AM5600: Computational Methods in Mechanics (July-Nov. 2018)

Assignment #2

Due: At the beginning of class on Sep 3, 2018

1. Find a polynomial degree $N = 5$ for $f(x) = 1/(1+x)$ expanded about $x_0 = 0$. Find the error term $E_5(x)$ for the polynomial.
2. Let $f(x) = 2\sin(\pi x/6)$, where *x* is in radians.
   a. Use quadratic Lagrange and Newton interpolation methods for nodes $x_0 = 0, x_1 = 1$ and $x_2 = 3$ to approximate $f(2)$ and $f(2.25)$.
   b. Use cubic Lagrange and Newton interpolation methods for nodes $x_0 = 0, x_1 = 1$, $x_2 = 3$ and $x_3 = 5$ to approximate $f(4)$ and $f(4.5)$.
3. Consider the *M+1* points (x_0, y_0),…, (x_M, y_M).
   a. If the *(N+1)*th divided difference are zero, then show that the *(N+2)*th up to the *M*th divided difference are zero.
   b. If the *(N+1)*th divided difference are zero, then show that there exist a polynomial $P_N(x)$ of degree *N* such that
   $$P_N(x_k) = y_k \text{ for } k = 0, 1, \ldots, M$$
4. Let $L_0(x), L_1(x), \ldots, L_N(x)$ be the Lagrange coefficients for the *N*th order polynomial based on the *N+1* nodes x_0,…, x_N. Show that $\sum_{k=0}^{N} L_k(x) = 1$ for any real number *x*.
5. For the given data:

x	1	2	3	5	7	8
$f(x)$	3	6	19	99	291	444

   Calculate $f(4)$ using Newton's interpolation for polynomials of the order 1 through 4. Choose your base points to attain good accuracy. What is the order of polynomial could have been used to generate this data?

6. For the given data:

x	4.0	5.0	6.0	7.0	8.0
$f(x)$	2.0000	2.3607	2.44949	2.64575	2.82843

   a. Compute the divided difference table for the Newton's interpolation.
   b. Write down the Newton polynomials $P_1(x), P_2(x), P_3(x)$ *and* $P_4(x)$.
   c. Using the 4th order polynomial, approximate the value of $f(4.5)$.
   d. Compare the findings in (b) with $f(x) = x^{0.5}$
7. Runge's function is written as $f(x) = 1/(1+25x^2)$
   a. Develop the plot of the function for *x* in [-1,1].
   b. Generate and plot the fourth order Lagrange polynomial using *x* = -1, -0.5, 0, 0.5 and 1.
   c. Using the fourth order Lagrange polynomial estimate $f(0.8)$ and discuss your results.
8. For the given data:

x	0	1	2	3	4	5
$f(x)$	0	0.5	0.8	0.9	0.941176	0.961538

   Determine the value of *x* for which $f(x) = 0.93$

AM5801/AM5810: Computational Lab (optional for students crediting AM5600)

Due: At the end of lab on Sep 12, 2018

I. Write a MATLAB code to find the Lagrange polynomial for problem 6 from the previous section. Plot the graph of each of the polynomials $P_1(x), P_2(x), P_3(x)\ and\ P_4(x)$ and compare with the actual function. Furthermore, compute the total number of mathematical operations required for each of the polynomials. (10 pts.)

II. Repeat problem II for Newton's polynomial interpolation. Furthermore, compute the total number of mathematical operations required for each of the polynomials.

Note: *The MATLAB codes should be general enough to perform any order of interpolation.*