x_2 + (1/5)x_3 + (1/6)x_4 = (57/60)$$
$$(1/4)x_1 + (1/5)x_2 + (1/6)x_3 + (1/7)x_4 = (319/420)$$

Use only three significant digits in your arithmetic operations to find the solution. Next, find the solution using 6 significant digits and compare with earlier results.

III. Write a program for triangular factorization ($A = LU$) algorithm of the matrix below:

$$A = \begin{bmatrix} 2 & -1 & 3 & 2 \\ 2 & 2 & 0 & 4 \\ 1 & 1 & -2 & 2 \\ 1 & 3 & 4 & -1 \end{bmatrix}$$