AM5600: Computational Methods in Mechanics (July-Nov. 2018)

Assignment #5

Due: At the beginning of class on Oct. 16, 2018

1. Use the Taylor series expansions for equally spaced points along x with step size h. Derive the central difference formula that is $O(h^2)$.

   $f^{(3)}(x_0) = [f_2 - 2f_1 + 2f_{-1} - f_{-2}]/(2h^3)$, where $f_1 = f(x_0 + h)$ and so on.

2. Find $f'(x)$ for $f(x) = e^{-2x} - x$ @ $x = 2$ for a step size of $h = 0.5$ using central difference method. Repeat calculation by decreasing the step size in increments of 0.05 till $h = 0.05$. Comment on the optimal step size for approximating $f'(x)$ @ $x = 2$ by finding the absolute error during numerical differentiation.

3. Compute the first-order central difference approximations of $O(h^4)$ for the following function at $x = 0$ for a step size of $h = 0.25$:

   $$y = x^3 + 4x - 15$$

   Compare your findings with the analytical solution.

4. Evaluate the integral for the following data using composite trapezoidal and Simpson's 1/3rd rule:

x	-2	0	2	4	6	8	10
$f(x)$	35	5	-10	2	5	3	20

   Compare and comment on the findings.

5. Determine the number of sub-intervals (M) and width (h) such that the composite trapezoidal rule can be used to compute the integral below with an accuracy of 5×10^{-9}.

   $$I = \int_0^2 x exp(-x) dx$$

6. Estimate the error involved in approximating using composite trapezoidal and Simpson's 1/3 rule with 100 sub-intervals (M)

   $$I = \int_1^3 \frac{1}{\sqrt{1 + x^3}} dx$$

7. Evaluate the integral below using Gauss 2- and 3-point quadrature methods

   $$I = \int_0^2 \sqrt{1 + 4x} dx$$

   Compare the results with exact values.

AM5801/AM5810: Computational Lab (optional for students crediting AM5600)

Due: At the end of lab on Oct. 17, 2018

I. Write the MATLAB code for finding the numerical derivate of any general function using central difference approximation of $O(h^4)$. The implementation should incorporate a numerical scheme to find the optimum step size (h) (*Hint: Carryout numerical differentiation with decreasing step size till absolute error reaches the minima*). Compare with forward and backward difference approximation of $O(h)$. Note: $f'(x)$ should be returned as a vector for analysis.

II. Write the MATLAB codes for carrying out numerical integration using composite trapezoidal and Simpson's 1/3 methods, adaptive quadrature using Simpson's 1/3 method and 2- *and* 3- point Gauss quadrature (including the change of integration limits). Comment on number of function evaluations, error and optimum step size (h).